



Linear Models

STAT 551

Course Notes by Meredith Bartley



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Part One

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1. Linear Regression

1.1 Overview

- projection
- orthongonal decomposition
- Gaussian Linear Regression
- prediction (generally of \hat{y})
- different types of errors
- influence
- lack of fit
- R^2
- Multicollinearity

1.2 Projection in Euclidean Space

Let Euclidian Space be denoted by \mathbb{R}^P .

$\mathbb{R}X \dots X\mathbb{R} = \{(x_1, \dots, x_p) : x_1 \in \mathbb{R}, \dots, x_p \in \mathbb{R}\}$

$a \in \mathbb{R}^P, b \in \mathbb{R}^P$

$$a^T b = \sum_{i=1}^P a_i b_i$$

$a^T b = \langle a, b \rangle$ inner product

$\{\mathbb{R}^P, \langle, \rangle\}$ inner product, Hilbert space

Let $\Sigma \in \mathbb{R}^{P \times P}$ set of all p x p matrices

Assume Σ positive definite matrix

$x^T \Sigma x > 0 \forall x \in \mathbb{R}^P, x \neq 0$

Then $a^T \Sigma b$ also satisfies the conditions for inner product.

$a^T \Sigma b = \langle a, b \rangle_{\Sigma}$

$a^T b = a^T I b = \langle a, b \rangle_I$

$\{\mathbb{R}^P, \langle, \rangle_{\Sigma}\}$ is a more general inner product space

A matrix, $A, \in \mathbb{R}^{P \times P}$ can be viewed as linear transformation

$$T_A : \mathbb{R}^P \rightarrow \mathbb{R}^P, x \mapsto Ax$$

Asside: He will denote T_A as A .


If $A : \mathbb{R}^P \rightarrow \mathbb{R}^P$,

$$\ker(A) = \{x \in \mathbb{R}^P, Ax = 0\} \quad \text{ran}(A) = \{Ax : x \in \mathbb{R}^P\}$$

A linear transformation is idempotent if

$$A = A^2 \quad Ax = A(A(x)) \quad \forall x \in \mathbb{R}^P$$

If A were a number it could only be 1 or 0.



2. ANOVA (1-way)

2.1 Overview

- General linear models
- Scheffe's simultaneous confidence
- Singular decomposition
- Non Gaussian error

3. Mutiway ANOVA

3.1 Overview

- Orthogonal design
- Additive 2 way ANOVA
- simultaneous intervals
- nonadditive
- decomposition of sum of squares
- Latin square
- nested design

4. Nonorthogonal Design

4.1 Overview

- $\bar{X}_j - \bar{X}_i$



5. Random Effects Model

5.1 Overview



Part Two

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6. Basic Concepts

6.1 Overview



7. Estimation

7.1 Overview

8. Inference

8.1 Overview

- deviance \leftrightarrow sum of squares



9. Residuals

9.1 Overview



10. Categorical Prediction

10.1 Overview



11. Some Important GLM

11.1 Overview



12. Multivariate GLM

12.1 Overview



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13. Principle Component Analysis

13.1 Overview



14. Canonical Correlation Analysis

14.1 Overview



15. Independent Component Analysis

15.1 Overview



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