An Introduction to Deep Reinforcement Learning

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Motivation

Can we create Artificial Intelligence?

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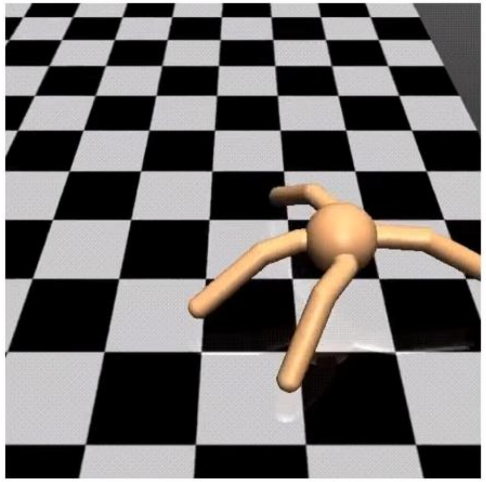
Can we create Artificial Intelligence?

We can create programs that LEARN!!

Deep Reinforcement Learning (Deep RL)

Deep Learning

Deep RL



- **What is it?** Framework for learning to solve sequential decision making problems.
- **How?** Trial and error in a world that provides occasional rewards
- **Deep?** Deep RL = RL + Neural Networks

Classes of Learning Problems

Supervised Learning

Data: (x, y)
x is data, y is label

Goal: Learn function to map
$$x \rightarrow y$$

Apple example:

This thing is an apple.

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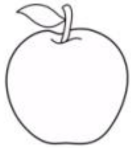

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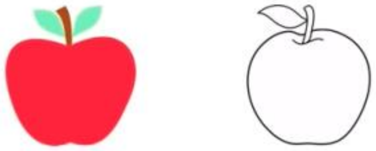
Unsupervised Learning

Data: x
x is data, no labels!

Goal: Learn underlying structure

Apple example:



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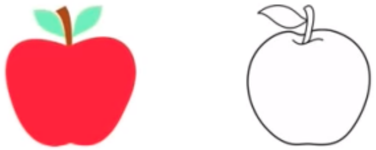

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Reinforcement Learning

Data: state-action pairs

Goal: Maximize future rewards over many time steps

Apple example:

Eat this thing because it will keep you alive.

Classes of Learning Problems

RL: our focus today!

Reinforcement Learning

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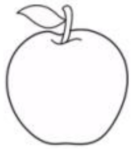
Apple example:



Eat this thing because it will keep you alive.

Reinforcement Learning (RL): Key Concepts

Agent: take actions.

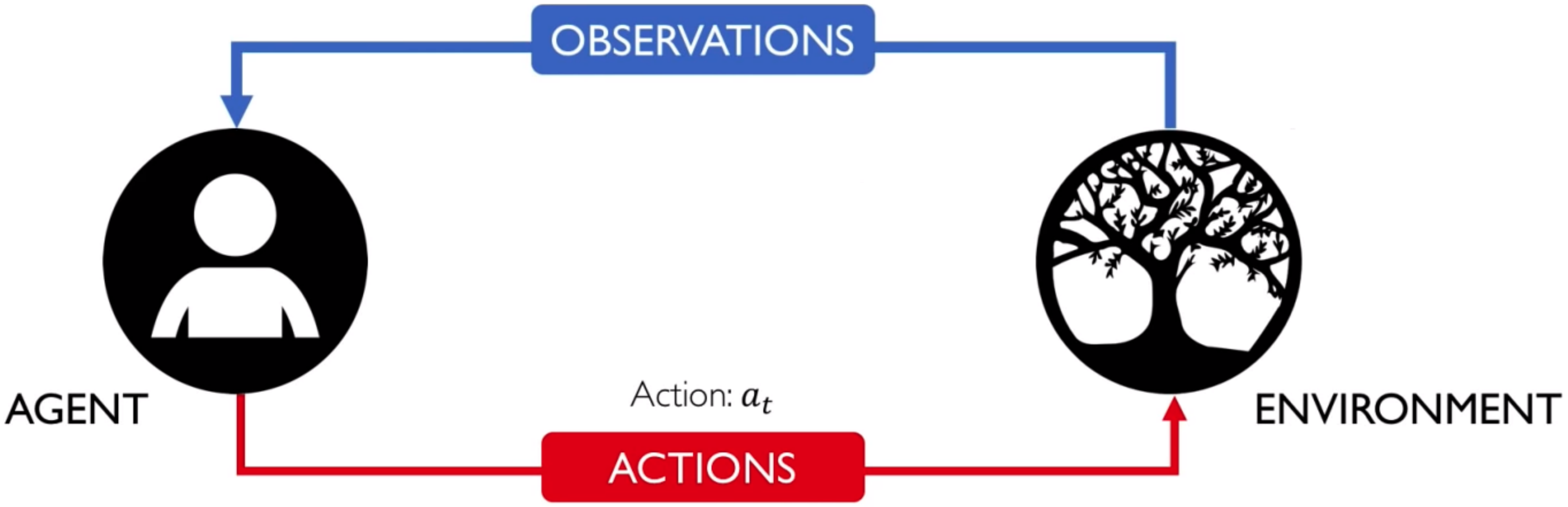
Reinforcement Learning (RL): Key Concepts

Environment: the wold in which the agent exist and operates.

Reinforcement Learning (RL): Key Concepts

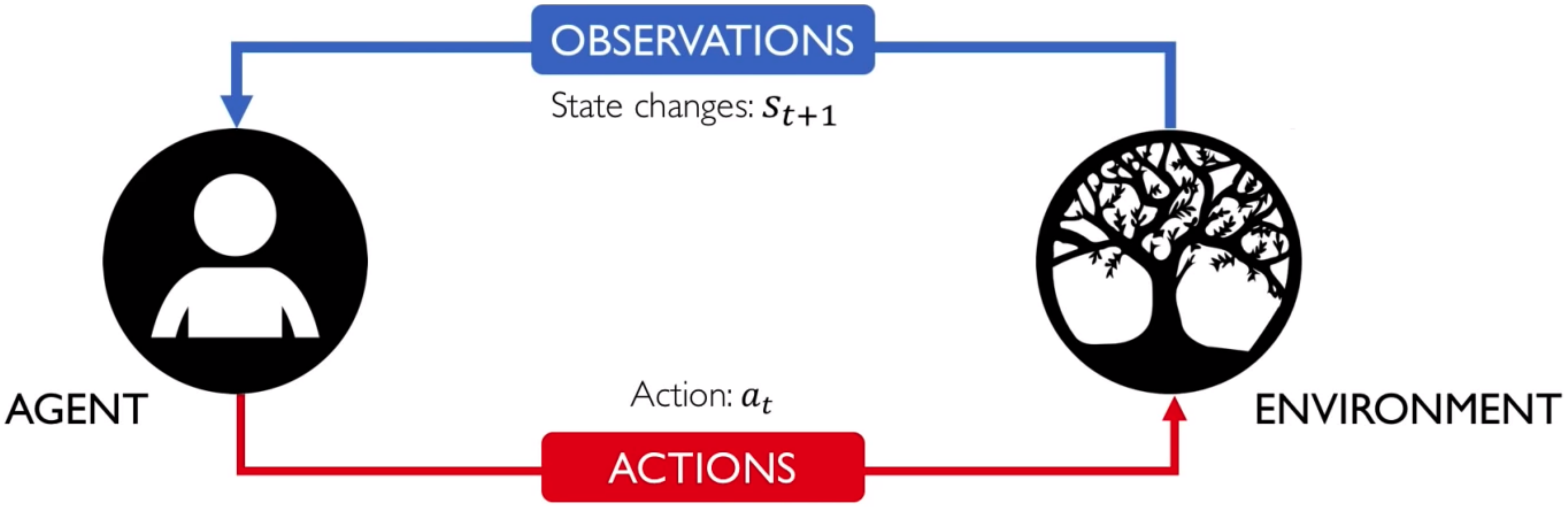
Action: a move the agent can make in the environment.

