OIC: An Online Incentivized and Context-Aware Scheme for Task Assignment: Technical Report

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***Abstract*—Online task assignment plays a central role in applications such as crowdsourcing and online advertising, where both context awareness and worker incentives are essential for ensuring quality and long-term participation. However, existing studies either ignore context-aware task-worker matching or overlook incentive design, failing to adapt when both factors interact under changing conditions. To bridge the gap, we propose an Online Incentivized and Context-aware (OIC) task assignment scheme that jointly addresses the interdependence between context awareness and worker incentives in dynamic environments. We first formulate the task assignment problem as a Stackelberg game and derive the optimal incentive allocation under an oracle setting. Then we develop an online learning algorithm within OIC that approximates the oracle solution to address the unknown and dynamic task-worker relationships. We provide theoretical guarantees by proving that our OIC scheme achieves sublinear regret. Extensive experiments demonstrate that OIC consistently outperforms baseline methods in terms of effective real-time task assignment under changing conditions and strategic worker behaviors.**

***Index Terms*—Task assignment, online learning, context aware, incentive mechanism**

I. Introduction

Online task assignment plays a vital role in many real-world applications [1], [2]. For example, in crowdsourcing platforms such as Amazon Mechanical Turk, assigning tasks to workers with appropriate expertise in real time can significantly reduce completion time and improve output quality. In online advertising, dynamic ad placement based on ad types and advertiser characteristics can increase click-through rates and revenue. Effective online task assignment not only boosts productivity but also helps maintain long-term worker engagement.

Related work. Two critical challenges in online task assignment are: (i) context-aware matching of task requirements with worker capabilities under non-stationary conditions; and (ii) designing incentive mechanisms that ensure non-negative utility and preserve workers' self-interest. Contextual bandit approaches have been used to solve the first challenge, aligning task and worker feature vectors [3]–[5]. For example, Liu *et al.* [3] consider contextual information including task type, base station location, and service provider, proposing a contextual sleeping bandit learning algorithm with sublinear regret. The second challenge involves game-theoretic and auction-based incentive mechanisms [6]–[8]. These mechanisms typically fall into two modes: *worker-selected tasks* and *server-assigned tasks* [9]. In the *worker-selected tasks* mode, tasks are published openly, and workers choose tasks based on individual preferences or utility [10]. In contrast, the *server-assigned tasks* mode features centralized assignment to optimize system-level objectives such as social welfare [11], fairness [12], or truthfulness [13]. For instance, Wang *et al.* [14] consider a server-centric crowdsensing model, incorporating data quality for federated learning and accounting for selfish worker behavior.

However, these studies often overlook strategic worker behavior (e.g., [5], [15]) or assume static or fully known environments, limiting their adaptability in dynamic and uncertain settings (e.g., [6], [14]). Joint optimization of context-aware task-worker matching and incentive allocation under dynamic conditions remains largely unexplored.

In this paper, we propose an Online Incentivized and Context-aware (OIC) task assignment scheme that considers context-aware task-worker matching and heterogeneous strategic workers, addressing both above challenges and achieving sublinear regret. We model the one-to-many task assignment problem under the *server-assigned tasks* mode as a Stackelberg game. Due to the context-dependent and unknown service quality of workers, computing the exact game equilibrium is intractable. To address this, we design an online learning algorithm to approximate the equilibrium, introducing an exploration–exploitation tradeoff. We manage this tradeoff by establishing an upper confidence bound on the server's utility and show that our scheme achieves strong long-term performance.

Our main contributions are as follows: (i) We propose a novel incentivized and context-aware task assignment scheme that accounts for dynamic task contexts and worker heterogeneity. (ii) We formulate the incentive mechanism design as a Stackelberg game and derive the optimal solution in an oracle setting with full knowledge of workers' service quality. (iii) Based on the oracle strategy, we develop the online scheme OIC in an online setting with no prior knowledge of workers' service quality. OIC addresses the estimation of unknown service quality and finds an approximately optimal task assignment scheme that achieves sublinear regret. (iv) We evaluate the proposed algorithm through extensive simulations, demonstrating its effectiveness in approximating the oracle solution and maximizing the server's long-term utility.

The remainder of this paper is organized as follows. Section II introduces our model and problem formulation. Section III

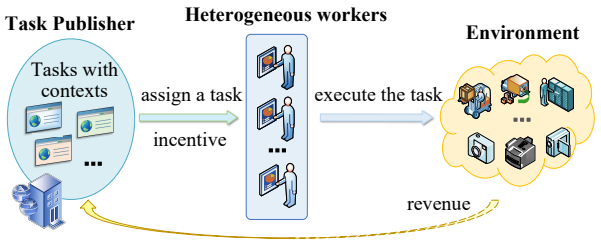


Fig. 1. Online incentivized and context-aware task assignment model.

presents our proposed scheme and its theoretical analysis. We evaluate the performance through numerical experiments in Section IV, and finally conclude the paper in Section V.

II. Model and Problem Formulation

In this section, we introduce our online task assignment model, define the utilities of both the server and clients, and formally present the objective of our proposed scheme.

Notations. Let $[n]$ denotes the index set $\{1, 2, \ldots, n\}$ for any positive integer n. Bold symbols represent vectors (column vectors by default) and matrices throughout the paper.

A. Online Task Assignment Model

Our online task assignment model, shown in Fig. 1, models a dynamic interaction between a task publisher (server) and multiple heterogeneous workers (clients). Formally, the model consists of a server and a set of clients $\mathcal{N}$, where $|\mathcal{N}| = N$. Each client $i \in \mathcal{N}$ incurs a unit cost c_i to execute a task At round t, the server selects one task from a set of K candidate tasks that maximizes its own utility and assigns it to clients. Each client provides service with quality $q_{i,t}^k > 0$ for task k, which represents metrics such as click-through rate or revenue in the real world. This model operates in an online manner over a total of T rounds. At round $t \in [T]$, the following steps occur:

(1) The server observes a set of context vectors of the K candidate tasks: $\mathcal{A}_t = \{\boldsymbol{x}_t^1, \cdots, \boldsymbol{x}_t^k, \cdots, \boldsymbol{x}_t^K\} \subset \mathbb{R}^d$, where the d-dimensional vector $\boldsymbol{x}_t^k$ represents the context vector of task $k \in [K]$ at round t.

(2) The server selects a task k_t to be executed at round t, determines the total incentive payment P_t, and assigns required resource usage $r_{i,t} \geq 0$ to client $i \in \mathcal{N}$ based on client i's cost and service quality (defined later). Client i would participate in the task if $r_{i,t} > 0$.

(3) Let $\mathcal{S}_t$ denote the set of participating clients. The client $i \in \mathcal{S}_t$ receives an incentive and executes the task k_t according to its required resource usage $r_{i,t}$.

In real-world applications, the server does not have prior knowledge of each client's service quality. Hence, we assume that the service quality of each client is an unknown but fixed linear function of the task context, with parameters that can be learned over time. Specifically, the actual service quality $q_{i,t}^k$ of client i for task k at round t is defined as:

$$q_{i,t}^k = \langle \boldsymbol{x}_t^k, \boldsymbol{\theta}_i \rangle + \eta_{i,t}^k, \tag{1}$$

where $\boldsymbol{\theta}_i \in \mathbb{R}^d$ is the unknown model parameter for client i and $\eta_{i,t}^k$ is a zero-mean σ-subgaussian noise term. Over time, client i can estimate its parameter $\boldsymbol{\theta}_i$ using historical data and thus compute an estimate $\hat{q}_{i,t}^k$, which enables the server to assign tasks properly in the absence of complete information.

B. Utility Definitions

We define the utility functions of both the clients and the server as the metric used to design an effective incentive mechanism.

1) Client's utility: The utility of a client is defined as the incentive received from the server minus the incurred cost. When the server assigns task k to client i at round t, the client's utility is:

$$u_{i,t}^k = \frac{q_{i,t}^k r_{i,t}^k}{\sum_{j \in \mathcal{N}} q_{j,t}^k r_{j,t}^k} P_t^k - r_{i,t}^k c_i, \tag{2}$$

where P_t^k is the total incentive payment to all clients if task k is selected at round t, and $r_{i,t}^k c_i$ represents the total cost incurred to execute task k for client i in round t.

2) Server's utility: If the server requires clients to execute task k, its utility is defined as

$$u_{0,t}^k = \gamma \log(1 + \sum_{i \in \mathcal{N}} q_{i,t}^k r_{i,t}^k) - P_t^k, \tag{3}$$

where γ is a system parameter representing the ratio between virtual task execution performance and actual monetary payment. The logarithmic function captures diminishing returns with respect to the total contributed service quality.

C. Objective

Our online task assignment model is server-centric, aiming to maximize the server's utility while ensuring clients are incentivized to participate. Specifically, at round t, for each task $k \in [K]$, we compute the Nash Equilibrium (NE) of the clients' strategies and then determine the server's utility under that equilibrium. NE in our model is defined as follows.

Definition 1 (Nash Equilibrium (NE))**.** *A Nash equilibrium is a strategy profile* $\boldsymbol{r}_t^{k^{\mathrm{NE}}} = (r_{1,t}^{k^{\mathrm{NE}}}, r_{2,t}^{k^{\mathrm{NE}}}, ..., r_{N,t}^{k^{\mathrm{NE}}})$ *in which no client can increase its utility by unilaterally deviating from its strategy. Formally, for each client* $i \in \mathcal{N}$ *and any* $r_{i,t}^k \geq 0$*,*

$$u_{i,t}^k(r_{i,t}^{k^{\mathrm{NE}}}, r_{-i,t}^{k^{\mathrm{NE}}}) \geq u_{i,t}^k(r_{i,t}^k, r_{-i,t}^{k^{\mathrm{NE}}}), \tag{4}$$

where $r_{-i,t}^{k^{\mathrm{NE}}}$ *denotes the equilibrium strategy profile of all clients except* i*.*

Given the NE for each task, the server then selects the task that maximizes its utility. In the online learning literature, it is equivalent to the long-term objective of minimizing the cumulative regret, which is defined as:

$$R_T = \sum_{t=1}^{T} u_{0,t}^{k_t^*} - u_{0,t}^{k_t}, \tag{5}$$

where k_t^* is the optimal task at round t, and k_t is the task chosen by the server. Regret captures the performance gap between the oracle and the actual task selections across all rounds, so a lower regret implies better performance.

