Introduction to Stationary Time Series

What is time series?

- Discrete observations ordered over time

What is time series?

- Discrete observations ordered over time
- Hard

What is time series?

- Discrete observations ordered over time
- Hard
- Messy

What is time series?

- Discrete observations ordered over time
- Hard
- Messy
- Important

What makes time series different?

- The goal is the same: Model outcomes from an underlying data generating process. However…

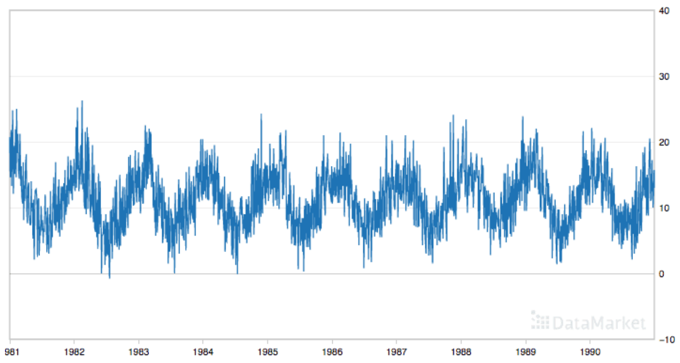
What makes time series different?

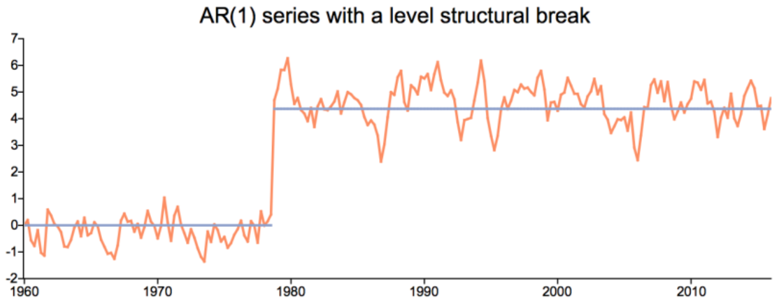
- The goal is the same: Model outcomes from an underlying data generating process. However…
- OLS assumes independent random variables
  - $E(\varepsilon | X) = 0$ (The expected value of the error term given our regressors is zero)
  - Also known as “strict exogeneity”

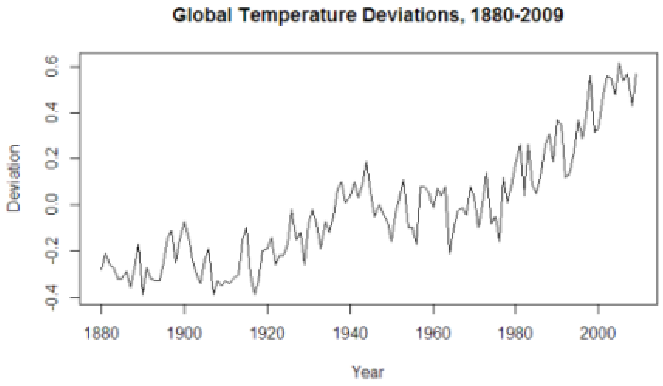
What makes time series different?

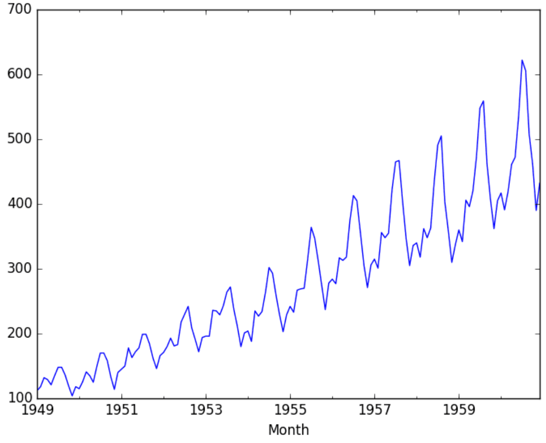
- The goal is the same: Model outcomes from an underlying data generating process. However…
- OLS assumes independent random variables
  - E(ε|X) = 0 (The expected value of the error term given our regressors is zero)
  - Also known as "strict exogeneity"
- Time series data usually violates this assumption because y_t is a function of y_{t-1}, thus OLS is no longer BLUE.
  - If y_t is a function of y_{t-1}, then the errors compound over time and the expected value of the error term is no longer zero.
  - $y_t = y_{t-1} + X_t + \varepsilon_t$
  - $y_{t-1} = y_{t-2} + X_{t-1} + \varepsilon_{t-1}$
- As a result, time series must model system dynamics