Reinforcement Learning (RL): Key Concepts



Observations: of the environment after taking actions.

Reinforcement Learning (RL): Key Concepts



State: an situation which the agent perceives.

Reinforcement Learning (RL): Key Concepts

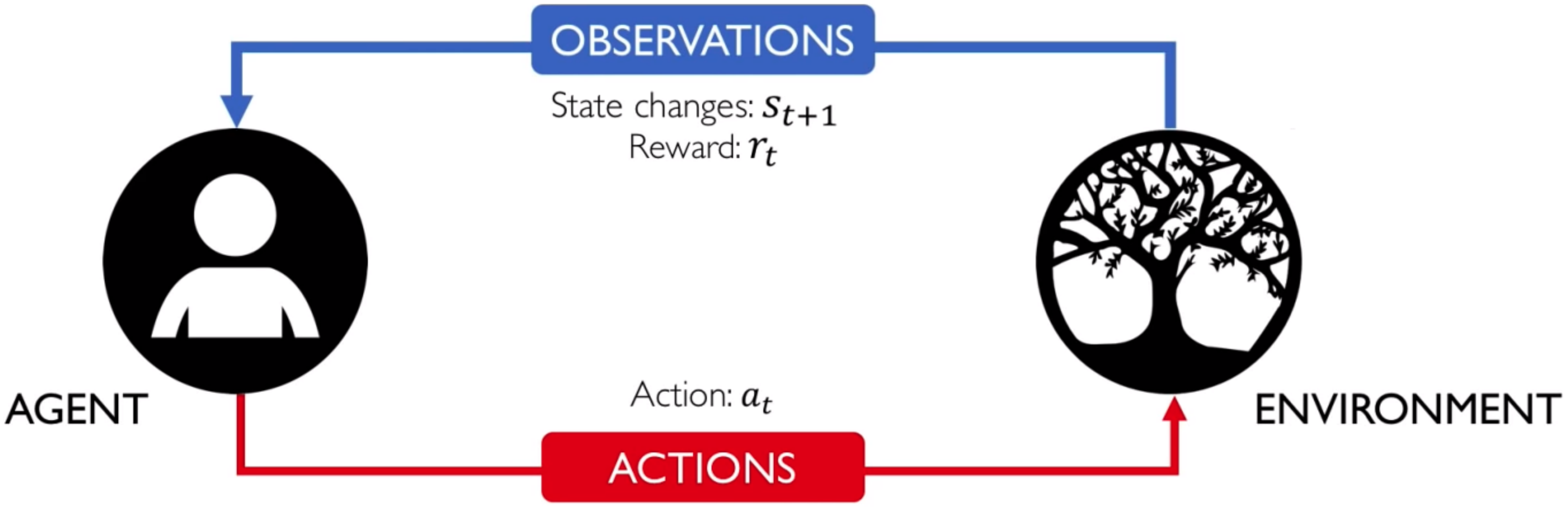
Reward: feedback that measure the success or failure of the agent’s action.

Reinforcement Learning (RL): Key Concepts

Total Reward

$$R_t = \sum_{i=t}^{\infty} r_i$$

Reinforcement Learning (RL): Key Concepts



Total Reward

$$R_t = \sum_{i=t}^{\infty} r_i = r_t + r_{t+1} \ldots + r_{t+n-1} + r_{t+n} + \cdots$$

Reinforcement Learning (RL): Key Concepts

Discounted
Total Reward

$$R_t = \sum_{i=t}^{\infty} \gamma^i r_i$$

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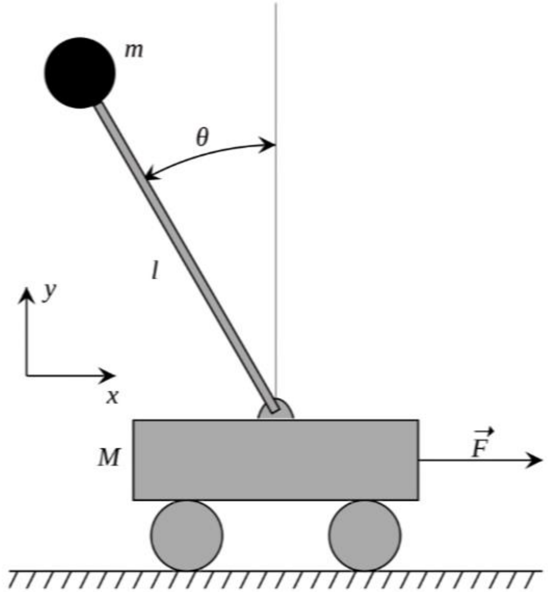
Discounted
Total Reward

$\boldsymbol{\gamma}$: discout factor

$$R_t = \sum_{i=t}^{\infty} \gamma^i r_i = \gamma^t r_t + \gamma^{t+1} r_{t+1} \ldots + \gamma^{t+n-1} r_{t+n-1} + \gamma^{t+n} r_{t+n} + \cdots$$

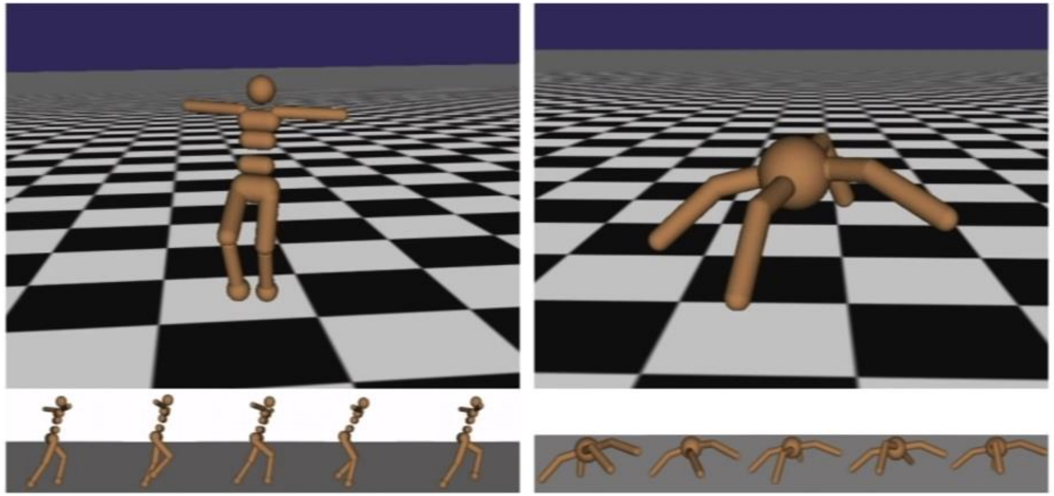
Example: Cart-Pole Problem



Objective: Balance a pole on top of a movable cart

State: angle, angular speed, position, horizontal velocity
Action: horizontal force applied on the cart
Reward: 1 at each time step if the pole is upright