III. OIC: Online Incentivized and Context-aware Task Assignment Scheme

We formulate the task assignment problem as a Stackelberg game, where the server is the leader and the clients are followers. To derive the optimal solution, we decompose the problem into the following two sub-problems:

P1: NE with known service quality. In this oracle setting, the server has full knowledge of each client's service quality $q_{i,t}^k$ for all tasks. For a given task k and total payment P_t^k, the server needs to determine a unique NE strategy profile $\boldsymbol{r}_t^{k^{\mathrm{NE}}}$ for resource usages allocation. (Discussed in Section III-A)

P2: Online task selection with approximate NE. Without prior knowledge of service quality, the server needs to (i) estimate it via online learning; (ii) use the estimate to compute an strategy profile $\hat{\boldsymbol{r}}_t^{k^{\mathrm{NE}}}$ approximating the NE from **P1**; and (iii) estimate the optimal payment $\hat{P}_t^k$ that maximizes its utility. The server then evaluates its estimated utility $\hat{u}_{0,t}^k$ for each task and selects the task with the highest estimated utility, i.e., $k_t = \arg\max_k \hat{u}_{0,t}^k$. (Discussed in Section III-B)

In the following paragraphs, we present detailed solutions to these sub-problems and provide a performance analysis of the proposed online scheme in Section III-C.

A. NE with Known Service Quality

We first derive the NE under the oracle setting with perfect prior knowledge of service quality. To compute the value of $r_{i,t}^k$ that satisfies the NE, we compute the derivative of $u_{i,t}^k$ with respect to $r_{i,t}^k$:

$$\frac{\partial u_{i,t}^k}{\partial r_{i,t}^k} = \frac{q_{i,t}^k \sum_{j\in\mathcal{N}\setminus\{i\}} q_{j,t}^k r_{j,t}^k}{\left(\sum_{j\in\mathcal{N}} q_{j,t}^k r_{j,t}^k\right)^2} P_t^k - c_i.$$

The second derivative of $u_{i,t}^k$ with respect to $r_{i,t}^k$ is: $\frac{\partial^2 u_{i,t}^k}{\partial (r_{i,t}^k)^2} = -\frac{2{q_{i,t}^k}^2 \Sigma_{j\in\mathcal{N}\setminus\{i\}} q_{j,t}^k r_{j,t}^k}{(\Sigma_{j\in\mathcal{N}} q_{j,t}^k r_{j,t}^k)^3} P_t^k < 0$, which implies that $u_{i,t}^k$ is concave with respect to $r_{i,t}^k$. Therefore, the optimal strategy of client i for task k is the solution of $\frac{\partial u_{i,t}^k}{\partial r_{i,t}^k} = 0$, i.e.,

$$r_{i,t}^k = \frac{\sqrt{\frac{q_{i,t}^k P_t^k \sum_{j\in\mathcal{N}\setminus\{i\}} q_{j,t}^k r_{j,t}^k}{c_i}} - \sum_{j\in\mathcal{N}\setminus\{i\}} q_{j,t}^k r_{j,t}^k}{q_{i,t}^k}.$$

If the right-hand side of the above equation is negative, we set $r_{i,t}^k = 0$ to prevent negative resource usage. Therefore, the optimal strategy of client i for task k is:

$$\beta_{i,t}^k(r_{-i,t}^k) = \begin{cases} 0 & P_t^k \le \frac{c_i}{q_{i,t}^k}\Sigma, \\ \frac{\sqrt{\frac{q_{i,t}^k P_t^k \Sigma}{c_i}} - \Sigma}{q_{i,t}^k} & \text{otherwise}, \end{cases} \tag{6}$$

where $\Sigma = \sum_{j\in\mathcal{N}\setminus\{i\}} q_{j,t}^k r_{j,t}^k$

However, the solution given in (6) is not explicit, as it contains terms $r_{j,t}^k$ for all clients $j \in \mathcal{N} \setminus \{i\}$ that remain unsolved. To facilitate the design of an algorithm for computing an NE, we establish that the NE strategy profile satisfies four statements in Theorem 1 as follows.

Algorithm 1 Oracle: NE computation of task k

1: **Input:** P_t^k, $\{c_i, q_{i,t}^k\}_{i\in\mathcal{N}}$
2: **Output:** $\boldsymbol{r}_t^{k^{\mathrm{NE}}} = (r_{1,t}^{k^{\mathrm{NE}}}, r_{2,t}^{k^{\mathrm{NE}}}, ..., r_{N,t}^{k^{\mathrm{NE}}})$
3: Sort clients according to their cost/quality ratio, $\frac{c_{i(1)}}{q_{i^{(1)},t}^k} < \frac{c_{i(2)}}{q_{i^{(2)},t}^k} < \cdots < \frac{c_{i(N)}}{q_{i^{(N)},t}^k}$
4: $\mathcal{S}_t^k \leftarrow \{i^{(1)}, i^{(2)}\}, n \leftarrow 3, i \leftarrow i^{(n)}$
5: **while** $n \le N$ and $\frac{c_{i(n)}}{q_{i^{(n)},t}^k} < \frac{\frac{c_{i(n)}}{q_{i^{(n)},t}^k} + \sum_{j\in\mathcal{S}_t^k} \frac{c_j}{q_{j,t}^k}}{|\mathcal{S}_t^k|}$ **do**
6: $\quad \mathcal{S}_t^k \leftarrow \mathcal{S}_t^k \cup \{i^{(n)}\}, n \leftarrow n+1, i \leftarrow i^{(n)}$
7: **for all** $i \in \mathcal{N}$ **do**
8: $\quad$ **if** $i \in \mathcal{S}_t$ **then**
9: $\quad\quad r_{i,t}^{k^{\mathrm{NE}}} \leftarrow \frac{(|\mathcal{S}_t^k|-1)P_t^k}{q_{i,t}^k \sum_{j\in\mathcal{S}_t^k} \frac{c_j}{q_{j,t}^k}} \left(1 - \frac{(|\mathcal{S}_t^k|-1)c_i}{q_{i,t}^k \sum_{j\in\mathcal{S}_t^k} \frac{c_j}{q_{j,t}^k}}\right)$
10: $\quad$ **else**
11: $\quad\quad r_{i,t}^{k^{\mathrm{NE}}} \leftarrow 0$

Theorem 1. *Assume that P_t^k is given. Let $\tilde{\boldsymbol{r}}_t^k = (\tilde{r}_{1,t}^k, \tilde{r}_{2,t}^k, ..., \tilde{r}_{N,t}^k)$ be the strategy profile of an NE for the Stackelberg game of task k, and let $\tilde{\mathcal{S}}_t^k$ denote the set of selected clients, i.e., $\tilde{\mathcal{S}}_t^k = \{i \in \mathcal{N} | \tilde{r}_{i,t}^k > 0\}$. The following four statements hold:*

(a) $|\tilde{\mathcal{S}}_t^k| \ge 2$.

(b) $\tilde{r}_{i,t}^k = \begin{cases} 0 & i \notin \tilde{\mathcal{S}}_t^k, \\ \frac{(|\tilde{\mathcal{S}}_t^k|-1)P_t^k}{q_{i,t}^k \sum_{j\in s_t^k} \frac{c_j}{q_{j,t}^k}} \left(1 - \frac{(|\tilde{\mathcal{S}}_t^k|-1)c_i}{q_{i,t}^k \sum_{j\in s_t^k} \frac{c_j}{q_{j,t}^k}}\right) & \text{otherwise}. \end{cases}$

(c) If $\frac{c_i}{q_{i,t}^k} \le \max_{j\in\tilde{\mathcal{S}}_t^k} \{\frac{c_j}{q_{j,t}^k}\}$, *then* $i \in \tilde{\mathcal{S}}_t^k$.

(d) Assume that the clients are ordered such that $\frac{c_{i(1)}}{q_{i^{(1)},t}^k} < \frac{c_{i(2)}}{q_{i^{(2)},t}^k} < \cdots < \frac{c_{i(N)}}{q_{i^{(N)},t}^k}$. *Let h be the largest integer in $[2, N]$ such that* $\frac{c_{i(h)}}{q_{i^{(h)},t}^k} < \frac{\sum_{j=1}^h \frac{c_{i(j)}}{q_{i^{(j)},t}^k}}{h-1}$. *Then* $\tilde{\mathcal{S}}_t^k = \{i^{(1)}, i^{(2)}, ..., i^{(h)}\}$.

We refer interested readers to Appendix A of our technical report [16] for the proof of Theorem 1. Theorem 1 provides client selection criteria and an explicit form of the NE strategy profile. It indicates that at least two clients are required to form an equilibrium, implies a preference for high-quality, low-cost clients in the equilibrium, and confirms the uniqueness of the Stackelberg equilibrium. A unique strategy profile ensures that clients consistently adopt the same equilibrium behavior.

With the help of Theorem 1, we design Algorithm 1 to compute the NE for task k. Algorithm 1 first sorts clients by their cost-to-quality ratio and then selects clients with the lowest ratios until the condition in Line 5 of Algorithm 1 is satisfied. We prove that the strategy profile $\boldsymbol{r}_t^{k^{\mathrm{NE}}}$ computed by Algorithm 1 is the NE for task k in Theorem 2 and provide the proof in Appendix B in our technical report [16].

Theorem 2. *The strategy profile $\boldsymbol{r}_t^{k^{\mathrm{NE}}}$ computed by Algorithm 1 is the NE of the Stackelberg game of task k, which corresponds to the solution in* (6).

Algorithm 2 OIC for task assignment

1: **[Clients]** Each client $i \in \mathcal{N}$ initializes $\boldsymbol{V}_{i,0} = \lambda \boldsymbol{I}$, $\boldsymbol{v}_{i,0} = \boldsymbol{0}^d$ and sends c_i to the server.
2: **for all** $t \in [T]$ **do**
/ 1) Optimistic estimate of service quality */*
3: [Server] Observe task set $\mathcal{A}_t$ from the environment and publish tasks to clients.
4: [Clients] Receive $\mathcal{A}_t$ from the server, compute optimistic estimate $\hat{q}_{i,t}^k$ according to (9) and (10), and send $\hat{q}_{i,t}^k$ to the server.
5: [Server] Receive $\hat{q}_{i,t}^k$ from clients $i \in \mathcal{N}$.
/ 2) Approx. optimal payment & resource usage */*
6: **for all** $k \in [K]$ **do**
7: [Server] Obtain $\hat{u}_{0,t}^k$, $\hat{P}_t^k$ and $\hat{\boldsymbol{r}}_t^{k^{\mathrm{NE}}} \leftarrow$ Algorithm 3.
/ 3) Optimal task selection */*
8: [Server] Select the optimal task $k_t \leftarrow \arg\max_k \hat{u}_{0,t}^k$
9: [Server] Assign task k_t to clients with resource usage $\hat{\boldsymbol{r}}_t \leftarrow \hat{\boldsymbol{r}}_t^{k_t^{\mathrm{NE}}}$ and payment $\hat{P}_t \leftarrow \hat{P}_t^{k_t}$.
/ 4) Incentivized task execution */*
10: [Clients] Receive incentives $I_{i,t}$ according to (14), participate in the assigned task according to $\hat{\boldsymbol{r}}_t$ and observe the true service quality $q_{i,t}$.
11: [Server] Get revenue from the environment.
12: [Clients] Update $\boldsymbol{V}_{i,t}$ and $\boldsymbol{v}_{i,t}$ for each client $i \in \mathcal{N}$ according to (7) and (8).

