# Introduction to Time Series

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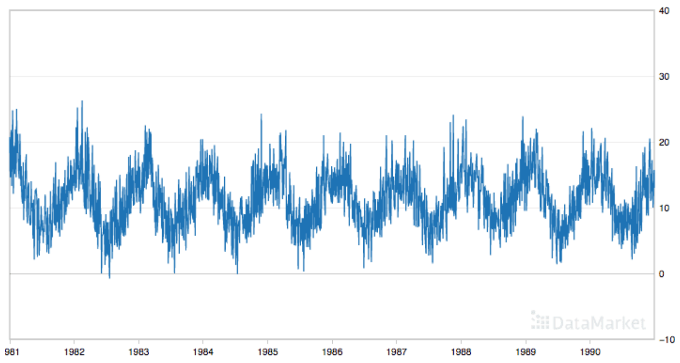
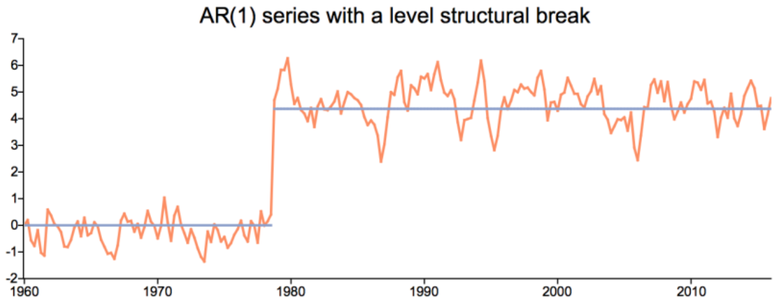
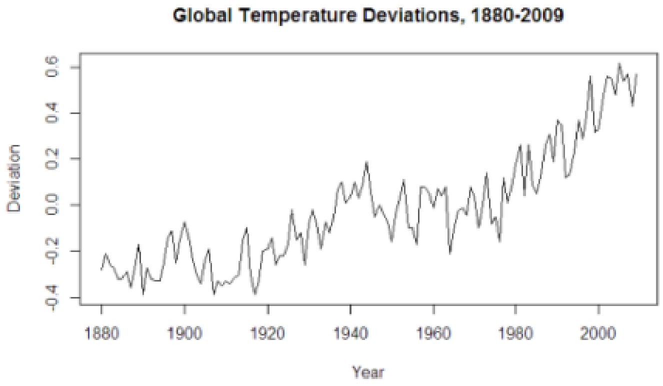
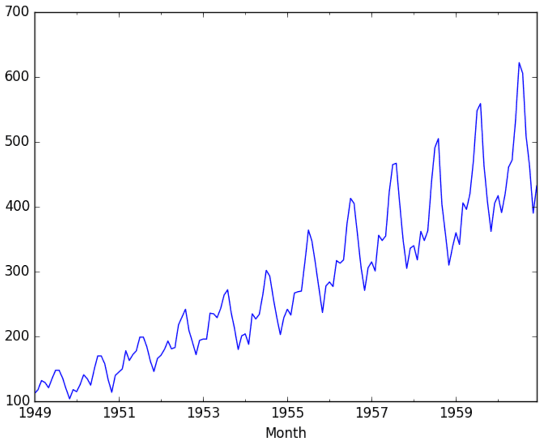
  - Also known as "strict exogeneity"
- Time series data usually violates this assumption because $y_t$ is a function of $y_{t-1}$, thus OLS is no longer BLUE.
  - If $y_t$ is a function of $y_{t-1}$, then the errors compound over time and the expected value of the error term is no longer zero.
  - $y_t = y_{t-1} + X_t + \varepsilon_t$
  - $y_{t-1} = y_{t-2} + X_{t-1} + \varepsilon_{t-1}$
- As a result, time series must model system dynamics

