Name:
Student #:

Question [2 marks]

a. Fill each box in a 3-by-3 arrangement of boxes with either 1, 0, or −1. For any such arrangement show that of the eight row, column, and diagonal sums, two sums must be equal. [1 mark]

There are 7 different possible sums (pigeonholes) considering the range [-3, 3], whereas there are 8 instances (pigeons) for 3 rows, 3 columns and 2 diagonals. Applying the pigeonhole principle, we know there must be two sums that are equal.

b. Show that if any 14 integers are selected from the set S = {1, 2, 3, . . ., 25}, there are at least two whose sum is 26. [1 mark]
Hint: Let n be a positive integer. If n + 2 integers are selected from the set S = {1, 2, 3, . . ., 2n + 1}, there are at least 2 whose sum is 2n + 2 (generalized version).

Considering the number pairs which add up to 26: {1, 25}, {2, 24},… , {12, 14} in total 12 sets plus a single number set {13}, these form up a partition in S and give 13 pigeonholes.
If 14 integers (pigeons) are selected from S, with pigeonhole principle we know that there are at least two numbers to be selected from the same set which makes the sum equals 26.