**SURVEY**

# NISQ Computers: A Path to Quantum Supremacy

## MUHAMMAD ABUGHANEM[1] AND HICHEM ELEUCH[2,3,4]

[1]Faculty of Science, Ain Shams University, Cairo 11566, Egypt
[2]Department of Applied Physics and Astronomy, University of Sharjah, Sharjah, United Arab Emirates
[3]College of Arts and Sciences, Abu Dhabi University, Abu Dhabi, United Arab Emirates
[4]Institute for Quantum Science and Engineering, Texas A&M University, College Station, TX 77843, USA

Corresponding author: Muhammad AbuGhanem (gaa1nem@gmail.com)

**ABSTRACT** The quest for quantum advantage, wherein quantum computers surpass the computational capabilities of classical computers executing state-of-the-art algorithms on well-defined tasks, represents a pivotal race in the domain of quantum computing. NISQ (Noisy Intermediate-Scale Quantum) computing has witnessed remarkable advancements, culminating in significant milestones on the journey towards the realization of universal fault-tolerant quantum computers. This transformative turning point, known as quantum supremacy, has been achieved amid a series of breakthroughs, signifying the dawn of the quantum era. Quantum hardware has undergone substantial integration and architectural evolution, contrasting with its nascent stages. In this review, we critically examine the quantum supremacy experiments conducted thus far, shedding light on their implications and contributions to the evolving landscape of quantum computing. Additionally, we endeavor to illuminate a range of cutting-edge proof-of-principle investigations in the realm of applied quantum computing, providing an insightful overview of the current state of applied quantum research and its prospective influence across diverse scientific, industrial, and technological frontiers.

**INDEX TERMS** Quantum supremacy, quantum computers, quantum artificial intelligence, Google quantum AI, Xanadu, Zuchongzhi, Jiuzhang, applied quantum computing, quantum machine learning, drug discovery. PACS: $03.67.Lx$, $03.67.Dd$ $03.67.-a$, $03.65.Ca$.

## I. INTRODUCTION

Since its creation, a hundred years ago, quantum mechanics has had something unexpected to offer nearly each single year. The last few decades in particular have seen a dramatic increase in theoretical and practical advances [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18]. Among the most compelling frontiers of quantum mechanics is the quest to engineer genuine quantum computers [1], [19], [20], [21], [22], [23], [24], [25], machines poised to perform computational tasks far surpassing the capabilities of classical computing counterparts [26], [27], [28], [29], [30], [31], [32], [33].

At the heart of quantum computing lies the concept of "computational advantage" or "quantum supremacy" [34], [35]. This arises when quantum computers surpass classical computers by executing state-of-the-art algorithms on well-defined tasks [31], [32]. Notably, only a select few

The associate editor coordinating the review of this manuscript and approving it for publication was Alessio Giorgetti.

experiments have ventured into the domain of quantum devices with the explicit objective of tackling computational problems that stand beyond the reach of contemporary classical computers [36], [37], [38], [39], [40]. These groundbreaking experiments centered around the sampling of probability distributions recognized as exponentially challenging for classical computation to simulate effectively. Key milestones include the demonstration of a 53-qubit programmable superconducting processor [36] and the realization of a non-programmable photonic system tailored for Gaussian boson sampling, involving 50 squeezed states introduced into a static random 100-mode interferometer [37]. Subsequent iterations of these experiments have witnessed the introduction of additional qubits [40], [41] and the refinement of control over various parameters [39]. In each case, a crucial benchmark was the comparison of the duration of quantum sampling experiments to the anticipated runtime and scalability of the most advanced classical algorithms, unequivocally situating these quantum platforms within the realm of quantum computational advantage.

**FIGURE 1.** The developmental trajectory of quantum information processing. This development can be conceptualized as a progression through seven distinct stages, each of which relies on and complements the preceding stages while requiring continuous refinement in parallel with the others. Over time, the hope is to advance from the earlier stages to the more advanced ones, building upon the foundations laid by the preceding phases. Presently, quantum computing platforms are primarily situated between the third stage, involving (QND) Quantum Non-Demolition measurements, and the fourth stage dedicated to logical memory. In this transitional phase, computers are becoming capable of implementing error-correcting codes to construct logical qubits from physical qubits. It's important to note that, with the exception of the last two stages that mark the beginning of the fault-tolerant era, the earlier stages mostly belong to what is known as the NISQ era. Additionally, it's crucial to emphasize that progressing from a lower to a higher stage does not negate the ongoing need for improving the techniques required for the lower stages. For instance, enhancing the control of individual physical qubits remains an essential goal throughout the entire developmental timeline, even after logical qubits replace physical ones in computational applications.

These quantum computing milestones stand as remarkable achievements, furnishing researchers and scientists with substantial programmability and control [36], [37], [38], [39], [40]. However, it is paramount to recognize that these systems have yet to attain fault tolerance [18], [42], [43], [44]. Presently, their operational scope is constrained to executing circuits of modest depth, constrained by notable error rates and decoherence challenges, particularly when managing a relatively low number of qubits (approximately 50 qubits). In spite of commendable efforts to scale up qubit counts, such as IBM's recent unveiling of a quantum computer housing 433 universal and controllable qubits [45], [46], and notable progress in advancing photonic quantum computing scalability demonstrated by Xanadu computing [47], these systems continue to fall short of the necessary scale required for executing error-correcting codes.

In response to these transformative developments, the new term "NISQ computing" emerged to characterize the prevailing quantum computing era [48], differentiating it from the anticipated "fault-tolerant era" that lies in the distant future. It is imperative to underscore that the term "NISQ" primarily conveys a hardware-centric definition and does not inherently imply a temporal dimension. A visual illustration of the developmental trajectory of quantum information processing, with seven distinct stages [49], is presented in Fig. 1.

As we navigate this quantum odyssey, let us not forget that each step taken in the NISQ era propels us further towards the realization of full fault tolerance and the advancement of increasingly potent quantum devices. While NISQ devices may not yet fully unlock the boundless potential of quantum computing, they provide a fertile ground for exploration and progress. It is imperative to acknowledge that this era may endure for a considerable duration, underscoring the pivotal importance of research during the NISQ era [48], given the uncertainties surrounding the timeline for achieving full fault tolerance.

Additionally, considering the future scalability of quantum computing, interconnecting multiple quantum cores via a quantum network could potentially enhance computational capabilities in the short term [50]. While the feasibility and practicality remain to be fully realized. NISQ technology holds substantial promise, equipping us with innovative tools to address a multitude of complex challenges [51], [52]. Moreover, the knowledge and expertise cultivated during the NISQ era are destined to play a pivotal role in propelling us towards the fault-tolerant era, where the true potential of quantum computing awaits realization.

This paper is structured into two main parts to explore interconnected aspects of quantum computing. The first part (The Path To Quantum Supremacy) delves into the frontier of quantum supremacy, where recent experiments

have demonstrated quantum computers' ability to perform computations beyond classical capabilities, showcasing their computational power and advancements. Building upon these achievements, the paper presents practical applications of quantum computing. In the second part (Applied Quantum Computing), we analyze recent proof-of-principle experiments across various domains in the field of quantum computing, such as quantum machine learning, quantum simulation and material science, quantum natural language processing, quantum computing in finance, graph theory, quantum numerical solvers, drug discovery, pharmaceutical research, and logistics optimization.

By tracing this trajectory from attaining quantum supremacy to applied innovations, this review assumes significant relevance, as it aims to provide an in-depth examination of recent quantum supremacy experiments and their far-reaching implications. By dissecting the intricacies and consequences of these groundbreaking achievements, we seek to foster a deeper comprehension of the present landscape of quantum computing and its transformative potential.

In preparing this review, we examined recent literature and experimental findings in the field. Our methodology included integrating insights from peer-reviewed research, experimental outcomes, and expert analyses, aiming to present a balanced and informed perspective.

## II. PRIMARY HOLDERS OF QUANTUM COMPUTING PATENTS

According to the 2023 report by the European Patent Office (EPO) [53], a significant metric for assessing the effectiveness and strategic direction of patent filing strategies–in the realm of second-generation quantum computing (2GQC)– is the ratio of granted intellectual property (IP) rights in specific countries or regions. Fig. 2 illustrates the portion of "International Patent Families" within this domain for which IP rights were granted in particular jurisdictions. Notably, over 37% of these International Patent Families secured at least one IP right in the United States, underscoring its pivotal role in the field of quantum simulation. In comparison, other regions also exhibit substantial proportions, though notably lower than that of the United States. Regions in this regard include Japan (14%), Australia (12%), China (9%), and Europe (8%, based on granted European patents).

Table 1 presents a compilation of leading holders of quantum computing patents, within the past two decades according to the United States (USPTO) and Europe (EPO), as reported in [54]. Among the top ten patent holders who have secured more than 25 patents in quantum computing, prominent USA-based multinational enterprises concentrating on both hardware and software platforms (such as IBM, Google, Microsoft, Intel) emerge. Additionally, we identify Northrop Grumman, a U.S. defense technology company, as well as Rigetti, a California-based venture-backed firm that was established in 2013 with a focus on delivering scalable quantum processor technology utilizing
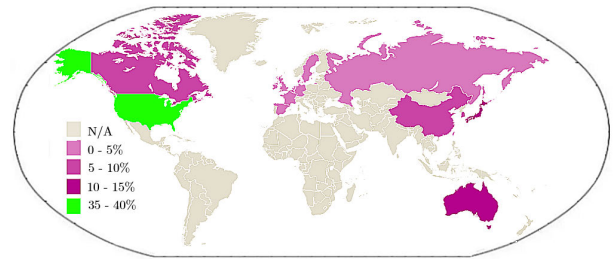


**FIGURE 2.** Proportion of patent applications in the field of 2G quantum simulation for which a patent was granted, highlighting the United States' significant role with over 37% of International Patent Families securing IP rights. It also notes other regions' substantial but relatively lower proportions, including Japan (14%), Australia (12%), China (9%), and Europe (8%, based on granted European patents). Data sourced from [53].

superconducting chips. Further contributors to this list include Honeywell International and the USA Government. Turning to the international arena, we encounter D-Wave Systems from Canada, which is particularly dedicated to specialized quantum annealing computers, and Toshiba from Japan. In the realm of academia, noteworthy institutions with substantial portfolios in quantum computing encompass MIT (located in the United States), Oxford (in the United Kingdom), Yale (in the United States), Harvard (in the United States), Caltech (in the United States), Stanford (in the United States), the University of Maryland (in the United States), the University of Wisconsin (in the United States), and the Technical University of Delft in the Netherlands.

## PART ONE: THE PATH TO QUANTUM SUPREMACY
## III. QUANTUM SUPREMACY

In the pursuit of achieving quantum supremacy [34], [35], significant efforts have been dedicated to developing quantum computers capable of performing tasks previously deemed impossible. Researchers have made notable strides in demonstrating quantum advantage using both cryogenic and photonic-based approaches. A number of experiments have harnessed quantum devices to perform computational tasks that exceed the capabilities of today's classical computers, as documented in various studies [36], [37], [38], [39], [40]. In all of these experiments, the computational challenges revolved around sampling from probability distributions that are widely acknowledged to be exceedingly difficult for classical computers to simulate efficiently.

As previously mentioned, the concept of quantum supremacy signifies the point at which a quantum computer achieves tasks previously considered beyond the reach of classical computing systems. Accordingly, two distinct quantum computing paradigms employed for this purpose are Boson Sampling (BS) and Gaussian Boson Sampling (GBS). Both BS [55] and GBS [56] paradigms exploit the principles of quantum interference among indistinguishable particles, typically bosons, to perform specialized computational tasks.