# System Dynamics

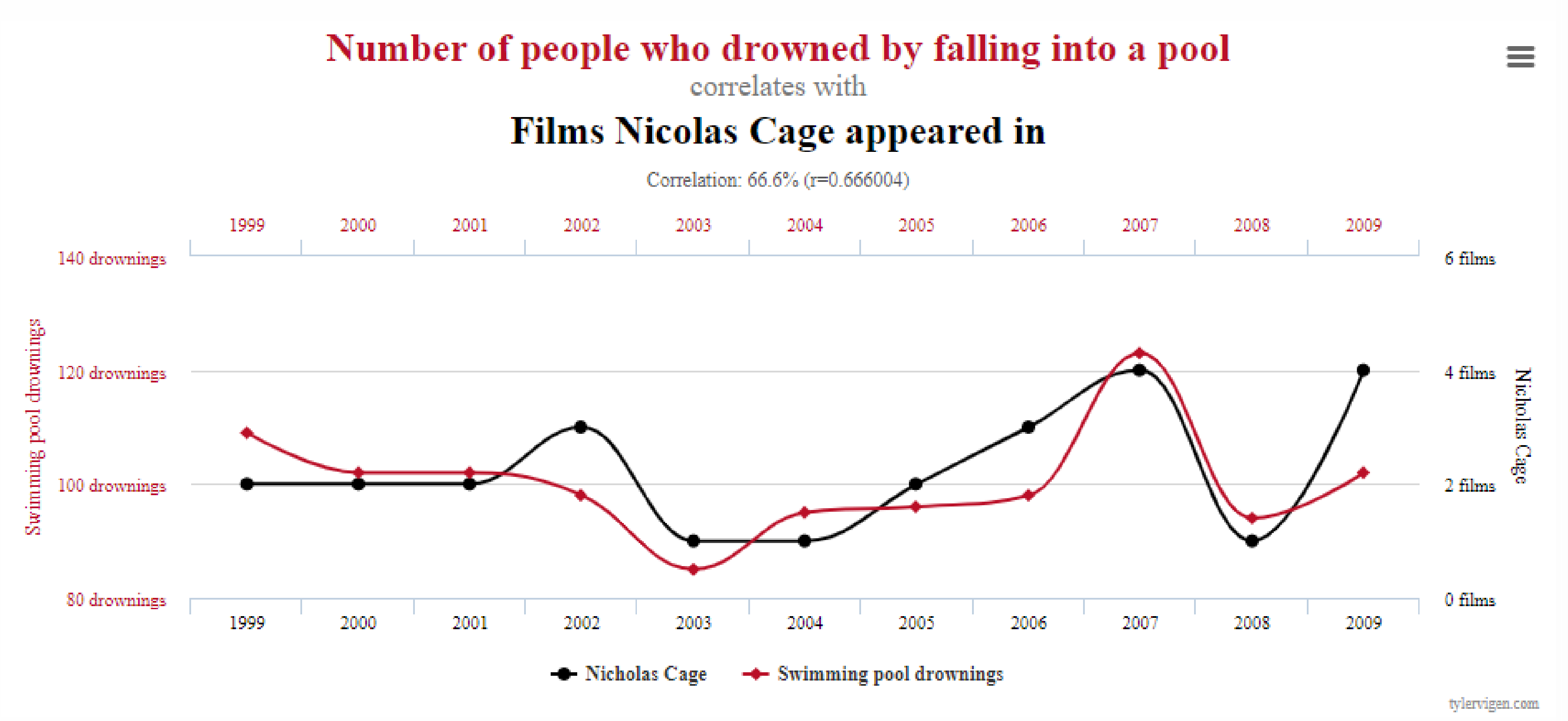


# Spurious correlations

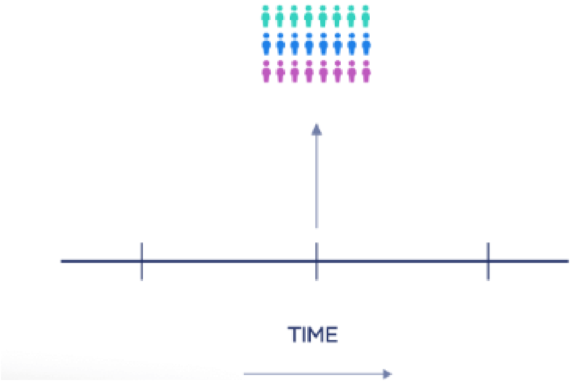


Data sources: Centers for Disease Control & Prevention and Internet Movie Database

