**Physics IA**

**Introduction**

As an avid user of mechanical pencils, I find it irritating for the pencil lead to snap when I am writing. As a result, whenever I had a choice between 0.5 mm, 0.7 mm, and 0.9 mm mechanical pencils, I would always choose the type with thicker lead because it would be more resistant to snapping. However, I’ve always been curious about just how increases in lead diameter affect the force needed to snap it. Having found internet searches fruitless, I have thus decided to answer that question in this paper:

**How does the diameter of a piece of lead from a mechanical pencil affect the amount of force needed to snap it?**

Background

To find this relationship, I will suspend a container from elevated mechanical pencil leads of various diameters with a string. Mass will then be steadily added into the container until the lead snaps. The only force acting on the pencil lead will be the weight force of the mass, which can be found with the following equation:

Where is the weight force, is the total mass of the object causing the force, and is Earth’s gravitational field strength, or approximately (Nave). The mass that will be added into the container is water because it can be measured continuously. I will not be able to directly mass the container and water because this experiment will be carried out at home due to COVID-19. Instead, because water has a set density at room temperature, I will use the volume of the water to determine its mass. Specifically, water at a room temperature of has a density of approximately (USGS). The mass in grams of a volume in mL of water is therefore

The water’s mass will then be added to the mass of the string and container that holds the water, to obtain the total mass of everything exerting a weight force on the lead.

As this experiment is a study of forces on a small scale and only utilizes water and household items, there are no significant ethical or safety concerns to account for.

Variables

The independent variable in this experiment will be the diameter of the mechanical pencil lead – I will test leads of diameter 0.3 mm, 0.5 mm, 0.7 mm, 0.9 mm, and 1.3 mm. These diameters will be used because they are the only readily attainable variations of diameter from one manufacturer. The dependent variable is the force needed to snap the lead, which is equivalent to the weight force exerted on the lead by the hanging mass.

There are several controlled variables. Firstly, the distance between the ledges on which the lead is suspended is 1.4 cm, the maximum width for which an empty container can hang on the 0.3 mm lead without snapping it. Secondly, the string will contact the lead at a point that is approximately 0.7 cm away from both supports, so that the lead can experience maximum force. Thirdly, the same container and string will be used throughout all trials. Lastly, all leads are of the brand June Gold, so they originate from the same manufacturer and have similar material compositions.

**Experimental Setup**

The materials used for this investigation are varying sizes of mechanical pencil lead, a string and container, high ledges to elevate the lead, water, a 5 mL syringe for measuring volume, and a device for recording data. A scale is used later for massing.

The procedure for this experiment will be as follows:

1. Obtain needed materials
2. Place the 0.3 mm pencil lead between the high ledges and carefully hang the container from it with a string
3. Slowly inject water into the container with the syringe until the lead snaps, keeping track of the volume of water added
4. Record the volume of water added
5. Repeat steps 2-4 four more times, for a total of five trials
6. Repeat steps 2-5 for the 0.5 mm, 0.7 mm, 0.9 mm, and 1.3 mm pencil leads
7. Mass the container, string, and any extra masses used in the process to snap the lead when water was not enough
8. Plot and analyze the data

An image of the experimental setup is depicted above.

**Experimental Data**

Following is my experimental data, with 5 trials for each variation of the independent variable.

*Diameter of lead (mm) to volume of water put into container (mL)*

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Diameter of Lead | 0.3 mm | 0.5 mm | 0.7 mm | 0.9 mm | 1.3 mm |
| Trial 1 (mL) | 5.6±0.24 | 20.0±0.80 | 135.0±5.40 | 284.8±11.40 | 300.6±12.04 |
| Trial 2 (mL) | 6.8±0.28 | 33.6±1.36 | 138.4±5.56 | 328.2±13.16 | 265.8±10.64 |
| Trial 3 (mL) | 5.0±0.20 | 24.4±1.00 | 143.2±5.76 | 352.2±14.12 | 315.0±12.60 |
| Trial 4 (mL) | 9.2±0.40 | 38.2±1.56 | 134.4±5.40 | 310.0±12.40 | 340.0±13.60 |
| Trial 5 (mL) | 5.4±0.24 | 39.6±1.60 | 147.4±5.92 | 367.4±14.72 | 332.4±13.32 |

I would like to first note that the experiment did not go entirely as expected – in the trials with the 1.3 mm lead, the lead did not snap even when the container was full of water. Hence, for the 1.3 mm trials only, I added small metal bars into the container to serve as extra weights.

