A Hidden Markov Model for Animal Networks

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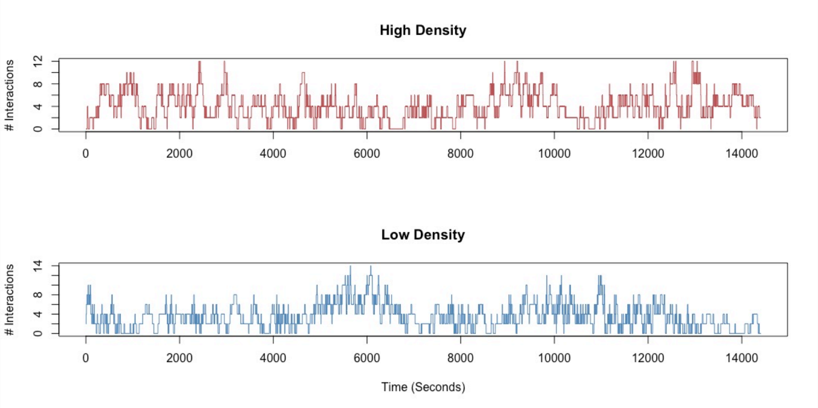
Carpenter Ant

- Trophallaxis - a mutual exchange of liquid nutrients between two ants
- Inactive forager ants may play an important role as brokers of trophallaxis in colonies, and active foragers are interacting with nest workers more than might be expected. (Quevillon et al., 2015)

Behavior Changes with Density

Does ant trophallaxis behavior change with colony density?



Analysis of Ant Trophallaxis

Data: $\{N_t, t = 1, 2, \ldots\}$
Number of ants engaging in trophallaxis at time t.

Goals:

1. Accurate/useful model of ant trophallaxis interactions
2. Inference on rates of starting and ending trophallaxis interactions

CTMC "Hidden" Markov Model

Unobserved: $\{X_t, t = 1, \ldots, T\}$
Rate (high/low) of trophallaxis in colony.

α: Rate of starting/ending trophallaxis event

$\alpha_{N,N+2} = \gamma_{X_t}$

$\alpha_{N,N-2} = \lambda_{X_t} \frac{N}{2}$

$$R^{(X_t)} = \begin{array}{c c} & \begin{array}{cccc} 0 & 2 & 4 & 6 \end{array} \\ \begin{array}{c} 0 \\ 2 \\ 4 \\ 6 \end{array} & \begin{pmatrix} 0 & \gamma_{X_t} & 0 & 0 \\ \lambda_{X_t} & 0 & \gamma_{X_t} & 0 \\ 0 & 2\lambda_{X_t} & 0 & \ddots \\ 0 & 0 & \ddots & 0 \end{pmatrix} \end{array}$$

$Q = R - diag(R1')$

$P(N_{t+1}|N_t, \gamma_{X_t}, \lambda_{X_t}, X_t) = e^{Q^{(X_t)} * \Delta t} = P^{(X_t)}$

Model Details

Data Likelihood

$$[\{N_t\}|\theta = (\gamma, \lambda, X_t)] = \prod_{t=2}^{T} P^{X_t}_{N_t, N_{t+1}}$$

Markov State Switching

$X_t | X_{t-1} \sim \text{Multinom}(M_{X_{t-1}})$

M: Probability transition matrix for X_t

Priors

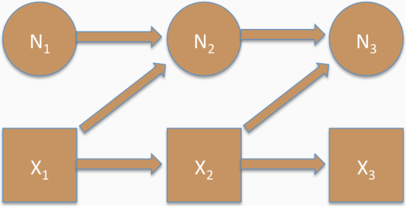
$\tilde{\gamma}_H \sim \text{Gamma}(a, b)$

$\gamma_L \sim \text{Gamma}(c, d)$

$\gamma_H = \tilde{\gamma}_H + \gamma_L$

$\lambda_{X_t} \sim \text{Gamma}(r, q)$

$M_{X_t} \sim \text{Direchlet}(\underline{\theta})$



MCMC Algorithm

Metropolis Hastings Updates

$$[\tilde{\gamma}_H|\cdot] \propto \prod_{t=2}^{T} P^{(H)}_{N_{t-1},N_t} \left[\tilde{\gamma}_H^{a-1} e^{-b\tilde{\gamma}_H}\right]$$

$$[\gamma_L|\cdot] \propto \prod_{t=2}^{T} P^{(L)}_{N_{t-1},N_t} \left[\gamma_L^{c-1} e^{-d\gamma_L}\right]$$

$$[\lambda_H|\cdot] \propto \prod_{t=2}^{T} P^{(H)}_{N_{t-1},N_t} \left[\lambda_H^{r-1} e^{-q\lambda_H}\right]$$

$$[\lambda_L|\cdot] \propto \prod_{t=2}^{T} P^{(L)}_{N_{t-1},N_t} \left[\lambda_L^{r-1} e^{-q\lambda_L}\right]$$

Gibbs Updates

$$P(X_t = H|\cdot) \propto \left[P^{(H)}_{N_{t-1},N_t}\right] \left[M_{X_{t-1}H}\right] \left[M_{HX_{t+1}}\right]$$

$$P(X_t = L|\cdot) \propto \left[P^{(L)}_{N_{t-1},N_t}\right] \left[M_{X_{t-1}L}\right] \left[M_{LX_{t+1}}\right]$$

$$[M_L|\cdot] \sim \text{Direchlet}(\Sigma_L + \theta_L)$$

$$[M_H|\cdot] \sim \text{Direchlet}(\Sigma_H + \theta_H)$$

Results

blah

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blah

Summary

summary

Future Research

- Covariates
    - Distance to nearest ant
    - Time since Forager entrance
    - Ant Functional Groups
- Disease research (e.g. Zika, Malaria, etc)

Questions?

Backup slides

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