# Key Concepts and Nomenclature

# Types of Data

Cross Sectional



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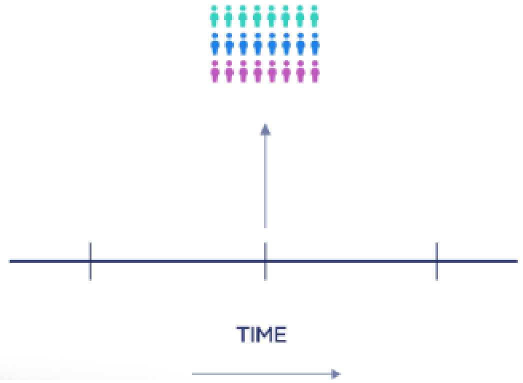
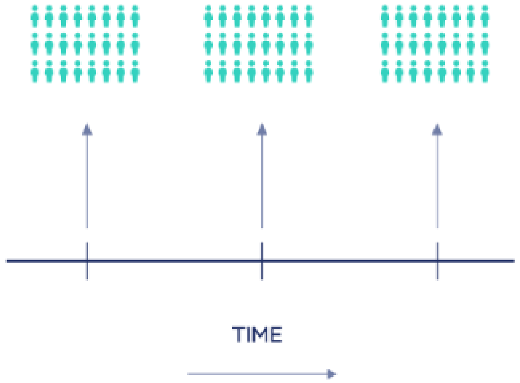



Time Series



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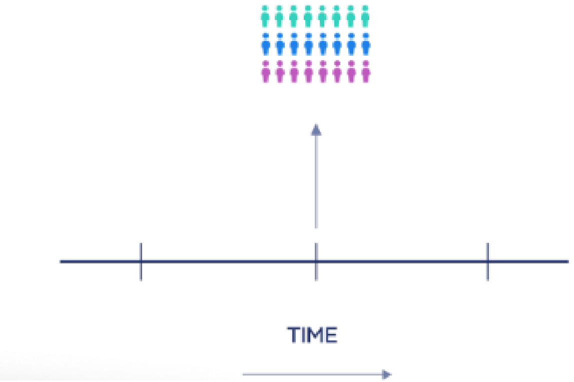
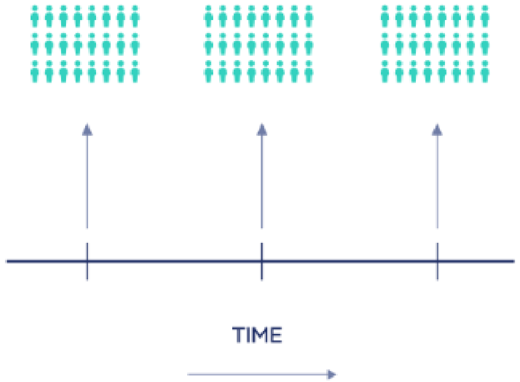
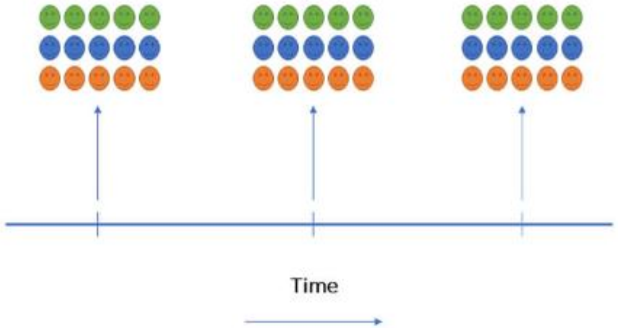
### Cross Sectional




