Optimization_professor

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1 Optimización

```
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Versión: 0.1 (2019/09/13)
         0.2 (2019/10/02): Solutions added
```

1.1 Exercise: compute the minimum of a real-valued function

The goal of this exercise is to implement and test optimization algorithms for the minimization of a given function. Gradient descent and Newton's method will be explored.

Our goal it so find the minimizer of the real-valued function

$$f(w) = -wexp(-w)$$

but the whole code will be easily modified to try with other alternative functions.

You will need to import some libraries (at least, `numpy` and `matplotlib`). Insert below all the imports needed along the whole notebook. Remind that numpy is usually abbreviated as np and `matplotlib.pyplot` is usually abbreviated as `plt`.

```
In [28]: # <SOL>
         import numpy as np
         import matplotlib.pyplot as plt
         # </SOL>
```

1.1.1 Part 1: The function and its derivatives.

Question 1.1: Implement the following three methods:

- Method `f`: given w, it returns the value of function $f(w)$.
- Method `df`: given w, it returns the derivative of f at w
- Medhod `d2f`: given w, it returns the second derivative of f at w

```
In [29]: # Funcion f
         # <SOL>
         def f(w):
             y = - w * np.exp(-w)
```

```
    return y
# </SOL>

# First derivative
# <SOL>
def df(w):
    y = (w -1) * np.exp(-w)
    return y
# </SOL>

# Second derivative
# <SOL>
def d2f(w):
    y = (2 - w) * np.exp(-w)
    return y
# </SOL>
```

1.1.2 Part 2: Gradient descent.

Question 2.1: Implement a method `gd` that, given `w` and a learning rate parameter `rho` applies a single iteration of the gradien descent algorithm

In [30]:
```
# <SOL>
def gd(w0, rho):
    y = w0 - rho * df(w)
    return y
# </SOL>
```

Question 2.2: Apply the gradient descent to optimize the given function. To do so, start with an initial value $w = 0$ and iterate 20 times. Save two lists:

- A list of succesive values of w_n
- A list of succesive values of the function $f(w_n)$.

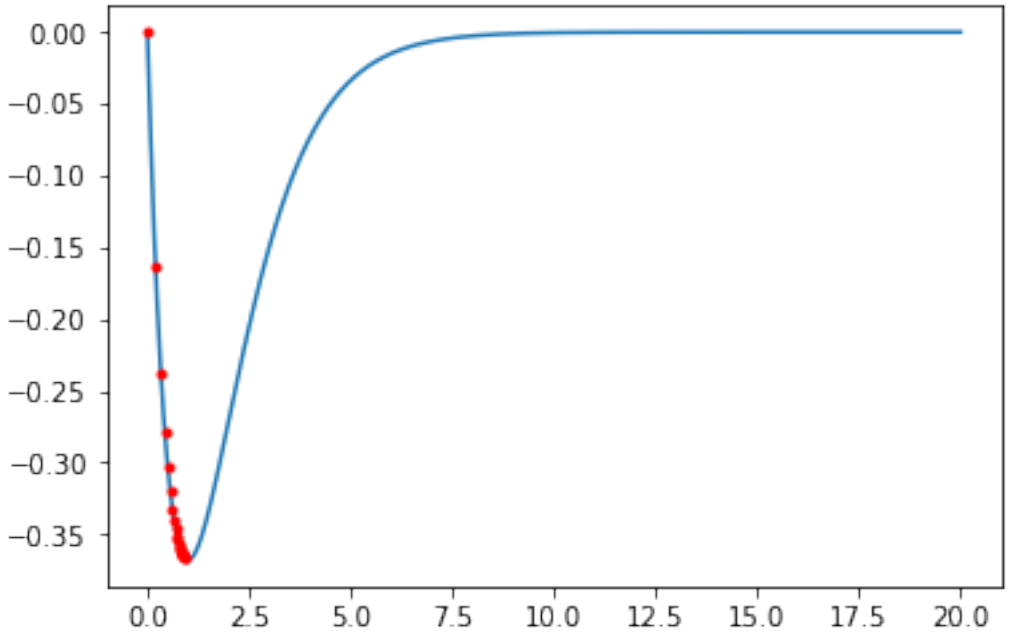
In [31]:
```
# <SOL>
wn = []
fwn = []
niter = 20
rho = 0.2
w = 0
wn.append(w)
fwn.append(f(w))

for k in range(niter):
    w = gd(w,rho)
    wn.append(w)
    fwn.append(f(w))
# </SOL>
```

