University of Victoria
Department of Economics
ECON 457
Computational Economics
Spring Term 2018/19

Instructor:
Marco Cozzi
Take Home Exam[1]
April 13th 2019
*Due: on **April 15$^{\textbf{th}}$** 2019 **before** 1.30pm, in the **ECON 457 Drop Box**, 3rd Floor BEC.*

Instructions:

- The exam consists of two questions: you have to answer both, using Python codes.
- The total number of marks is 100. Question 1 is worth 40 marks, while Question 2 is worth 60 marks.
- Include your name and student number on the front page of your answers.
- Answer clearly and concisely. Devote some time to give the graphs, plots and tables a format easy to understand. Also the way you present your answers matters for the final grade.
- Since the https://uvic.syzygy.ca/ web interface can be unreliable when several users are trying to access it at the same time, please try not to use it. Use a version of Python installed on your computer or in a computer lab.
- Please print your answers and submit them before the deadline. You must also submit the print-outs of the codes you have used to generate your results (together with this page, dated and signed).
- Tables and graphs must be printed, clearly labeled and referenced in the text.
- No late, group, or electronic (e.g., by email) submissions will be accepted.
- This is an exam, so neither myself nor Sheri will provide suggestions on how to solve the questions, or preliminary feedback on the quality of your answers.
- You are asked to work by yourselves on each question. UVic's rules about cheating and plagiarism apply also to this exam. Please date and sign the bottom of this page as an acknowledgment of understanding and observing the exam rules.
- On April 15th, starting from 2.00pm, each student will have five minutes to explain their answers to Ms. Sheri Liu and Prof. Graham Voss. For privacy considerations, and not to give any students an advantage, these meetings will take place in office BEC 371. Please refer to the schedule attached to this document to know when you should go to BEC 371 (do not be late).
- The scheduled exam room, DTB A120, will not be used. You will submit your answers in the ECON 457 Drop Box by 1.30pm, and go to BEC 371 at the appropriate time.

Date ______________________________

Name ______________________________

Student Number ______________________________

Signature ______________________________

Question 1: Ordinary Least Squares and Instrumental Variables (40 Marks)

Consider the classical linear econometric model. We analyze some properties of the Ordinary Least Squares (OLS) and Instrumental Variables (IV) estimators. We will rely on Monte Carlo simulation methods. In terms of notation, the index $i = 1, ..., I$ denotes the units of observation in our (simulated) cross-sectional sample, and I is the sample size. As the initial seed number, use your date of birth (e.g., if you were born on April the 13^{th}, use the number 13 as the seed).

(a) Set the sample size to $I = 10,000$. First consider the explanatory variable (x_i). Generate it by drawing I independent observations from the standard normal distribution. Now consider the dependent variable (y_i). For every data point, you are asked to generate it using the following model, setting the true parameter values to $\beta_0 = 0$, $\beta_1 = 1$ and $\sigma_\varepsilon^2 = 2$:

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i, \ \varepsilon_i \sim N(0, \sigma_\varepsilon^2)$$

where the errors ε_i are drawn from the normal distribution.

(a.1) Compute the OLS estimates $\hat{\beta}_0^{OLS}$ and $\hat{\beta}_1^{OLS}$. Comment. You can find the relevant formulas here:

`https://en.wikipedia.org/wiki/Simple_linear_regression`

(a.2) Now reset the seed to its original value, and set the sample size to $I = 100$. Create 100 different datasets, generating the values for x_i and y_i using the same procedure as above. Use a for-loop and set the seed to `seed=seed+itr`, where `itr` stands for the iteration number. On each sample, compute the 100 OLS estimates $\hat{\beta}_0^{OLS}$ and $\hat{\beta}_1^{OLS}$ and plot them with two histograms. Comment.

(a.3) Now we move to analyze an IV estimator. Reset the seed to its original value and generate three sets of $I = 10,000$ independent error terms, denoting them as η_1, η_2 and η_3. These shocks are drawn from independent normal distributions $N(0, \sigma_\eta^2 = 2)$.