### A. BOSON SAMPLING
BS has emerged as a valuable tool for exploring the quantum advantages over classical computing, particularly because

**TABLE 1.** Top primary holders of quantum computing patents in the USPTO and EPO [54].

| No. | Holder | Country | Patents | No. | Holder | Country | Patents |
|---|---|---|---|---|---|---|---|
| 1 | IBM | USA | 254 | 29 | STMicroelectronics | Switzerland | 9 |
| 2 | D-Wave Systems | Canada | 183 | 30 | Harvard College | USA | 8 |
| 3 | Northrop Grumman | USA | 120 | 31 | Magiq Tech Inc | USA | 8 |
| 4 | Microsoft Corp | USA | 111 | 32 | California Inst of Tech | USA | 7 |
| 5 | Alphabet Inc | USA | 59 | 33 | HP Inc | USA | 7 |
| 6 | Rigetti & Co Inc | USA | 53 | 34 | NEC Corp | Japan | 7 |
| 7 | Toshiba Corp | Japan | 37 | 35 | Stanford University | USA | 7 |
| 8 | Intel Corp | USA | 32 | 36 | University Sys of Maryland | USA | 7 |
| 9 | Honeywell Int Inc | USA | 26 | 37 | University Wisconsin Warf | USA | 7 |
| 10 | US Government | USA | 26 | 38 | Kyndryl Inc | USA | 6 |
| 11 | HP Enterprise | USA | 23 | 39 | Mitre Corp | USA | 6 |
| 12 | Newsouth Innovations | Australia | 22 | 40 | Parallel Investment | USA | 6 |
| 13 | Mass. Inst. of Tech. MIT | USA | 20 | 41 | QC Ware Corp | USA | 6 |
| 14 | Equal1labs Inc | USA | 16 | 42 | Technische Universiteit Delft | Netherlands | 6 |
| 15 | Hitachi Ltd | Japan | 15 | 43 | University Johns Hopkins | USA | 6 |
| 16 | Japan Science & Tech Ag | Japan | 15 | 44 | Wells Fargo Bank | USA | 6 |
| 17 | 1QB Information Tech | Canada | 14 | 45 | Corning Corp | USA | 5 |
| 18 | Accenture Public Ltd | Ireland | 12 | 46 | Dartmouth College | USA | 5 |
| 19 | IonQ Inc | USA | 12 | 47 | Lockheed Martin Corp | USA | 5 |
| 20 | Nokia Corp | Finland | 12 | 48 | Phoenix Co of Chicago | USA | 5 |
| 21 | Bank of America | USA | 11 | 49 | Quantum Valley Invest | Canada | 5 |
| 22 | Element Six SA | Luxembourg | 11 | 50 | Samsung Electronics | South Korea | 5 |
| 23 | University Oxford | UK | 11 | 51 | University California | USA | 5 |
| 24 | Yale University | USA | 10 | 52 | Bull SA | France | 4 |
| 25 | STMicroelectronics | Switzerland | 9 | 53 | DWSI Holdings Inc | USA | 4 |
| 26 | Commissariat Atomique | France | 9 | 54 | IMEC | Belgium | 4 |
| 27 | Raytheon Tech Corp | USA | 9 | 55 | Korea Elect. Res Inst | South Korea | 4 |
| 28 | SEEQC Inc | USA | 9 | 56 | Quintessencelabs Pty | Australia | 4 |

it does not necessitate universal control over the quantum system. This characteristic aligns with the capabilities of existing photonic experimental platforms. BS [55] has captivated both theorists and experimentalists by showcasing the superior computational power of quantum systems and challenging the Extended Church-Turing (ECT) thesis without relying on the full capabilities of a universal quantum computer. In BS, identical photons traverse a network of optical elements, generating complex interference patterns, and the output distribution encodes information about the underlying quantum states and network parameters. The goal is to sample from this distribution, a task believed to be computationally challenging for classical computers, thus showcasing quantum computational superiority [55]. Nevertheless, the original BS approach encounters significant challenges when it comes to scalability and experimental implementation, chiefly due to the demanding requirement of a reliable source of numerous indistinguishable photons.

### B. GAUSSIAN BOSON SAMPLING

In response to these experimental constraints, a novel approach known as Gaussian Boson Sampling emerged in 2017 [56]. GBS tackles a classically hard-to-solve problem by utilizing squeezed states as a non-classical resource. This innovative approach addresses questions regarding the complexity of sampling from a general squeezed state. In GBS, a novel expression has been derived, connecting the probability of measuring a specific output pattern of photons from a general Gaussian state to the *hafnian* matrix function [56]. GBS extends this concept to continuous-variable

quantum states and operations, utilizing Gaussian states and transformations. This approach maintains the fundamental principles of interference while dealing with continuous variables like position and momentum.

Both BS and GBS represent key quantum supremacy experiments, highlighting the potential computational advantages of quantum systems in specific, though challenging, computational scenarios. For a more in-depth study of GBS, readers are encouraged to refer to [57], and for more understanding of GBS applications, a wealth of information can be found in [58], [59], [60], [61], [62], [63], [64], and [65].

### C. QUANTUM RANDOM CIRCUIT SAMPLING

The pursuit of quantum advantage on noisy quantum devices represents a crucial objective in contemporary quantum computing research. One prominent strategy for achieving this goal involves random circuit sampling (RCS) [66], which entails sampling from probability distributions generated by randomly selected quantum circuits.

In RCS, the task is to sample a bit string from a distribution closely resembling the output distribution of a random quantum circuit. It's important to note that some error allowance is inherent in this problem, as even a fault-tolerant quantum computer can only execute RCS up to a certain small error threshold.

From an experimental perspective, executing RCS on noisy quantum computers without error correction is relatively straightforward, with the quality of the samples depending on the fidelity characteristics of the device [36], [67], [68]. Additionally, theoretical investigations into RCS have contributed

to our understanding of its computational complexity [69], [70]. Notably, the RCS supremacy conjecture posits that no efficient classical algorithm can feasibly perform RCS within polynomial time for a certain small yet reasonable error margin.

In historical context, the study of circuit sampling began with the examination of quantum circuits from specific families with structured characteristics. For example, instantaneous quantum polynomial time (IQP) circuits were found to be challenging to simulate classically under reasonable average-case conjectures [71], [72]. Furthermore, RCS is recognized to be computationally challenging for noiseless, fully randomly drawn quantum circuits, based on widely accepted conjectures within the field [66], [73], [74].

RCS has demonstrated its ability to meet an average-case hardness condition [69], [70], [71], which is crucial for establishing computational difficulty in the presence of experimental noise [66]. Moreover, existing findings suggest that RCS also satisfies an anti-concentration property, indicating that errors in estimating output probabilities are relatively minor compared to the probabilities themselves [74]. These characteristics position RCS as the most promising avenue for achieving quantum advantage, driving its implementation in experiments aimed at demonstrating quantum supremacy [66]. For a deeper insight into RCS and its computational complexity arguments regarding its hardness, readers are encouraged to explore [74], [75], [76].

## IV. ATTAINING QUANTUM SUPREMACY

### A. GOOGLE'S QUANTUM AI

#### 1) OVERVIEW

Random quantum circuits (RQCs) are widely recognized for their complexity, posing significant challenges for classical simulation. The utilization of RQCs played a pivotal role in the pursuit of achieving quantum supremacy [34], [35]. In a notable experiment [36], a programmable quantum processor comprising 54 superconducting qubits performed computations within a Hilbert space of size $2^{53}$. The comparative performance of this task on *Summit*, the world's most powerful supercomputer at the time of estimation, was initially projected to require approximately 10,000 years for completion. However, the same computation was achieved in a mere 200 seconds using the *Sycamore* quantum processor.

Shortly after the publication, a discussion ensued regarding the possibility of overestimating the time needed to solve the identical problem on a supercomputer. Within this debate, a significant concern emerged. It is the question, whether reliable methods exist to accurately assess the true capabilities of quantum computing [77], [78], [79]. Subsequently, it was suggested that an equivalent task could be executed with high fidelity on a classical computer, specifically the *Summit* supercomputer at Oak Ridge National Laboratories, within a matter of days [77]. Additionally, an alternative classical simulation algorithm based on tensor networks was introduced in [78] capable of performing the task in less than 20 days.

Furthermore, an additional tensor network approach was proposed in [79] to address the classical simulation of *Sycamore*'s quantum circuits. This computational task, when executed on a cluster equipped with 512 GPUs, was successfully completed within a span of 15 hours. These findings collectively underscore the immense computational challenges posed by RQCs and the significant strides made in both quantum and classical computing to tackle them. A calculation based on computational complexity arguments determined the minimum number of qubits required for achieving quantum supremacy in [80], providing a rigorous benchmark for assessing the attainment of quantum supremacy. whereas, in [81], the study investigated the scaling of entanglement in different quantum supremacy benchmarks.
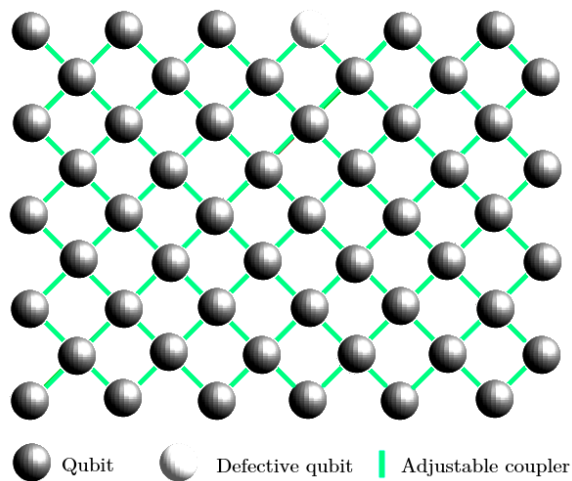
#### 2) GOOGLE'S SYCAMORE QUANTUM PROCESSOR

The Google's *Sycamore* quantum processor [36] is composed of a two-dimensional array consisting of 54 programmable Superconducting transmon qubits [82]. Each qubit is configured to have tunable connections with its four nearest neighbors, forming a rectangular lattice as depicted in Fig. 3.

In pursuit of achieving a state of quantum supremacy [34], [35], the *Sycamore* processor introduced significant technological advancements, as detailed in [36]. These innovations enabled the execution of random quantum circuits on the 54-qubit *Sycamore* quantum device, introducing a novel class of random quantum circuits. These circuits are characterized by alternating layers of single-qubit and two-qubit gates, collectively forming a "cycle." Each random circuit comprises *m* such cycles, with a cycle defined as a combination of a layer of single-qubit gates followed by a layer of two-qubit gates, concluding with another layer of one-qubit gates before the final measurement step, as illustrated in Fig. 4.

The *Sycamore* processor is notable for its capability to execute high-fidelity one-qubit and two-qubit quantum gates. These gates are not only performed individually but also simultaneously across multiple qubits during practical computations [36].

The *Sycamore* quantum computer employs transmon qubits [82], which can be regarded as nonlinear superconducting resonators operating at frequencies between 5 and 7 GHz. Each qubit is encoded using the two lowest quantum eigenstates of the resonant circuit. Furthermore, each transmon qubit is equipped with two controls: a microwave drive for qubit excitation and a magnetic flux control for tuning its frequency. For qubit state readout, each qubit is associated with a linear resonator [83]. Additionally, each qubit is interconnected with its neighboring qubits through adjustable couplers [84], [85], facilitating dynamic tuning of qubit-qubit coupling, ranging from fully disengaged to 40 MHz. During the quantum supremacy experiment, due to one qubit's malfunction, the device operated with 53 qubits and 86 couplers.

**FIGURE 3.** Google's *Sycamore* quantum processor layout. This processor composed of a rectangular-array of 54 programmable Superconducting transmon qubits. Each qubit linked to the four nearest neighbors via couplers. As one qubit did not operate perfectly during the supremacy experiment, the quantum processor has 53-qubits and 86-couplers.

Google Quantum AI and its collaborators unveiled a demonstration that transformed quantum computing from a theoretical concept into a practical reality [36]. While Google Quantum AI had not yet achieved the ability to create physical qubits with the desired longevity, it did manage to develop qubits that remained stable long enough to outperform the world's most powerful supercomputer in certain calculations [36]. This transition ushered us into the era of NISQ computing, marking a pivotal shift away from pure classical computing [48]. A comprehensive characterization of Google's *Sycamore* quantum AI is presented in [86] and [87].

### 3) RCS EXPERIMENTS ON 67- AND 70- QUBITS PROCESSORS

In December 2023, Google Quantum AI and its collaborators conducted experiments on two quantum machines featuring 67 and 70 qubits, aiming at quantum supremacy regimes [88]. Their experiment yields direct insights into the interaction between quantum dynamics and noise. The observed phase boundaries offer quantitative guidance on the specific conditions under which noisy quantum devices can effectively harness their computational capabilities. Furthermore, they present new RCS results with an estimated fidelity of $1.5 \times 10^{-3}$ at 67 qubits and 32 cycles or 880 entanglement gates. This represents a significant increase in circuit volume compared to previous work [36] while maintaining the same fidelity level.

A practical assessment of the computational resources necessary for simulating RCS must consider the finite computational efficiency of a supercomputer in terms of FLOPS (floating-point operations per second), as well as its memory constraints and other limitations like finite bandwidth. Table 2 outlines estimates of the runtime for the approximate simulation of the largest RCS instances from

previous studies [36], [40], [41] and the study detailed in [88]. These estimations encompass the sampling of 1 million uncorrelated bit-strings at a fidelity comparable to the experiment, leveraging the top-performing supercomputer, *Frontier*. *Frontier* boasts a theoretical peak performance of $1.7 \times 10^{18}$ single precision FLOPS, distributed across GPUs equipped with 128 GB of RAM each.