### Time Series




### Panel

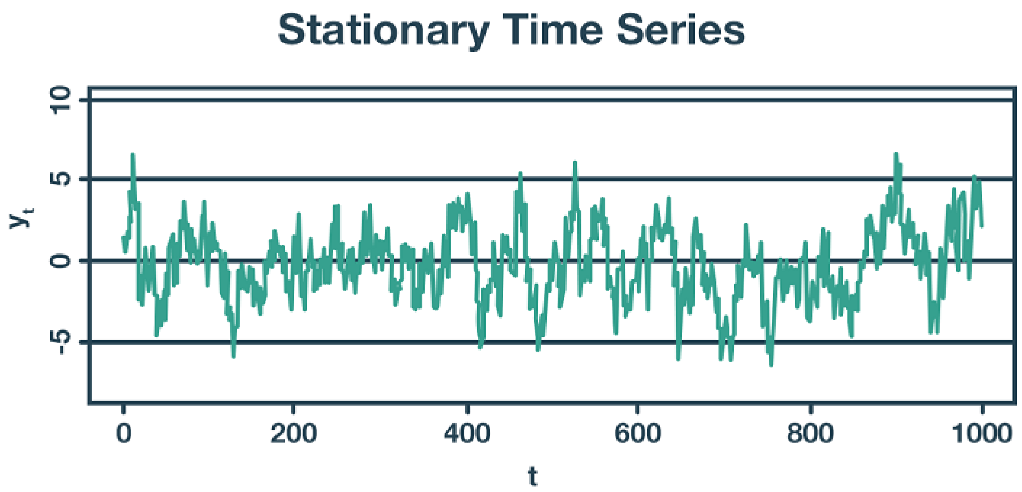


# Sample window and interval

- Window: The total period of time over which your data are collected
- Interval: The discrete period of time you use to measure your data (e.g. hours, days, years)

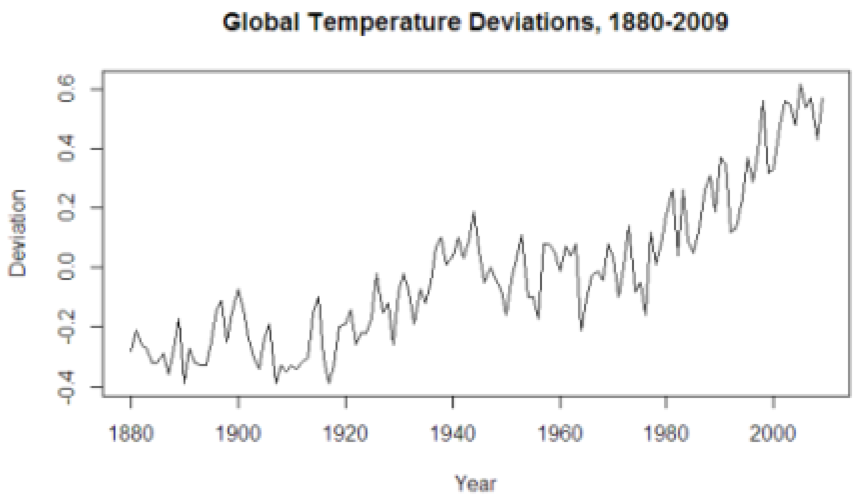
# Stationarity

- A time series is *covariance stationary* if its mean and variance are independent of time and its covariances are finite and depend only on the number of periods separating observations
  - $\mu_t = \mu_{t-s}$ or $\mathrm{E}(y_t) = \mathrm{E}(y_{t-s})$
  - $\mathrm{V}(y_t) = \mathrm{V}(y_{t-s})$
  - $\mathrm{cov}(y_t, y_{t-s}) = \mathrm{cov}(y_{t-j}, y_{t-j-s})$