Algorithm 3 Approx. optimal payment & resource usage of task k

1: **Input:** $\{c_i, \hat{q}_{i,t}^k\}_{i\in\mathcal{N}}$
2: **Output:** $\hat{u}_{0,t}^k$, $\hat{P}_t^k$ and $\hat{\boldsymbol{r}}_t^{k^{\mathrm{NE}}} = (\hat{r}_{1,t}^{k^{\mathrm{NE}}}, \hat{r}_{2,t}^{k^{\mathrm{NE}}}, ..., \hat{r}_{N,t}^{k^{\mathrm{NE}}})$
3: Sort clients according to their cost/quality ratio, $\frac{c_{i(1)}}{\hat{q}_{i^{(1)},t}^k} < \frac{c_{i(2)}}{\hat{q}_{i^{(2)},t}^k} < \cdots < \frac{c_{i(N)}}{\hat{q}_{i^{(N)},t}^k}$
4: $\hat{\mathcal{S}}_t^k \leftarrow \{i^{(1)}, i^{(2)}\}, n \leftarrow 3, i \leftarrow i^{(n)}$
5: **while** $n \le N$ and $\frac{c_{i^{(n)}}}{\hat{q}_{i^{(n)},t}^k} < \frac{\frac{c_{i^{(n)}}}{\hat{q}_{i^{(n)},t}^k} + \sum_{j\in\hat{\mathcal{S}}_t^k} \frac{c_j}{\hat{q}_{j,t}^k}}{|\hat{\mathcal{S}}_t^k|}$ **do**
6: $\hat{\mathcal{S}}_t^k \leftarrow \hat{\mathcal{S}}_t^k \cup \{i^{(n)}\}, n \leftarrow n+1, i \leftarrow i^{(n)}$
7: Compute $\hat{X}_t^k \leftarrow \frac{(|\hat{\mathcal{S}}_t^k|-1)}{\sum_{j\in\hat{\mathcal{S}}_t^k} \frac{c_i}{\hat{q}_{i,t}^k}}$, $\hat{P}_t^k \leftarrow \gamma - \frac{1}{\hat{X}_t^k}$ and $\hat{u}_{0,t}^k \leftarrow \gamma \log(\gamma \hat{X}_t^k) + \frac{1}{\hat{X}_t^k} - \gamma$
8: **for all** $i \in \mathcal{N}$ **do**
9: **if** $i \in \hat{\mathcal{S}}_t$ **then**
10: $\hat{r}_{i,t}^{k^{\mathrm{NE}}} \leftarrow \frac{(|\hat{\mathcal{S}}_t^k|-1)\hat{P}_t^k}{\hat{q}_{i,t}^k \sum_{j\in\hat{\mathcal{S}}_t^k} \frac{c_j}{\hat{q}_{j,t}^k}} (1 - \frac{(|\hat{\mathcal{S}}_t^k|-1)c_i}{\hat{q}_{i,t}^k \sum_{j\in\hat{\mathcal{S}}_t^k} \frac{c_j}{\hat{q}_{j,t}^k}})$
11: **else**
12: $\hat{r}_{i,t}^{k^{\mathrm{NE}}} \leftarrow 0$

B. Online Task Selection with Approximate NE

Building upon the oracle solution provided in Algorithm 1, we propose our online scheme OIC for task assignment. The detailed procedures of OIC are presented in Algorithms 2 and 3. Specifically, OIC consists of four core components, described as follows.

1) Optimistic estimate of service quality: To estimate service quality, we first estimate the model parameters $\boldsymbol{\theta}_i$ and then estimate the expected service quality according to the linear model defined in (1). To estimate the model $\boldsymbol{\theta}_i$, client i needs to maintains a covariance matrix $\boldsymbol{V}_{i,t}$ and a reward vector $\boldsymbol{v}_{i,t}$, which store the client's action and reward history: $\boldsymbol{V}_{i,t} = \lambda \boldsymbol{I} + \sum_{\tau\in\{\tau\le t | r_{i,\tau}>0\}} \boldsymbol{x}_\tau \boldsymbol{x}_\tau^\top$, $\boldsymbol{v}_{i,t} = \sum_{\tau\in\{\tau\le t | r_{i,\tau}>0\}} \boldsymbol{x}_\tau q_{i,\tau}$. At the beginning of the first round, we initialize $\boldsymbol{V}_{i,t} = \lambda \boldsymbol{I}$ and $\boldsymbol{v}_{i,t} = \boldsymbol{0}^d$, where λ the regularization parameter and $\boldsymbol{I}$ is the identity matrix (see Line 1 in Algorithm 2). The task context and service quality observed after task execution are used to iteratively update the covariance matrix and reward vector at each round t (see Line 12 of Algorithm 2):

$$\boldsymbol{V}_{i,t} = \begin{cases} \boldsymbol{V}_{i,t-1} + \boldsymbol{x}_t \boldsymbol{x}_t^\top, & \hat{r}_{i,t} > 0, \\ \boldsymbol{V}_{i,t-1}, & \hat{r}_{i,t} = 0. \end{cases} \tag{7}$$

$$\boldsymbol{v}_{i,t} = \begin{cases} \boldsymbol{v}_{i,t-1} + \boldsymbol{x}_t q_{i,t}, & \hat{r}_{i,t} > 0, \\ \boldsymbol{v}_{i,t-1}, & \hat{r}_{i,t} = 0. \end{cases} \tag{8}$$

Using the historical data, we estimate $\boldsymbol{\theta}_i$ via ridge regression:

$$\hat{\boldsymbol{\theta}}_{i,t} = \boldsymbol{V}_{i,t-1}^{-1} \boldsymbol{v}_{i,t-1}. \tag{9}$$

With the estimated model $\hat{\boldsymbol{\theta}}_{i,t}$, client i can estimate its expected service quality as $\langle \boldsymbol{x}_t^k, \hat{\boldsymbol{\theta}}_{i,t} \rangle$.

However, directly using the expected service quality for client selection may exclude clients with high actual quality but limited exploration. In such cases, the variance of the estimate may be large, resulting in an underestimated service quality. To balance exploration and exploitation, we use an optimistic estimate of service quality by computing an upper confidence bound (UCB) (derived in [17], [18]) of the service quality (see Line 4 in Algorithm 2):

$$\hat{q}_{i,t}^k = \langle \boldsymbol{x}_t^k, \hat{\boldsymbol{\theta}}_{i,t} \rangle + \alpha_{i,t} \| \boldsymbol{x}_t^k \|_{\boldsymbol{V}_{i,t-1}^{-1}}, \tag{10}$$

where $\alpha_{i,t} = \sigma \sqrt{\log \frac{|\boldsymbol{V}_{i,t-1}|}{|\lambda \boldsymbol{I}|} + 2 \log \frac{2}{\delta}} + \sqrt{\lambda}$ and δ denotes the confidence level. Such a UCB estimation includes a confidence interval of the actual service quality with probability at least $1-\delta$. Using optimistic estimates, our scheme naturally prioritizes clients with high expected quality (exploitation) or high uncertainty (exploration).

2) Approximately optimal payment and resource usage: After obtaining optimistic estimates of service quality, we compute approximate NE resource usages $\hat{\boldsymbol{r}}_t^{k^{\mathrm{NE}}}$ and the corresponding optimal payment $\hat{P}_t^k$ to maximize the server's utility in the online setting. To extend the NE in the oracle setting to the online setting, we replace $q_{i,t}^k$ in Algorithm 1 with its optimistic counterpart $\hat{q}_{i,t}^k$ from (10), yielding Algorithm 3 for computing $\hat{\boldsymbol{r}}_t^{k^{\mathrm{NE}}}$ (see Line 10 in Algorithm 3).

Once $\hat{\boldsymbol{r}}_t^{k^{\mathrm{NE}}}$ is determined, we proceed to calculate the optimal payment $\hat{P}_t^k$ that maximizes the utility of the server. Applying the value of $\hat{\boldsymbol{r}}_t^{k^{\mathrm{NE}}}$ to the server's utility function, we have $\hat{u}_{0,t}^k = \gamma \log(1 + \frac{(|\hat{\mathcal{S}}_t^k|-1)\hat{P}_t^k}{\sum_{j\in\hat{\mathcal{S}}_t^k} \frac{c_i}{\hat{q}_{i,t}^k}}) - \hat{P}_t^k$. Let $\hat{X}_t^k = \frac{(|\hat{\mathcal{S}}_t^k|-1)}{\sum_{j\in\hat{\mathcal{S}}_t^k} \frac{c_i}{\hat{q}_{i,t}^k}}$, the utility can be rewritten as $\hat{u}_{0,t}^k = \gamma \log(1 + \hat{X}_t^k \hat{P}_t^k) - \hat{P}_t^k$.

We can analyze the server's utility $\hat{u}_{0,t}^k$ as a function of $\hat{P}_t^k$. The first derivative of $\hat{u}_{0,t}^k$ with respect to $\hat{P}_t^k$ is:

$$\frac{\partial \hat{u}_{0,t}^k}{\partial \hat{P}_t^k} = \frac{\gamma}{1 + \hat{X}_t^k \hat{P}_t^k} - 1. \tag{11}$$

The second derivative of $\hat{u}_{0,t}^k$ with respect to $\hat{P}_t^k$ is $\frac{\partial^2 \hat{u}_{0,t}^k}{\partial (\hat{P}_t^k)^2} = -\frac{\gamma \hat{X}_t^k}{(1+\hat{X}_t^k \hat{P}_t^k)^2} < 0$. Hence, $\hat{u}_{0,t}^k$ is a concave function of $\hat{P}_t^k$. We set (11) to zero and obtain the optimal $\hat{P}_t^k$:

$$\hat{P}_t^k = \gamma - \frac{1}{\hat{X}_t^k}. \tag{12}$$

Applying (12) to compute the approximate server's utility:

$$\hat{u}_{0,t}^k = \gamma \log(\gamma \hat{X}_t^k) + \frac{1}{\hat{X}_t^k} - \gamma. \tag{13}$$

We apply (12) and (13) in Line 7 of Algorithm 3 to compute the approximately optimal payment and approximate server utility for each task.