After my trials, all relevant equipment was massed at school. The container and the string have a total mass of g, while the masses put into the container to help snap the 1.3 mm lead have a total mass of g.

Uncertainties

The uncertainties for my equipment and in the raw data are justified as follows:

The pencil leads are factory manufactured, so the diameters of the lead can be assumed to have a negligible uncertainty.

The scale is of the brand Scout Pro and is specific to a hundredth of a gram. The uncertainty of its measurements is thus its unit of lowest significance, or 0.01 g.

The unit of lowest significance on the syringe is 0.2 mL, so the uncertainty in one measurement by the syringe is 0.2 mL. However, because the syringe can only hold a maximum of 5 mL at a time, the volume for each trial needs to be split into multiple 5 mL measurements as appropriate. Therefore, given a trial’s volume of water in mL, the uncertainty of the volume for that trial is:

**Analysis**

To plot this data, the arithmetic average of all the trials per data point will be used. To find the average of the volumes,  , across the five trials for a certain diameter, the following formula will be used:

For example, the average volume of water needed to snap the 0.3 mm lead is:

When the data for each diameter is averaged, the following table is obtained:

|  |  |
| --- | --- |
| Diameter of Lead (mm) | Average Water put into Container (mL) |
| 0.3 | 6.4±0.27 |
| 0.5 | 31.2±1.26 |
| 0.7 | 139.7±5.61 |
| 0.9 | 328.5±13.16 |
| 1.3 | 310.8±12.44 |

To find the force acting on the lead, the mass of the water added is first calculated, and then added to the mass of the container and string, to obtain the total mass of everything hanging from the lead. In the case of the 1.3 mm lead, the mass of the extra weights added will be summed into the calculation too. Next, because the weight force on the mass is equivalent to the net force experienced by the lead, and the weight force , the force experienced by the pencil lead can be found with the equation:

Data Processing

A sample calculation is shown below for the force exerted on the 1.3 mm lead:

The density of water at room temperature is approximately . Hence, the mass of the water added is:

The mass hanging from the lead for the 1.3 mm trials include the water, the container holding the water, the string attached to the lead, and the extra masses that were added in for only the 1.3 mm trials. The mass of the container and the string combined is , and the mass of the extra weights is in total. Hence, the total mass of everything hanging from the lead is:

To find the force withstood by the lead, the aforementioned equation can be applied. However, since the mass in the equation must be in kilograms, but my previous results are in grams, a conversion rate of is used. Thus, the force exerted on the lead is:

When this calculation is applied to all diameters, the following table is obtained:

|  |  |
| --- | --- |
| Diameter of Lead (mm) | Average Force to Snap (N) |
| 0.3 | 0.346±0.00276 |
| 0.5 | 0.588±0.0125 |
| 0.7 | 1.65±0.0550 |
| 0.9 | 3.50±0.129 |
| 1.3 | 5.88±0.122 |

Data Visualization

When plotted, the following graph is obtained. An equation of best fit has been added to model a likely relationship between lead diameter and snapping force.

Firstly, it should be noted that the plotted uncertainties are extremely small, and the trendline does not satisfactorily fit within the error bars of all points.

Nonetheless, from this graph, it is evident that as the diameter of the lead increases, more force is required to snap it. Specifically, the snapping force increases at an increasing rate as the diameter increases at a constant rate. The relevant domain of this graph is all diameters of lead above zero, as it is not possible to have negative length. If extrapolated towards the left, the trendline passes through the origin, which makes sense – as the diameter of the lead approaches zero, the force required to snap it approaches zero as well. On the other hand, as the diameter of the lead approaches infinity, the force needed to snap it approaches infinity too – which also makes physical sense.

Additionally, as seen through the trendline, the relationship between lead diameter and snapping force most closely resembles that of a second-degree power function.

