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①

Q3) $A = \begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix}$

Finding Eigen value :-

$$A - \lambda I = 0$$

$$\left| \begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} -\lambda & -1 \\ 2 & 3-\lambda \end{bmatrix} \right| = 0$$

$$-3\lambda + \lambda^2 + 2 = 0$$

$$\lambda^2 - 3\lambda + 2 = 0$$

$$\therefore \lambda = 2, 1$$

Eigen values = 2, 1

(2)

Finding eigen vector.

$$(A - \lambda I) \vec{v} = 0$$

$$\left(\begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \right) \vec{v} = 0$$

$$\left(\begin{bmatrix} -2 & -1 \\ 2 & 1 \end{bmatrix} \right) \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0$$

$$-2v_1 - v_2 = 0$$

$$2v_1 + v_2 = 0$$

$$\boxed{v_1 = v_2 / 2} \text{ --- (1)}$$

$$\left(\begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \vec{v} = 0$$

$$\begin{bmatrix} 1 & -1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0$$

$$v_1 - v_2 = 0$$

$$2v_1 + 2v_2 = 0$$

$$\boxed{v_1 = -v_2} \text{ --- (2)}$$

(2)

Ques 2) Let $\{v_1, v_2, \dots, v_n\}$ be an orthonormal basis for a subspace of \mathbb{R}^n

for each element in this subspace, the weights in the linear combination is

$$\vec{v} = c_1 \vec{v}_1 + c_2 \vec{v}_2 + \dots + c_n \vec{v}_n$$

this can be given as

$$c_i = \frac{\vec{v} \cdot \vec{v}_i}{(\vec{v}_i)^2} \quad \text{where } i = 1, \dots, n$$

As orthonormal set is a set of orthogonal set of unit vectors. This formula holds true for constants in terms of \vec{v} .