By showcasing a RCS experiment with 67 qubits at 32 cycles, Google Quantum AI and its collaborators demonstrate that the computational demands of their experiment surpass the capabilities of existing classical supercomputers, even when considering the presence of inevitable noise. Moreover, the dominance of global correlations over XEB (cross-entropy benchmarking) in the weak noise phase serves as a protective measure for RCS against "spoofing" attacks. They assert that this finding differs from Boson-Sampling experiments [55], where recent experiments' metrics [37], [39], [47] are predominantly affected by local correlations [58].

### B. THE ZUCHONGZHI QUANTUM COMPUTERS
### 1) OVERVIEW

In another investigation, researchers delved into the capabilities of the *Zuchongzhi* 2.0 quantum computing system, which comprises 56 superconducting qubits [40]. Their study underscored *Zuchongzhi*'s remarkable efficiency in executing complex sampling tasks. Notably, *Zuchongzhi* achieved a sampling task in approximately 1.2 hours, a feat that would require the *Summit* supercomputer, the most powerful classical computer at that time, a minimum of 8 years to complete. Building upon these achievements, the researchers subsequently introduced an enhanced iteration known as *Zuchongzhi* 2.1, featuring 66 qubits and delivering substantial improvements in quantum advantage compared to its predecessor. These results illuminate the quantum information processing prowess of *Zuchongzhi*, marking a significant stride in quantum computing hardware and its computational superiority over classical counterparts.

### 2) ZUCHONGZHI 2.0

*Zuchongzhi* 2.0 is a novel two-dimensional programmable superconducting quantum processor comprising 66 functional qubits and 110 couplers [40]. To evaluate its performance, researchers conducted a benchmarking task involving quantum RCS, utilizing 56 qubits and 20 cycles. The computational cost of classically simulating this task is estimated to be 2 to 3 orders of magnitude higher compared to prior work with the 53-qubit Sycamore processor [36]. Remarkably, *Zuchongzhi* completed the sampling task in approximately 1.2 hours, a task that would require the most powerful supercomputer at least 8 years to accomplish [40].

*Zuchongzhi*'s achievements extend to high-fidelity single-qubit gate operations (averaging 99.86% fidelity), two-qubit gate operations (averaging 99.41% fidelity), and high-fidelity readout processes (averaging 95.48% fidelity).
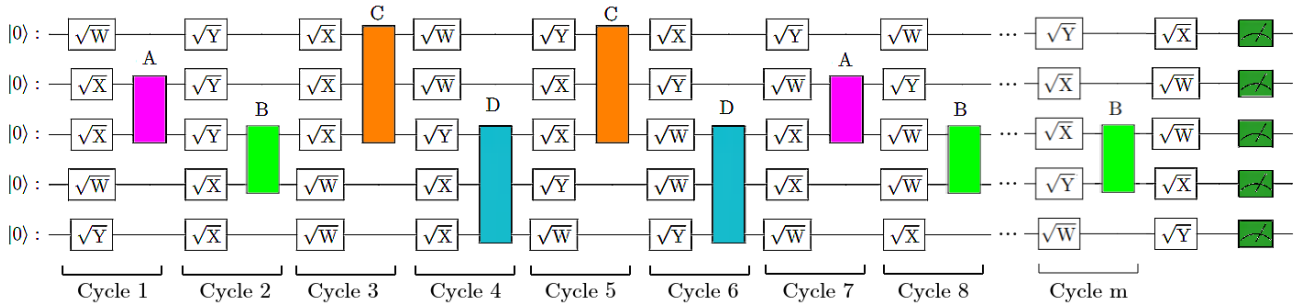
**FIGURE 4.** A *m*-cycle circuit schematic diagram of the *Sycamore* 53-qubit RQCs. Each cycle joining a layer of random single-qubit gates (chosen at random from the gate set {$\sqrt{X}$, $\sqrt{Y}$, $\sqrt{W}$}), followed by a layer of 2-qubit gates, labeled A, B, C, or D. Layers in longer circuits repeat in the order A; B; C; D – C; D; A; B. It is worth noting that single-qubit gates do not repeat sequentially, also there is an additional layer of one-qubit gates prior measurements.

**TABLE 2.** Estimated computational cost of simulation by Google Quantum AI and collaborators: The fourth column illustrates the FLOP count requisite for computing a single output amplitude under no memory constraints. The last 3 columns delineate the expenditure associated with simulating noisy sampling of one million bit-strings. The estimations by Google Quantum AI and collaborators are grounded on the specifications of *Frontier*, the current top-performing supercomputer, featuring a theoretical peak performance of $1.685 \times 10^{18}$ single precision FLOP per second. A 20% FLOP efficiency is assumed [78], [79], [89], with consideration given to the diminished target fidelity of the simulation in the computational cost [68], [78], [89], [91]. Each single precision complex FLOP necessitates eight machine FLOPs. Regarding *Sycamore* (67-qubits), the cost estimation, assuming memory distribution over all RAM$^+$ or even all secondary storage$^{++}$, is encompassed, disregarding realistic bandwidth constraints. All other rows employ tensor contraction algorithms that are embarrassingly parallelizable across each GPU [78], [79], [90]. The table presented here has been adapted from [88] under a Creative Commons license, https://creativecommons.org/licenses/by/4.0/. It's important to note that the views expressed in this table belong to Google Quantum AI and its collaborators, and not necessarily to the authors of this work.

| Experiment | Single amplitude | | | One million noisy samples | | |
|---|---|---|---|---|---|---|
| | Qubits | Year | FLOPs [a] | FLOPs | XEB fidelity [b] | Time |
| *Sycamore*-53 [36] | 53 | 2019 | $6.0 \times 10^{17}$ | $2.0 \times 10^{17}$ | $2.0 \times 10^{-3}$ | 6 seconds |
| *Zuchongzhi* 2.0 [40] | 56 | 2021 | $6.0 \times 10^{19}$ | $6.0 \times 10^{19}$ | $6.0 \times 10^{-4}$ | 20 minutes |
| *Zuchongzhi* 2.1 [41] | 60 | 2022 | $1.0 \times 10^{21}$ | $1.0 \times 10^{23}$ | $3.0 \times 10^{-4}$ | 40 days |
| *Sycamore*-70 [88] | 70 | 2023 | $5.0 \times 10^{23}$ | $6.0 \times 10^{25}$ | $2.0 \times 10^{-3}$ | 50 years |
| *Sycamore*-67 [88] | 67 | 2023 | $2.0 \times 10^{23}$ | $2.0 \times 10^{37}$ | $1.0 \times 10^{-3}$ | $1.0 \times 10^{13}$ years |
| | | | | $2.0 \times 10^{28}$ | | $1.0 \times 10^{4}$ years$^+$ |
| | | | | $2.0 \times 10^{25}$ | | 12 years$^{++}$ |

[a] FLOPS: floating-point operations per second.
[b] $\mathcal{F}_{XEB}$: a metric utilized in cross-entropy benchmarking (XEB) to precisely specify the computational objective for achieving quantum supremacy through random circuit sampling.

These milestones firmly establish a quantum computational advantage that is unattainable through classical computation within a reasonable timeframe.

### 3) ZUCHONGZHI 2.1

In February 2022, a significant milestone in quantum supremacy was reached with the introduction of *Zuchongzhi* 2.1, a two-dimensional superconducting quantum processor featuring 66 qubits [41]. Compared to its predecessor, *Zuchongzhi* 2.0 [40], *Zuchongzhi* 2.1 [41] showcases remarkable improvements in readout performance, achieving an average fidelity of 97.74%.

This enhanced quantum processor empowers the execution of larger-scale random quantum circuit sampling, accommodating a system scale of up to 60 qubits and 24 cycles. The complexity of the achieved sampling task surpasses previous benchmarks, being approximately six orders of magnitude more challenging than that tackled by Google's Sycamore quantum processor [36] and three orders of magnitude more complex than the task performed on *Zuchongzhi* 2.0 [40].

It was estimated that, state-of-the-art classical algorithms and supercomputers would require approximately 48,000 years to simulate the RCS experiment, while *Zuchongzhi*

2.1 accomplishes it in just around 4.2 hours. This substantial reduction in time underscores the significant quantum computational advantage offered by *Zuchongzhi* 2.1 [41].

Table 3 presents a comprehensive comparison of system characteristics (mean values) between the *Zuchongzhi* 2.0 and *Zuchongzhi* 2.1 superconducting quantum computers. Furthermore, Table 3 provides a performance evaluation of tensor network algorithm runtimes for various circuits executed on the *Summit* supercomputer. It also includes estimates for the computational resources required for classical simulations of the quantum RCS experiment conducted on the *Sycamore* quantum processor [36], *Zuchongzhi* 2.0 [40], and *Zuchongzhi* 2.1 [41] quantum processors.

The findings reported in [40] and [41] underscore the remarkable capabilities of *Zuchongzhi* in quantum information processing, marking a substantial leap forward in quantum computing hardware and its computational superiority when compared to classical counterparts.

### C. THE PHOTONIC QUANTUM COMPUTERS, JIUZHANG
### 1) OVERVIEW
Researchers at the University of Science and Technology in China (USTC) have accomplished a remarkable feat in the

**TABLE 3.** Performance comparison of tensor network algorithm runtimes for various circuits on *Summit* supercomputer. With estimated classical simulation consumption for the random quantum circuit sampling experiment on *Google's Sycamore*, *Zuchongzhi* 2.0, and *Zuchongzhi* 2.1 quantum processors is included [41].

| Parameters | Google's *Sycamore* Quantum AI vs *Zuchongzhi* Quantum Processors | | | |
| | *Sycamore* [36] | *Zuchongzhi* 2.0 [40] | *Zuchongzhi* 2.1 [41] | *Zuchongzhi* 2.1 [41] |
| --- | --- | --- | --- | --- |
| Quantum Circuit | 53-qubit 20-cycle | 56-qubit 20-cycle | 60-qubit 22-cycle | 60-qubit 24-cycle |
| Estimated Runtime on *Summit* | ~~10,000 years~~ 15.9 days [a] | 8.2 years | 4, 800 years | 48, 000 years |
| Runtime on Quantum Computer | 3 minutes [b] | 72 minutes | 60 minutes | 252 minutes |
| Fidelity | 0.224% | 0.0662% | 0.0758% | 0.0366% |

[a] Google's estimations projected that the task would demand an astounding 10,000 years for *Summit*, the most powerful supercomputer. Subsequently, IBM provided a contrasting perspective, asserting that *Summit* could accomplish the same task in a matter of days [41], [77].
[b] As reported by Google's Quantum AI and Collaborators in [36], while the time estimated for performing the task in [41] was 600 seconds.
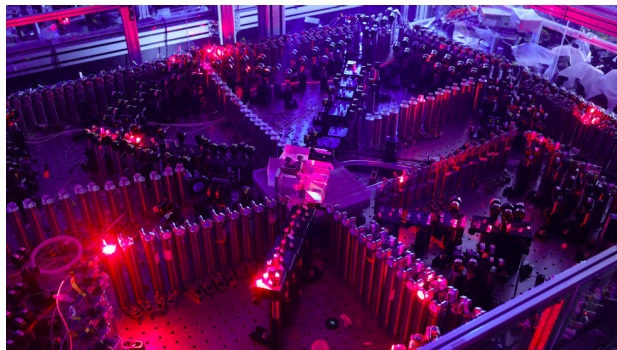


**FIGURE 5.** The photonics-based quantum computing device developed by USTC, known as *Jiuzhang*, operates by intricately controlling light through an array of optical devices. A photographic representation of the *Jiuzhang* photonic network reveals the experimental configuration, which is situated on an optical table spanning approximately three square meters. Within this setup, 25 Two-Mode Squeezed States (TMSSs) are introduced into the photonic network, resulting in the collection of 25 phase-locked light signals. To further elaborate, the output modes of the *Jiuzhang* photonic network are meticulously separated into 100 distinct spatial modes through the utilization of mini-mirrors and Polarizing Beam Splitters (PBSs). This achievement marks the second quantum computer claiming to attain quantum computational advantages, following Google's Sycamore quantum processor. Reproduced under a creative commons license https://creativecommons.org/licenses/by/4.0/ from [37].

realm of quantum computing by achieving quantum computational advantage using novel photonic quantum computers. These cutting-edge quantum computing systems harness the power of photons, or particles of light, to perform calculations that surpass the capabilities of traditional computers [33].