Question 2.3: Plot, in a single figure:

- The given function, for values ranging from 0 to 20.
- The sequence of points $(w_n, f(w_n))$.

```
In [32]: # <SOL>
         npoints = 1000
         w_grid = np.linspace(0,20,npoints)
         plt.plot(w_grid, f(w_grid))
         plt.plot(wn,fwn,'r.')
         plt.show()
         # </SOL>
```



You can check the effect of modifying the value of the learning rate.

1.1.3 Part 2: Newton's method.

Question 3.1: Implement a method **`newton`** that, given `w` and a learning rate parameter `rho` applies a single iteration of the Newton's method

```
In [33]: # <SOL>
         def newton(w0, rho):
             y = w0 - rho * df(w) / d2f(w)
             return y
         # </SOL>
```

Question 3: Apply the Newton's method to optimize the given function. To do so, start with an initial value $w = 0$ and iterate 20 times. Save two lists:

- A list of succesive values of w_n
- A list of succesive values of the function $f(w_n)$.

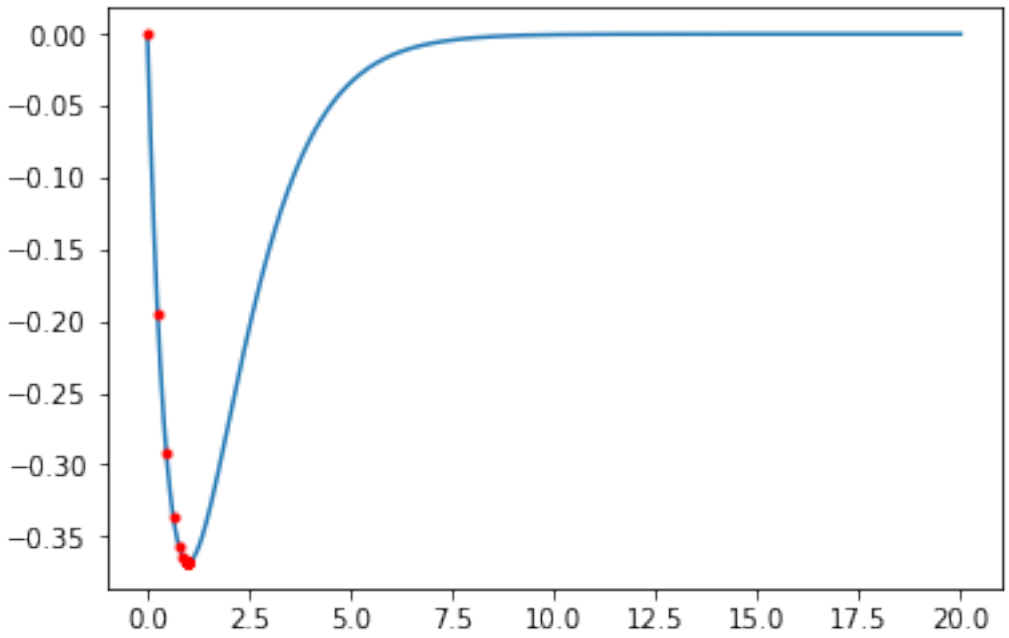
```
In [34]: # <SOL>
         wn = []
         fwn = []
         niter = 20
         rho = 0.5
         w = 0
         wn.append(w)
         fwn.append(f(w))

         for k in range(niter):
             w = newton(w,rho)
             wn.append(w)
             fwn.append(f(w))
         # </SOL>
```

Question 4: Plot, in a single figure:

- The given function, for values ranging from 0 to 20.
- The sequence of points $(w_n, f(w_n))$.

```
In [35]: # <SOL>
         npoints = 1000
         w_grid = np.linspace(0,20,npoints)
         plt.plot(w_grid, f(w_grid))
         plt.plot(wn,fwn,'r.')
         plt.show()
         # </SOL>
```



You can check the effect of modifying the value of the learning rate.

1.1.4 Part 3: Optimize other cost functions

Now you are ready to explore these optimization algorithms with other more sophisticated functions. Try with them.

In [ ]: