ML- QUIZ 1

AKARSH GUPTA, 800969888

Q3 a) $A = \begin{bmatrix} 0, & -1 \\ 2, & 3 \end{bmatrix}$

Sol.

$$A\vec{x} = \lambda \vec{x}$$

$$(A - \lambda I)\vec{x} = \vec{0}$$

$$|A - \lambda I| = 0$$

$$\left| \begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} -\lambda & -1 \\ 2 & 3-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \quad -\lambda(3-\lambda) + 2 = 0$$

$$\Rightarrow \quad -3\lambda + \lambda^2 + 2 = 0$$

$$\Rightarrow \quad \lambda = 1, 2$$

for $\lambda = 1$

$(A - \lambda I)\vec{x} = \vec{0}$, let $\vec{x} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$

$$\left(\begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 & -1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\Rightarrow -v_1 - v_2 = 0 \Rightarrow v_1 = -v_2$

therefore, $\vec{x} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

for $\lambda = 2$

$(A - \lambda I)\vec{x} = \vec{0}$

$$\left(\begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \right) \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -2 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\Rightarrow -2v_1 - v_2 = 0.$

$\Rightarrow v_2 = -2v_1 \Rightarrow \vec{x} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

Q 2.

~~for~~ given $S = \{v_1, v_2 \ldots v_n\}$ is an orthonormal basis of $\mathbb{R}^n$.

$\therefore$ any vector $v = \sum_{i=1}^{n} c_i v_i$

for some constants c_i $i = 1, 2, \ldots n$.

~~for some vector~~ v_1

$v = c_1 v_1 + c_2 v_2 + \ldots c_i v_i$

lets take the dot product of v_i on both sides.

$v \cdot v_i = c_i v_i \cdot v_i$

$$\boxed{c_i = \frac{v \cdot v_i}{\|v_i\|^2}}$$