Example: Robot Locomotion



Objective: Make the robot move forward

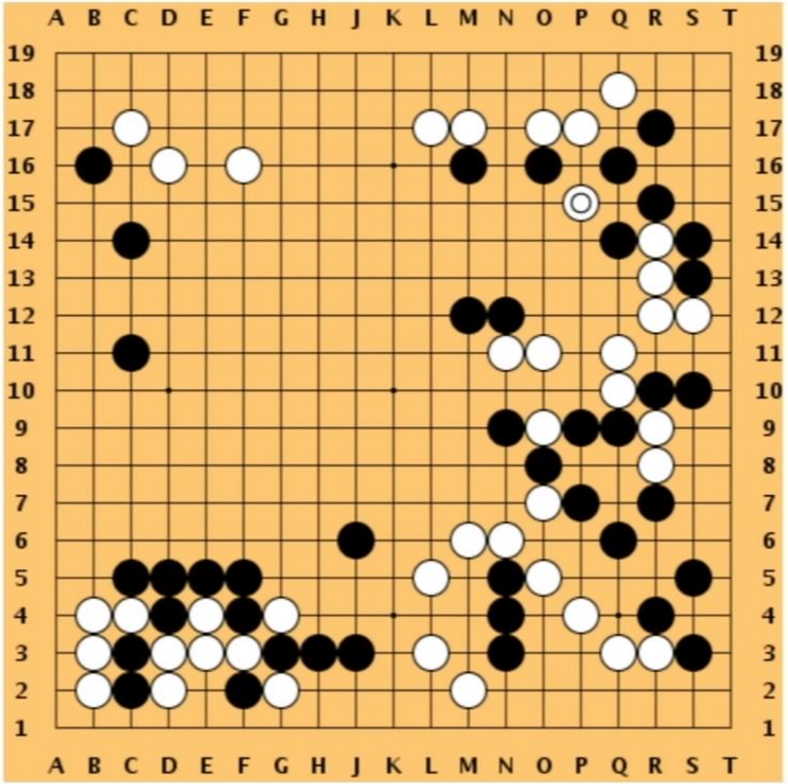
State: Angle and position of the joints
Action: Torques applied on joints
Reward: 1 at each time step upright + forward movement

Example: Atari games

Objective: Complete the game with the highest score

State: Raw pixel inputs of the game state
Action: Game controls e.g. Left, Right, Up, Down
Reward: Score increase/decrease at each time step

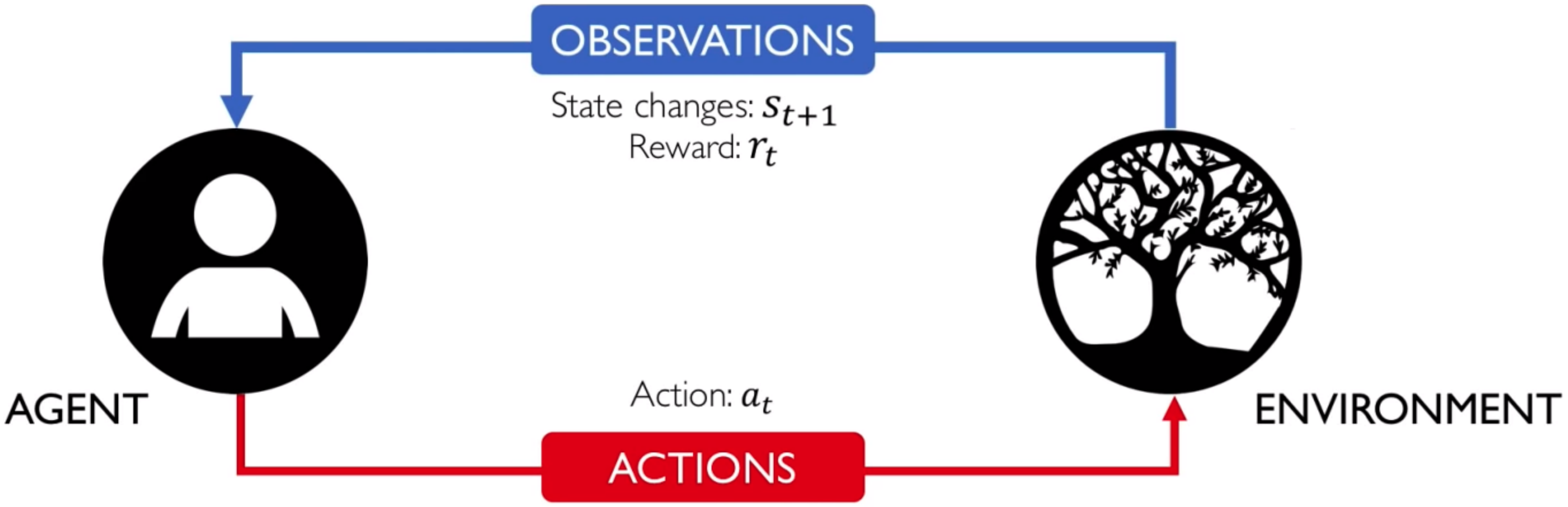
Example: Go



Objective: Win the game!

State: Position of all pieces
Action: Where to put the next piece down
Reward: 1 if win at the end of the game, 0 otherwise

How can we mathematically formalize the RL problem?