3) Optimal task selection: After computing the approximate NE resource usage and payment for each task k, the server selects the one with the highest UCB of utility to balance exploration and exploitation. Specially, we prove in Theorem 3 that $\hat{u}_{0,t}^k$ computed by (13) upper bounds the actual utility $u_{0,t}^k$ with high probability. Therefore, the task with the highest UCB (i.e., the largest $\hat{u}_{0,t}^k$) is assigned to clients, together with resource usage $\hat{\boldsymbol{r}}_t = \hat{\boldsymbol{r}}_{i,t}^{k_t^{\mathrm{NE}}}$ and payment $\hat{P}_t = \hat{P}_t^{k_t}$ (see Lines 8-9 in Algorithm 2).

Theorem 3 (UCB of Server's Utility)**.** *If $\gamma \geq (1 + \frac{1}{N-1})\frac{c_{\max}}{q_{\min}}$ holds, where $q_{\min}$ is the minimum value of data quality and $c_{\max}$ is the maximum value of cost, then $\hat{u}_{0,t}^k$ computed by* (13) *is a valid UCB of $u_{0,t}^k$ with probability at least $(1-\delta)^N$.*

Proof sketch: First, we prove that $\hat{X}_t^k$ is an upper confidence bound of $X_t^k = \frac{(|\mathcal{S}_t^k|-1)}{\sum_{j \in \mathcal{S}_t^k} \frac{c_i}{q_{i,t}^k}}$, where $\mathcal{S}_t^k$ is the set of selected clients in the oracle setting. Then, under the condition that $\gamma \geq (1 + \frac{1}{N-1})\frac{c_{\max}}{q_{\min}}$, we reduce the problem of bounding $\hat{u}_{0,t}^k$ to that of bounding $\hat{X}_t^k$ being a UCB of X_t^k, thus completing the proof. The complete proof is provided in Appendix C in our technical report [16].

4) Incentivized Task Execution: As shown in Lines 10-11 of Algorithm 2, once the server assigns task k_t, clients execute the task with corresponding incentives:

$$I_{i,t} = \frac{\hat{q}_{i,t}^{k_t} \hat{r}_{i,t}^{k_t}}{\sum_{j \in \mathcal{N}} \hat{q}_{j,t}^{k_t} \hat{r}_{j,t}^{k_t}} \hat{P}_t^{k_t}. \tag{14}$$

Clients then execute the task according to $\hat{\boldsymbol{r}}_t$, observe their actual service quality $q_{i,t}$, and the server receives revenue from the environment based on the client's service quality.

C. Performance Analysis

We analyze the performance of OIC and establish that its regret grows sublinearly with the number of rounds T, indicating that the scheme becomes increasingly effective over time. This result is formalized in Theorem 4.

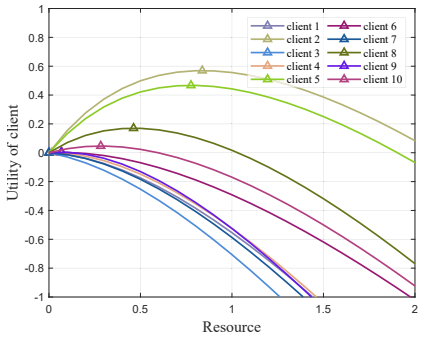


Fig. 2. Verification of NE computation.

Theorem 4 (Sub-linear Regret Upper Bound)**.** *Suppose that $||\boldsymbol{x}_t||_2 \leq L$ and $\gamma \geq (1 + \frac{1}{N-1})\frac{c_{\max}}{q_{\min}}$. Then, the regret of OIC is upper bounded as follows with probability at least $(1-\delta)^N$:*

$$R_T \leq O(\frac{\gamma N \sigma d}{q_{\min}} \sqrt{T} \log(TL^2/\lambda d)). \tag{15}$$

Proof Sketch: The proof proceeds in two steps. First, we bound the single-round regret by showing that the regret of the server's utility at each round can be bounded by the UCB of the clients' service quality estimates, linking the server's performance to the quality of client-side task assignments. Second, we bound the cumulative regret by leveraging the confidence intervals on service quality to form a martingale sequence whose sum converges over time. Combining these two steps yields the overall regret bound. The complete proof is provided in Appendix D in our technical report [16].

IV. Experiments

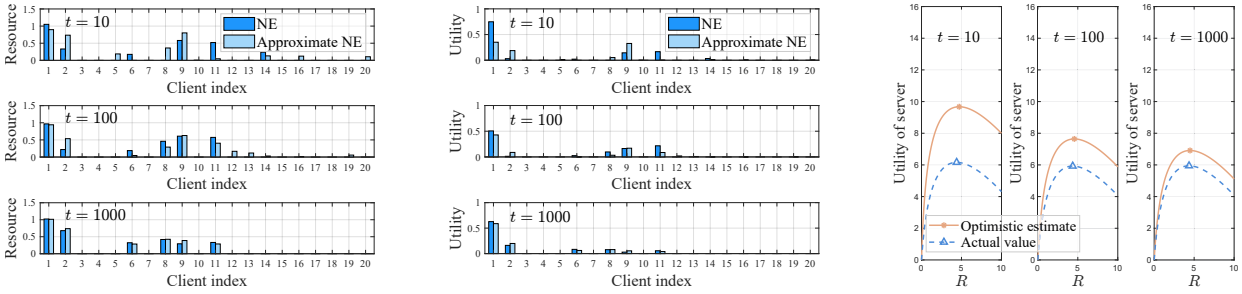
In this section, we conduct experiments using synthetic and real-world datasets to evaluate the performance of OIC. We first give the experiment settings and then show our results.

A. Experiment Settings

We conduct experiments on two datasets: a synthetic dataset and the real-world Yahoo "Learning to Rank Challenge Dataset" [19]). We set the parameters as $T = 10^3$, $K = 10$, $N = 20$, $\gamma = 5$, $d = 9$, $L = 9$, $\sigma = 1$ and $\delta = 0.05$.

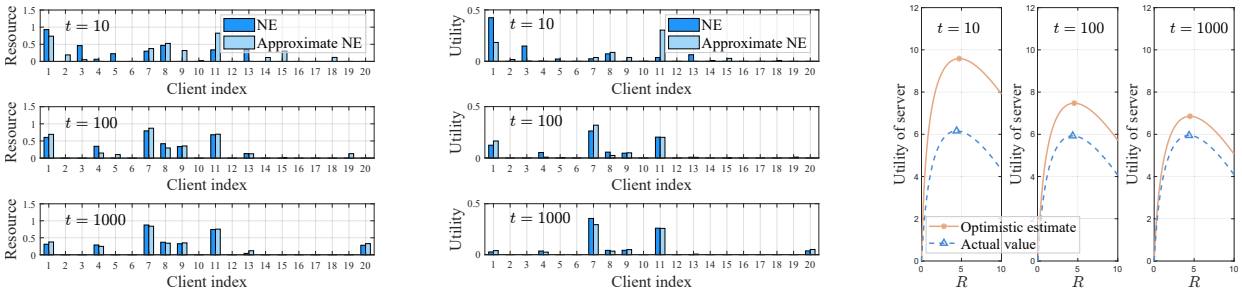
Synthetic dataset: Task contexts ($\boldsymbol{x}_t^k$) and client's model parameters ($\boldsymbol{\theta}_i$) are sampled independently from a uniform distribution over $[0, 1]^d$. The true service quality is calculated as the inner product between the task context and the client's model parameter. Costs are sampled uniformly from $[0.1, 1.1]$.

Yahoo dataset: We use Yahoo dataset to extract model parameters reflecting real-world scenarios. Specifically, we use the "set2.test.txt" subset, which provides ratings and normalized feature values. We select top $d = 9$ feature columns with the highest number of valid entries. To simulate client heterogeneity, these features are clustered into 500 groups using K-means, from which $N = 20$ clusters are randomly chosen to generate client model parameters. For client $i \in [N]$, the true model parameter is derived through linear regression, where ratings are regressed on the corresponding feature



(a) Resource usage (b) Client's utility (c) Server's utility

Fig. 3. Gap between the approximate NE in OIC and the exact NE in oracle (synthetic dataset).



(a) Resource usage (b) Client's utility (c) Server's utility

Fig. 4. Gap between the approximate NE in OIC and the exact NE in oracle (Yahoo dataset).

vectors. The remaining feature vectors form the task contexts over T rounds. Each client's actual service quality in a given round is computed as the inner product of its model parameter and the current task context. As the dataset lacks data about costs, we sample costs uniformly from $[0.1, 1.1]$.

B. Verification of NE Computation of Oracle

To verify the NE computation in Algorithm 1, we compute each client's utility as a function of its resource usage while fixing other clients' resource usages at their equilibrium values computed by Algorithm 1. The result is shown in Fig. 2, and the equilibrium points identified by Algorithm 1 are marked with triangles. Each client's utility curve is concave and reaches its maximum precisely at the equilibrium points computed by Algorithm 1. When a client deviates, either increasing or decreasing its resource usage, its utility decreases. This confirms that no client benefits from deviating unilaterally, thus satisfying the NE condition. Therefore, OIC effectively rationalizes clients' participation in the assigned task in alignment with the server's expectations.

C. Convergence of Approximate NE in OIC

We evaluate the convergence of the approximate NE computed by Algorithm 3 (OIC) against the exact NE obtained by Algorithm 1 (oracle). The comparison is conducted over three metrics: (i) resource usage of each client; (ii) utility of each client; (iii) server utility (exact in the oracle; optimistic estimate in OIC). A smaller discrepancy between the OIC and oracle results indicates a better convergence of our online scheme. As shown in Fig. 3 and Fig. 4, all three metrics in OIC progressively converge toward those in the oracle over time, validating the effectiveness of OIC in approximating the optimal solution under the oracle setting. Additionally, the optimistic estimates of the server utility consistently upper bound the actual utility values, empirically supporting Theorem 3.

D. Long-term Performance

We evaluate the long-term performance of OIC via regret analysis. As predicted by Theorem 4, regret increases with both the number of clients N and the parameter γ in Fig. 5. In Fig. 5(a,c), regret grows with N, but not linearly. This is because the theoretical regret bound is derived by relaxing the actual number of selected clients and the number of times each client is selected. Due to the dynamic nature of service quality, the number of selected clients per round varies and is difficult to bound precisely. Thus, we conservatively bound the regret using the total number of clients N and total rounds T, leading to a loose N dependency. Fig. 5(b,d) confirms the linear growth with γ. Since γ represents the ratio between virtual task execution performance and actual monetary payment, a higher γ amplifies the regret bound, reflecting increased sensitivity to errors in task performance estimation.

