Akush

2) Orthonormal basis are dependent & normalized & they span all the vector in the space

we have $V = \sum_{i=1}^{n} C_i V_i$

i.e. $V = C_1 V_1 + C_2 V_2 + \cdots C_i V_i$

By dot product

$V V_i = C_i V_i \cdot V_i$

$V V_i = C_i \|V_i\|^2$

$$\boxed{C_i = \frac{V \cdot V_i}{\|V_i\|^2}}$$

3) a) $A = \begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix}$

To find eigen values

$$|A - \lambda I| = \begin{Bmatrix} 0-\lambda & -1 \\ 2 & 3-\lambda \end{Bmatrix} = 0$$

$$-\lambda(3-\lambda) + 2 = 0$$

$$-3\lambda + \lambda^2 + 2 = 0$$

$$\lambda^2 - 3\lambda + 2 = 0$$

$$\lambda^2 - 2\lambda - \lambda + 2 = 0$$

$$\lambda(\lambda-2) - 1(\lambda-2) = 0$$

$$\lambda = 2 \quad \lambda = 1$$

To find Eigen vectors

for $\lambda = 2$

$$\begin{bmatrix} -2 & -1 \\ 2 & 3-2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} -2 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} -2 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$-2x_1 - x_2 = 0$$

$$x_1 = x_2/2 \Rightarrow v_1 = \begin{pmatrix} 1/2 \\ 1 \end{pmatrix}$$

for $\lambda = 1$

$$\begin{bmatrix} -1 & -1 \\ 2 & 3-1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} -1 & -1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} -1 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$-x_1 - x_2 = 0$

$x_1 = -x_2$

$\Rightarrow v_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\therefore$ Eigen vectors are

$$\begin{bmatrix} 1/2 \\ 1 \end{bmatrix} \quad \& \quad \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

for $\lambda = 2$ & $\lambda = 1$

3b) Eigen values were same as ~~new~~ manually calculated but not eigen ~~ei~~ vectors, ~~becau~~ because eigen vectors are normalized (made unit length)