Contents

List of Figures

Chapter 1

Introduction

1.1 Interesting Quantum Programming Languages

Qiskit, which is the most prominent and based around Python. q1tsim, which is based on Rust and the docmentation is mostly in Rust but apparently can be used in Python as well.

1.2 Quantum Fourier Transform

The quantum Fourier transform (QFT) transforms between two bases, the computational (Z) basis, and the Fourier basis. It can be written as a mapping

$$|x\rangle \mapsto \frac{1}{\sqrt{N}} \sum_{y=0}^{N-1} \omega_N^{xy} |y\rangle \tag{1.1}$$

or as an unitary matrix

$$U_{\mathrm{QFT}} = \frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} \omega_N^{xy} |y\rangle \langle x| \,. \tag{1.2}$$

The H-gate is the signle-qubit QFT, and it transfroms between the Z-basis states $|0\rangle$ and $|1\rangle$ to the X-basis states $|+\rangle$ and $|-\rangle$. In the same way, all multi-qubit states in the computational basis have corresponding states in the Fourier basis. The QFT is simply the function that transforms between these bases.

$$|\text{State in Computational Basis}\rangle \xrightarrow{QFT} |\text{State in Fourier Basis}\rangle \tag{1.3}$$
$$QFT\,|x\rangle = |\tilde{x}\rangle\,. \tag{1.4}$$
$$\tag{1.5}$$

In the computational basis, we store numbers in binary, just like a Johnson Counter. The first qubit has the highest frequency and changes every iteration, while the last one has the lowest frequency. In the Fourier basis, we store numbers using different rotations around the Z-axis. To encode the state $|\tilde{5}\rangle$ on 4 qubits, we rotate the leftmost qubit by $\frac{5}{2^n} = \frac{5}{2^4} = \frac{5}{16} \times 2\pi$radians. For the next qubit the numerator is doubled, thus $\frac{10}{16} \times 2\pi$. This leads to the result, that now the first qubit has the lowest frequency, while the fourth qubit has the quickest. Note that in the Fourier basis, we are no longer counting binary.