# Non-stationary

- Mean, variance, and covariance change over time



# Autocorrelation

- The dependence between observations of the same time series observed at different points in time.
- Formally, the autocorrelation coefficient is:
  - $\rho_s = \mathrm{cov}(y_t\ ,\ y_{t-s}\ )/\ \mathrm{V}(y_t\ )$

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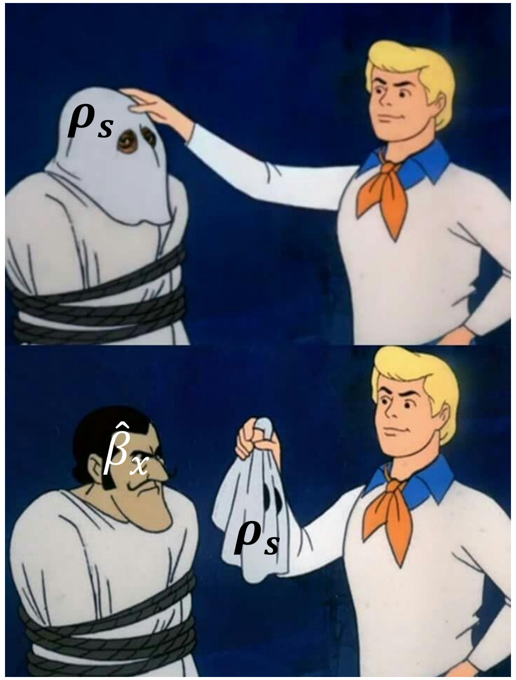
- Autocorrelation is just a bivariate regression coefficient!
  - $\hat{\beta}_x = \text{cov}(y_i\ ,\ x_i\ )/\ \text{V}(x_i\ )$

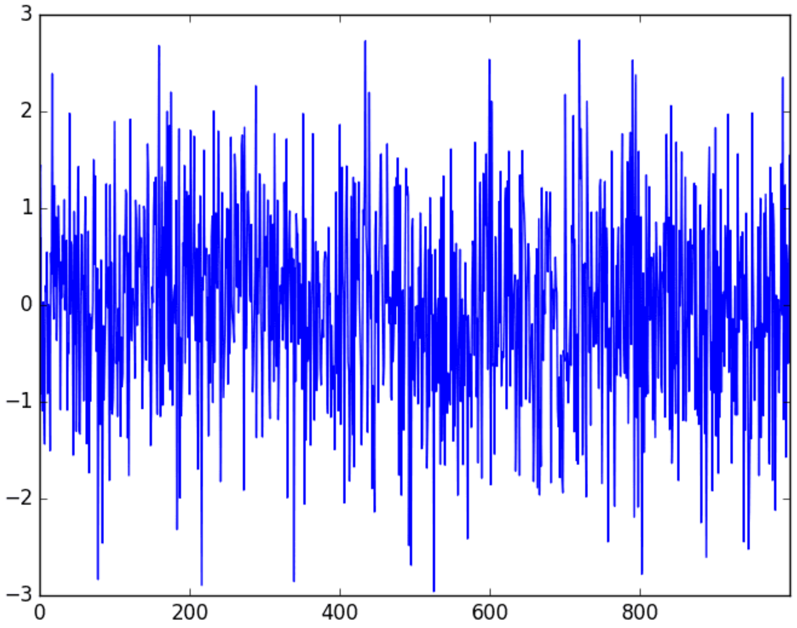


# Partial autocorrelation

- The correlation between $y_t$ and $y_{t-s}$ after controlling for the intervening lags.
- If autocorrelation is a slope coefficient in a bivariate regression, partial autocorrelation is a slope coefficient in a multiple regression.
- Generally represented as $\varphi$ (fi)