Consider the explanatory variable (x_i), and generate it as $x_i = \beta_2 \eta_{1,i} + \eta_{2,i}$, with $\beta_2 = 1$. Now consider an instrumental variable (z_i), and generate it as $z_i = \beta_3 \eta_{2,i} + \eta_{3,i}$, with $\beta_3 = 1$.

To conclude with, generate the dependent variable (y_i), using the following model and setting the true parameter values to $\beta_0 = 0$, $\beta_1 = 1$:

$$y_i = \beta_0 + \beta_1 x_i + \eta_{1,i}$$

Compute both the OLS estimate $\hat{\beta}_1^{OLS}$ and the IV (2SLS) estimate $\hat{\beta}_1^{IV}$. To get the latter, first apply the OLS formula to the model:

$$x_i = \gamma_0 + \gamma_1 z_i + \nu_i$$

.

Then, compute the predicted values $\hat{x}_i = \hat{\gamma}_0 + \hat{\gamma}_1 z_i$. Finally, apply the OLS formula to the model:

$$y_i = \beta_0 + \beta_1 \hat{x}_i + u_i$$

Comment.

(a.4) Now reset the seed to its original value, and set the sample size to $I = 100$. Create 100 different datasets, generating the values for x_i, z_i and y_i using the same procedure as in part (a.3). Use a for-loop and set the seed to `seed=seed+itr`, where `itr` stands for the iteration number. On each sample, compute the 100 OLS estimates $\hat{\beta}_0^{OLS}$ and the 100 IV estimates $\hat{\beta}_1^{IV}$. Plot them with two histograms and comment.

(a.5) Redo parts (a.3) and (a.4) when $\beta_3 = 0.1$. Redo parts (a.3) and (a.4) when $\beta_3 = 0$. Comment.

(b) Now consider a multivariate case with two explanatory variables and two instruments.
Generate five sets of $I = 10,000$ independent error terms, denoting them as η_1, η_2, η_3, η_4 and η_5. These shocks are drawn from independent normal distributions $N(0, \sigma_\eta^2 = 2)$.

Consider the two explanatory variables $(x_{1,i}, x_{2,i})$. Generate the first one as $x_{1,i} = \beta_3\eta_{1,i} + \eta_{2,i}$, with $\beta_3 = 1$. Generate the second one as $x_{2,i} = \beta_4\eta_{1,i} + \eta_{3,i}$, with $\beta_4 = -1$.

Now consider two instrumental variables $(z_{1,i}, z_{2,i})$. Generate the first one as $z_{1,i} = \beta_5\eta_{2,i} + \eta_{4,i}$, with $\beta_5 = 1$. Generate the second one as $z_{2,i} = \beta_6\eta_{3,i} + \eta_{5,i}$, with $\beta_6 = 1$.

To conclude with, generate the dependent variable (y_i), using the following model and setting the true parameter values to $\beta_0 = 0$, $\beta_1 = 1$, $\beta_2 = -1$:

$$y_i = \beta_0 + \beta_1 x_{1,i} + \beta_2 x_{2,i} + \eta_{1,i}$$

(b.1) Get both the OLS estimates and the IV estimates. For this part, you must use the matrix formulation of the model, and obtain the vectors $\hat{\beta}^{OLS}$ and $\hat{\beta}^{IV}$ with matrix operations on the matrices X, Z, and the vector y. The columns of the matrix X consists of the constant, the variable x_1, and the variable x_2. The columns of the matrix Z consists of the constant, the variable z_1, and the variable z_2.

Compute the OLS estimates $\hat{\beta}_0^{OLS}$, $\hat{\beta}_1^{OLS}$ and $\hat{\beta}_2^{OLS}$ as:

$$\hat{\beta}^{OLS} = (X'X)^{-1} X'y$$

Compute the IV estimates $\hat{\beta}_0^{IV}$, $\hat{\beta}_1^{IV}$ and $\hat{\beta}_2^{IV}$ as:

$$\hat{\beta}^{IV} = (Z'X)^{-1} Z'y$$

Comment.