Such a conjecture can be supported by a linearization, in which the square roots of the snapping force are taken. For example, the data point of the average force needed to snap the 1.3 mm lead can be linearized as such:

When all data points are linearized, the following table and graph are obtained:

|  |  |
| --- | --- |
| Diameter of Lead (mm) | Root of Average Snapping Force (N0.5) |
| 0.3 | 0.59±0.01 |
| 0.5 | 0.77±0.03 |
| 0.7 | 1.28±0.09 |
| 0.9 | 1.87±0.14 |
| 1.3 | 2.42±0.10 |

In this graph, a linear line of best fit is plotted. Note that it is not possible to draw a straight line that passes within the error bounds of all points – thus, it is not possible to plot minimum and maximum slope lines the conventional way. Nevertheless, what would otherwise likely be minimum and maximum slope lines are still plotted – a max line from the top bound of the 1.3 mm point to the bottom bound of the 0.3 mm point, and a min line from the bottom bound of the 1.3 mm point to the top bound of the 0.3 mm point. The inability of any of these lines to properly fit through all the points may suggest that this linearization is not optimal. However, the points on this graph still roughly resemble a linear relationship, supporting the hypothesis of a square relationship between snapping force and lead diameter. None of the data points are really outliers because all points in both the original graph and the linearization lie reasonably close to the trendline. It thus follows that the deviation of certain points from the line should be attributed to random errors in the data, caused either by suboptimal execution of the procedure or by unforeseen factors that were not accounted for in uncertainty calculations.

**Conclusion**

The relationship between the diameter of mechanical pencil lead and the force needed to snap it is definitely not linear – from the original graph, it can be seen that as the diameter of the lead increases, the force needed to snap it increases at an increasing rate. The line of best fit seems to suggest a square relationship between snapping force and lead diameter – such a conjecture is also supported by a linearization of the data. Furthermore, a square relationship would also make physical sense – diameter bears a square relationship with cross-sectional area, and cross-sectional area is directly proportional to the number of chemical bonds in the lead that are broken when the lead snaps. The major caveat with this is that the line of best fit does not pass within the error bounds of all data points. However, as a square trendline fits the data relatively well, deviations from the line of best fit should be attributed to random errors. Thus, the following conclusion is reached:

*An increase in the diameter of mechanical pencil lead correlates with an increase in the force needed to snap the lead in a square relationship*

Experimental Evaluation

Overall, this experiment had both strengths and limitations. The biggest strength was that the data made physical sense – for example, the line of best fit passed through the origin, and the data consistently demonstrated that as lead diameter increases, the force needed to snap it increases as well.

However, this experiment also had weaknesses. The most glaring shortfall was that no line of best fit could be regressed within the error bars of all points. This can be attributed to random errors not accounted for by the uncertainties. In my data, these errors are noted by the deviation of points both above and below my trendline. Such errors could have taken several forms in the execution of my experiment. First, I tried to place the string on the lead at a position equidistant between the supports, or 0.7 cm away from both of them. However, because I carried out the procedure alone and without special equipment, I was not able to measure and ensure preciseness in this positioning. Additionally, when carrying out trials for thicker leads, the water splashed everywhere when the lead snapped, causing the string to soon get very wet. Because of this, the string was especially hard to handle near the end, allowing inaccurate positioning to possibly be a major source of random error between my trials. Furthermore, because the composition of pencil leads includes a sizable amount of clay, the wet string may have made the lead more prone to snap than normal, introducing another source of inconsistency between my trials (Pencils.com).

If I could redo this experiment, there are several things that I would change. First, I would use masses other than water to build up most of the weight needed to snap the thicker leads, and use water only to finish off the process, similar to what I did with the 1.3 mm lead in this experiment. Most likely, I would use rocks to supply most of the weight. Rocks have a high density, meaning that I could add a lot of weight without taking up much volume. Additionally, rocks are very common and come in various sizes, so obtaining suitable masses should be easy. Using rocks for most of the weight will reduce the inaccuracy from repeated measurements, and would also make the experimental process less messy as the water would not splash as much. Secondly, I would use a lighter container, so that an empty container weighs less and I could thus place the supports for the lead further apart – the distance between the supports is the maximum distance at which an empty container can hang from the 0.3 mm lead and not snap it. Such a change would greatly ease the process of snapping the thicker leads, and would also allow for better precision when positioning the string in the exact middle between the supports. Lastly, I would hope that I have access to better equipment for carrying out my experiment – namely, a graduated pipette would be much more suitable for measuring volumes than the syringe that I used this time.

**Bibliography**

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