Markov Decision Problem

- Mathematical formulation of the RL problem
- **Markov property**: Current state completely characterises the state of the world

Defined by: $(\mathcal{S}, \mathcal{A}, \mathcal{R}, \mathbb{P}, \gamma)$

$\mathcal{S}$: set of possible states
$\mathcal{A}$: set of possible actions
$\mathcal{R}$: distribution of reward given (state, action) pair
$\mathbb{P}$: transition probability i.e. distribution over next state given (state, action) pair
γ : discount factor

Markov Decision Problem

- At time step t=0, environment samples initial state $s_0 \sim p(s_0)$
- Then, for t=0 until done:
  - Agent selects action a_t
  - Environment samples reward $r_t \sim R(. \mid s_t, a_t)$
  - Environment samples next state $s_{t+1} \sim P(. \mid s_t, a_t)$
  - Agent receives reward r_t and next state s_{t+1}

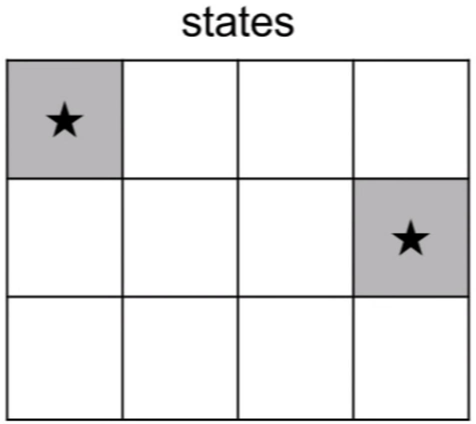
- A policy **π** is a function from S to A that specifies what action to take in each state
- **Objective**: find policy **π*** that maximizes cumulative discounted reward: $\sum_{t \geq 0} \gamma^t r_t$

A simple MDP: Grid World

actions = {

1. right
2. left
3. up
4. down

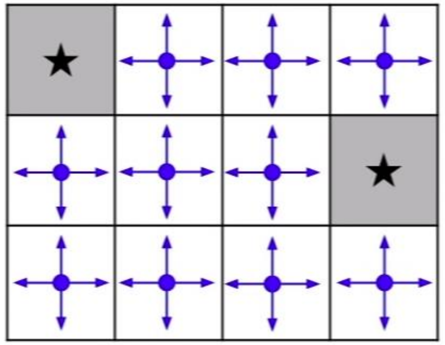
}



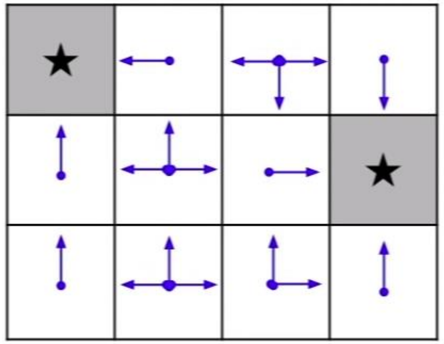
Set a negative "reward" for each transition (e.g. r = -1)

Objective: reach one of terminal states (greyed out) in least number of actions

A simple MDP: Grid World



Random Policy



Optimal Policy

Defining the Q-function

$$\boldsymbol{R_t = r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \cdots}$$

Total reward, $\boldsymbol{R_t}$, is the discounted sum of all rewards obtained from time $\boldsymbol{t}$

$$Q(s, a) = \mathbb{E}[R_t]$$

The Q-function captures the **expected total feature reward** an agent in state, s, can receive by executing a certain action, a

How to take actions given a Q-function?

$$Q(s, a) = \mathbb{E}[R_t]$$

$$(state, action)$$

Ultimately, the agent needs a **policy $\boldsymbol{\pi(s)}$**, to infer the **best action to take** at its state, s

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Strategy: the policy should choose an action that maximizes futures reward

$$\pi^*(s) =$$

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$$\uparrow \quad \uparrow$$

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$$\pi^*(s) = \underset{a}{argmax}\, Q(s, a)$$

Deep Reinforcement Learning Algorithms

Value Learning

Find $Q(s, a)$

$$a = \underset{a}{argmax}\ Q(s, a)$$

Policy Learning

Find $\pi(s)$

Sample $a \sim \pi(s)$

Digging deeper into the Q-function

Example: Atari Breakout

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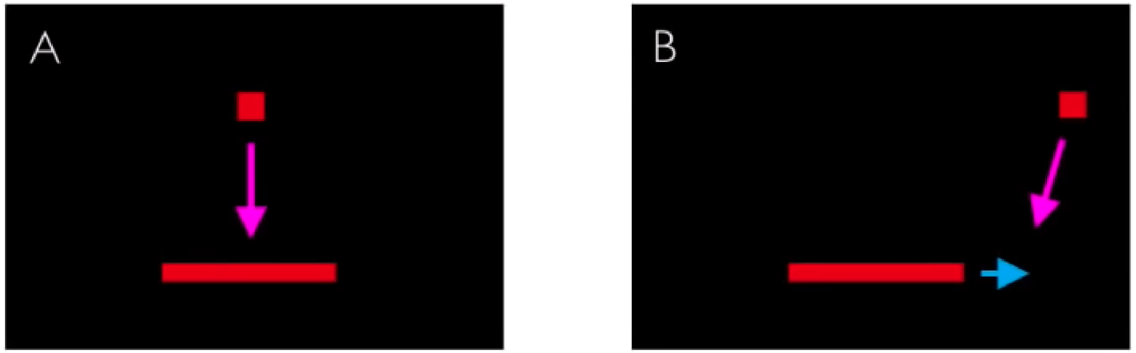
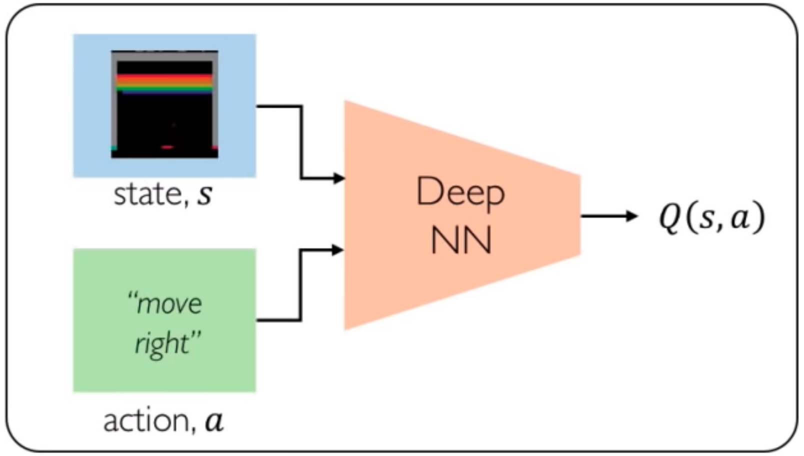
It can be very difficult for humans to accurately estimate Q-values



Which (**s**, **a**) pair has a higher Q-value? 🤔

Deep Q Networks (DQN): Training

How can we use deep neural networks to model Q-functions?