We further compare OIC with two baselines: (i) OIC without UCB (OIC w/o UCB), which uses ridge regression for service quality estimation but does not incorporate exploration; and (ii) ϵ-greedy, which explores with probability ϵ by randomly assigning tasks to all clients, and exploits the best estimated task otherwise. We set $\epsilon = \min\{1, \frac{K}{t}\}$ following [20] to

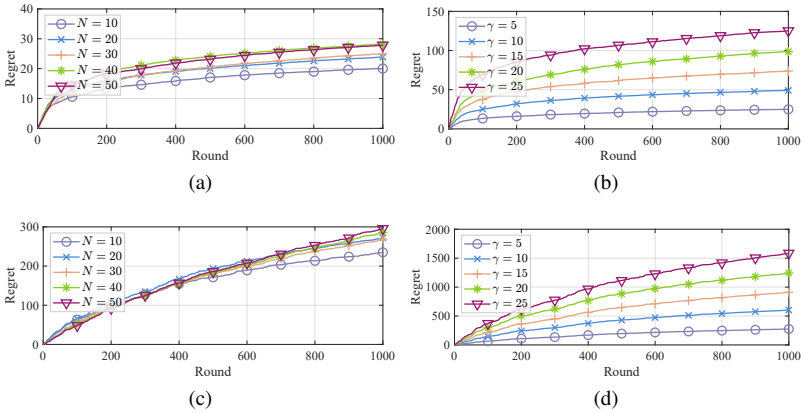


Fig. 5. Regret performance. Experiments using synthetic dataset: (a) regret versus N, (b) regret versus γ; experiments using Yahoo dataset: (c) regret versus N, (d) regret versus γ.

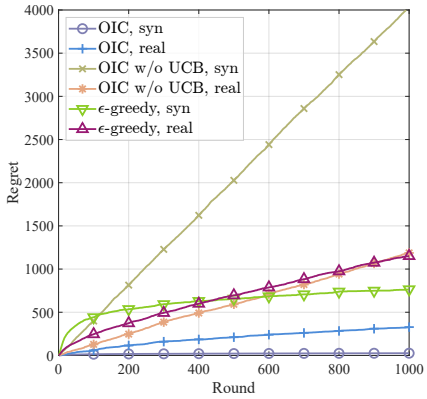


Fig. 6. Regret comparison.

ensure sublinear regret. As shown in Fig. 6, OIC outperforms both baselines. OIC w/o UCB performs the worst due to the absence of exploration, which leads to the exclusion of potentially high-quality clients based on initial poor performance. ϵ-greedy performs better by incorporating exploration, but its naive strategy incurs linear regret growth during exploratory rounds. In contrast, OIC achieves a more efficient exploration-exploitation tradeoff, resulting in consistently lower regret.

V. Conclusion

In this paper, we proposed OIC, an online incentivized and context-aware task assignment scheme designed for environments with unknown, context-dependent service quality. We formulated the task assignment problem as a Stackelberg game and applied online learning to approximate the optimal scheme. By deriving an upper confidence bound on the server's utility, OIC effectively balances exploration and exploitation, achieving a provably sublinear regret bound and strong long-term performance. Extensive experiments on both synthetic and real-world datasets validated the effectiveness of OIC in incentivizing client participation and maximizing the server's expected utility over time.

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APPENDIX A
PROOFS OF THEOREM 1

Proof. **Proof of (1)**: We prove this statement by contradiction. Suppose that $|\tilde{\mathcal{S}}_t^k| = 0$, then client i can increase its utility from 0 to $\frac{P_t^k}{2}$ by unilaterally changing its computational resource from 0 to $\frac{P_t^k}{2c_i}$, contradicting the NE assumption. This proves that $|\tilde{\mathcal{S}}_t^k| \geq 1$. Suppose that $|\tilde{\mathcal{S}}_t^k| = 1$. This means $\tilde{r}_{i,t}^k > 0$ for some $i \in \mathcal{N}$, and $\tilde{r}_{j,t}^k = 0$ for all $j \in \mathcal{N} \setminus \{i\}$. The current utility of client i is $P_t^k - \tilde{r}_{i,t}^k c_i$. Client i can increase its utility by unilaterally changing its sensing time from $\tilde{r}_{i,t}^k$ to $\frac{\tilde{r}_{i,t}^k}{2}$, again contradicting the NE assumption. Therefore $|\tilde{\mathcal{S}}_t^k| \geq 2$.

Proof of (2): The optimal strategy can be obtained by letting the derivative of the utility of client be 0, i.e.,

$$\begin{aligned}\frac{\partial u_{i,t}^k}{\partial r_{i,t}^k} &= \frac{q_{i,t}^k}{\sum_{j\in\mathcal{N}} q_{j,t}^k r_{j,t}^k} P_t^k - \frac{q_{i,t}^k}{(\sum_{j\in\mathcal{N}} q_{j,t}^k r_{j,t}^k)^2} P_t^k - c_i \\ &= \frac{q_{i,t}^k}{\sum_{j\in\tilde{\mathcal{S}}_t^k} q_{j,t}^k r_{j,t}^k} P_t^k - \frac{q_{i,t}^k}{(\sum_{j\in\tilde{\mathcal{S}}_t^k} q_{j,t}^k r_{j,t}^k)^2} P_t^k - c_i = 0.\end{aligned} \tag{16}$$

Summing up over the clients in $\tilde{\mathcal{S}}_t^k$, we have

$$n_0 P_t^k - P_t^k = \sum_{j\in\tilde{\mathcal{S}}_t^k} q_{j,t}^k r_{j,t}^k \sum_{j\in\tilde{\mathcal{S}}_t^k} \frac{c_j}{q_{j,t}^k}, \tag{17}$$

thus we have

$$\sum_{j\in\tilde{\mathcal{S}}_t^k} q_{j,t}^k r_{j,t}^k = \frac{(n_0-1)P_t^k}{\sum_{j\in\tilde{\mathcal{S}}_t^k} \frac{c_j}{q_{j,t}^k}}. \tag{18}$$

Substituting (18) into (16) and considering $\tilde{r}_{i,t}^k = 0$ for any $j \in \mathcal{N} \setminus \{\tilde{\mathcal{S}}_t^k\}$, we obtain the optimal strategy:

$$\tilde{r}_{i,t}^k = \frac{(n_0-1)P_t^k}{q_{i,t}^k \sum_{j\in\tilde{\mathcal{S}}_t^k} \frac{c_j}{q_{j,t}^k}} \left(1 - \frac{(n_0-1)c_i}{q_{i,t}^k \sum_{j\in\tilde{\mathcal{S}}_t^k} \frac{c_j}{q_{j,t}^k}}\right). \tag{19}$$

Therefore, the statement (2) is proved.

Proof of (3): We prove this statement by contradiction. By definition of $\tilde{\mathcal{S}}_t^k$, we know that $\tilde{r}_{i,t}^k > 0$ for every $i \in \tilde{\mathcal{S}}_t^k$. From statement (2), $\tilde{r}_{i,t}^k > 0$ implies $\frac{(n_0-1)c_i}{q_{i,t}^k \sum_{j\in\tilde{\mathcal{S}}_t^k q_{j,t}^k}} < 1$, i.e., $\forall i \in \tilde{\mathcal{S}}_t^k$, $\frac{c_i}{q_{i,t}^k} < \frac{\sum_{j\in\tilde{\mathcal{S}}_t^k} \frac{c_j}{q_{j,t}^k}}{n_0-1}$. Therefore we have $\max_{j\in\tilde{\mathcal{S}}_t^k}\{\frac{c_j}{q_{j,t}^k}\} < \frac{\sum_{j\in\tilde{\mathcal{S}}_t^k} \frac{c_j}{q_{j,t}^k}}{n_0-1}$. Assume that $\frac{c_i}{q_{i,t}^k} \leq \max_{j\in\tilde{\mathcal{S}}_t^k}\{\frac{c_j}{q_{j,t}^k}\} < \frac{\sum_{j\in\tilde{\mathcal{S}}_t^k} \frac{c_j}{q_{j,t}^k}}{n_0-1}$ but $i \notin \tilde{\mathcal{S}}_t^k$, i.e., $\tilde{r}_{i,t}^k = 0$.

$$\begin{aligned}\frac{\partial u_{i,t}^k}{\partial r_{i,t}^k} &= \frac{q_{i,t}^k \sum_{j\in N\setminus\{i\}} q_{j,t}^k r_{j,t}^k}{(\sum_{j\in N} q_{j,t}^k r_{j,t}^k)^2} P_t^k - c_i \\ &= \frac{q_{i,t}^k \sum_{j\in\tilde{\mathcal{S}}_t^k} q_{j,t}^k r_{j,t}^k}{(\sum_{j\in\tilde{\mathcal{S}}_t^k} q_{j,t}^k r_{j,t}^k)^2} P_t^k - c_i \\ &= \frac{q_{i,t}^k}{\sum_{j\in\tilde{\mathcal{S}}_t^k} q_{j,t}^k r_{j,t}^k} P_t^k - c_i \\ &= \frac{q_{i,t}^k \sum_{j\in\tilde{\mathcal{S}}_t^k} \frac{c_j}{q_{j,t}^k}}{n_0 - 1} - c_i \\ &> q_{i,t}^k \max_{j\in\tilde{\mathcal{S}}_t^k}\{\frac{c_j}{q_{j,t}^k}\} - c_i \geq 0,\end{aligned}$$

where the third equation comes from (18). This means that client i can increase its utility by unilaterally increasing its sensing time from $\tilde{r}_{i,t}^k = 0$, contradicting the NE assumption.

Proof of (4): Statements (1) and (3) imply that $\tilde{\mathcal{S}}_t^k = \{i^{(1)}, i^{(2)}, ..., i^{(g)}\}$ for some integer g in $[2, N]$. From $\frac{c_{i(g)}}{q_{i(g),t}^k} < \frac{\sum_{j\in\tilde{\mathcal{S}}_t^k} \frac{c_j}{q_{j,t}^k}}{n_0-1}$, we have $g \leq h$. Assume that $g < h$, then $\frac{c_{i(g+1)}}{q_{i(g+1),t}^k} \geq \frac{\sum_{j=1}^{g+1} \frac{c_{i(j)}}{q_{i(j),t}^k}}{g}$, which implies $\sum_{j=1}^{g} \frac{c_{i_j^{Sort}}}{q_{i_j^{Sort},t}^k} \frac{c_{i(g+1)}}{q_{i(g+1),t}^k} > 0$.

$$\begin{aligned}\frac{\partial u_{i(g+1),t}^k}{\partial r_{i(g+1),t}^k} &= \frac{q_{i(g+1),t}^k \sum_{j\in\tilde{\mathcal{S}}_t^k} q_{j,t}^k r_{j,t}^k}{(\sum_{j\in\tilde{\mathcal{S}}_t^k} q_{j,t}^k r_{j,t}^k)^2} P_t^k - c_{i(g+1)} \\ &= \frac{q_{i(g+1),t}^k}{\sum_{j\in\tilde{\mathcal{S}}_t^k} q_{j,t}^k r_{j,t}^k} P_t^k - c_{i(g+1)} \\ &= \frac{q_{i_{g+1}^{sort},t}^k \sum_{j=1}^{g} q_{i(j),t}^k}{g-1} - c_{i(g+1)} > 0,\end{aligned}$$

where the third equation comes from (18). This contradiction proves $g = h$, and thus proves statement (4).

