Second Degree Price Discrimination

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Monopoly Discrimination

We have seen first degree discrimination and third degree discrimination

- In the first case, the monopoly is able to distinct between each consumer and knows their valuation.
- In the second case, the monopoly can distinguish between group of consumers, and knows the valuation of each group.

But what about second degree discrimination? Which is the big difference between second degree and the two previous cases?

Information

What does this mean?

In the second degree discrimination, the monopoly is able to recognize that there exist two types of consumers, but it can't recognize which consumer he's fronting.

Analogous

Insurance companies faced a similar situation when they identified groups of consumers with high and low probabilities of accidents. However, they struggled to accurately determine whether a consumer belonged to the high-risk or low-risk category.

Solving Second Degree Discrimination

1. Participation Constraint

$$CS^i(x^i) - T^i \geq 0$$

where $i \in [H, L]$.

This constraint implies that consumers are better off consuming rather than not consuming at all.

Example

An internet company (monopolist) knows pretty well that there exist two types of consumers:

- Those who surf a lot.
- Those who surf little.

The variable cost of the firm is zero. And the firm has estimated the following inverse of the demand functions:

$$p^H = 12–3q^H$$

$$p^L = 12–4q^L$$

Example

The profit maximization problem is:

$$\max_{q^H, q^L} \pi = T^L + T^H$$

$s.t.$ $\int_0^{q^L} 12–4q \ dq - T^L \geq 0$
$s.t.$ $\int_0^{q^H} 12–3q \ dq - T^H \geq 0$

Substituting

$$\max_{q^H, q^L} \pi = \int 12–4q^L dq^L + \int 12–3q^H dq^H$$

FOC

$\frac{\partial \pi}{\partial q^L} = 12 - 4q^L = 0$
$\frac{\partial \pi}{\partial q^H} = 12 - 3q^H = 0$

Example

Solving the first-order conditions we arrive to:

$$q^{L^*} = 3 \implies T^{L^*} = 18$$
$$q^{H^*} = 4 \implies T^{H^*} = 24$$

But does the low-demand consumer have sufficient incentive to **signal** their type?

If he pretends to be a high-demand consumer, his net willingness to pay is:

$\int_0^{q^H} 12–4q \; dq \geq T^H$

$$16 - 24 \not\geq 0$$

Low demand consumers will never pretend to be high demand consumers.

Example

What about high-demand consumers? Do they have sufficient incentive to **signal** their type?
If he pretends to be a low-demand consumer, his net willingness to pay is:

$\int_0^{q^L} 12–3q \; dq \geq T^L$

$$22.5 - 18 \geq 0$$

It's evident that they have incentives to simply pretend to be low-consumers and consuming slightly less than they would if there were **symmetric information**, meaning, if the monopoly were able to determine their type.

Solving Second Degree Discrimination (cont)

But then, how those the monopoly solves this signalling problem? We need to add one more constraint:

2. Incentive Compatibility Constraint

$$CS^i(x^i) - T^i \geq CS^i(x^j) - T^j$$

This new constraint implies that consumers are better off consuming quantities consistent with their type rather than consuming as if they belonged to the other group.

Example

Now the profit maximization problem is:

$$\max_{q^H, q^L} \pi = T^L + T^H$$

$$
\begin{aligned}
s.t.\quad & \int_0^{q^L} 12–4q \; dq - T^L \geq 0 \\
s.t.\quad & \int_0^{q^H} 12–3q \; dq - T^H \geq 0 \\
s.t.\quad & \int_0^{q^L} 12–4q \; dq - T^L \geq \int_0^{q^H} 12–4q \; dq - T^H \\
s.t.\quad & \int_0^{q^H} 12–3q \; dq - T^H \geq \int_0^{q^L} 12–3q \; dq - T^L
\end{aligned}
$$

Example

Firstly, we must avoid redundant constraints. From the previous analysis, it's evident that the high-demand consumer always benefits if they act as a low-demand consumer. In other words:

$$\int_0^{q^L} 12\text{–}3q \; dq - T^L \geq 0$$

This indicates that our participation constraint is irrelevant for high consumers, and only the incentive compatibility constraint will be binding.

Using the same reasoning:

$$\int_0^{q^H} 12\text{–}4q \; dq - T^H \leq 0$$

This shows that our incentive compatibility constraint is irrelevant for low consumers, and only the participation constraint will be binding.

Example

Thus, our profit maximization problem is:

$$\max_{q^H, q^L} \pi = T^L + T^H$$

$$s.t. \quad \int_0^{q^L} 12–4q \; dq - T^L \geq 0$$
$$s.t. \quad \int_0^{q^H} 12–3q \; dq - T^H \geq \int_0^{q^L} 12–3q \; dq - T^L$$

Because these conditions are binding:

$$\max_{q^H, q^L} \pi = \int_0^{q^L} 12–4q \; dq + T^H$$

$$s.t. \quad \int_0^{q^H} 12–3q \; dq - T^H = \int_0^{q^L} 12–3q \; dq - \int_0^{q^L} 12–4q \; dq$$

Example

Replacing:

$$\max_{q^H,q^L} \pi = \int_0^{q^H} 12–3q\ dq - \int_0^{q^L} 12–3q\ dq + 2\int_0^{q^L} 12–4q\ dq$$

FOC

$\frac{\partial \pi}{\partial q^L} = 24 - 8q^L - 12 + 3q^L = 0$

$\frac{\partial \pi}{\partial q^H} = 12 - 3q^H = 0$

Solving the first-order conditions we arrive to:

$$q^{L^*} = 2.4 \implies T^{L^*} = 17.28$$

$$q^{H^*} = 4 \implies T^{H^*} = 21.12$$

After that you can check, if you doubt, that we have a **separating** equilibria.

Generalized

To solve a problem of this kind, the following optimization framework can be applied:

$$\max_{q^H, q^L} \pi = T^L + T^H - c(q^H + q^L)$$

$$\begin{aligned} s.t. \quad & CS^L(q^L) - T^L \geq 0 \\ s.t. \quad & CS^H(q^H) - T^H \geq CS^H(q^L) - T^L \end{aligned}$$

Nevertheless, grasping how these conditions are derived is crucial for a comprehensive understanding.