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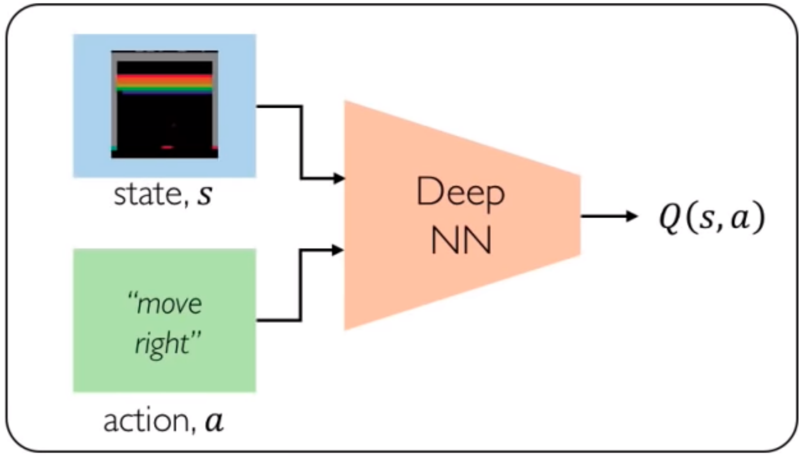

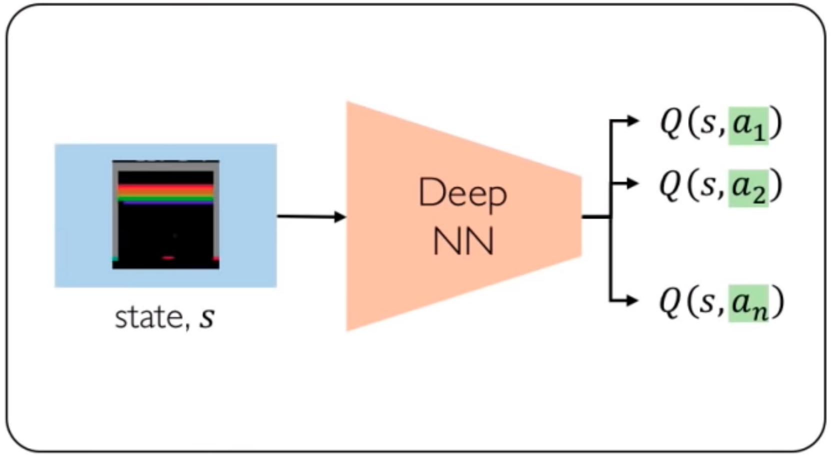

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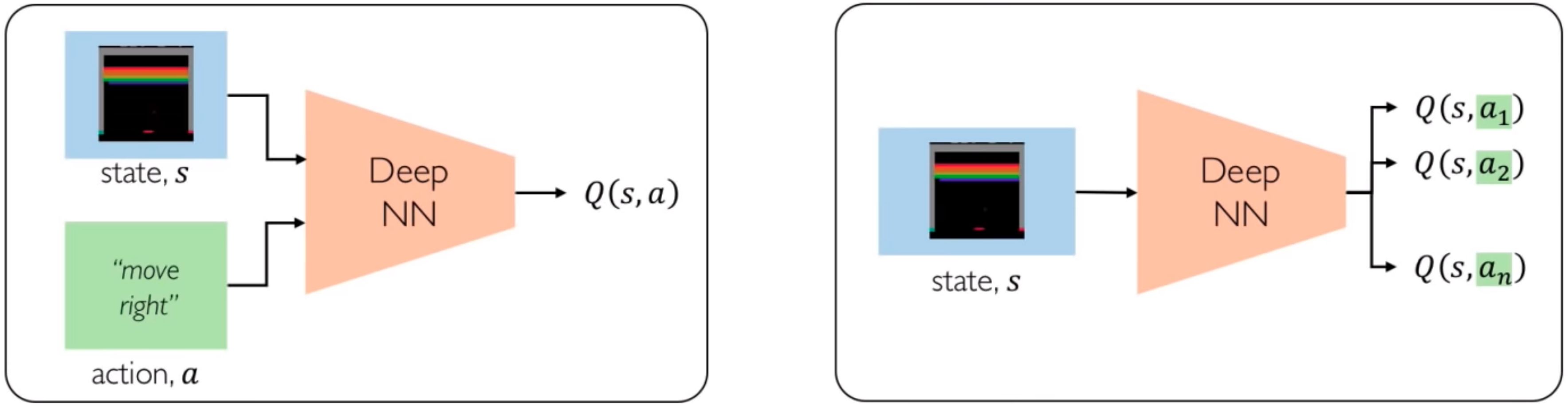
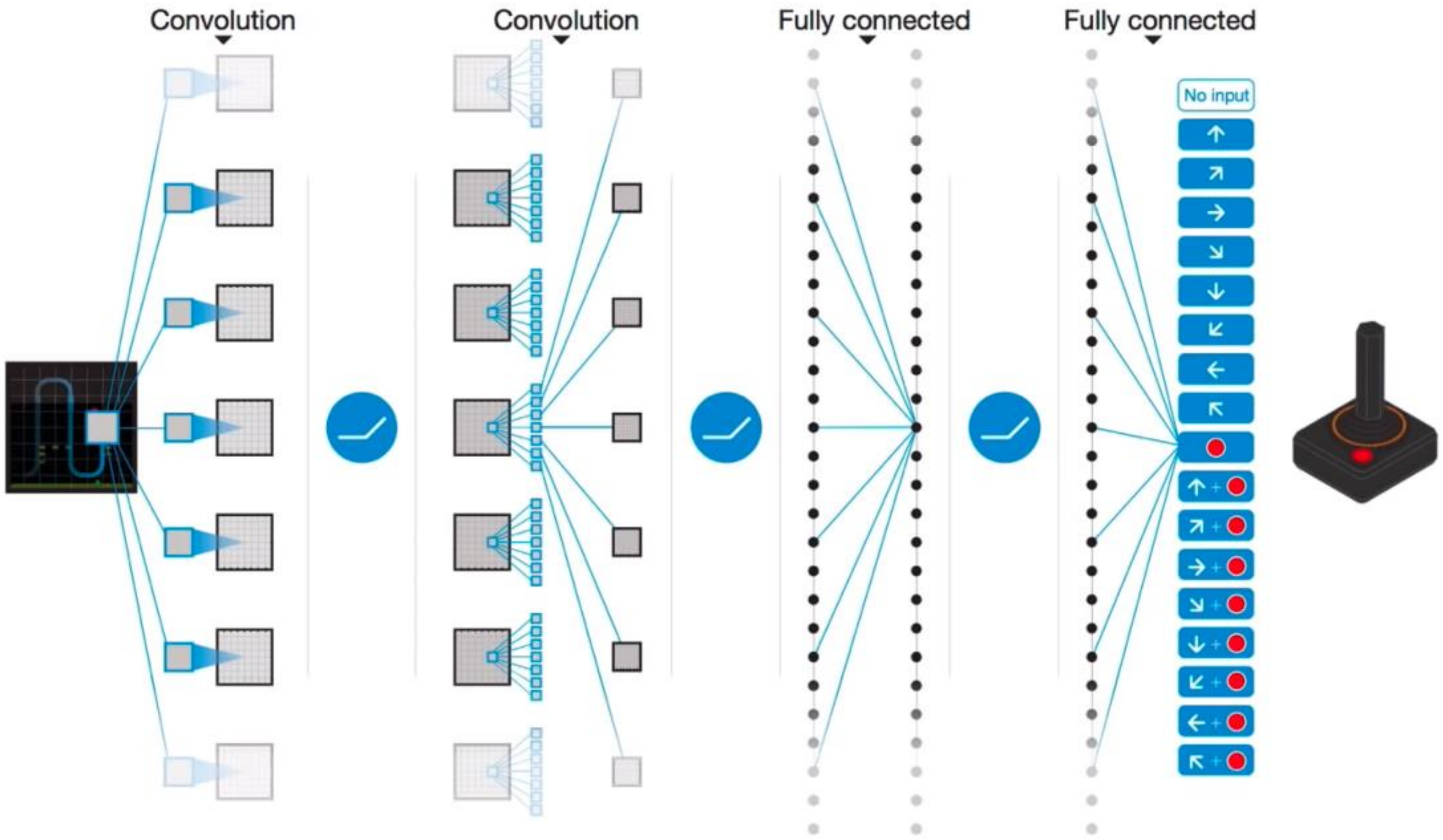


