$X \in \mathbb{R}^{m \times n}$ $\quad x_{ij} \in X \quad x_j^{(i)} \in X \quad i \in 1, \dots, m \quad j \in 1, \dots, n$

$p = 1 \quad (p \ll n) \quad x^{(i)} \in \mathbb{R}^{1 \times n}$

$u_1 \in \mathbb{R}^{n \times 1}$ PROJECTION OF $x^{(i)}$ $\quad x^{(i)} \cdot u_1 \in \mathbb{R}$

MEAN $\bar{x}$ PROJECTION OF MEAN $\bar{x}$ $\quad \bar{x} \cdot u_1 \in \mathbb{R}$

VARIANCE OF PROJECTED DATA $\delta_p^2 = \frac{1}{m-1} \sum_{i=1}^{m} \left(x^{(i)} \cdot u_1 - \bar{x} \cdot u_1 \right)^2 =$

S_0 COVARIANCE MATRIX OF ORIGINAL DATA $= \boxed{u_1^T \cdot S_0 \cdot u_1}$

MAXIMIZE THE VARIANCE

$$\underset{u_1}{\operatorname{argmax}} \, \delta_p^2 = \underset{u_1}{\operatorname{argmax}} \underbrace{u_1^T \cdot S_0 \cdot u_1} \in \mathbb{R}$$

$\cancel{\|u_1\|} \to \infty \qquad \|u_1\| = 1 \qquad u_1^T \cdot u_1 = 1 = \sum_{i=1}^{j} u_{ij}$

$$\underset{u_1}{\operatorname{argmax}} \left(u_1^T S_0 u_1 + \lambda (1 - u_1^T u_1) \right)$$

$\downarrow$ DERIVATIVE WRT u_1

$$(2 u_1^T S + 0 - 2\lambda u_1^T) = 0$$

$\downarrow$

$$u_1^T S = \lambda u_1^T$$

λ EIGENVALUE

u_1 EIGENVECTOR

RIGHT MULTIPLY BY u_1 $\quad u_1^T S u_1 = \lambda u_1^T u_1$

$$\boxed{u_1^T S u_1} = \lambda$$

$$\delta_p^2 = \lambda$$

$\max \delta_p^2 = \max_i \lambda_{(i)}$

$\downarrow$

CORRESPONDING EIGENVECTOR

$p = 2$

$\vdots$

$p = n$