

## Machine Learning Quiz 1

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Prob 1:

```
Covariance Matrix for Dataset 1:  
[[ 0.08060992  0.40242878 -0.0025104 ]  
 [ 0.40242878  2.09900159 -0.01439466]  
 [-0.0025104  -0.01439466  0.08058254]]  
Eigenvalues for Dataset 1:
```

1)

$\text{Var}(x) = 0.08060992$

$\text{Var}(y) = 2.09900159$

$\text{Var}(z) = 0.08058254$

2)

$\text{Cov}(x, y) = 0.40242878$

$\text{Cov}(y, z) = -0.01439466$

3) Do pca (see code)

Below is the three PCs

```
PCA for dataset 1:  
[[ -1.75681915e+00  2.20257289e-03 -8.97784291e-02]  
 [ -8.88177567e-01  1.64657996e-02 -2.54949633e-02]  
 [ 5.26487022e-02 -4.46783426e-01  3.83101057e-02]  
 ...,  
 [ 3.60428154e-01  3.45422947e-01  4.99235344e-02]  
 [ -2.86808081e+00 -3.85070743e-01  6.02112236e-02]  
 [ 1.52319700e+00 -1.45746184e-01 -3.05903854e-02]]
```

Prob2:

$C_i = V_i \cdot V_i$

Prob 3:

1)

Prob 3.

(1)  $A = \begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix}$

characteristic equation:  $A - \lambda I = 0$  iff  $\det(A - \lambda I) = 0$

$$A - \lambda I = \begin{bmatrix} 0 - \lambda & -1 \\ 2 & 3 - \lambda \end{bmatrix} = \begin{bmatrix} -\lambda & -1 \\ 2 & 3 - \lambda \end{bmatrix}$$
$$\det(A - \lambda I) = (-\lambda)(3 - \lambda) - (-1)2 = -3\lambda + \lambda^2 + 2$$
$$= (\lambda - 2)(\lambda - 1) = 0$$
$$\Rightarrow \lambda_1 = 2 \quad \lambda_2 = 1$$
$$\begin{bmatrix} -2 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
$$\Rightarrow \vec{v}_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$
$$\begin{bmatrix} -1 & -1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
$$\Rightarrow \vec{v}_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Then eigenvalues  $\lambda_1 = 2$   $\lambda_2 = 1$

corresponding eigenvector of  $\lambda_1$  is  $\vec{v}_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

corresponding eigenvector of  $\lambda_2$  is  $\vec{v}_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

(2) See code 1

The results are consistent.

2)

```
eigenvalues for a
[ 1.  2.]
eigenvectors for a
[[-0.70710678  0.4472136 ]
 [ 0.70710678 -0.89442719]]
```