System Dynamics

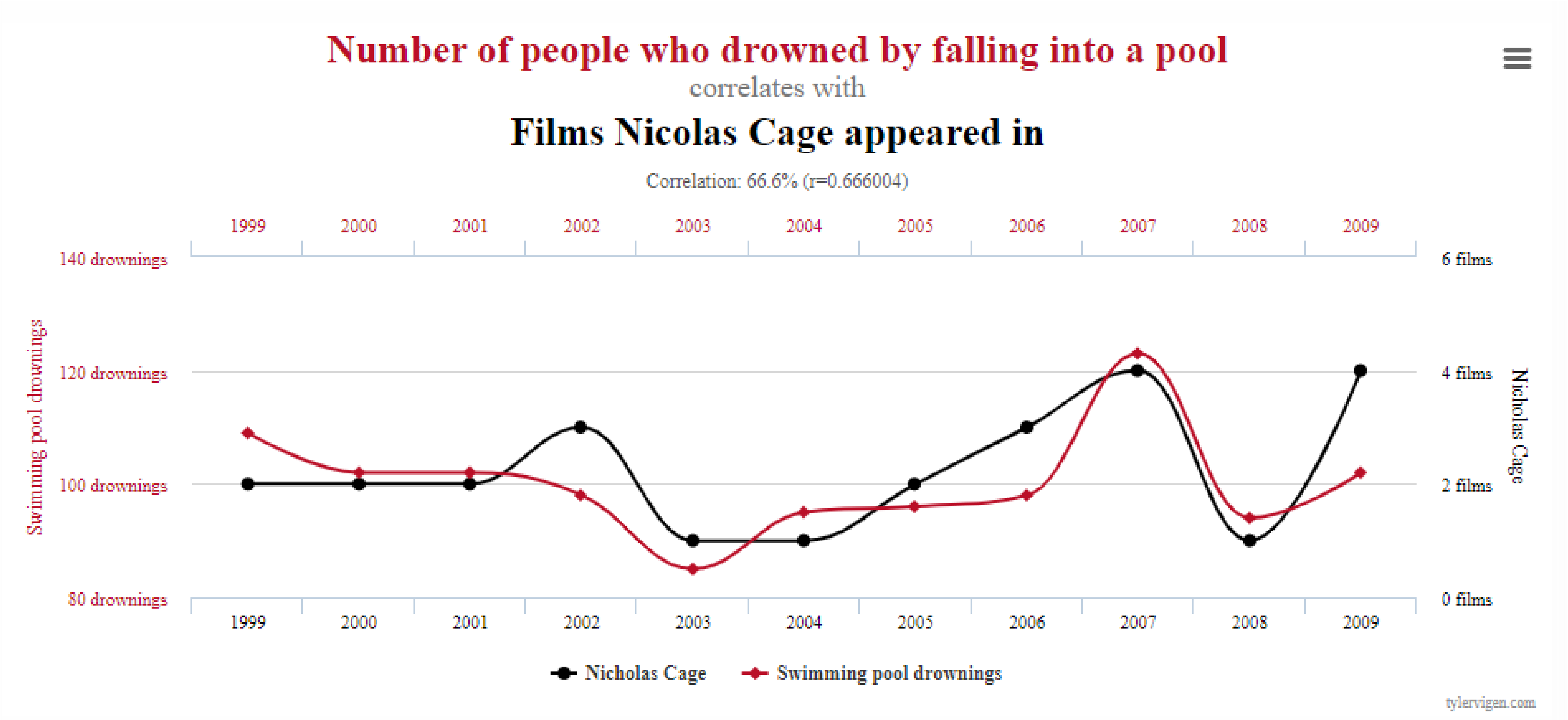








Spurious correlations

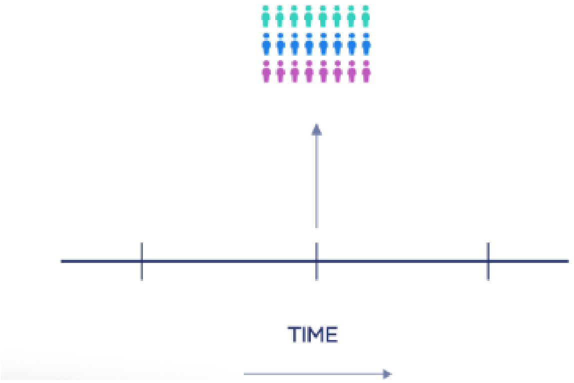


Data sources: Centers for Disease Control & Prevention and Internet Movie Database