$$\mathcal{L} = \mathbb{E}\left[\left\| \overbrace{\left(r + \gamma \max_{a'} Q(s', a')\right)}^{\text{target}} - \overbrace{Q(s, a)}^{\text{predicted}} \right\|^2\right]$$

DQN Atari results



Downsides of Q-learning

Complexity:

- Can model scenarios where the action space is discrete and small
- Cannot handle continuous action spaces

Flexibility:

- Cannot learn stochastic policies since policy is deterministically computed from the Q function

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IMPORTANT:
Imagine you want to predict steering wheel angle of a car!

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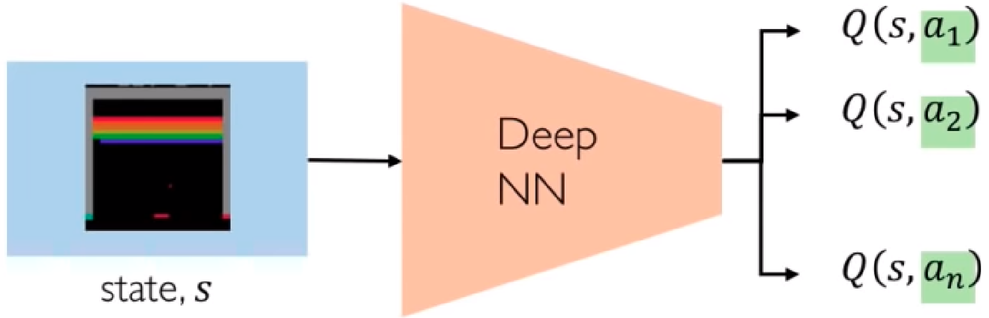
To overcome, consider a new class of RL training algorithms:
Policy gradient methods

Policy Gradient (PG): Key Idea

DQN (before): Approximating Q and inferring the optimal policy,



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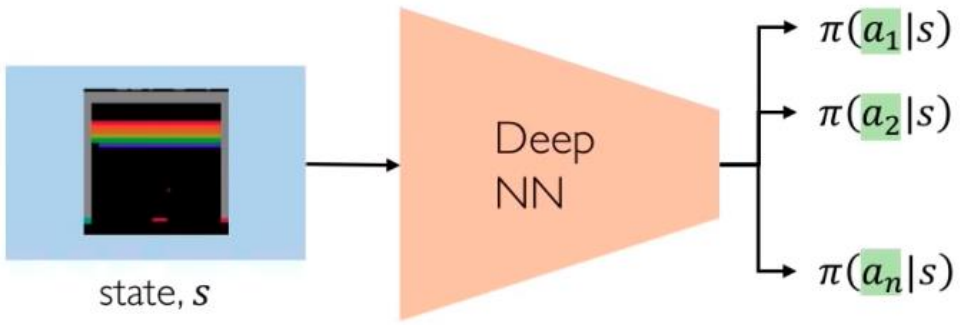
Policy Gradient: Directly optimize the policy!



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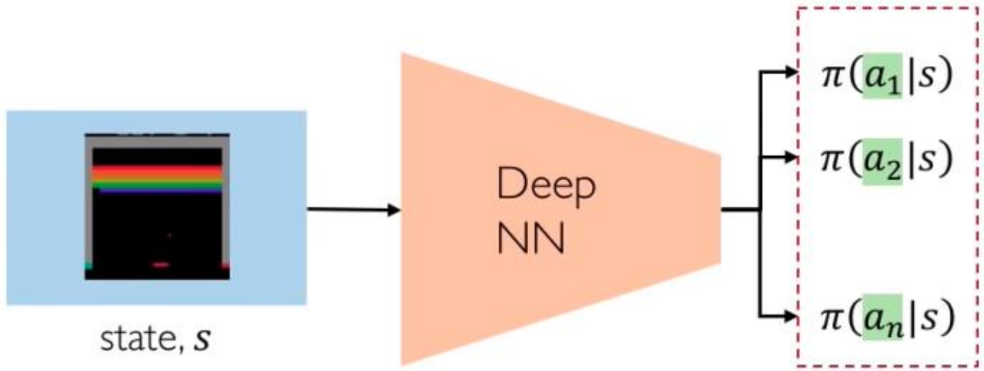
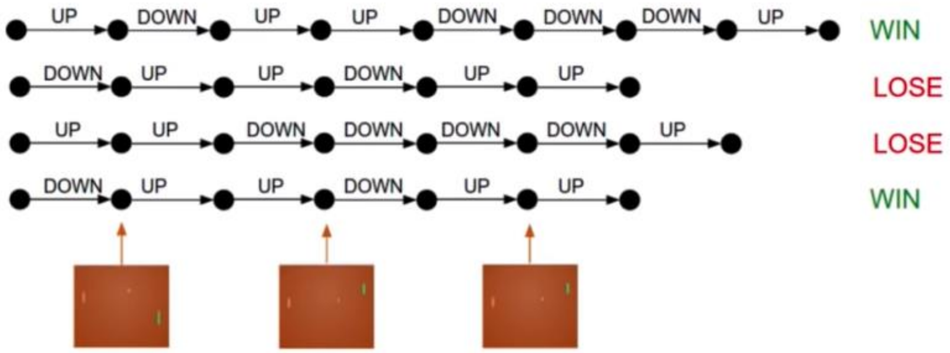


$$\sum_{a_i \in A} \pi(a_i|s) = 1$$

$$\pi(a|s) = P(action|state)$$

Policy Gradient (PG): Training



1. Run a policy for a while
2. Increase probability of actions that lead to high rewards
3. Decrease probability of actions that lead to low/no rewards

function REINFORCE

  Initialize θ

  for $episode \sim \pi_\theta$

    $\{s_i, a_i, r_i\}_{i=1}^{T-1} \leftarrow episode$

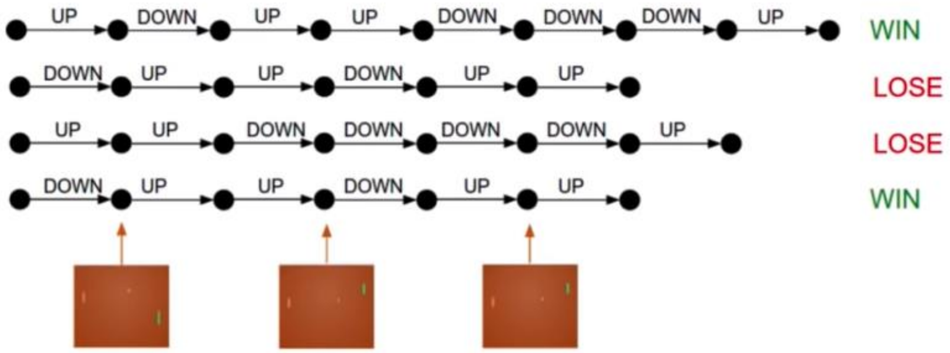
    for t = 1 to T-1

      $\boldsymbol{\nabla} \leftarrow \boldsymbol{\nabla_\theta \log \pi_\theta (a_t | s_t)\, R_t}$