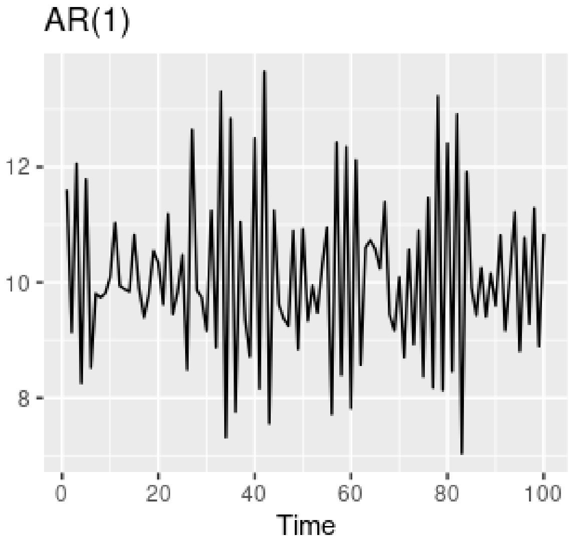
# White noise process

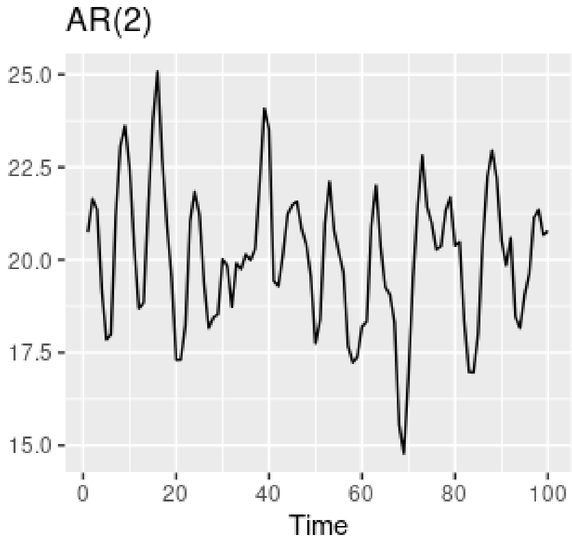
- No predictable pattern over time
  - $E(y_t) = E(\varepsilon_t)$
  - Mean of zero
  - Constant variance
  - All covariances equal zero



# Autoregressive process (AR)

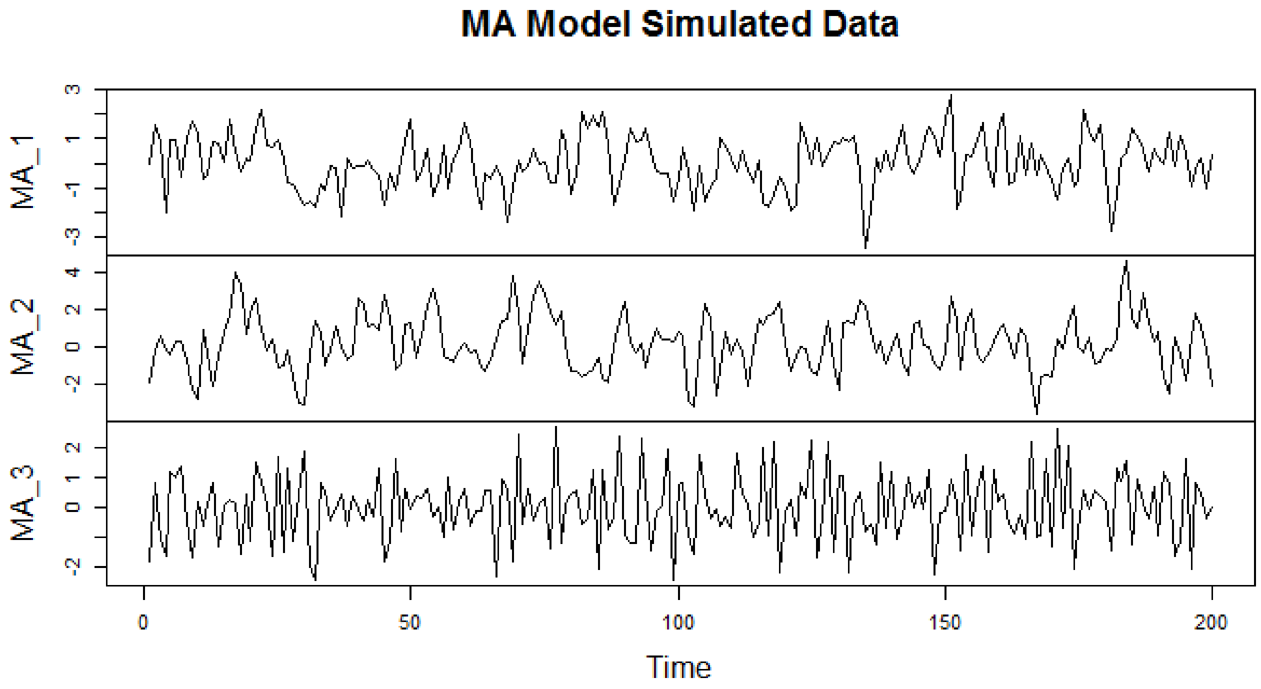
- A process in which the current value of $y_t$ is a function of previous values of $y_t$
- $y_t = c + \varphi y_{t-1} + \varepsilon_t$
- $y_{t-1} = c + \varphi y_{t-2} + \varepsilon_{t-1}$
- Useful to think of AR as inertia in the process regressing back to its mean.