APPENDIX B
PROOF OF THEOREM 2

Proof. The following three observations are used to prove Algorithm 1 finds the Nash equilibrium of task k.

(1) For any $i \in \mathcal{S}_t^k$, $\frac{c_i}{q_{i,t}^k} \geq \frac{\Sigma_{j\in\mathcal{S}_t^k} \frac{c_j}{q_{j,t}^k}}{n_0-1}$, where $n_0 = |\mathcal{S}_t^k|$.

(2) $\sum_{j\in S_t^k} q_{j,t}^k r_{j,t}^{k\ \mathrm{NE}} = \frac{(n_0-1)P_t^k}{\Sigma_{j\in S_t^k} \frac{c_j}{q_{j,t}^k}}$.

(3) $\sum_{j\in\mathcal{S}_t^k\setminus\{i\}} q_{j,t}^k r_{j,t}^{k\ \mathrm{NE}} = \frac{(n_0-1)^2 c_i P_t^k}{q_{j,t}^k (\Sigma_{j\in S_t^k} \frac{c_j}{q_{j,t}^k})^2}$

Based on the above observations, we prove two cases respectively: client $i \in \mathcal{S}_t^k$ and client $i \notin \mathcal{S}_t^k$. For any $i \notin \mathcal{S}_t^k$,

$$\begin{aligned}\frac{c_i}{q_{i,t}^k}\sum_{j\in\mathcal{S}_t^k} q_{j,t}^k r_{j,t}^{k\ \mathrm{NE}} &= \frac{c_i}{q_{i,t}^k}\sum_{j\in\mathcal{S}_t^k} q_{j,t}^k r_{j,t}^{k\ \mathrm{NE}} \\ &\overset{(2)}{=} \frac{c_i}{q_{i,t}^k}\frac{(n_0-1)P_t^k}{\sum_{j\in\mathcal{S}_t^k}\frac{c_j}{q_{j,t}^k}} \\ &\overset{(1)}{\geq} \frac{\sum_{j\in\mathcal{S}_t^k}\frac{c_j}{q_{j,t}^k}}{n_0-1}\frac{(n_0-1)P_t^k}{\sum_{j\in\mathcal{S}_t^k}\frac{c_j}{q_{j,t}^k}} = P_t^k, \quad (20)\end{aligned}$$

thus $\beta_{i,t}^k(r_{-i,t}^{k\ \mathrm{NE}}) = 0$. For any $i \in \mathcal{S}_t^k$,

$$\sum_{j\in\mathcal{N}\setminus\{i\}} q_{j,t}^k r_{j,t}^{k\ \mathrm{NE}} = \sum_{j\in\mathcal{S}_t^k\setminus\{i\}} q_{j,t}^k r_{j,t}^{k\ \mathrm{NE}} \quad (21)$$

$$\overset{(3)}{=} \frac{c_i}{q_{i,t}^k}\frac{(n_0-1)^2 c_i P_t^k}{q_{j,t}^k(\Sigma_{j\in\mathcal{S}_t^k}\frac{c_j}{q_{j,t}^k})^2} = (\frac{c_i}{q_{i,t}^k})^2\frac{(n_0-1)^2 P_t^k}{(\Sigma_{j\in\mathcal{S}_t^k}\frac{c_j}{q_{j,t}^k})^2}, \quad (22)$$

$$\begin{aligned}(n_0-1)\frac{c_i}{q_{i,t}^k} &= (i-1)\frac{c_i}{q_{i,t}^k} + (n_0-i)\frac{c_i}{q_{i,t}^k} \\ &< \sum_{j=1}^{i}\frac{c_i}{q_{j,t}^k} + \sum_{j=i+1}^{n_0}\frac{c_i}{q_{j,t}^k} = \sum_{j=1}^{n_0}\frac{c_i}{q_{j,t}^k}, \quad (23)\end{aligned}$$

where the second inequality holds because $\frac{c_i}{q_{i,t}^k} < \frac{\frac{c_i}{q_{i,t}^k}+\sum_{j\in S_t^k}\frac{c_j}{q_{j,t}^k}}{|\mathcal{S}_t^k|}$ holds for client $i \in \mathcal{S}_t^k$, which comes from Line 5 of Algorithm 1. Therefore, we have

$$\begin{aligned}&\frac{c_i}{\hat{q}_{i,t}^k}\sum_{j\in\mathcal{N}\setminus\{i\}}\hat{q}_{j,t}^k r_{j,t}^{k\ \mathrm{NE}} \\ <&(\frac{\sum_{j=1}^{n_0}\frac{c_j}{\hat{q}_{j,t}^k}}{n_0-1})^2\frac{(n_0-1)^2P_t^k}{(\Sigma_{j\in S_t^k}\frac{c_j}{q_{j,t}^k})^2} = P_t^k, \quad (24)\end{aligned}$$

and thus the strategy of client $i \in \mathcal{S}_t^k$ is

$$\begin{aligned}\beta_{i,t}^k(r_{-i,t}^{k\ \mathrm{NE}}) &= \frac{\sqrt{\frac{q_{i,t}^k P_t^k\sum_{j\in\mathcal{N}\setminus\{i\}} q_{j,t}^k r_{j,t}^k}{c_i}} - \sum_{j\in\mathcal{N}\setminus\{i\}} q_{j,t}^k r_{j,t}^k}{q_{i,t}^k} \\ &= \frac{(|S_t^k|-1)P_t^k}{q_{i,t}^k\sum_{j\in S_t^k}\frac{c_j}{q_{j,t}^k}}(1 - \frac{(|S_t^k|-1)c_i}{q_{i,t}^k\sum_{j\in S_t^k}\frac{c_j}{q_{j,t}^k}}) = r_{i,t}^{k\ \mathrm{NE}}. \quad (25)\end{aligned}$$

Therefore, the strategy in Algorithm 1 is the NE of the Stackelberg game of task k, which corresponds to the solution in (6). □

□

APPENDIX C
PROOFS OF THEOREM 3

In this section, we provide the proof of Theorem 4.

Lemma 1. *Suppose that there are three sequences:*

1. $\{a_1, a_2, \cdots, a_n\}$, *where* $a_1 < a_2 < \cdots < a_n$,
2. $\{b_1, b_2, \cdots, b_n\}$, *where* $b_1 \leq a_1$, $b_2 \leq a_2$, $\cdots$, $b_n \leq a_n$,
3. *Sort sequence 2 in ascending order and denoted as* $\{b'_1, b'_2, \cdots, b'_n\}$, *where* $b'_1 < b'_2 < \cdots < b'_n$.

For sequence 1 and sequence 3, we select elements according to the following rules:

- *First select the two smallest elements in the sequence 1 and 3 and denote the selected sets of sequence 1 and 3 as $\mathcal{S}_a$ and $\mathcal{S}_b$ respectively (note that sequences 1 and 3 are sorted in ascending order, so the first two elements are the two smallest elements).*
- *Then for the elements in sequences 1, we verify in order if $a_i < \frac{a_i+\sum_{j\in\mathcal{S}_a} a_j}{|\mathcal{S}_a|}$. If the condition is satisfied, then a_i will be selected into $\mathcal{S}_a$ and $\mathcal{S}_a$ will be updated. If not, then the selection stops.*
- *The same rule $b'_h < \frac{b'_h+\sum_{j\in\mathcal{S}_b} b'_j}{|\mathcal{S}_b|}$ is used to determine whether to select b'_h and update $\mathcal{S}_b$.*

Finally, we denote the size of final result of $\mathcal{S}_a$ and $\mathcal{S}_b$ as $n_a = |\mathcal{S}_a|$ and $n_b = |\mathcal{S}_b|$ respectively. We have $\frac{\sum_{j=1}^{n_a} a_j}{n_a-1} \geq \frac{\sum_{j=1}^{n_b} b'_j}{n_b-1}$.

Proof. We prove this lemma in three cases: (1) $n_a = n_b$, (2) $n_a < n_b$, and (3) $n_a > n_b$.

We first prove the case of $n_a = n_b$. Obviously, we have

$$\frac{\sum_{j=1}^{n_b} b'_j}{n_b-1} \leq \frac{\sum_{j=1}^{n_b} b_j}{n_b-1} \leq \frac{\sum_{j=1}^{n_b} a_j}{n_b-1} = \frac{\sum_{j=1}^{n_a} a_j}{n_a-1}.$$

Then we prove the case of $n_a < n_b$. For any $h < n_b$, we have $b'_h < \frac{\sum_{j=1}^{h-1} b'_j}{h-2}$ from the selection condition $b'_h < \frac{b'_h+\sum_{j\in\mathcal{S}_b} b'_j}{|\mathcal{S}_b|}$. Thus we have

$$\begin{aligned}\frac{\sum_{j=1}^{n_b} b'_j}{n_b-1} &= \frac{b'_{n_b}+\sum_{j=1}^{n_b-1} b'_j}{n_b-1} < \frac{\frac{\sum_{j=1}^{n_b-1} b'_j}{n_b-2}+\sum_{j=1}^{n_b-1} b'_j}{n_b-1} \\ &= \frac{\sum_{j=1}^{n_b-1} b'_j}{n_b-2} = \frac{b'_{n_b-1}+\sum_{j=1}^{n_b-2} b'_j}{n_b-2} \\ &< \frac{\frac{\sum_{j=1}^{n_b-2} b'_j}{n_b-3}+\sum_{j=1}^{n_b-2} b'_j}{n_b-2} = \frac{\sum_{j=1}^{n_b-2} b'_j}{n_b-3} \\ &< \cdots < \frac{\sum_{j=1}^{n_b-\Lambda n} b'_j}{n_b-\Delta n-1} = \frac{\sum_{j=1}^{n_a} b'_j}{n_a-1} \\ &\leq \frac{\sum_{j=1}^{n_a} b_j}{n_a-1} \leq \frac{\sum_{j=1}^{n_a} a_j}{n_a-1}. \quad (26)\end{aligned}$$

