

Covariance matrix is:

```
[[ 0.08060992  0.40242878 -0.0025104 ]  
 [ 0.40242878  2.09900159 -0.01439466]  
 [-0.0025104  -0.01439466  0.08058254]]
```

Variance of x: 0.0806099157997998

Covariance matrix for x,y:

```
[[ 0.08060992  0.40242878]  
 [ 0.40242878  2.09900159]]
```

Covariance matrix for y,z:

```
[[ 2.09900159 -0.01439466]  
 [-0.01439466  0.08058254]]
```

eigvalues are [2.17638133 0.00333122 0.0804815]

eigenVectors are

```
[[ 0.18857784  0.982048  0.00448705]  
 [ 0.98203351 -0.18860355  0.00623651]  
 [-0.00697082 -0.00323037  0.99997049]]
```

transform matrix is:

```
[[ 0.18857784  0.98203351 -0.00697082]  
 [ 0.00448705  0.00623651  0.99997049]  
 [ 0.982048  -0.18860355 -0.00323037]]
```

Q3

3.

$$A = \begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix}$$

$$|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} -\lambda & -1 \\ 2 & 3-\lambda \end{vmatrix} = -\lambda(3-\lambda) - (-1)(2) = 0$$

$$\Rightarrow \lambda(\lambda-3) + 2 = 0 \Rightarrow \lambda^2 - 3\lambda + 2 = 0 \Rightarrow (\lambda-2)(\lambda-1) = 0$$

$$\Rightarrow \lambda_1 = 1, \quad \lambda_2 = 2$$

for λ_1 :

$$\begin{bmatrix} 0-1 & -1 \\ 2 & 3-1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -v_1 - v_2 \\ 2v_1 + 2v_2 \end{bmatrix}$$

We have many eigenvectors, with this format $\begin{bmatrix} +1 \\ -2 \end{bmatrix}$
 e.g. $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$

for λ_2 :

$$\begin{bmatrix} 0-2 & -1 \\ 2 & 3-2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -2v_1 - v_2 \\ 2v_1 + v_2 \end{bmatrix} = \begin{bmatrix} -(2v_1 + v_2) \\ (2v_1 + v_2) \end{bmatrix}$$

eigenvectors are like $\begin{bmatrix} -b \\ b \end{bmatrix}$ as an example: $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

```
In [84]: A=[[0,-1],[2,3]]
```

```
In [85]: A
```

```
Out[85]: [[0, -1], [2, 3]]
```

```
In [86]: la.eig(A)
```

```
Out[86]:
```

```
(array([ 1.,  2.]), array([[ -0.70710678,  0.4472136 ],
 [ 0.70710678, -0.89442719]]))
```

Q2

(c)

$$S = \{V_1, V_2, \dots, V_n\}$$

$$V = \sum_{i=1}^n C_i V_i$$

$$V_i \perp V_j$$

$$V_i \cdot V_j = 0$$

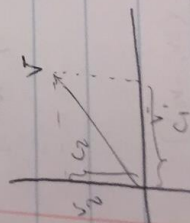
$$V_i \cdot V_i = 1$$

$$V = (a_1, a_2, \dots, a_n)$$

$$V = C_1 V_1 + C_2 V_2 + \dots + C_n V_n$$

$$\Rightarrow C_1 = a_1, C_2 = a_2, \dots, C_n = a_n$$

C_1, \dots, C_n are basically the components of V



e_1

e_2

$V = (1, 2, 4, 7)$
 $\Rightarrow C_1 = 1, C_2 = 2, C_3 = 4, C_4 = 7$

