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Quiz -1

Q3) $A = \begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix}$

$\cancel{(A - \lambda I)}$

$$A\vec{x} = \lambda \vec{x}$$

$$(A - \lambda I)\vec{x} = \vec{0} \Rightarrow |A - \lambda I| = 0$$

$$\left| \begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} -\lambda & -1 \\ 2 & 3-\lambda \end{bmatrix} \right| = 0$$

$$\Rightarrow -\lambda(3-\lambda) + 2 = 0$$

$$\Rightarrow -3\lambda + \lambda^2 + 2 = 0$$

$$\Rightarrow \lambda^2 - 3\lambda + 2 = 0$$

$$\Rightarrow \lambda = 1, 2$$

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Putting the values back in original equation,

for $\lambda = 1$

$(A - \lambda I)\vec{x} = \vec{0}$, Let $\vec{x} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$

$$\left(\begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 & -1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow -v_1 - v_2 = 0 \rightarrow \boxed{v_1 = -v_2}$$

So, $\vec{x} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

for $\lambda = 2$

$(A - \lambda I)\vec{x} = \vec{0}$

$$\left(\begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \right) \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -2 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow -2v_1 - v_2 = 0 \Rightarrow v_2 = -2v_1 \Rightarrow \vec{x} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$\Rightarrow$ This is same as the programmatic value

Q2.) for some vector v,

$$v = \sum_{i=1}^{n} c_i v_i$$

$$v = c_1 v_1 + c_2 v_2 + \cdots + c_i v_i$$

~~Multiply~~ Taking dot product of v_i on both sides

$$v \cdot v_i = c_i v_i \cdot v_i$$

$$v \cdot v_i = c_i \|v_i\|^2$$

$$\boxed{c_i = v \cdot v_i / \|v_i\|^2}$$