Linear Models

STAT 551

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Part One

1. Linear Regression

1.1 Overview

- projection
- orthongonal decomposition
- Gaussian Linear Regression
- prediction (generally of $\hat{y}$)
- different types of errors
- influence
- lack of fit
- R^2
- Multicollinearity

1.2 Projection in Euclidean Space

Let Euclidian Space be denoted by $\mathbb{R}^P$.

$\mathbb{R}X \ldots X\mathbb{R} = \{(x_1, \ldots, x_p) : x_1 \in \mathbb{R} \ldots, x_p \in \mathbb{R}^P\}$

$a \in \mathbb{R}^P, b \in \mathbb{R}^P$

$$a^T b = \sum_{i=1}^{P} a_i b_i$$

$a^T b = < a, b >$ inner product

$\{\mathbb{R}^P, <, >\}$ inner product, Hilbert space

Let $\Sigma \in \mathbb{R}^{PxP}$ set of all pxp matrices

Assume Σ positive definite matrix

$x^T \Sigma x < 0 \forall x \in \mathbb{R}^P, x \neq 0$

Then $a^T \Sigma b$ also satisfies the conditions for inner product.

$a^T \Sigma b = < a, b >_\Sigma$

$a^T b = a^T I b = < a, b >_I$

$\{\mathbb{R}^P, <, >_\Sigma\}$ is a more general inner product space

A matrix, $A, \in \mathbb{R}^{PxP}$ can be viewed as linear transformation

$T_A : \mathbb{R}^P \to \mathbb{R}^P, x \mapsto A$
Asside: He will denote T_A as A.
If $A : \mathbb{R}^P \to \mathbb{R}^P$,
$ker(A) = \{x \in \mathbb{R}^P, Ax = 0\}$ $ran(A) = \{Ax : x \in \mathbb{R}^P\}$
A linear transformation is idempotent if
$A = A^2$ $Ax = A(A(x))$ $\forall x \in \mathbb{R}^P$
If A were a number it could only be 1 or 0.

2. ANOVA (1-way)

2.1 Overview

- General linear models
- Scheffe's simulteaneous confidence
- Singular decomposition
- Non Gaussian error

3. Mutiway ANOVA

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- Orthogonal design
- Additive 2 way ANOVA
- simultaneous intervals
- nonadditive
- decomposition of sum of squares
- Latin square
- nested design

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- $\bar{X}_{i} - \bar{X}_{i}$

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