(b.2) Now reset the seed to its original value, and set the sample size to $I = 100$. Create 100 different datasets, generating the values for x_1, x_2, z_1, z_2 and y using the same procedure as above. Use a for-loop and set the seed to `seed=seed+itr`, where `itr` stands for the iteration number. On each sample, compute the 100 OLS estimates $\hat{\beta}^{OLS}$ and the 100 IV estimates $\hat{\beta}^{IV}$ and plot them with histograms. Comment.

(b.3) Redo part (b.2) when $\beta_5 = 1$ and $\beta_6 = 0.01$, and when $\beta_5 = 0.01$ and $\beta_6 = 1$. Comment.

(b.4) Redo part (b.2) when $\beta_5 = 0$ and $\beta_6 = 0$. Comment.

Question 2: Optimal Taxation Policies (60 Marks)

In this question, we want to study the optimal mix of labor income taxes (T_l) and corporate taxes (T_f), in a situation where the tax revenues are used to finance total government expenditure (G). The government runs a balanced budget, namely:

$$G = T_l + T_f$$

Total government expenditure consists of public investment (I), which is exogenous and constant, and subsidies (S), which are variable and depend on the economy's outcomes, with $G = I + S$. In order to balance the budget, a proportional tax rate (τ_l) is imposed on workers' income and a proportional tax rate (τ_f) is imposed on corporate income.

In the economy, there are L identical potential workers. The unemployed individuals, if any, do not pay taxes and do not produce any output. The employed ones they all earn the same wage (w), and their after-tax income is:

$$(1 - \tau_l)w$$

Firms maximize their profits (π) and rely on one input, labor (l), to produce a homogeneous good (y). All firms have access to the same technology, a decreasing return to scale production function (with parameter $0 < \phi < 1$), but they differ in their productivity, captured by the variable a:

$$y = al^{\phi}$$

Productivity is assumed to be log-normally distributed across firms, with probability density function $f(a)$ and parameters μ_a and σ_a^2: $a \sim LN(\mu_a, \sigma_a^2)$. The price of the final good is normalized to 1. It follows that total pre-tax revenues ($R(a)$), net of labor costs, are:

$$R(a) = al^{\phi} - wl$$

The government subsidizes job creation, giving each firm a proportional subsidy (s) per employed worker. Total profits for a firm ($\pi(a)$), inclusive of the government interventions (corporate taxes and labor subsidies), are:

$$\pi(a) = (1 - \tau_f)R + sl = (1 - \tau_f)(al^{\phi} - wl) + sl$$

(a) For this part of the question assume that $\phi = 0.6$, $\tau_f = 0.4$, $\tau_l = 0.2$, $s = 0.1$, $w = 1$, $\mu_a = 0$ and $\sigma_a^2 = 2$. Notice also that here the government does not run a balanced budget and L is to be interpreted as a large number that is not binding.

Solve the problem of the firm and obtain the labor demand functions at the level of the individual firm ($l(a)$). Calculate total output (Y), which is defined as follows:

$$Y = \int_0^{\infty} y(a)f(a)da = \int_0^{\infty} a[l(a)]^{\phi} f(a)da$$

Explain how you implemented this computation.

Now compute total output using different parameter values. In one case they are:

$$\phi = 0.7, \tau_f = 0.4, \tau_l = 0.2, s = 0.1, w = 1, \mu_a = 0, \sigma_a^2 = 2.$$

In another case they are:

$$\phi = 0.6, \tau_f = 0.6, \tau_l = 0.2, s = 0.1, w = 1, \mu_a = 0, \sigma_a^2 = 2.$$

In a final case they are:

$$\phi = 0.6, \tau_f = 0.6, \tau_l = 0.2, s = 0.1, w = 2, \mu_a = 0, \sigma_a^2 = 2.$$

Report your results in a table and comment on your findings, stressing the economic channels at play.