Key Concepts and Nomenclature

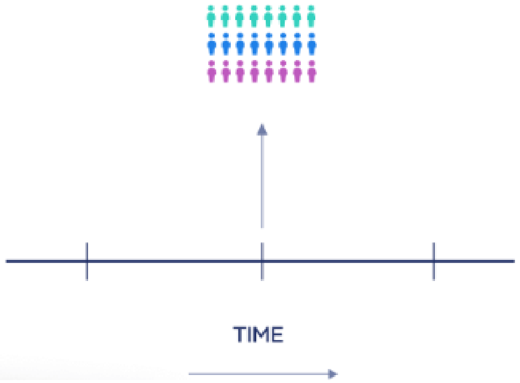
Types of Data

Cross Sectional

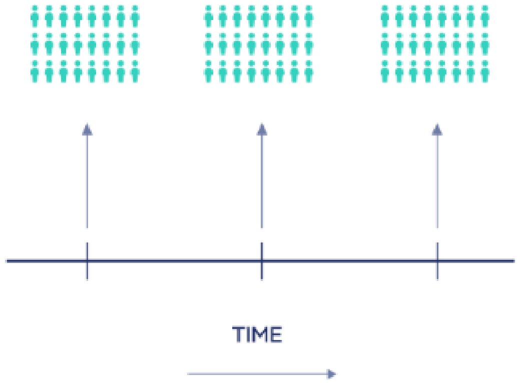


Types of Data

Cross Sectional

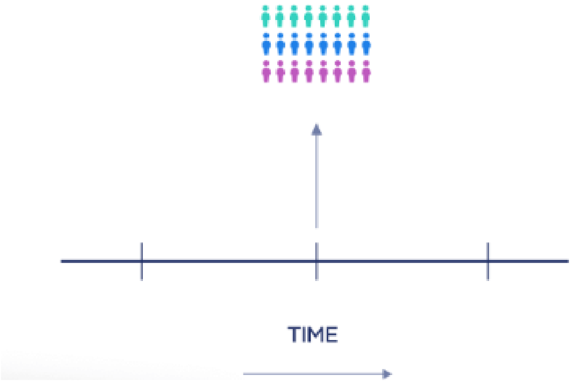


Time Series

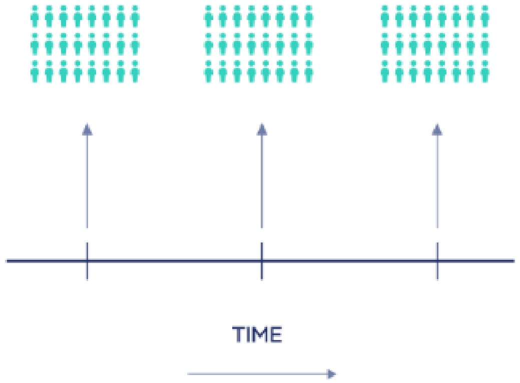


Types of Data

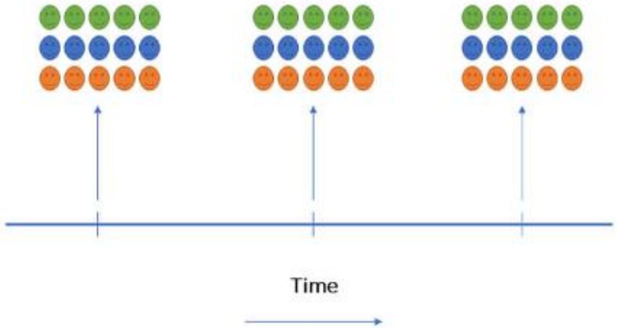
Cross Sectional



Time Series



Panel

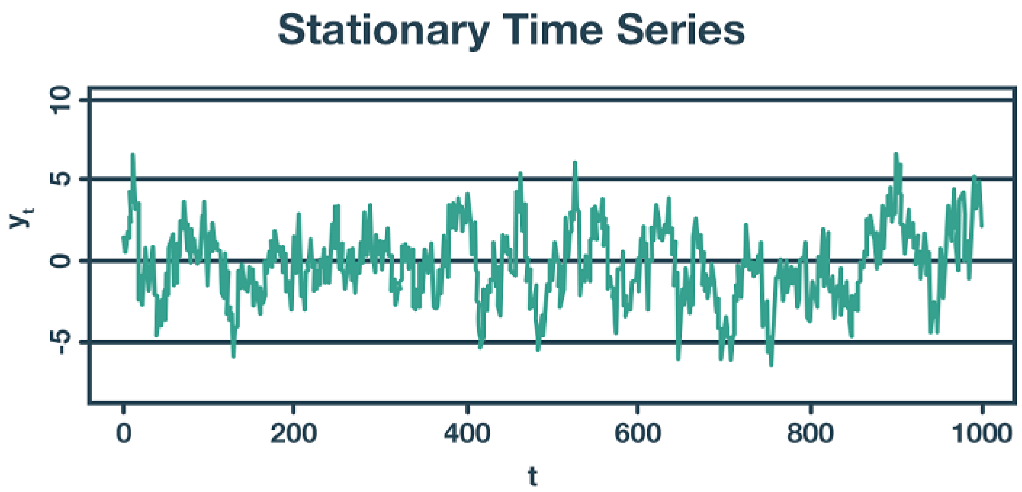


Sample window and interval

- Window: The total period of time over which your data are collected
- Interval: The discrete period of time you use to measure your data (e.g. hours, days, years)

Stationarity

- A time series is *covariance stationary* if its mean and variance are independent of time and its covariances are finite and depend only on the number of periods separating observations