Finally, we prove the case of $n_a > n_b$. For any $i \leq n_a$, we have $a_i \leq a_{n_a} < \frac{a_{n_a}+\sum_{j=1}^{n_a-1} a_j}{n_a-1} < \frac{\sum_{j=1}^{n_a} a_j}{n_a-1}$ from the selection condition $a_i < \frac{a_i+\sum_{j\in\mathcal{S}_a} a_j}{|\mathcal{S}_a|}$. For any $h > n_b$, we have $b'_h \geq \frac{\sum_{j=1}^{n_b} b'_j}{n_b-1}$ since it does not satisfy the selection condition $b'_h < \frac{b'_h+\sum_{j\in\mathcal{S}_b} b'_j}{|\mathcal{S}_b|}$. Since $n_a > n_b$, there must be at least one element in sequence 1 which is selected into $\mathcal{S}_a$, but its corresponding lower bound in sequence 3 is not selected into $\mathcal{S}_b$ Denote any of these special elements in sequence 1 as a_{i_0}, and denote its corresponding lower bound in sequence 2 and sequence 3 as b_{i_0} and b'_{h_0}, we have $a_{i_0} \geq b_{i_0} = b'_{h_0}$. With the help of this special element, we have $\frac{\sum_{j=1}^{n_b} b'_j}{n_b-1} \leq b'_{h_0} \leq a_{i_0} < \frac{\sum_{j=1}^{n_a} a_j}{n_a-1}$. □

Proof.

$$u_{0,t}^k = \gamma \log(\gamma X_t^k) + \frac{1}{X_t^k} - \gamma, X_t^k = \frac{n_0 - 1}{\sum_{j \in \mathcal{S}_t^k} \frac{c_j}{q_{j,t}^k}} \quad (27)$$

$$\hat{u}_{0,t}^k = \gamma \log(\gamma \hat{X}_t^k) + \frac{1}{\hat{X}_t^k} - \gamma, \hat{X}_t^k = \frac{\hat{n}_0 - 1}{\sum_{j \in \hat{\mathcal{S}}_t^k} \frac{c_j}{q_{j,t}^k}}. \quad (28)$$

For function $f(x) = \gamma \log(\gamma x) + \frac{1}{x} - \gamma$, we compute the first derivative of $f(x)$ as $\frac{\partial f(x)}{\partial x} = \frac{\gamma}{x} - \frac{1}{x^2}$. Let $\frac{\partial f(x)}{\partial x} = 0$, we have $x = \frac{1}{\gamma}$. Since $\frac{\partial f(x)}{\partial x}|_{x=\frac{2}{\gamma}} = \frac{\gamma^2}{4} > 0$ and $\frac{\partial f(x)}{\partial x}|_{x=\frac{1}{2\gamma}} = -2\gamma^2 < 0$, we have $f(x)$ is a convex function and $f(x)$ reaches its minimum at $x = \frac{1}{\gamma}$. Therefore, for $x_1 \geq x_2 \geq \frac{1}{\gamma}$, we have $f(x_1) \geq f(x_2)$. Therefore, proving $\hat{u}_{0,t}^k \geq u_{0,t}^k$ is equivalent to proving $\hat{X}_t^k \geq X_t^k$.

We utilizes Lemma 1 to prove $\hat{X}_t^k \geq X_t^k$, where sequence 1 can be referred to as the sorted sequence $\{\frac{c_1}{q_{1,t}^k}, \frac{c_2}{q_{2,t}^k}, \cdots, \frac{c_i}{q_{i,t}^k}\}$, where $\frac{c_1}{q_{1,t}^k} \leq \frac{c_2}{q_{2,t}^k} \leq \cdots \leq \frac{c_i}{q_{i,t}^k}$. Sequence 2 can be referred to as $\{\frac{c_1}{\hat{q}_{1,t}^k}, \frac{c_2}{\hat{q}_{2,t}^k}, \cdots, \frac{c_i}{\hat{q}_{i,t}^k}\}$ and $\frac{c_1}{\hat{q}_{1,t}^k} \leq \frac{c_1}{q_{1,t}^k}, \frac{c_2}{\hat{q}_{2,t}^k} \leq \frac{c_1}{q_{1,t}^k}, \cdots, \frac{c_i}{\hat{q}_{i,t}^k} \leq \frac{c_1}{q_{1,t}^k}$ with probability at least $(1-\delta)^N$. Sequence 3 can be referred to as an ascending sort of $\{\frac{c_1}{\hat{q}_{1,t}^k}, \frac{c_2}{\hat{q}_{2,t}^k}, \cdots, \frac{c_i}{\hat{q}_{i,t}^k}\}$.

According to Lemma 1, we have

$$\frac{\sum_{j \in \hat{\mathcal{S}}_t^k} \frac{c_j}{q_{j,t}^k}}{\hat{n}_0 - 1} \leq \frac{\sum_{j \in \mathcal{S}_t^k} \frac{c_j}{q_{j,t}^k}}{n_0 - 1}. \quad (29)$$

Since $X_t^k = \frac{n_0 - 1}{\sum_{j \in \mathcal{S}_t^k} \frac{c_j}{q_{j,t}^k}}$ and $\hat{X}_t^k = \frac{\hat{n}_0 - 1}{\sum_{j \in \hat{\mathcal{S}}_t^k} \frac{c_j}{q_{j,t}^k}}$, we have $\hat{X}_t^k \geq X_t^k$ with probability at least $(1-\delta)^N$. Thus we prove Lemma 3. □

APPENDIX D
PROOFS OF THEOREM 4

In this section, we provide the proof of Theorem 4. First, we bound the **single-round regret**. Specifically, we show in Lemma 2 that the regret in the server's utility in a given round can be bounded by the uncertainty—i.e., the confidence bounds—of the clients' utility estimates in that round. This result establishes a crucial connection between the server's performance and the quality of client-side task assignments.

Lemma 2. *For a single round t, the regret of Algorithm 2 is upper bounded by*

$$r_t \leq \gamma \log(1 + \frac{1}{\hat{n}_0 - 1} + \frac{\sum_{j \in \hat{\mathcal{S}}_t^k} \alpha_{j,t} ||\boldsymbol{x}_t||_{\boldsymbol{V}_{j,t-1}^{-1}}}{q_{\min}}), \quad (30)$$

where $q_{\min}$ denotes the minimum of data quality.

Proof.

$$\begin{aligned} r_t &= u_{0,t}^{k^*} - u_{0,t}^{k_t} \\ &= (u_{0,t}^{k^*} - \hat{u}_{0,t}^{k^*}) + (\hat{u}_{0,t}^{k^*} - \hat{u}_{0,t}^{k_t}) + (\hat{u}_{0,t}^{k_t} - u_{0,t}^{k_t}) \\ &\leq \hat{u}_{0,t}^{k_t} - u_{0,t}^{k_t}, \end{aligned}$$

where the inequality is due to the fact that $\hat{u}_{0,t}^{k^*} \geq u_{0,t}^{k^*}$ according to Lemma 3 and $\hat{u}_{0,t}^{k_t} \geq u_{0,t}^{k_t}$ since the server selects the estimated optimal arm k_t.

Next, we analyze the difference between $\hat{u}_{0,t}^{k_t}$ and $u_{0,t}^{k_t}$. To simplify the notation, we analyze the difference between $\hat{u}_{0,t}^k$ and $u_{0,t}^k$ in the following steps, which is also the bound of the difference between $\hat{u}_{0,t}^{k_t}$ and $u_{0,t}^{k_t}$.

$$\begin{aligned} &\hat{u}_{0,t}^k - u_{0,t}^k \\ &= \gamma \log(\gamma \hat{X}_t^k) + \frac{1}{\hat{X}_t^k} - \gamma - (\gamma \log(\gamma X_t^k) + \frac{1}{X_t^k} - \gamma) \\ &= \gamma \log(\frac{\gamma \hat{X}_t^k}{\gamma X_t^k}) + \frac{1}{\hat{X}_t^k} - \frac{1}{X_t^k} \\ &\leq \gamma \log(\frac{\hat{X}_t^k}{X_t^k}), \end{aligned}$$

where the inequality is due to the fact that $\hat{X}_t^k \geq X_t^k$ we proved in Lemma 3. Therefore, we need to analyze the bound of $\frac{\hat{X}_t^k}{X_t^k}$. We analyze the bound from three cases: $\mathcal{S}_t^k = \hat{\mathcal{S}}_t^k$, $\mathcal{S}_t^k \subset \hat{\mathcal{S}}_t^k$, and $\mathcal{S}_t^k \not\subset \hat{\mathcal{S}}_t^k$.

For the case of $\mathcal{S}_t^k = \hat{\mathcal{S}}_t^k$, we have

$$\begin{aligned} \frac{\hat{X}_t^k}{X_t^k} &= \frac{\frac{\hat{n}_0 - 1}{\sum_{h \in \hat{\mathcal{S}}_t^k} \frac{c_h}{\hat{q}_{h,t}^k}}}{\frac{n_0 - 1}{\sum_{j \in \mathcal{S}_t^k} \frac{c_j}{q_{j,t}^k}}} = \frac{\sum_{j \in \mathcal{S}_t^k} \frac{c_j}{q_{j,t}^k}}{\sum_{h \in \hat{\mathcal{S}}_t^k} \frac{c_h}{\hat{q}_{h,t}^k}} \\ &= \sum_{j \in \mathcal{S}_t^k} \frac{c_j}{q_{j,t}^k \sum_{h \in \hat{\mathcal{S}}_t^k} \frac{c_h}{\hat{q}_{h,t}^k}} \leq \sum_{j \in \hat{\mathcal{S}}_t^k} \frac{c_j}{q_{j,t}^k \frac{c_j}{\hat{q}_{j,t}^k} (\hat{n}_0 - 1)} \\ &= \frac{1}{\hat{n}_0 - 1} \sum_{j \in \hat{\mathcal{S}}_t^k} \frac{\hat{q}_{j,t}^k}{q_{j,t}^k} = \frac{1}{\hat{n}_0 - 1} \sum_{j \in \hat{\mathcal{S}}_t^k} \frac{q_{j,t}^k + \alpha_{j,t} ||\boldsymbol{x}_t||_{\boldsymbol{V}_{j,t-1}^{-1}}}{q_{j,t}^k} \\ &= \frac{1}{\hat{n}_0 - 1} (\hat{n}_0 + \frac{\sum_{j \in \hat{\mathcal{S}}_t^k} \alpha_{j,t} ||\boldsymbol{x}_t||_{\boldsymbol{V}_{j,t-1}^{-1}}}{q_{\min}}) \\ &\leq \frac{\hat{n}_0}{\hat{n}_0 - 1} + \frac{\sum_{j \in \hat{\mathcal{S}}_t^k} \alpha_{j,t} ||\boldsymbol{x}_t||_{\boldsymbol{V}_{j,t-1}^{-1}}}{q_{\min}} \\ &= 1 + \frac{1}{\hat{n}_0 - 1} + \frac{\sum_{j \in \hat{\mathcal{S}}_t^k} \alpha_{j,t} ||\boldsymbol{x}_t||_{\boldsymbol{V}_{j,t-1}^{-1}}}{q_{\min}}. \end{aligned}$$