In December 2020, the photonic quantum computer "*Jiuzhang*" achieved quantum advantage [37]. However, it lacked programmability due to its reliance on fixed optical components. Whereas, in 2021, researchers in USTC reported on the improved photonic quantum computer "*Jiuzhang* 2.0," capable of large-scale GBS [39]. Additionally, in 2023, a new GBS experiment employing pseudo-photon-number-resolving detection was detailed by Deng et al. in [96], recording photon-click events of up to 255 using the photonic quantum computer, *Jiuzhang* 3.0 [96]. This successful demonstration of quantum supremacy by photonic quantum computers represents a significant leap forward in the field of quantum computing.

### 2) JIUZHANG

The photonic quantum computers, *Jiuzhang* [37] and *Jiuzhang* 2.0 [39] mark significant milestones in the advancement of photonic quantum computing technology. *Jiuzhang* achieved the distinction of being the first to attain quantum supremacy, while *Jiuzhang* 2.0 has made substantial progress in terms of programmability and speed, particularly in executing large-scale GBS tasks [55], [56], [93], [94]. As research and development in the field of quantum computing continue to advance, further enhancements and refinements in both systems are expected to shape the landscape of quantum information processing.

Zhong et al. conducted an experiment [37] wherein they introduced 50 identical single-mode squeezed states into a 100-mode ultralow-loss interferometer, and subsequently, they captured the outcomes using 100 highly efficient single-photon detectors. By achieving up to 76-photon coincidences, corresponding to a state space dimension of approximately $10^{30}$, they ascertained a sampling rate approximately $10^{14}$ times faster compared to contemporary classical simulation techniques and supercomputing systems.

To provide context, they also computed the time required by two supercomputers to accomplish the same GBS task. Zhong et al. projected that the *TaihuLight* supercomputer would necessitate around $8 \times 10^{16}$ seconds (which is 2.5 billion years), while *Fugaku* would take $2 \times 10^{16}$ seconds (which is 0.6 billion years) [37]. The light-based quantum computer *Jiuzhang* is shown in Fig. 5.

### 3) JIUZHANG 2.0

In 2021, researchers from the USTC introduced an advanced photonic quantum computer called "*Jiuzhang* 2.0," designed specifically for large-scale GBS [39]. This quantum computer utilized a 144-mode photonic circuit, achieving up to 113 photon detection events. A notable achievement was the development of a novel and scalable quantum light source based on stimulated emission of squeezed photons, which exhibited both near-unity purity and efficiency.

The remarkable photonic quantum computing capabilities of *Jiuzhang* 2.0 correspond to an impressive Hilbert space dimension of approximately $10^{43}$, resulting in a sampling rate
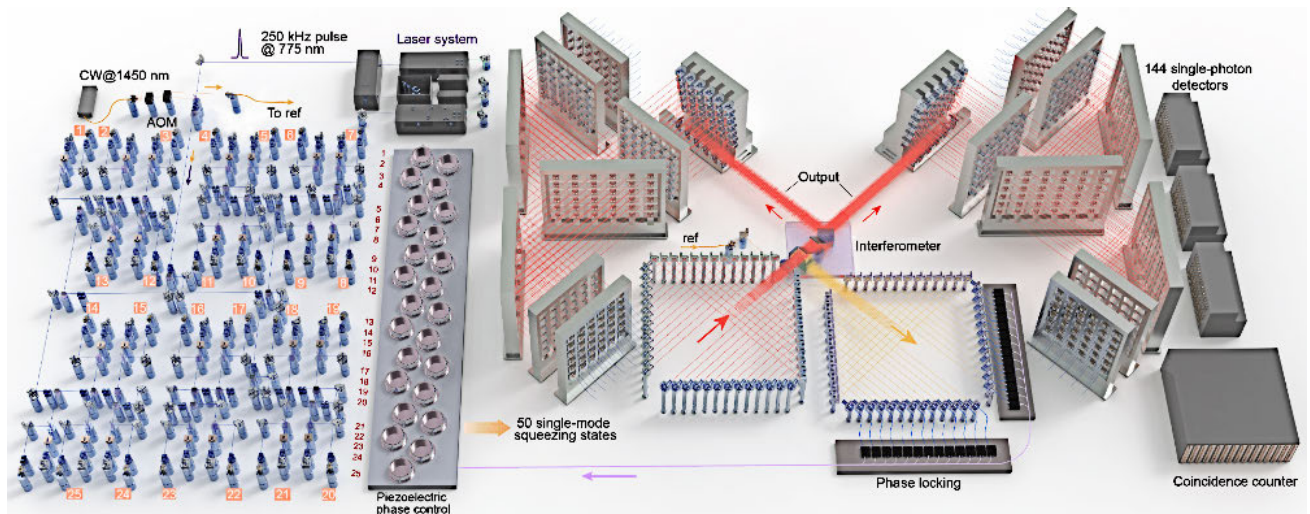
**FIGURE 6.** The *Jiuzhang* light-based quantum computer. In the upper-left quadrant of its configuration, a high-intensity pulse laser emitting light with a wavelength of 775 nm is utilized to stimulate 25 sources of Two-Mode Squeezed States (TMSS), indicated by the orange label in the left section. Simultaneously, a continuous-wave laser emitting light at 1450 nm is directed to co-propagate alongside the aforementioned 25 TMSS sources. The resulting output is two-mode squeezed light at 1550 nm, which is then guided into a single-mode fiber known for its temperature insensitivity. Notably, a 5-meter segment of this fiber is wrapped around a piezo-electric cylinder, allowing for precise control over the source phase, situated in the central region of the setup. Transitioning to the center-right section, an optical arrangement involving collimators and mirrors facilitates the injection of the 25 TMSSs into a photonic network. In this context, 25 light beams corresponding to these TMSSs (depicted in yellow) with a wavelength of 1450 nm and an intensity power of approximately 0.5 $\mu$W are employed for the purpose of phase synchronization. The output modes resulting from this setup, totaling 144 modes, are divided into four segments through arrays of adjustable periscopes and mirrors. Ultimately, these output modes are subjected to detection using 144 superconducting nanowire single-photon detectors and subsequently processed through a 144-channel ultra-fast electronics coincidence unit. Reproduced under a creative commons license https://creativecommons.org/licenses/by/4.0/ from [39].

that is approximately $10^{24}$ times faster than brute-force simulations performed on supercomputers. These advancements represent a significant leap forward in the field of photonic quantum computing, opening the door to groundbreaking applications.

The core concept involved the generation of photon pairs through spontaneous emission, resonating with a pump laser, to stimulate the emission of a second photon pair within a gain medium [95]. The experiment employed transform-limited laser pulses with a wavelength of 775 nm, focused on PPKTP crystals, to create two-mode squeezed states (TMSS). A concave mirror played a crucial role by reflecting both the pump laser and the collinear TMSS photons, acting as seeds for the second parametric process. To compensate for birefringence walk-off between horizontally and vertically polarized TMSS photons, a quarter-wave plate was utilized. Furthermore, the dispersion between the pump laser and the TMSS was mitigated by designing the PPKTP crystals to eliminate frequency correlation. Visual representations of the experimental setups are thoughtfully provided in Fig. 6.

In their research, the team achieved high collection efficiency (0.918 at a waist of 125 $\mu$m and 0.864 at 65 $\mu$m) along with simultaneous high photon purity (0.961 and 0.946) without the need for narrowband filtering. Their double-pass approach demonstrated scalability to higher orders, making it possible to generate quantum light sources with greater brightness, which are nearly optimal for a range of applications.

### 4) JIUZHANG 3.0

In 2023, a new experiment in GBS utilizing pseudo-photon-number-resolving detection was outlined in [96], documenting photon-click events reaching a maximum of 255, as depicted in Fig. 7. This investigation integrates considerations for partial photon distinguishability and introduces an extensive model for characterizing noisy GBS. Bayesian tests and correlation function analysis are applied within the context of quantum computational advantage to validate the samples against established classical spoofing simulations. Comparative assessments with state-of-the-art classical algorithms reveal that generating a single ideal sample from the identical distribution on the supercomputer *Frontier* would demand approximately 600 years using exact methods, whereas the quantum computer, *Jiuzhang* 3.0, achieves this feat in merely 1.27$\mu$s. Computing the most challenging sample from the experiment utilizing an exact algorithm would necessitate *Frontier* approximately $3.1 \times 10^{10}$ years [96].

### D. XANADU'S PHOTONIC QPUS

### 1) OVERVIEW

Recently, the Canadian quantum computing company Xanadu [98] introduced its revolutionary photonic quantum computer, named "*Borealis*." Notably, *Borealis* demonstrated its quantum advantage by accomplishing a computational task in just 36 microseconds, a feat that would have
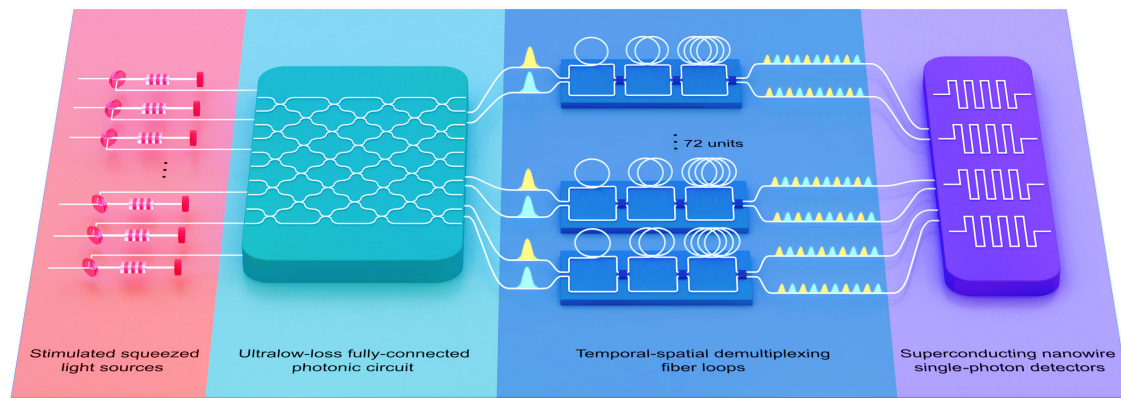
**FIGURE 7.** The experimental arrangement comprises 25 stimulated two-mode squeezed state photon sources, synchronized in phase, and directed into a 144-mode ultralow-loss fully-connected optical interferometer. These photons traverse 72 units of fiber loop setups for temporal-spatial demultiplexing before reaching 144 superconducting nanowire single-photon detectors, constituting a pseudo-photon-number resolving detection scheme. Each fiber loop setup, distinguished by distinct colors, accommodates two input modes. Within these setups, photons undergo temporal demultiplexing via fiber beam splitters and delay lines, resulting in four time bins. Furthermore, each time bin is partitioned into two path bins at the terminal fiber beam splitter. Discrimination between photons from the two input modes within the same fiber loop setup is facilitated by assessing their temporal parity through a coincidence event analyzer (not depicted). Reproduced from [96] under a creative commons licenses https://creativecommons.org/licenses/by/4.0.

taken a conventional supercomputer over 9,000 years to complete [47].

### 2) THE PHOTONIC QUANTUM COMPUTER BOREALIS

Xanadu's quantum technology primarily relies on Silicon quantum photonic chips [97], forming the core of a room-temperature-operating quantum computer distinguished by its scalability. The realization of quantum computational advantage through Xanadu's photonic quantum computer, *Borealis*, is comprehensively examined in [47]. In this context, Xanadu's *Borealis* stands out as a photonic processor renowned for its dynamic programmability across all implemented gates [58], demonstrates quantum computational advantage [47].

The research employs GBS [56], utilizing *Borealis* with 216 squeezed modes entangled through a three-dimensional connectivity configuration [38], achieved through a time-multiplexed and photon-number-resolving architecture. Notably, *Borealis* completes this task within an astonishing 36 microseconds, while the most advanced classical algorithms and supercomputers would require over 9,000 years to accomplish the same feat [47]. This performance improvement is claimed to surpassing the advantage demonstrated by earlier photonic machines by a factor exceeding 50 million [47].

The study represents a substantial GBS experiment, capturing events involving up to 219 photons with a mean photon number of 125. This significant achievement marks a pivotal step toward the realization of practical quantum computing and underscores the viability of photonics as a platform for advancing toward this goal. Fig. 8 present an illustration of the fully programmable photonic processor, *Borealis*. Furthermore, Fig. 9 provides a comparative analysis of the quantum computational capabilities achieved by *Borealis* alongside *Jiuzhang* 2.0 quantum computers.