  - $\mu_t = \mu_{t-s}$ or $E(y_t) = E(y_{t-s})$
  - $V(y_t) = V(y_{t-s})$
  - $cov(y_t, y_{t-s}) = cov(y_{t-j}, y_{t-j-s})$

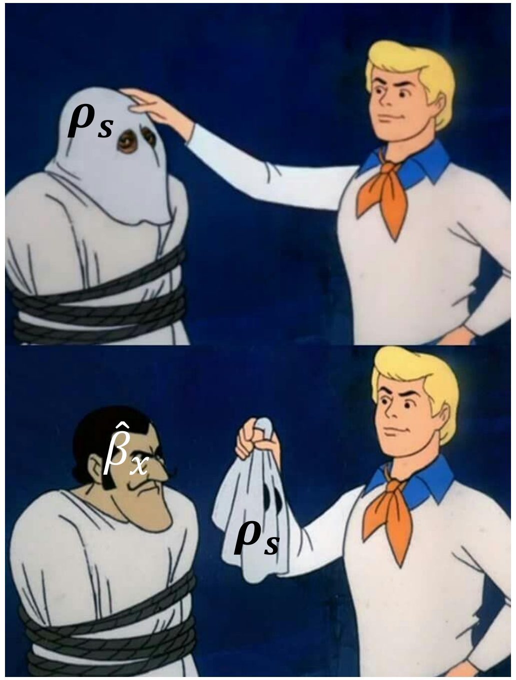


Autocorrelation

- The dependence between observations of the same time series observed at different points in time.
- Formally, the autocorrelation coefficient is:
  - ρ_s = cov(y_t , y_{t-s})/ V(y_t)

Autocorrelation

- The dependence between observations of the same time series observed at different points in time.
- Formally, the autocorrelation coefficient is:
  - $\rho_s = \text{cov}(y_t , y_{t-s})/ \text{V}(y_t)$
- Autocorrelation is just a bivariate regression coefficient!
  - $\hat{\beta}_x = \text{cov}(y_i , x_i)/ \text{V}(x_i)$