For the case of $\hat{\mathcal{S}}_t^k \subset \mathcal{S}_t^k$, we obtain $\frac{\sum_{j \in \mathcal{S}_t^k} \frac{c_j}{q_{j,t}^k}}{n_0 - 1} \leq \frac{\sum_{j \in \hat{\mathcal{S}}_t^k} \frac{c_j}{q_{j,t}^k}}{\hat{n}_0 - 1}$ by using the same method as (26) when proving case (2) in proof of lemma 1. Thus we have

$$\begin{aligned} \frac{\hat{X}_t^k}{X_t^k} &= \frac{\frac{\hat{n}_0 - 1}{\sum_{h \in \hat{\mathcal{S}}_t^k} \frac{c_h}{\hat{q}_{h,t}^k}}}{\frac{n_0 - 1}{\sum_{j \in \mathcal{S}_t^k} \frac{c_j}{q_{j,t}^k}}} \leq \frac{\frac{\hat{n}_0 - 1}{\sum_{h \in \hat{\mathcal{S}}_t^k} \frac{c_h}{\hat{q}_{h,t}^k}}}{\frac{\hat{n}_0 - 1}{\sum_{j \in \hat{\mathcal{S}}_t^k} \frac{c_j}{q_{j,t}^k}}} = \frac{\sum_{j \in \hat{\mathcal{S}}_t^k} \frac{c_j}{q_{j,t}^k}}{\sum_{h \in \hat{\mathcal{S}}_t^k} \frac{c_h}{\hat{q}_{h,t}^k}} \\ &\leq 1 + \frac{1}{\hat{n}_0 - 1} + \frac{\sum_{j \in \hat{\mathcal{S}}_t^k} \alpha_{j,t} ||\boldsymbol{x}_t||_{\boldsymbol{V}_{j,t-1}^{-1}}}{q_{\min}}. \end{aligned}$$

For the case of $\hat{\mathcal{S}}_t^k \not\subset \mathcal{S}_t^k$, there must be at least one client that is selected into $\hat{\mathcal{S}}_t^k$, but is not selected into $\mathcal{S}_t^k$. Thus we can find the special client $j_0 = h_0$ that satisfies $j_0 \in \hat{\mathcal{S}}_t^k$ and

$h_0 \notin \mathcal{S}_t^k$. We have $\frac{c_{j_0}}{\hat{q}_{j_0,t}^k} < \frac{\sum_{j\in\hat{\mathcal{S}}_t^k}\frac{c_j}{\hat{q}_{j,t}^k}}{\hat{n}_0-1}$ and $\frac{c_{h_0}}{q_{h_0,t}^k} \geq \frac{\sum_{j\in\mathcal{S}_t^k}\frac{c_j}{q_{j,t}^k}}{n_0-1}$ from the selection condition. Therefore, we have

$$\begin{aligned}\frac{\hat{X}_t^k}{X_t^k} &= \frac{\frac{\hat{n}_0-1}{\sum_{h\in\hat{\mathcal{S}}_t^k}\frac{c_h}{\hat{q}_{h,t}^k}}}{\frac{n_0-1}{\sum_{j\in\mathcal{S}_t^k}\frac{c_j}{q_{j,t}^k}}} \leq \frac{\frac{\hat{q}_{j_0,t}^k}{c_{j_0}}}{\frac{q_{h_0,t}^k}{c_{h_0}}} = \frac{\frac{\hat{q}_{j_0,t}^k}{c_{j_0}}}{\frac{q_{j_0,t}^k}{c_{j_0}}} = \frac{\hat{q}_{j_0,t}^k}{q_{j_0,t}^k} \\ &= 1 + \frac{\alpha_{j_0,t}||\boldsymbol{x}_t||_{\boldsymbol{V}_{j_0,t-1}^{-1}}}{q_{\min}} \\ &\leq 1 + \frac{1}{\hat{n}_0-1} + \frac{\sum_{j\in\hat{\mathcal{S}}_t^k}\alpha_{j,t}||\boldsymbol{x}_t||_{\boldsymbol{V}_{j,t-1}^{-1}}}{q_{\min}}.\end{aligned}$$

Therefore, the regret in single round t is bounded by

$$\begin{aligned}r_t &\leq \gamma\log(\frac{\hat{X}_t^k}{X_t^k}) \\ &\leq \gamma\log(1 + \frac{1}{\hat{n}_0-1} + \frac{\sum_{j\in\hat{\mathcal{S}}_t^k}\alpha_{j,t}||\boldsymbol{x}_t||_{\boldsymbol{V}_{j,t-1}^{-1}}}{q_{\min}}).\end{aligned}$$

□

Next, we bound the **cumulative regret** of the server's utility. This is done by leveraging the confidence bounds on service quality to construct a martingale sequence, which enables us to show that the sum of the confidence bounds over time converges, as shown in Lemma 3.

Lemma 3 (Lemma 11 in [17])**.** *Let $\boldsymbol{x}_1, \boldsymbol{x}_2, ..., \boldsymbol{x}_n \in \mathbb{R}^d$ be a sequence in $\mathbb{R}^d$, $\boldsymbol{U}_0 \in \mathbb{R}^{d\times d}$ a positive defnite matrix and define $\boldsymbol{U}_n = \boldsymbol{U}_0 + \sum_{t=1}^n \boldsymbol{x}_t\boldsymbol{x}_t^\top$ for all t.Then, we have that*

$$\log\left(\frac{\det(\boldsymbol{U}_n)}{\det(\boldsymbol{U}_0)}\right) \leq \sum_{t=1}^n ||\boldsymbol{x}_t||_{\boldsymbol{U}_{t-1}^{-1}}^2$$

Further, if $||\boldsymbol{x}_t||_2 \leq L$, then

$$\sum_{t=1}^n \min\{1, ||\boldsymbol{x}_t||_{\boldsymbol{U}_{t-1}^{-1}}^2\} \leq 2(\log\det(\boldsymbol{U}_t) - \log\det\boldsymbol{U}_0),$$

and finally, if $\lambda_{\min}(\boldsymbol{U}_0) \geq \max(1, L^2)$ then

$$\sum_{t=1}^n ||\boldsymbol{x}_t||_{\boldsymbol{U}_{t-1}^{-1}}^2 \leq 2\log\frac{\det(\boldsymbol{U}_n)}{\det(\boldsymbol{U}_0)}.$$

With the help of Lemma 2 and 3, we can analyze the regret of Algorithm 2 in multiple rounds.

Proof. According to Lemma 2, the regret in single round t is bounded by

$$r_t \leq \gamma\log(1 + \frac{1}{\hat{n}_0-1} + \frac{\sum_{j\in\hat{\mathcal{S}}_t^k}\alpha_{j,t}||\boldsymbol{x}_t||_{\boldsymbol{V}_{j,t-1}^{-1}}}{q_{\min}}). \quad (31)$$

Therefore, the total regret in T rounds is bounded by

$$\begin{aligned}R_T &= \sum_{t=1}^T r_t \\ &\leq \sum_{t=1}^T \gamma\log(1 + \frac{1}{\hat{n}_0-1} + \frac{\sum_{j\in\hat{\mathcal{S}}_t^k}\alpha_{j,t}||\boldsymbol{x}_t||_{\boldsymbol{V}_{j,t-1}^{-1}}}{q_{\min}})\end{aligned}$$

Denote $Y_t^k = \frac{\sum_{j\in\hat{\mathcal{S}}_t^k}\alpha_{j,t}||\boldsymbol{x}_t||_{\boldsymbol{V}_{j,t-1}^{-1}}}{q_{\min}}$. Since $\frac{1}{\hat{n}_0-1}$ is small enough, it is resonable to assume that Y_t^k is large enough so that $1 + \frac{1}{\hat{n}_0-1} \leq e^{Y_t^k} - Y_t^k$. Therefore, we can safely assume that $\log(1 + \frac{1}{\hat{n}_0-1} + Y_t^k) \leq Y_t^k$. Thus, the total regret is bounded by

$$\begin{aligned}R_T \leq& \gamma\sum_{t=1}^T \frac{\sum_{j\in\hat{\mathcal{S}}_t^k}\alpha_{j,t}||\boldsymbol{x}_t||_{\boldsymbol{V}_{j,t-1}^{-1}}}{q_{\min}} \\ =& \frac{\gamma}{q_{\min}}\sum_{j\in\mathcal{N}}\sum_{t\in\mathcal{T}_j}\alpha_{j,t}||\boldsymbol{x}_t||_{\boldsymbol{V}_{j,t-1}^{-1}} \\ \leq& \frac{\gamma\alpha_T}{q_{\min}}\sum_{j\in\mathcal{N}}\sqrt{T\sum_{t\in\mathcal{T}_j}||\boldsymbol{x}_t||_{\boldsymbol{V}_{j,t-1}^{-1}}^2} \\ \leq& \frac{\gamma\alpha_T}{q_{\min}}\sum_{j\in\mathcal{N}}\sqrt{2Td\log(\frac{\det(\boldsymbol{V}_{j,t-1})}{\det(\boldsymbol{V}_{j,0})})},\end{aligned}$$

where the last inequality is due to Lemma 3. Obviously, $\det(\boldsymbol{V}_{j,0}) = \lambda^d$ and thus $\log(\det(\boldsymbol{V}_{j,0})) = d\log(\lambda)$. Denote the eigenvalues of $\boldsymbol{V}_{j,t-1}$ as λ_s. With the fact that $\det(\boldsymbol{V}_{j,t-1}) = \prod_s \lambda_s \leq (\frac{\text{trace}(\boldsymbol{V}_{j,0})+tL^2}{d})^d$ and $\text{trace}(\boldsymbol{V}_{j,0}) = \lambda d$, we have $\log(\det(\boldsymbol{V}_{j,t-1})) \leq d\log(\frac{\lambda d+tL^2}{d})$. Therefore,

$$\begin{aligned}R_T \leq& \frac{\gamma\alpha_T}{q_{\min}}\sum_{j\in\mathcal{N}}\sqrt{2Td\log(\frac{\lambda d+TL^2}{\lambda d})} \\ \leq& \frac{\gamma\sigma N}{q_{\min}}\sqrt{2Td\log(1+\frac{TL^2}{\lambda d})} \\ &\cdot(\sqrt{2d\log(1+\frac{TL^2}{\lambda d}) + 2\log(\frac{2}{\delta})} + \sqrt{\lambda}) \\ =& O(\frac{\gamma\sigma dN}{q_{\min}}\sqrt{T}\log(TL^2/\lambda d)).\end{aligned}$$

□