### 3) XANADU'S QUANTUM CLOUD

Accessible through Xanadu's Quantum Cloud (XQC) platform, *Borealis* provides direct access to their photonic Quantum Processing Units (QPUs), enabling remote utilization of their capabilities. Moreover, Xanadu's commitment to advancing quantum research is evident through their open-source library, PennyLane, which seamlessly integrates quantum machine learning, computing, and chemistry. PennyLane can interface with various quantum processors, including those offered by IBM [99] and Rigetti [100]. The successful realization of quantum computational advantage through Borealis' dynamic programmability underscores the promise of photonic quantum computing and contributes significantly to the advancement of quantum computing technology.

## PART TWO: APPLIED QUANTUM COMPUTING
## V. PRACTICAL APPLICATIONS OF NISQ COMPUTERS

Quantum computing is steadily advancing, catching the attention of governments and industry leaders. Its transformative potential spans various domains, promising swift and precise solutions that currently elude classical computers. Its impact encompasses pharmaceutical research, drug design and discovery [101], financial modeling [102], artificial intelligence and machine learning [103], combinatorial optimization [104], logistics [105], cybersecurity [106], cryptographic and encryption methodologies [107], [108], [109], [110], [111], [112], [113], [114], [115], [116], [117]. This relentless potential extends into fields such as electronic materials' discovery, computational chemistry, weather forecasting and climate change, manufacturing and industrial design, and more [118], [119], [120]. The horizons of quantum computing are marked by endless possibilities, and its ascent holds the promise of reshaping our understanding of what is achievable across numerous scientific, industrial, and technological frontiers [118], [119], [120].
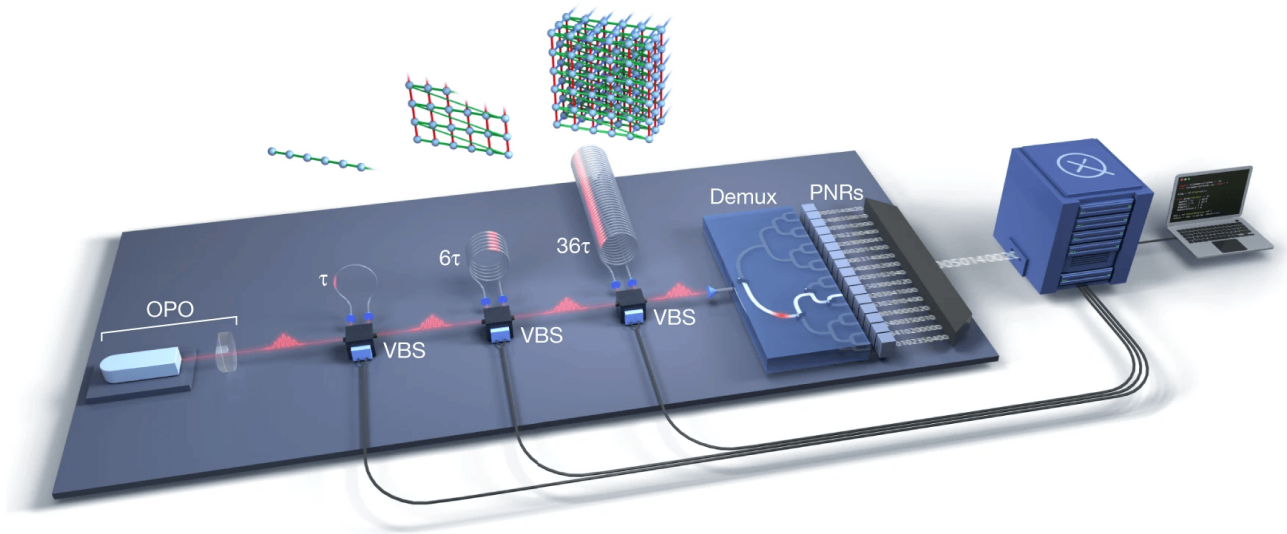
**FIGURE 8.** High-dimensional GBS from a fully programmable photonic processor, *Borealis*: A series of periodic pulses composed of single-mode squeezed states generated by a pulsed Optical Parametric Oscillator (OPO) is introduced into a sequence of three dynamically adjustable loop-based interferometers. Each of these loops consists of a Variable Beam Splitter (VBS) equipped with a programmable phase shifter, as well as an optical fiber delay line. The resulting Gaussian state at the interferometer's output is then directed towards a 1-to-16 binary switch tree (demux), which partially separates the output before it is measured by Photon Number Resolvers (PNRs). This recorded sequence comprises 216 photon numbers, and it is considered one sample, achieved in approximately 36 microseconds. The optical fiber delays, along with accompanying beam splitters and phase shifters, facilitate the implementation of gates between both temporally adjacent and distant modes, thereby enabling high-dimensional connectivity within the quantum circuit. Additionally, above each stage of the loop, there is a visual representation of the multipartite entangled Gaussian state being progressively generated. In the first stage ($\tau$), two-mode programmable gates (depicted in green) are applied to adjacent modes in one dimension. In contrast, the second ($6\tau$) and third ($36\tau$) stages create connections between modes separated by six and 36 time bins in the second and third dimensions, respectively (illustrated by red and blue connections). Each operation of this device necessitates the specification of 1,296 real parameters, corresponding to the settings for all VBS units in the sequence. Reproduced under a Creative Commons Attribution 4.0 International License https://creativecommons.org/licenses/by/4.0/ from [47].
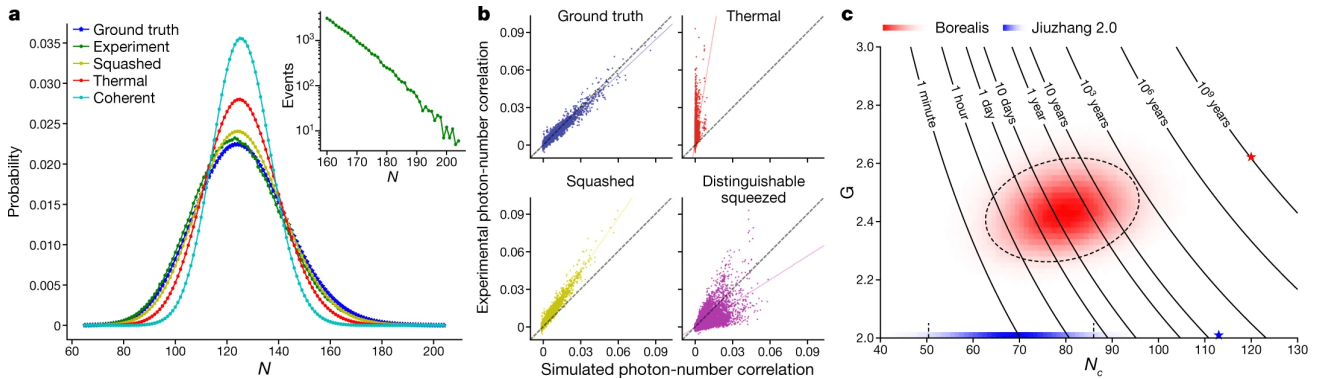


**FIGURE 9.** A comparative analysis of the quantum computational advantages achieved by Borealis vs Jiuzhang 2.0 quantum computers. a) The measured photon statistics from $10^6$ samples of a high-dimensional Gaussian state are compared with numerical simulations based on various hypotheses. The inset displays the same distribution using a logarithmic scale, revealing substantial support for photon counts extending beyond 160 photons, up to 219. b) A scatter plot illustrates the comparison between experimentally obtained two-mode cumulants $C_{ij}$ and those predicted by four different hypotheses. A perfect fit (depicted in plot) would result in experimentally obtained cumulants lying on a straight line at a 45-degree angle. Notably, only the ground truth hypothesis provides a good explanation for the cumulants. Furthermore, for a fair comparison, all hypotheses share identical first-order cumulants, i.e., the mean photon count in each mode. c) The distribution of classical simulation times is depicted for each sample obtained in this experiment, denoted as *Borealis* in red, and for *Jiuzhang* 2.0 in blue. For each sample from both experiments, a pair ($N_c$, $G$) is calculated, and a frequency histogram is constructed based on this two-dimensional space. It's important to note that all samples from *Jiuzhang* 2.0, being threshold samples, have $G = 2$, while samples from *Borealis*, which include collisions and photon-number resolution, have $G \geq 2$. The samples with the highest complexity in each experiment are marked with stars; these samples are found at the extreme end of their respective distributions and occur infrequently. The lines representing equal simulation times, based on $N_c$ and $G$, are overlaid for each experiment. Dashed lines delineate boundaries corresponding to two standard deviations in the plotted distributions to aid visualization. Reproduced under a Creative Commons Attribution 4.0 International License https://creativecommons.org/licenses/by/4.0/. from [47].

Herein, it is important to emphasize our deep appreciation for the dedicated research efforts that have contributed to the advancement of all disciplines within quantum computation and information processing. However, in this section, we specifically focus on a selection of cutting-edge quantum computing applications.
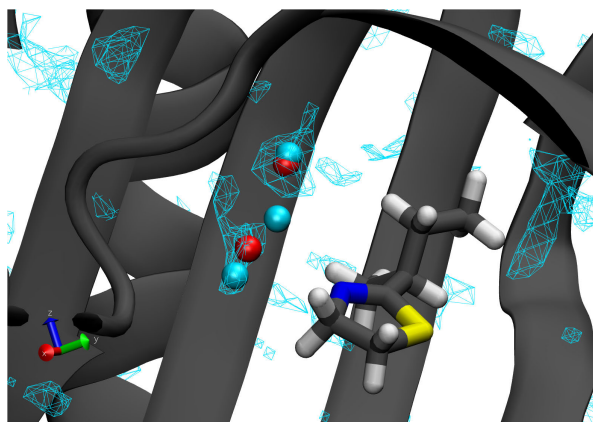
**FIGURE 10.** The MUP-I protein complexed with two-sec-butyl-4,5-dihydrothiazole, with an inside view of the protein binding pocket. Oxygen atoms of the crystal water molecules are depicted in red, while the three water molecules positioned by the HQC algorithm are in cyan. The 3D-RISM density isosurface (isovalue 5.5) is represented in a wireframe style, also in cyan. Reproduced from [168] under a creative commons licenses https://creativecommons.org/licenses/by/4.0/.

## A. QUANTUM MACHINE LEARNING

Recently in [103] a collaborative research effort involving scientists from prestigious institutions such as Harvard, Berkeley, Caltech, and Microsoft culminated in the project titled ''quantum advantage in learning from experiments.'' Within this initiative, the researchers conducted experiments employing a superconducting quantum processor, up to 40 qubits. Their findings unequivocally demonstrated a significant quantum advantage, even in the presence of noise, within contemporary NISQ platforms [48].

The primary focus of their investigation was the exploration of how quantum technology could augment our capacity to unveil hitherto unknown natural phenomena. These experimental endeavors showcased the potential of supervised and unsupervised machine learning models [121], [122] when fed with data derived from quantum-augmented experiments utilizing quantum memory and quantum processing. Remarkably, these models exhibited the capability to predict properties and unveil underlying structures within physical systems that lay beyond the purview of traditional experimentation.

Crucially, their research substantiated that quantum computing systems can acquire knowledge from a notably reduced number of experiments in comparison to conventional methodologies, a phenomenon that holds true across various tasks. This quantum advantage is most pronounced in the prediction of physical system properties, the execution of quantum principal component analysis on noisy states, and the approximation of models representing physical dynamics, all achievable with a significantly reduced experimental load.

Notably, while previous investigations into quantum advantage predominantly centered on computational tasks with predefined inputs, the researchers deviated from this paradigm by concentrating on learning tasks aimed at gaining insights into previously unknown physical systems.

This novel approach contributes fresh perspectives toward comprehending and attaining quantum advantage within the domains of Quantum Machine Learning (QML) [122], [123], [124] and quantum sensing [125].

In [126], the study synergistically melds Variational Quantum Algorithms (VQAs) with Variational Inference (VI) techniques to facilitate variational inference grounded in probability distributions that are classically challenging to compute. The practicality of this approach is substantiated through experimental validation on the five-qubit IBM quantum processor named ''*ibmq_rome*.'' In the academic literature, an extensive body of dedicated research efforts [127], [128], [129], [130], [131], [132], [133], [134], [135], [136], [137], [138], [139], [140], [141], [142], [143], [144], [145], [146], [147], [148], [149], [150], [151], [152], [153], [154], [155], [156], [157], [158], [159], [160], [161], [162], [163] has made substantial contributions to the progression of QML and Quantum Artificial Intelligence.