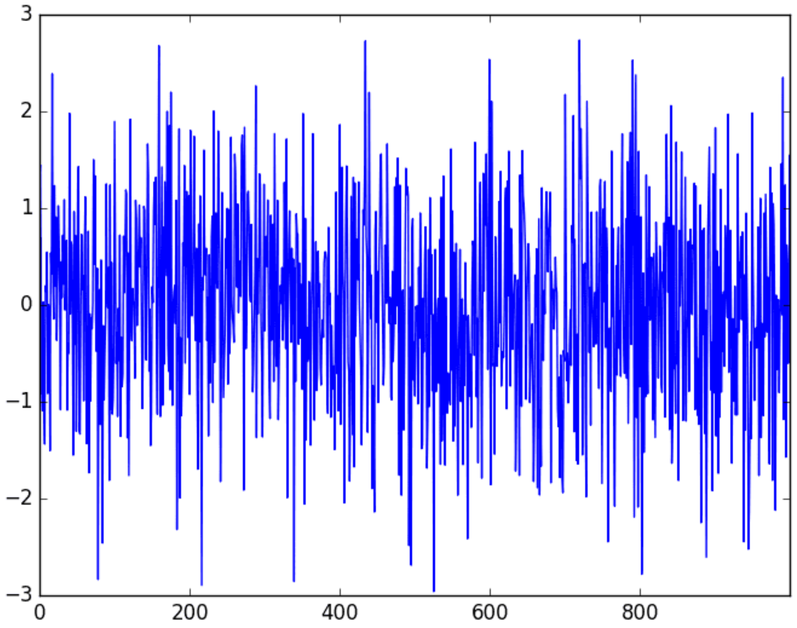


Partial autocorrelation

- The correlation between y_t and y_{t-s} after controlling for the intervening lags.
- If autocorrelation is a slope coefficient in a bivariate regression, partial autocorrelation is a slope coefficient in a multiple regression.
- Generally represented as φ (fi)

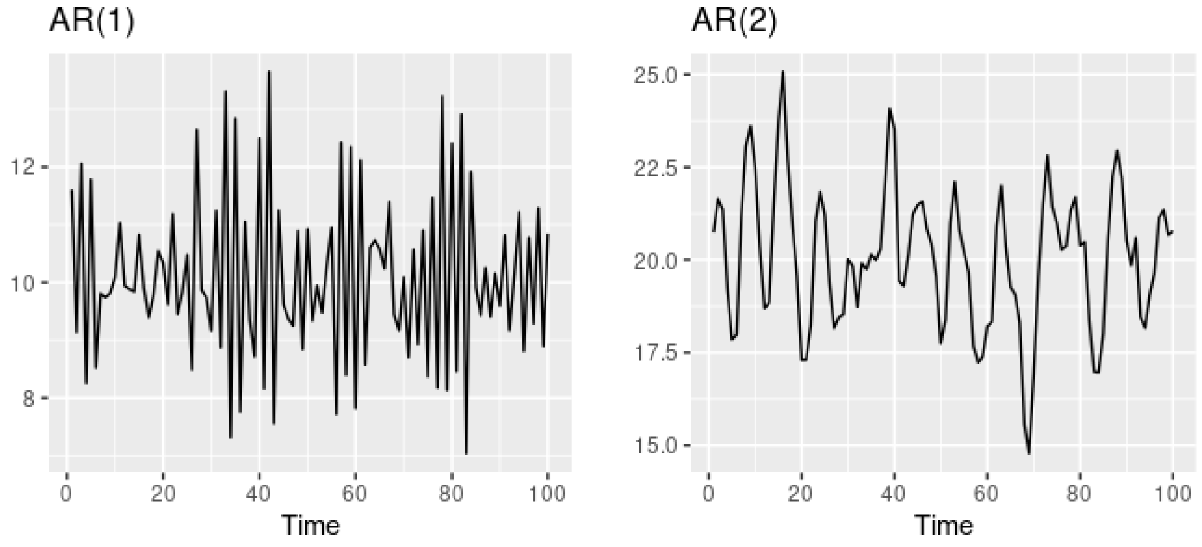
White noise process

- No predictable pattern over time
  - $E(y_t) = E(\varepsilon_t)$
  - Mean of zero
  - Constant variance
  - All covariances equal zero