(b) For this part of the question assume that $\phi = 0.6$, $\tau_f = 0.4$, $s = 0.1$, $I = 10$, $w = 1$, $\mu_a = 0$ and $\sigma_a^2 = 2$. Notice that here L is to be interpreted as a large number that is not binding, namely $L \geq l^d$, where l^d is total labor demand.

Find the labor income tax rate τ_l that allows the government to run a balanced budget. Explain how you implemented this computation and which numerical methods you have used.

(c) From now on, assume that labor supply (l^s) is flexible and represented by the following schedule:

$$l^s = [10 * (1 - \tau_l)w]^{\frac{1}{\varepsilon}}$$

Assume that $\phi = 0.6$, $\tau_f = 0.4$, $\tau_l = 0.2$, $s = 0.1$, $I = 10$, $\varepsilon = 0.5$, $\mu_a = 0$ and $\sigma_a^2 = 2$. Notice also that for this part of the question the government does not run a balanced budget, and L is to be interpreted as a large number that is not binding, namely $L \geq l^s$.

Find the wage w that clears the labor market. Explain how you implemented this computation, report your results and comment.
Hint: Use a guess and verify procedure. Guess the wage w_0 and obtain the labor demand functions at the level of the individual firm by solving their profit maximization problem. Compute the total labor demand $l^d(w_0)$ and the labor supply $l^s(w_0)$. If they are different, update the guess on the wage and iterate until convergence. For this problem, bisection would work well.

(d) For this part of the question retain the same labor supply function used in part (c).
Assume that $\phi = 0.6$, $s = 0.1$, $I = 10$, $\varepsilon = 0.5$, $\mu_a = 0$ and $\sigma_a^2 = 2$. Notice that here L is to be interpreted as a large number that is not binding, namely $L \geq l^s$. The government runs a balanced budget and the labor market has to be in equilibrium.

For a constant job creation (proportional) subsidy $s = 0.1$, find the combination of corporate tax rate (τ_f) and labor income tax rate (τ_l) that maximizes aggregate output. Explain how you implemented this computation, report your results and comment.

(e) Now the job creation (proportional) subsidy s is no longer fixed. Find the combination of corporate tax rate (τ_f), labor income tax rate (τ_l), and labor subsidy (s) that maximizes aggregate output. Explain how you implemented this computation, report your results and comment.

END OF THE EXAM

Student Name	Student ID	Time (BEC 371)	Notes Q1	Notes Q2
Al-Hashimi, Afnan		2.00pm		
Auchterlonie, Michael		2.05pm		
Ben Ammar, Youssef		2.10pm		
Blaney, Taryn		2.15pm		
Bruce, Nicole		2.20pm		
Cheng, Alex		2.25pm		
Fairweather, Connor		2.30pm		
Feng, Sina		2.35pm		
Ford, Hayden		2.40pm		
Hayer, Amrit		2.45pm		
Hu, Yuhao		2.50pm		
Hutchinson, Shaun		2.55pm		
Jin, Joyce	VXXXXXXX9	3.00pm		
Jin, Joyce	VXXXXXXX6	3.05pm		
Kochhar, Sneh		3.10pm		
Liao, Luying		3.15pm		
Liu, Chloe		3.20pm		
Liu, Kehan		3.25pm		
Mathew, Nishant		3.30pm		
Monks, Jordan		3.35pm		
Peng, Xiangning		3.40pm		
Qian, Xulin		3.45pm		
Reyerse, Robbie		3.50pm		
Schopper, Michael		3.55pm		
Scott, Ryan		4.00pm		
Su, Lisa		4.05pm		
Todd, Brendan		4.10pm		
Tolentino, Carlo		4.15pm		
Wang, Zixuan		4.20pm		
Yang, Candice		4.25pm		
Yao, Kaspar		4.30pm		
Zhang, Yihan		4.35pm		
Zhu, Mingjia		4.40pm		
Zhu, Yu		4.45pm		