## B. QUANTUM SIMULATION/MATERIAL SCIENCE

Quantum computing holds the potential to simulate complex quantum systems, offering valuable insights into the behavior of molecules and materials. This has profound implications for fields such as materials science and drug discovery. Traditional classical computers face limitations in accurately modeling intricate molecular interactions. In [164], [165], and [166], a notable application involves the accelerated development of chemical products, including catalysts and surfactants, within the chemical and petroleum industries. Quantum computers are being employed by companies in these sectors, such as IBM, to model substances like beryllium hydride ($BeH_2$) and lithium hydride (LiH). These quantum-powered simulations pave the way for the creation of new catalysts aimed at emissions reduction and enhanced surfactants to improve subsurface recovery [164], [165], [166], [167].

Researchers at PASQAL have effectively conducted a trial run of the innovative quantum water placement algorithm using *Fresnel* 1, the inaugural industrial neutral atoms quantum computer developed by PASQAL. This proof-of-concept experiment [168] holds promising implications for drug discovery, as it demonstrates the potential of quantum algorithms to contribute to pharmaceutical research. In this POC experiment, the scientists selected Major Urinary Protein-1 (MUP-1), a protein primarily synthesized in the liver. Studies suggest that this protein serves a significant function in pheromone-mediated communication, as well as in the regulation of energy expenditure and metabolic processes [169]. The optimal configuration of water molecules discovered by their algorithm (depicted by cyan atoms), corresponding to the best bitstring $|101001\rangle$ identified during optimization, is illustrated in Fig. 10.

In the field of chemistry, quantum computers offer the promising potential to enhance the precision of direct molecule simulations. This advancement holds the prospect

of enabling a more accurate investigation into critical reactions and systems that currently remain beyond the reach of classical computing methods. Initial assessments suggest that, in the fault-tolerant era, quantum computers will eventually have the capacity to address such complex challenges [170], [171], [172].

There have been proposed strategies for leveraging quantum computers to solve models involving strongly correlated electrons [175]. Among the molecules garnering significant attention is FeMoco [173], [174]. Preliminary applications have utilized Variational Quantum Eigensolver (VQE) techniques to estimate the energy levels of water molecules [176] and small hydrogen chains [177], [178]. For a more exploration of the applicability of VQE to chemical problems, detailed studies can be found here [179]. Additionally, VQAs have been proposed to simulate exciton dynamics within molecules [180], [181]. For a deeper understanding of the application of quantum computers in chemistry, comprehensive review papers are available in [182] and [183].

Moreover, within the realm of high-energy physics, initial proof-of-principle simulations have shown promise in studying the temporal dynamics of a simplified Quantum Chromodynamics (QCD) model. These simulations have been conducted using both trapped ions [184] and superconducting circuits [185]. Variational Quantum Simulators (VQS) have also been harnessed for such simulations, achieving success with system sizes of up to 20 qubits and 15 variational parameters [186]. Quantum neural networks (QNN) have been proposed as tools to assist in the analysis of data generated by experiments in high-energy physics [187], [188]. For comprehensive insights into this field, valuable review papers are available [189], [190].

In recent research efforts, as discussed in [191], hybrid quantum-classical algorithms have been developed to optimize variational quantum circuits for quantum simulations, resulting in a notable increase in accuracy compared to Trotter product formulas [192], [193]. Additionally, [194] introduced hardware-efficient VQAs tailored for time evolution. In [195], the VQE was applied to address quantum many-body problems, specifically focusing on the long-range XY model. Furthermore, the study presented in [196] applied variational quantum simulation techniques to tackle partial differential equations, primarily for applications related to colloidal transport. Whereas, [197] employed VQAs for simulating waveguide modes. Efforts have also been made to mitigate estimation runtime challenges on NISQ computers, particularly during the execution of algorithms such as the VQE, as discussed in [198] and [199].

Quantum simulation finds intriguing applications in diverse domains, including nuclear physics [200], [201], quantum control [202], and material design [203], among others. For a comprehensive overview of quantum simulators, readers are referred to [205] and [204].

## C. QUANTUM NATURAL LANGUAGE PROCESSING

Cambridge Quantum Computing (CQC) has recently unveiled an outstanding advancement [206], [207] in the field of Natural Language Processing (NLP) by introducing the pioneering quantum-based natural language processing (QNLP) toolkit and library [208], [209]. This innovative toolkit, known as "*lambeq*," [208] represents a seminal achievement as the world's inaugural software toolkit capable of translating sentences into quantum circuits. Its primary objective is to expedite the development of pragmatic QNLP with real-world relevance [212]. These applications span a spectrum of domains, encompassing automated dialogue systems, text mining, language translation, text-to-speech synthesis, language generation, and bioinformatics [210], [211], [212].
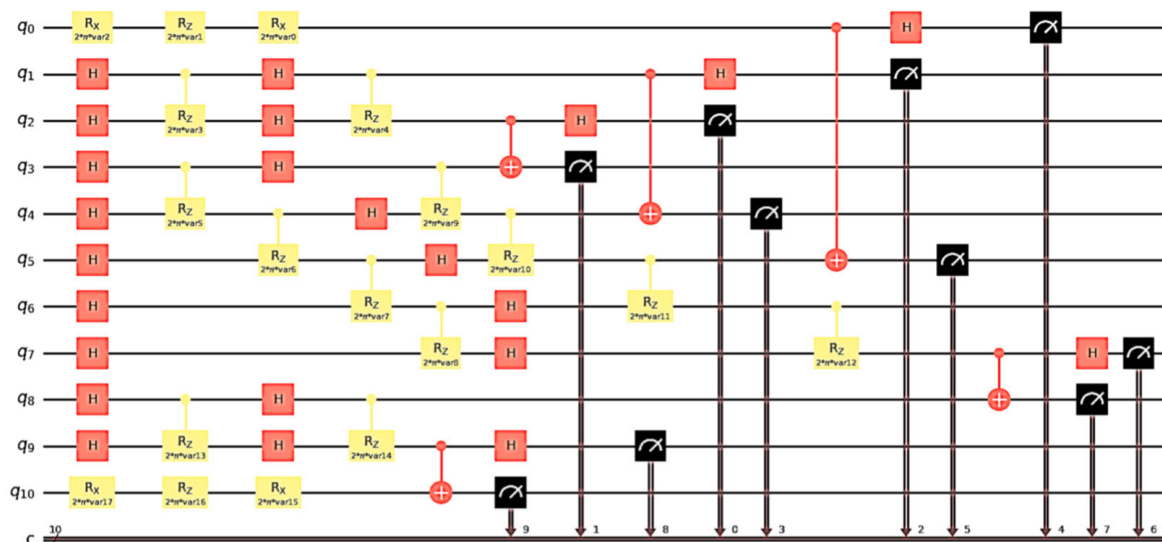
Fig. 11 presents a quantum circuit that corresponds to the text "John walks in the park" [208], affording readers a visual depiction of the toolkit's operational prowess. For an in-depth technical exposition, [208] furnishes a comprehensive exposition detailing the fundamental architecture of *lambeq*. Moreover, it offers a detailed explication of the toolkit's essential modules, highlighting their significance. Further insights into *lambeq*'s implementation details are explained in [210], [211], and [212].

## D. QUANTUM COMPUTING IN FINANCE

Quantum computing companies PASQAL [214] and Multiverse [215] Computing, in collaboration with Crédit Agricole [216], unveiled [213] the successful conclusion of a 1.5-year POC study [217]. This study aimed to assess the effectiveness of an algorithmic approach inspired by quantum computing, as well as the potential applications of quantum computers in two critical areas: the valuation of financial instruments and the evaluation of credit risks [217], [218]. The outcomes of their research highlight the development of the inaugural quantum-enhanced machine learning (QEML) algorithm designed for forecasting credit rating downgrades. These POCs [217], [218], [219] serve as tangible demonstrations of quantum computing's prospective and real-world utility within the realm of finance. In another investigation [220], quantum amplitude estimation was implemented using shallow variational quantum circuits, demonstrating superior efficiency compared to conventional classical Monte Carlo sampling within specific financial applications.

## E. SOLVING HARD GRAPH PROBLEMS

Combinatorial optimization, a prevalent challenge across scientific and technological fields, often involving problems that exhibit inherent computational complexity, serving as a cornerstone for understanding contemporary computer science complexity classes [221]. The utilization of quantum machines to expedite the solution of such problems has been explored theoretically for over two decades through various quantum algorithms [222], [223], [224]. Typically, a relevant

**FIGURE 11.** A ten-qubit quantum circuit representation that corresponds to the sentence "John walks in the park" in IBM's Qiskit format, with H stands for Hadamard gate and $R_Z$ denotes the rotation gates. Reproduced with formatting alterations under a Creative Commons license https://creativecommons.org/licenses/by/4.0/ from [208].

cost function is encoded within a quantum Hamiltonian [225], and the goal is to identify its lowest-energy state, initiated from a generic initial state. This is achieved either through adiabatic evolution [223] or a variational approach [222], involving iterative optimization processes [186], [226]. The computational performance of these algorithms has been examined both theoretically [224], [227], [228], [229], [230], [231], [232], [233] and experimentally [234], [235], [236], mostly in small quantum systems with shallow quantum circuits or systems lacking the presumed many-body coherence essential for achieving quantum advantage [237], [238]. However, these investigations offer limited insights into algorithm performance in the most interesting scenario, characterized by large system sizes and deep circuitry [239], [240].

Achieving quantum acceleration in addressing computationally challenging dilemmas stands as the primary imperative within the realm of quantum information science. Ebadi et al. [104] conducted a pioneering investigation employing Rydberg atom arrays, a neutral atom quantum processor comprising up to 289 coupled qubits in two spatial dimensions, to explore quantum optimization algorithms for tackling practical dilemmas, specifically the maximum independent set (MIS) problem. The MIS problem serves as a quintessential example of a combinatorial optimization problem that falls into the category of nondeterministic polynomial time-complex challenges.

In prior research, proposals had been advanced for the efficient encoding of specific challenging combinatorial optimization problems within neutral-atom quantum processors. However, in this groundbreaking publication [104], the authors not only executed the inaugural instance of effective quantum optimization on an operational quantum

computing platform but also demonstrated an unprecedented level of quantum hardware capabilities. These findings signify the initial stride towards harnessing valuable quantum advantages in addressing demanding optimization issues that hold relevance across various industrial sectors.

Additionally, one prominent problem in this realm is the NP-complete Max-Cut problem [241], which has garnered substantial attention in the quantum computing community. The Quantum Approximate Optimization Algorithm (QAOA) was introduced for solving the Max-Cut problem and has been demonstrated on instances involving up to 19 qubits [242]. Subsequent research has extended the QAOA's applicability to the Max-Cut problem [243], [244], [245], [246].

In [247], it is demonstrated that combining VQE with filtering operators (F-VQE) yields significantly enhanced efficiency and accuracy when compared to standard VQE and QAOA approaches, particularly in the context of solving Max-Cut problems. The experimental viability of these F-VQE algorithms is confirmed through implementation on a Honeywell trapped-ion quantum processor [248]. Whereas, a novel variant of QAOA known as XQAOA was introduced in [249], demonstrating its superior performance compared to traditional QAOA and other variants, such as multi-angle QAOA, on specific Max-Cut problem instances.

Quantum computers have also been investigated for tackling other combinatorial optimization problems, including the knapsack problem [250], protein folding [251], [252], [253], the k-clique problem [254], and graph coloring [255].

### F. LOGISTICS OPTIMIZATION
Quantum computing, a game-changing technology, is finding applications in logistics and manufacturing, promising to

revolutionize optimization tasks. Quantum-South, a quantum technology company, exemplifies this potential. In June 2022, Quantum-South showcased their breakthrough in cargo loading optimization using quantum annealers through Amazon Braket [256]. This approach improves cargo plans, potentially streamlining the allocation of passengers, weight, and volume on flights, boosting operational efficiency and reducing analysis time [257].

Quantum-South's pioneering work has not gone unnoticed in the industry. IAG Cargo [258], a division of International Airlines Group, trialed Quantum-South's quantum algorithms for air cargo optimization, highlighting growing industry interest. Additionally, Quantum-South explored ULD flight loading optimization through quantum computing, customizing algorithms to streamline the loading process in partnership with IAG Cargo [105], [257], [259], [260]. These developments underscore the transformative impact of quantum computing in logistics and manufacturing.