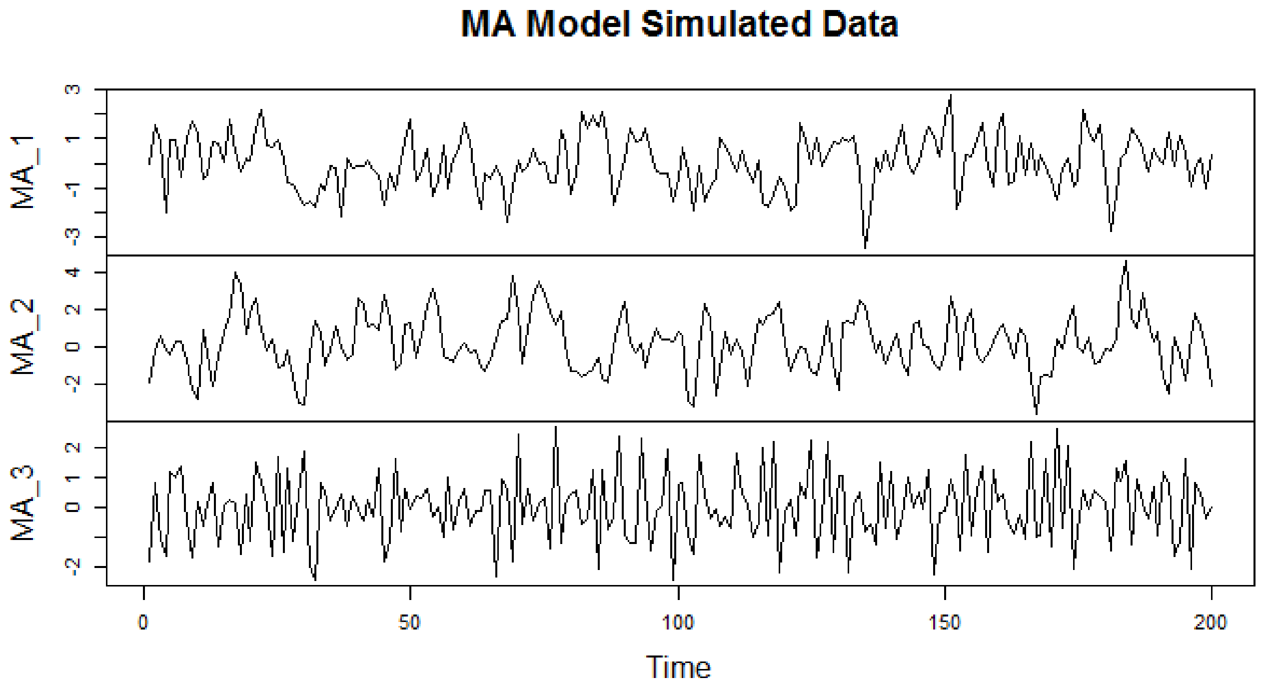
Autoregressive process (AR)

- A process in which the current value of y_t is a function of previous values of y_t
- $y_t = c + \varphi y_{t-1} + \varepsilon_t$
- $y_{t-1} = c + \varphi y_{t-2} + \varepsilon_{t-1}$
- Useful to think of AR as inertia in the process regressing back to its mean.



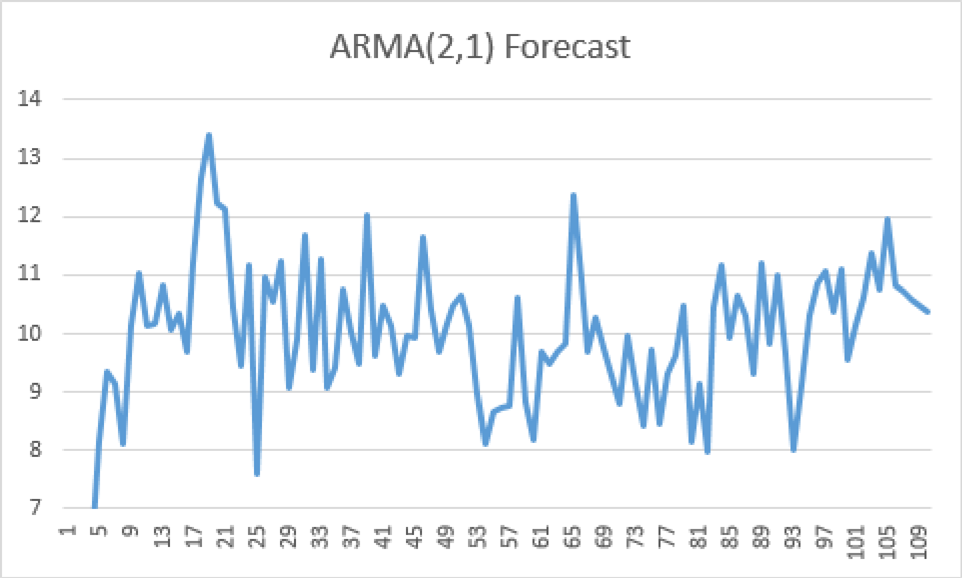
Moving average process (MA)

- The current values of y_t is a function of past values of the error term. The data generating process for y_{t+1} adjusts in response to ε_t
- Example: Polling accuracy



Autoregressive moving average process (ARMA)

- A process that has both AR and MA components.
- Example: Budgets.
  - AR: Start with previous years budget and then adjust
  - MA: Additional adjustments for unexpected windfalls and shortfalls



Seasonality

- Regular patterns that repeat over some number of time periods s.