      $\theta \leftarrow \theta + \alpha \nabla$

  return θ

Policy Gradient (PG): Training



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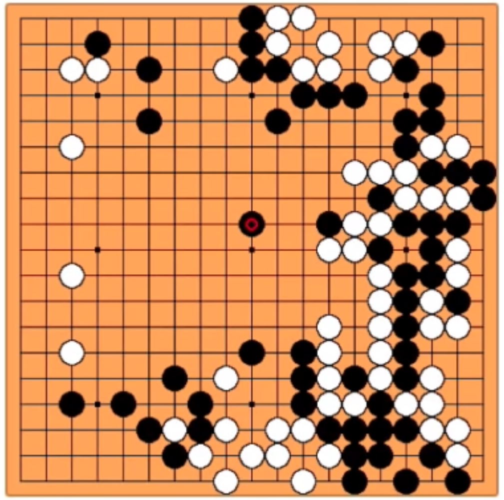
```
function REINFORCE
 Initialize θ
 for episode ~ π_θ
  {s_i, a_i, r_i}_{i=1}^{T-1} ← episode
  for t = 1 to T-1
    ∇ ← ∇_θ log π_θ(a_t|s_t) R_t
    θ ← θ + α∇
 return θ
```

$$\nabla_\theta \overbrace{\log \pi_\theta(a_t|s_t)}^{\text{log-likelihood of action}} \underbrace{R_t}_{\text{reward}}$$

The Game of Go

Aim: Get more board territory than your opponent.



Board Size n x n	Positions 3^{n^2}	% Legal	Legal Positions
1×1	3	33.33%	1
2×2	81	70.37%	57
3×3	19,683	64.40%	12,675
4×4	43,046,721	56.49%	24,318,165
5×5	847,288,609,443	48.90%	414,295,148,741
9×9	$4.434264882 \times 10^{38}$	23.44%	$1.039191487 91 \times 10^{38}$
13×13	$4.300233593 \times 10^{80}$	8.66%	$3.72497923077 \times 10^{79}$
19×19	$1.740896506 \times 10^{172}$	1.20%	$2.08168199382 \times 10^{170}$

Greater number of legal board positions than atoms in the universe.

Source: Wikipedia.

AlphaGo Beats Top Human Player at Go (2016)

Silver et al., *Nature* 2016.

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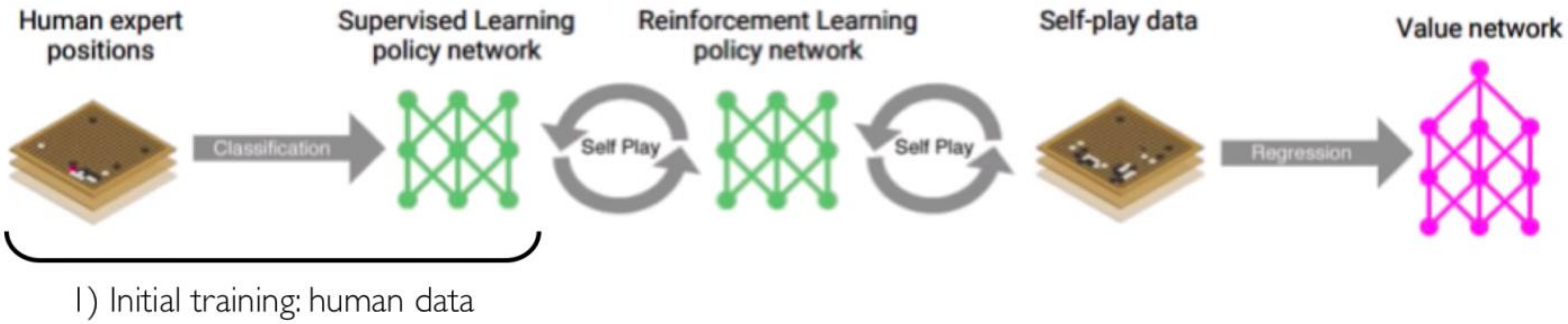

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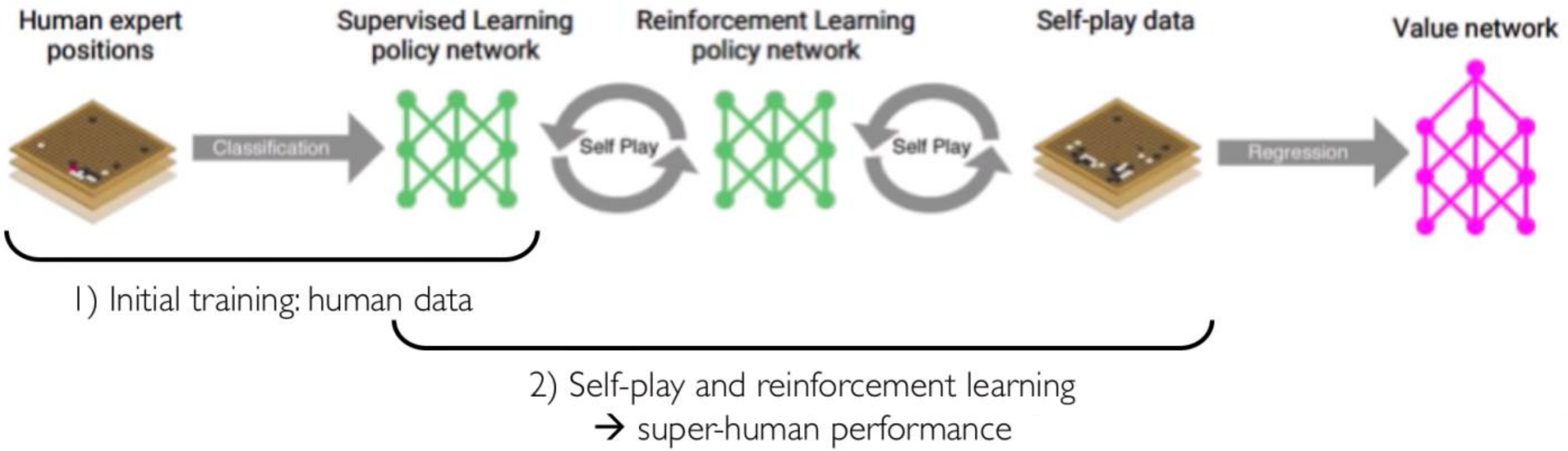

