Quiz-1

3) i)

$$A = \begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix}$$

$$|(A - \lambda I)| v = 0$$

$$\left| \begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} 0-\lambda & -1 \\ 2 & 3-\lambda \end{bmatrix} \right| = 0$$

$$(-\lambda)(3-\lambda) + 2 = 0$$

$$-3\lambda + \lambda^2 + 2 = 0$$

$$\lambda^2 - 3\lambda + 2 = 0$$

$$\lambda^2 - \lambda - 2\lambda + 2 = 0$$

$$\lambda(\lambda - 1) - 2(\lambda - 1) = 0$$

$$(\lambda - 1)(\lambda - 2) = 0$$

Eigen values $\lambda = 1$, $\lambda = 2$

Substituting $\lambda = 1$

$$\begin{bmatrix} -1 & -1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} V_{1,1} \\ V_{1,2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-V_{1,1} - V_{1,2} = 0$$

$$2V_{1,1} + 2V_{1,2} = 0$$

$V_{1,1} = -V_{12}$

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$$v_1 = \begin{bmatrix} v_{1,1} \\ v_{1,2} \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Substituting $\lambda = 2$

$$\begin{bmatrix} -2 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} v_{21} \\ v_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-2v_{21} - v_{22} = 0$$
$$2v_{21} + v_{22} = 0$$

$$2v_{21} = -v_{22} \qquad v_{21} = -\tfrac{1}{2} v_{22}$$

$$v_2 = \begin{bmatrix} -1/2 \\ 1 \end{bmatrix}$$

Eigen Vector

$$V = \begin{bmatrix} -1 & -1/2 \\ 1 & 1 \end{bmatrix}$$

2) Yes, the ans is ~~also~~ similar

Q2 $S = \{v_1, v_2 \,\ldots\, v_n\}$

we can write any vector v as

$$v = \sum_{i=1}^{n} c_i v_i$$

for some constants c_i + $i = 1, 2, \ldots n$

$$v_1 - (c_1 v_1 + c_2 v_2 + c_3 v_3 \ldots + c_n v_n)$$

$$\cancel{v_2 - (c_1 v_1 + c_2 v_2 + c_3 v_3 \ldots + c_n v_n)}$$

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Since $v_1 \perp v_2$

$v_1 \cdot v_1 \neq 0 \quad v_1 \cdot v_2 = 0 \quad v_1 \cdot v_n = 0$

$$v_1 \cdot v_i = (c_1 v_1 + c_2 v_2 + \dots c_n v_n) \cdot v_i$$

$$v_1 \cdot v_1 = c_1$$

$$v_2 \cdot v_2 = c_2$$

$$\vdots$$

$$v_n \cdot v_n = c_n$$

So, we can write c ~~in terms of s~~ as

$$\sum_{i=1}^{n} c_i = v \cdot v$$