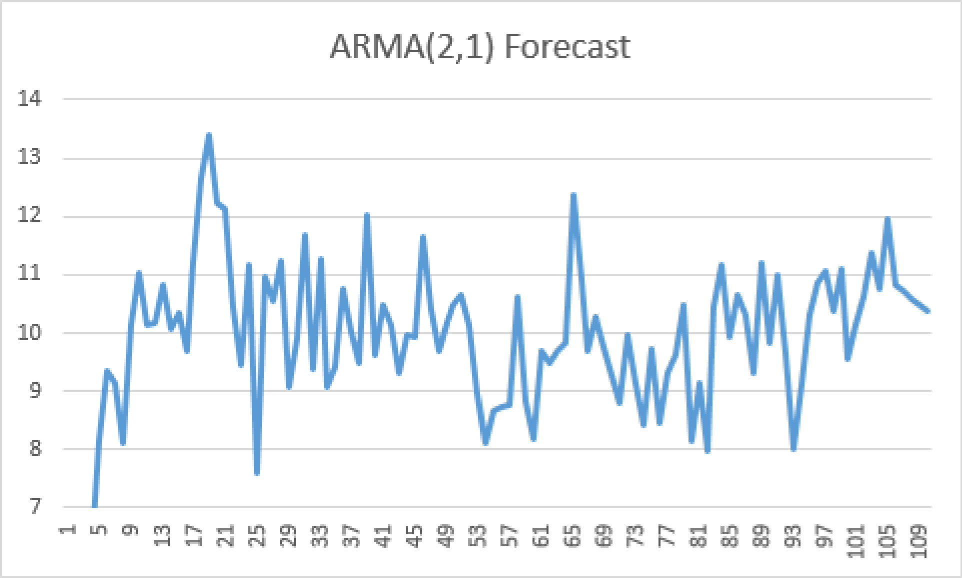
# Moving average process (MA)

- The current values of $y_t$ is a function of past values of the error term. The data generating process for $y_{t+1}$ adjusts in response to $\varepsilon_t$
- Example: Polling accuracy



# Autoregressive moving average process (ARMA)

- A process that has both AR and MA components.
- Example: Budgets.
  - AR: Start with previous years budget and then adjust
  - MA: Additional adjustments for unexpected windfalls and shortfalls



# Seasonality

- Regular patterns that repeat over some number of time periods $s$.