Akash

2) We have, in orthonormal basis

$$V = \sum_{i=0}^{n} c_i v_i$$

let $i = 3$, orthonormal basis are

$$e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

lets represent some random vector in R^3 $V = 2x + 4y + 3z = 0$, in orthonormal basis as following

$$2\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 4\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + 3\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0$$

i.e $2e_1 + 4e_2 + 3e_3 = 0$

for this case constants are 2, 4 & 3

in ~~one~~ general for n

$$a_1\begin{bmatrix} 1 \\ 0 \\ \vdots \end{bmatrix} + a_2\begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \end{bmatrix} - - - a_n\begin{bmatrix} 0 \\ 0 \\ \vdots \end{bmatrix} = 0$$

or $\boxed{a_1 V_1 + a_2 V_2 + \; --- \; a_n V_n = 0}$

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3) a) $A = \begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix}$

To find eigen values

$$|A - \lambda I| = \begin{Bmatrix} 0-\lambda & -1 \\ 2 & 3-\lambda \end{Bmatrix} = 0$$

$$-\lambda(3-\lambda) + 2 = 0$$

$$-3\lambda + \lambda^2 + 2 = 0$$

$$\lambda^2 - 3\lambda + 2 = 0$$

$$\lambda^2 - 2\lambda - \lambda + 2 = 0$$

$$\lambda(\lambda-2) - 1(\lambda-2) = 0$$

$$\lambda = 2 \quad \lambda = 1$$

To find Eigen vectors

for $\lambda = 2$

$$\begin{bmatrix} -2 & -1 \\ 2 & 3-2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} -2 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} -2 & -1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$-2x_1 = x_2 = 0$$

$$x_1 = x_2/2 \quad \Rightarrow v_1 = \begin{pmatrix} 1/2 \\ 1 \end{pmatrix}$$

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for $\lambda = 1$

$$\begin{bmatrix} -1 & -1 \\ 2 & 3-1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} -1 & -1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} -1 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$-x_1 - x_2 = 0$

$x_1 = -x_2$

$\Rightarrow v_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\therefore$ Eigen vectors are

$$\begin{bmatrix} 1/2 \\ 1 \end{bmatrix} \quad \& \quad \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

for $\lambda = 2$ & $\lambda = 1$

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3b) Eigen values were same as ~~new~~ manually calculated but not eigen ~~ei~~ vectors, ~~becau~~ because eigen vectors are normalized (made unit length)

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