### G. QUANTUM NUMERICAL SOLVERS

Quantum computing has demonstrated its theoretical capability to achieve remarkable speed improvements in specific numerical problem-solving tasks [261]. Notably, the HHL algorithm and subsequent enhancements promise an exponential acceleration in solving linear systems compared to classical computer solvers [262], [263]. In addition to these advancements, alternative approaches utilizing VQAs have been proposed to address linear system problems [264], [265], [266]. Furthermore, quantum-assisted methods have exhibited proficiency in solving classically solvable systems, including those with dimensions as large as $2^{300} \times 2^{300}$ [267].

Classical computers often require substantial computational resources to solve non-linear differential equations [268]. However, POC results from experiments conducted on an IBM Quantum's device suggest that variational quantum computing can efficiently tackle such non-linear problems, exemplified by the nonlinear Schrödinger equation [269]. Moreover, the scientific community is exploring differentiable quantum circuits as a means to address complex problems [270], with considerable interest directed towards employing near-term quantum algorithms for solving intricate equations like the Navier-Stokes equations [271], [272]. Quantum computers exhibit potential in several other numerical problem-solving applications, as evidenced by a body of literature encompassing [268], [269], [270], [271], [272], [273], [274], [275], [276], [277], [278].

The horizons of quantum computing are marked by endless possibilities, and its ascent holds the promise of reshaping our understanding of what is achievable across numerous scientific, industrial, and technological frontiers.

## VI. CONCLUSION AND OUTLOOK

Noisy intermediate-scale quantum computers have set the stage for a technological revolution, garnering significant attention and investment. This is due to its potential to tackle complex problems–more effectively than conventional computing schemes–with far-reaching implications across various sectors, from scientific breakthroughs to national security. This has prompted research agencies and tech giants to establish quantum computing departments. However, as we stand at the intersection of quantum dreams and real-world challenges, it is essential to acknowledge that errors, challenges in experimental control techniques, and the limited number of qubits remain substantial obstacles that hinder the realization of the full potential of current NISQ computers.

The road to fault-tolerant quantum (FTQ) computers, which are quantum computers capable of handling these intricate challenges, necessitates substantial advancements in refining experimental control techniques for qubit gate operations, qubit quality, error correction, and a significant increase in the number of available qubits. While recent experimental progress has sparked optimism, building a fault-tolerant quantum computer is a demanding endeavor, likely requiring at least another decade of tireless development.

In the era of rapid technological advancement, quantum computing appears to defy the trend by progressing more gradually. The presented proof-of-concept cases underscore the immense potential of quantum computing and quantum processors. Despite recent strides in quantum hardware and algorithms, practical applications remain limited. However, as quantum computing technologies continue to advance, we anticipate the emergence of more practical use cases. We should closely monitor developments in academia and eagerly await well-researched applications from leading organizations worldwide. These applications may soon bring quantum computing into our lives in transformative and positive ways.

The journey toward unlocking the full potential of quantum computing is ongoing, and its ultimate impact remains a promising prospect on the horizon. As we bid farewell to NISQ era and delve deeper into the FTQ realm, we find ourselves at the doorstep of an exceptional future, where things that seem impossible today may soon become not only possible but routine.

Raytheon Tech Corporation, SEEQC Inc., MIT, Oxford, Yale, Harvard, Caltech, Stanford, University of Maryland, University of Wisconsin, Technical University of Delft, or any affiliated organizations. "This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors."

## AUTHOR CONTRIBUTIONS

Muhammad AbuGhanem: Conceptualization, Literature Review, Methodology, Resources, Data Curation, Critical Analysis, Visualization, Investigation, Validation, Writing, Review and Editing. Hichem Eleuch: Investigation, Validation, Review and editing. The authors have approved the final manuscript.

## COMPETING INTERESTS

The authors declare no competing interests.

## REFERENCES

[1] D. P. DiVincenzo, "The physical implementation of quantum computation," *Fortschritte Der Physik*, vol. 48, nos. 9–11, pp. 771–783, Sep. 2000.

[2] D. P. DiVincenzo, D. Bacon, J. Kempe, G. Burkard, and K. B. Whaley, "Universal quantum computation with the exchange interaction," *Nature*, vol. 408, no. 6810, pp. 339–342, Nov. 2000, doi: 10.1038/35042541.

[3] D. DeMille, "Quantum computation with trapped polar molecules," *Phys. Rev. Lett.*, vol. 88, no. 6, Jan. 2002, Art. no. 067901.

[4] R. Raussendorf, D. E. Browne, and H. J. Briegel, "Measurement-based quantum computation on cluster states," *Phys. Rev. A, Gen. Phys.*, vol. 68, no. 2, Aug. 2003, Art. no. 022312.

[5] G. Vidal, "Efficient classical simulation of slightly entangled quantum computations," *Phys. Rev. Lett.*, vol. 91, no. 14, Oct. 2003, Art. no. 022312.

[6] A. Blais, R.-S. Huang, A. Wallraff, S. M. Girvin, and R. J. Schoelkopf, "Cavity quantum electrodynamics for superconducting electrical circuits: An architecture for quantum computation," *Phys. Rev. A, Gen. Phys.*, vol. 69, no. 6, Jun. 2004, Art. no. 022312.

[7] P. Walther, K. J. Resch, T. Rudolph, E. Schenck, H. Weinfurter, V. Vedral, M. Aspelmeyer, and A. Zeilinger, "Experimental one-way quantum computing," *Nature*, vol. 434, no. 7030, pp. 169–176, Mar. 2005.

[8] A. Sørensen and K. Mølmer, "Quantum computation with ions in thermal motion," *Phys. Rev. Lett.*, vol. 82, no. 9, pp. 1971–1974, Mar. 1999.

[9] S. L. Braunstein, C. M. Caves, R. Jozsa, N. Linden, S. Popescu, and R. Schack, "Separability of very noisy mixed states and implications for NMR quantum computing," *Phys. Rev. Lett.*, vol. 83, no. 5, pp. 1054–1057, Aug. 1999.

[10] P. Zanardi and M. Rasetti, "Holonomic quantum computation," *Phys. Lett. A*, vol. 264, nos. 2–3, pp. 94–99, Dec. 1999.

[11] R. Vrijen, E. Yablonovitch, K. Wang, H. W. Jiang, A. Balandin, V. Roychowdhury, T. Mor, and D. Divincenzo, "Electron-spin-resonance transistors for quantum computing in silicon-germanium heterostructures," *Phys. Rev. A, Gen. Phys.*, vol. 62, Sep. 2000, Art. no. 012306.

[12] L. B. Ioffe, V. B. Geshkenbein, M. V. Feigel'man, A. L. Fauchère, and G. Blatter, "Environmentally decoupled SDS-wave Josephson junctions for quantum computing," *Nature*, vol. 398, no. 6729, pp. 679–681, Apr. 1999, doi: 10.1038/19464.

[13] D. Kielpinski, C. Monroe, and D. J. Wineland, "Architecture for a large-scale ion-trap quantum computer," *Nature*, vol. 417, no. 6890, pp. 709–711, Jun. 2002, doi: 10.1038/nature00784.

[14] J. A. Jones, V. Vedral, A. Ekert, and G. Castagnoli, "Geometric quantum computation using nuclear magnetic resonance," *Nature*, vol. 403, no. 6772, pp. 869–871, Feb. 2000, doi: 10.1038/35002528.

[15] R. Raussendorf and H. J. Briegel, "A one-way quantum computer," *Phys. Rev. Lett.*, vol. 86, no. 22, pp. 5188–5191, May 2001.

[16] M. N. Leuenberger and D. Loss, "Quantum computing in molecular magnets," *Nature*, vol. 410, no. 6830, pp. 789–793, Apr. 2001, doi: 10.1038/35071024.

[17] E. Knill, R. Laflamme, and G. J. Milburn, "A scheme for efficient quantum computation with linear optics," *Nature*, vol. 409, no. 6816, pp. 46–52, Jan. 2001, doi: 10.1038/35051009.

[18] A. Y. Kitaev, "Fault-tolerant quantum computation by anyons," *Ann. Phys.*, vol. 303, no. 1, pp. 2–30, Jan. 2003.

[19] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information*. Cambridge, U.K.: Cambridge Univ. Press, 2011.

[20] D. P. D. Vincenzo, "Quantum computation," *Science*, vol. 270, Sep. 1995, Art. no. 5234.

[21] A. Barenco, C. H. Bennett, R. Cleve, D. P. DiVincenzo, N. Margolus, P. Shor, T. Sleator, J. A. Smolin, and H. Weinfurter, "Elementary gates for quantum computation," *Phys. Rev. A, Gen. Phys.*, vol. 52, no. 5, pp. 3457–3467, Nov. 1995.

[22] T. D. Ladd, F. Jelezko, R. Laflamme, Y. Nakamura, C. Monroe, and J. L. O'Brien, "Quantum computers," *Nature*, vol. 464, p. 45, Jul. 2010.

[23] M. AbuGhanem and H. Eleuch, "Two-qubit entangling gates for superconducting quantum computers," *Results Phys.*, vol. 56, Aug. 2024, Art. no. 107236.

[24] M. AbuGhanem, "Properties of some quantum computing models," M.S. thesis, Dept. Math., Fac. Sci., Ain Shams Univ., Cairo, Egypt, 2019.

[25] I. Chuang, "Building the building blocks," *Nature Phys.*, vol. 14, p. 974, Sep. 2018.

[26] R. P. Feynman, *Feynman and Computation. Simulating Physics With Computers*. Evanston, IL, USA: Routledge, 2018, p. 133153.

[27] P. Benioff, "The computer as a physical system: A microscopic quantum mechanical Hamiltonian model of computers as represented by Turing machines," *J. Stat. Phys.*, vol. 22, no. 5, pp. 563–591, May 1980.

[28] D. Deutsch and R. Jozsa, "Rapid solution of problems by quantum computation," *Proc. R. Soc. Lond. A Math. Phys. Sci.*, vol. 439, no. 1907, 1992, Art. no. 553558.

[29] D. Deutsch, "Quantum theory, the church Turing principle and the universal quantum computer," *Proc. R. Soc. Lond. A Math. Phys. Sci.*, vol. 400, no. 1818, 1985, Art. no. 97117.

[30] E. Bernstein and U. Vazirani, "Quantum complexity theory," *SIAM J. Comput.*, vol. 26, no. 5, Sep. 1997, Art. no. 14111473.

[31] D. R. Simon, "On the power of quantum computation," *SIAM J. Comput.*, vol. 26, no. 5, 1997, Art. no. 14741483.

[32] A. W. Harrow and A. Montanaro, "Quantum computational supremacy," *Nature*, vol. 549, no. 7671, pp. 203–209, Sep. 2017.

[33] M. AbuGhanem, *Information Processing At the Speed of Light*. Amsterdam, The Netherlands: Elsevier, 2024.

[34] J. Preskill, "Quantum computing and the entanglement frontier," 2012, *arXiv:1203.5813*.

[35] C. Neill et al., "A blueprint for demonstrating quantum supremacy with superconducting qubits," *Science*, vol. 360, no. 6385, pp. 195–199, Apr. 2018.

[36] F. Arute, K. Arya, R. Babbush, D. Bacon, and J. C. Bardin, "Quantum supremacy using a programmable superconducting processor," *Nature*, vol. 574, Jun. 2019, Art. no. 505510.

[37] H.-S. Zhong et al., "Quantum computational advantage using photons," 2020, *arXiv:2012.01625*.

[38] A. Deshpande, A. Mehta, T. Vincent, N. Quesada, M. Hinsche, M. Ioannou, L. Madsen, J. Lavoie, H. Qi, J. Eisert, D. Hangleiter, B. Fefferman, and I. Dhand, "Quantum computational advantage via high-dimensional Gaussian boson sampling," *Sci. Adv.*, vol. 8, no. 1, Jan. 2022, Art. no. eabi7894.

[39] H.-S. Zhong et al., "Phase-programmable Gaussian boson sampling using stimulated squeezed light," 2021, *arXiv:2106.15534*.

[40] Y. Wu et al., "Strong quantum computational advantage using a superconducting quantum processor," *Phys. Rev. Lett.*, vol. 127, no. 1, 2021, Art. no. 180501.

[41] Q. Zhu et al., "Quantum computational advantage via 60-qubit 24-cycle random circuit sampling," *Sci. Bull.*, vol. 67, no. 3, pp. 240–245, Feb. 2022.