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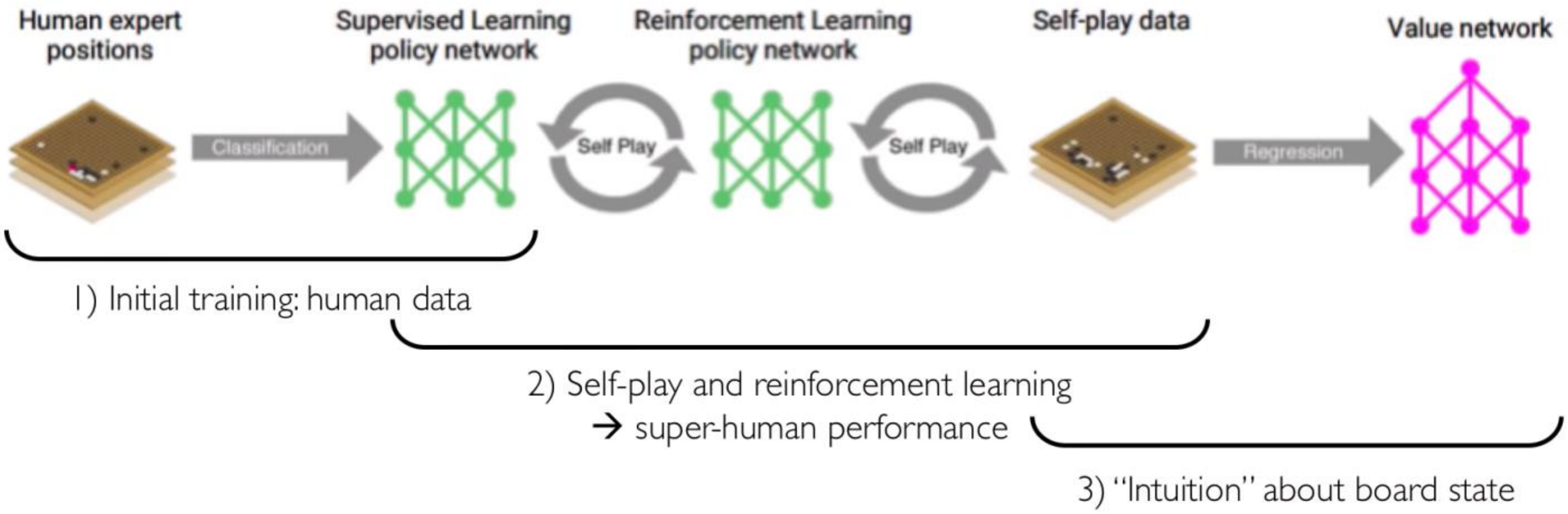

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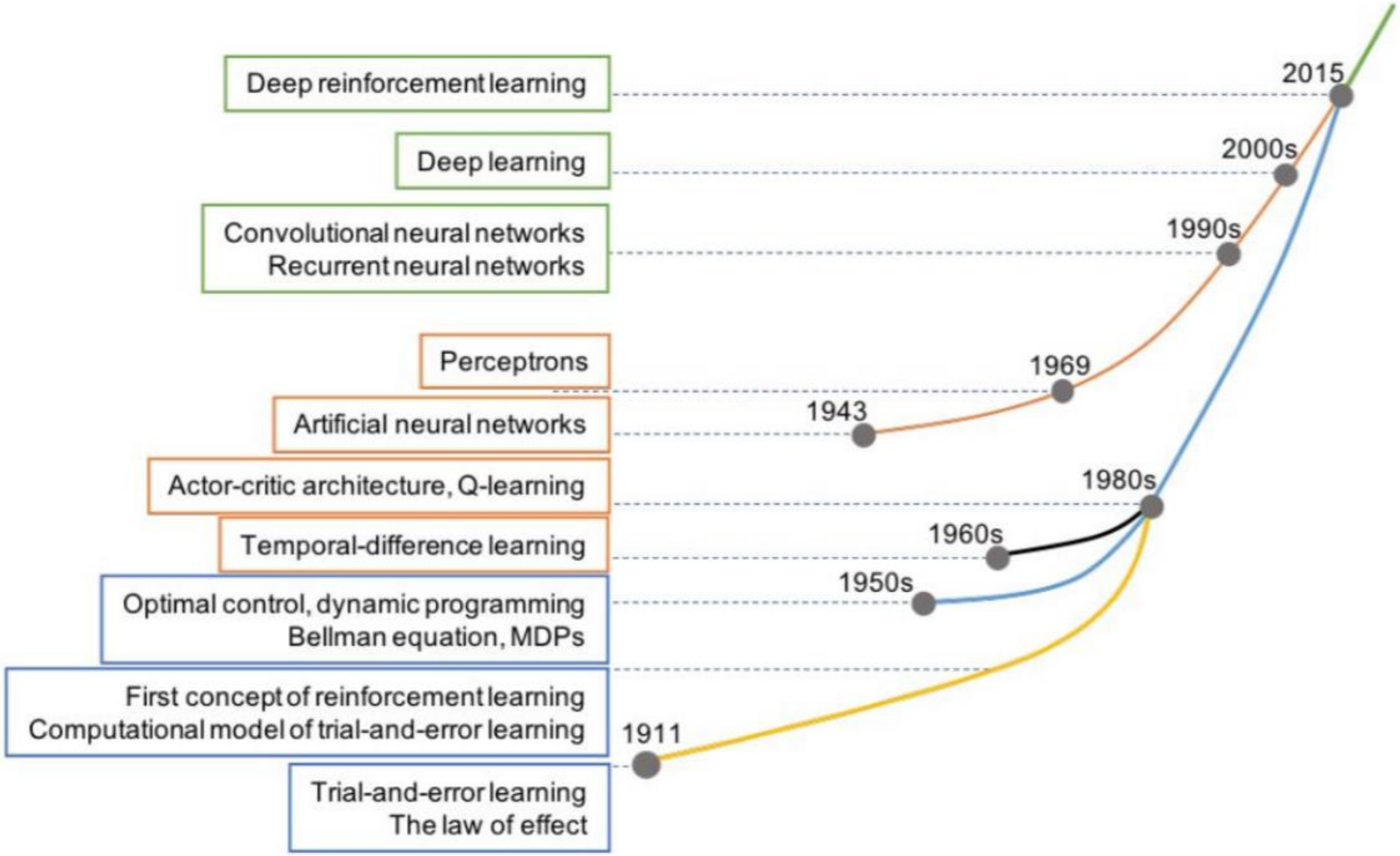
RL Milestones



Questions?