CSC 225 - SUMMER 2019
ALGORITHMS AND DATA STRUCTURES I
WRITTEN ASSIGNMENT 1
UNIVERSITY OF VICTORIA

Due: Wednesday, May 22nd, 2019 before noon. **Late assignments will not be accepted.**

Submit your answers on paper to the CSC 225 drop box on the second floor of ECS (in front of the elevators). You are expected to submit a typed solution (handwritten documents will not be marked). However, you are permitted to draw mathematical formulas or diagrams onto the typed copy by hand if that is more convenient than typesetting them.

Question 1: Review of Induction Proofs [5 marks]
Using induction, prove that

$$\sum_{i=0}^{n} 3 \cdot 4^i = 4^{n+1} - 1$$

for all integers $n \geq 0$.

Question 2: Asymptotic Analysis I [10 marks]
For the parts below, let

$$g(n) = \sum_{i=0}^{n} i^2 n$$

(a) Give a simple function (non-recursive, containing no summations) $f(n)$ such that

$$f(n) \in \Theta(g(n))$$

Hint: You do not need to find an exact closed form for the summation.

(b) Prove the asymptotic relationship

$$f(n) \in O(g(n))$$

where $f(n)$ is your answer from part (a).

(c) Prove the asymptotic relationship

$$f(n) \in \Omega(g(n))$$

where $f(n)$ is your answer from part (a).

Question 3: Asymptotic Analysis II [15 marks]

(a) Give a function $f(n)$ such that

$$f(n) \in o\left(\frac{n^2}{\log_2 n}\right)$$
$$f(n) \in \Omega(n \log_2 n)$$
$$f(n) \in \omega(n \log_2 \log_2 n)$$

(b) Prove the asymptotic relationship

$$f(n) \in o\left(\frac{n^2}{\log_2 n}\right)$$

where $f(n)$ is your answer from part (a).

(c) Prove the asymptotic relationship

$$f(n) \in \Omega(n \log_2 n)$$

where $f(n)$ is your answer from part (a).

(d) Prove the asymptotic relationship

$$f(n) \in \omega(n \log_2 \log_2 n)$$

where $f(n)$ is your answer from part (a).

Question 4: Algorithm Analysis [10 marks]
Consider the function `MysterySum` in the pseudocode below, which takes two integer arrays A and B as arguments. Assume that array indexing starts at 0 (so the indices of an array of length q are $0, 1, \ldots, q-1$).

```
procedure MysterySum(A, B)
    sum ← 0
    i ← 0
    while i < length(A) do
        k ← 1
        while k < length(B) do
            if A[i] < B[k] then
                k ← 2k
            else
                sum ← sum + 1
                k ← k + 1
            end if
        end while
        i ← i + 1
    end while
    return sum
end procedure
```

For the parts below, suppose that length$(A) = n$ and length$(B) = m$ for integers $n, m \geq 0$

(a) What is the asymptotic worst case running time of MysterySum in terms of the array sizes n and m? Give your answer as a big-theta expression (you do not need to derive an exact operation count). Briefly explain your answer (in one or two sentences).

(b) Give an example of a worst-case input when $n = 5$ and $m = 5$ (that is, where both A and B have five elements).

(c) What is the best case running time of MysterySum in terms of the array sizes n and m? Give your answer as a big-theta expression (you do not need to derive an exact operation count). Briefly explain your answer.

(d) Give an example of a best-case input when $n = 5$ and $m = 5$ (that is, where both A and B have five elements).