[42] B. W. Reichardt, "Fault-tolerant quantum computation," in *Encyclopedia of Algorithms*. Cham, Switzerland: Springer, 2008.

[43] A. Paler and S. J. Devitt, "An introduction to fault-tolerant quantum computing," 2015, *arXiv:1508.03695*.

[44] C. Q. Choi, "The race to build a fault-tolerant superconducting quantum computer: Amazon, Google, and IBM are all pursuing different strategies to reduce error rates," *IEEE Spectr.*, vol. 1, no. 1, pp. 1–15, Aug. 2022.

[45] D. L. Underwood et al., "Using cryogenic CMOS control electronics to enable a two-qubit cross-resonance gate," 2023, *arXiv:2302.11538*.

[46] (2022). *IBM Unveils 400 Qubit-Plus Quantum Processor and Next-Generation IBM Quantum System*. Accessed: Nov. 9, 2022. [Online]. Available: https://newsroom.ibm.com/2022-11-09-IBM-Unveils-400-Qubit-Plus-Quantum-Processor-and-Next-Generation-IBM-Quantum-System-Two

[47] L. S. Madsen, F. Laudenbach, M. F. Askarani, F. Rortais, T. Vincent, J. F. F. Bulmer, F. M. Miatto, L. Neuhaus, L. G. Helt, M. J. Collins, A. E. Lita, T. Gerrits, S. W. Nam, V. D. Vaidya, M. Menotti, I. Dhand, Z. Vernon, N. Quesada, and J. Lavoie, "Quantum computational advantage with a programmable photonic processor," *Nature*, vol. 606, no. 7912, pp. 75–81, Jun. 2022.

[48] J. Preskill, "Quantum computing in the NISQ era and beyond," *Quantum*, vol. 2, p. 79, Aug. 2018.

[49] M. H. Devoret and R. J. Schoelkopf, "Superconducting circuits for quantum information: An outlook," *Science*, vol. 339, no. 6124, pp. 1169–1174, Mar. 2013.

[50] J. Kim, D. Min, J. Cho, H. Jeong, I. Byun, J. Choi, J. Hong, and J. Kim, "A fault-tolerant million qubit-scale distributed quantum computer," in *Proc. 29th ACM Int. Conf. Architectural Support Program. Lang. Operating Syst.*, Apr. 2024, pp. 1–23, doi: 10.1145/3620665.3640388.

[51] J. S. Clarke, "An optimists view of the 4 challenges to quantum computing," *IEEE Spectr.*, vol. 1, no. 1, pp. 1–18, Nov. 2019.

[52] D. Franklin and F. T. Chong, "Challenges in reliable quantum computing," in *Nano, Quantum and Molecular Computing*. Cham, Switzerland: Springer, 2004, pp. 247–266.

[53] M. Grilli, A. Gröger, B. Jurgens, C. Soltmann, and K. Herreman, "Quantum simulation insight report," Eur. Pat. Office, Munich, Germany, 2023. [Online]. Available: https://link.epo.org/web/epo_patent_insight_report_quantum_simulation_en.pdf

[54] M. Aboy, T. Minssen, and M. Kop, "Mapping the patent landscape of quantum technologies: Patenting trends, innovation and policy implications," *Int. Review Intellectual Property Competition Law*, vol. 53, Oct. 2022, Art. no. 853882.

[55] S. Aaronson and A. Arkhipov, "The computational complexity of linear optics," in *Proc. 43rd Annu. ACM Symp. Theory Comput.*, Jun. 2011, pp. 333–342, doi: 10.1145/1993636.1993682.

[56] C. S. Hamilton, R. Kruse, L. Sansoni, S. Barkhofen, C. Silberhorn, and I. Jex, "Gaussian boson sampling," *Phys. Rev. Lett.*, vol. 119, no. 17, 2017, Art. no. 170501.

[57] R. Kruse, C. S. Hamilton, L. Sansoni, S. Barkhofen, C. Silberhorn, and I. Jex, "Detailed study of Gaussian boson sampling," *Phys. Rev. A, Gen. Phys.*, vol. 100, no. 3, Sep. 2019, Art. no. 032326.

[58] B. Villalonga, M. Yuezhen Niu, L. Li, H. Neven, J. C. Platt, V. N. Smelyanskiy, and S. Boixo, "Efficient approximation of experimental Gaussian boson sampling," 2021, arXiv:2109.11525.

[59] J. Huh, G. G. Guerreschi, B. Peropadre, J. R. McClean, and A. Aspuru-Guzik, "Boson sampling for molecular vibronic spectra," *Nature Photon.*, vol. 9, no. 9, pp. 615–620, Sep. 2015.

[60] T. R. Bromley, J. M. Arrazola, S. Jahangiri, J. Izaac, N. Quesada, A. D. Gran, M. Schuld, J. Swinarton, Z. Zabaneh, and N. Killoran, "Applications of near-term photonic quantum computers: Software and algorithms," *Quantum Sci. Technol.*, vol. 5, no. 3, May 2020, Art. no. 034010.

[61] J. M. Arrazola and T. R. Bromley, "Using Gaussian boson sampling to find dense subgraphs," *Phys. Rev. Lett.*, vol. 121, no. 3, Jul. 2018, Art. no. 030503.

[62] S. Jahangiri, J. M. Arrazola, N. Quesada, and N. Killoran, "Point processes with Gaussian boson sampling," *Phys. Rev. E, Stat. Phys. Plasmas Fluids Relat. Interdiscip. Top.*, vol. 101, no. 2, Feb. 2020, Art. no. 022134.

[63] L. Banchi, M. Fingerhuth, T. Babej, C. Ing, and J. M. Arrazola, "Molecular docking with Gaussian boson sampling," *Sci. Adv.*, vol. 6, no. 23, Jun. 2020, Art. no. eaax1950.

[64] L. Banchi, N. Quesada, and J. M. Arrazola, "Training Gaussian boson sampling distributions," *Phys. Rev. A, Gen. Phys.*, vol. 102, no. 1, Jul. 2020, Art. no. 012414.

[65] S. Jahangiri, J. M. Arrazola, N. Quesada, and A. Delgado, "Quantum algorithm for simulating molecular vibrational excitations," *Phys. Chem. Chem. Phys.*, vol. 22, no. 44, pp. 25528–25537, Nov. 2020.

[66] A. Bouland, B. Fefferman, C. Nirkhe, and U. Vazirani, "On the complexity and verification of quantum random circuit sampling," *Nature Phys.*, vol. 15, no. 2, pp. 159–163, Feb. 2019.

[67] S. Boixo, S. V. Isakov, V. N. Smelyanskiy, R. Babbush, N. Ding, Z. Jiang, M. J. Bremner, J. M. Martinis, and H. Neven, "Characterizing quantum supremacy in near-term devices," *Nature Phys.*, vol. 14, no. 6, pp. 595–600, Jun. 2018.

[68] I. L. Markov, A. Fatima, S. V. Isakov, and S. Boixo, "Quantum supremacy is both closer and farther than it appears," 2018, arXiv:1807.10749.

[69] A. Bouland, B. Fefferman, C. Nirkhe, and U. Vazirani, "Quantum supremacy and the complexity of random circuit sampling," 2018, arXiv:1803.04402.

[70] R. Movassagh, "Efficient unitary paths and quantum computational supremacy: A proof of average-case hardness of random circuit sampling," 2018, arXiv:1810.04681.

[71] M. J. Bremner, A. Montanaro, and D. J. Shepherd, "Average-case complexity versus approximate simulation of commuting quantum computations," *Phys. Rev. Lett.*, vol. 117, no. 8, Aug. 2016, Art. no. 080501.

[72] M. J. Bremner, A. Montanaro, and D. J. Shepherd, "Achieving quantum supremacy with sparse and noisy commuting quantum computations," *Quantum*, vol. 1, p. 8, Apr. 2017.

[73] R. Movassagh, "Quantum supremacy and random circuits," 2019, arXiv:1909.06210.

[74] A. Zlokapa, B. Villalonga, S. Boixo, and D. A. Lidar, "Boundaries of quantum supremacy via random circuit sampling," *Npj Quantum Inf.*, vol. 9, no. 1, pp. 1–36, Apr. 2023.

[75] M. P. A. Fisher, V. Khemani, A. Nahum, and S. Vijay, "Random quantum circuits," *Annu. Rev. Condens. Matter Phys.*, vol. 14, pp. 335–379, Oct. 2023.

[76] M. Zhang, C. Wang, and Y. Han, "Noisy random quantum circuit sampling and its classical simulation," *Adv. Quantum Technol.*, vol. 6, no. 7, Jul. 2023, Art. no. 2300030.

[77] E. Pednault, J. A. Gunnels, G. Nannicini, L. Horesh, and R. Wisnieff, "Leveraging secondary storage to simulate deep 54-qubit sycamore circuits," 2019, arXiv:1910.09534.

[78] C. Huang, F. Zhang, M. Newman, J. Cai, X. Gao, Z. Tian, J. Wu, H. Xu, H. Yu, B. Yuan, M. Szegedy, Y. Shi, and J. Chen, "Classical simulation of quantum supremacy circuits," 2020, arXiv:2005.06787.

[79] F. Pan, K. Chen, and P. Zhang, "Solving the sampling problem of the sycamore quantum circuits," *Phys. Rev. Lett.*, vol. 129, no. 9, Aug. 2022, Art. no. 090502.

[80] A. M. Dalzell, A. W. Harrow, D. E. Koh, and R. L. La Placa, "How many qubits are needed for quantum computational supremacy?" *Quantum*, vol. 4, p. 264, May 2020.

[81] J. D. Biamonte, M. E. S. Morales, and D. E. Koh, "Entanglement scaling in quantum advantage benchmarks," *Phys. Rev. A, Gen. Phys.*, vol. 101, no. 1, Jan. 2020, Art. no. 012349.

[82] J. Koch, T. M. Yu, J. Gambetta, A. A. Houck, D. I. Schuster, J. Majer, A. Blais, M. H. Devoret, S. M. Girvin, and R. J. Schoelkopf, "Charge-insensitive qubit design derived from the Cooper pair box," *Phys. Rev. A, Gen. Phys.*, vol. 76, no. 4, Oct. 2007, Art. no. 012349.

[83] A. Wallraff, D. I. Schuster, A. Blais, L. Frunzio, R.-S. Huang, J. Majer, S. Kumar S. M. Girvin, and R. J. Schoelkopf, "Strong coupling of a single photon to a superconducting qubit using circuit quantum electrodynamics," *Nature*, vol. 431, no. 7005, pp. 162–167, Sep. 2004.

[84] Y. Chen et al., "Qubit architecture with high coherence and fast tunable coupling," *Phys. Rev. Lett.*, vol. 113, no. 22, Nov. 2014, Art. no. 220502.

[85] F. Yan, P. Krantz, Y. Sung, M. Kjaergaard, D. L. Campbell, T. P. Orlando, S. Gustavsson, and W. D. Oliver, "Tunable coupling scheme for implementing high-fidelity two-qubit gates," *Phys. Rev. A, Gen. Phys.ppl.*, vol. 10, no. 5, Nov. 2018, Art. no. 054062.

[86] M. AbuGhanem, "Experimental characterization of Google's sycamore quantum AI on an IBM's quantum computer," *SSRN Electron. J.*, vol. 1, no. 1, pp. 1–27, 2139, doi: 10.2139/ssrn.4299338.

[87] M. AbuGhanem and H. Eleuch, "Full quantum tomography study of Google's sycamore gate on IBMs quantum computers," *EPJ Quantum Technol.*, vol. 11, no. 1, pp. 1–36, 2024.

[88] A. Morvan et al., "Phase transition in random circuit sampling," 2023, arXiv:2304.11119.

[89] J. Gray and S. Kourtis et al., "Hyper-optimized tensor network contraction," *Quantum*, vol. 5, p. 410, Oct. 2021.

[90] J. Chen, F. Zhang, C. Huang, M. Newman, and Y. Shi, "Classical simulation of intermediate-size quantum circuits," 2018, arXiv:1805.01450.

[91] B. Villalonga, S. Boixo, B. Nelson, C. Henze, E. Rieffel, R. Biswas, and S. Mandra, "A flexible high-performance simulator for verifying and benchmarking quantum circuits implemented on real hardware," *Npj Quantum Inf.*, vol. 5, pp. 1–86, Nov. 2019.

[92] H. Fu, J. Liao, J. Yang, L. Wang, Z. Song, X. Huang, C. Yang, W. Xue, F. Liu, F. Qiao, W. Zhao, X. Yin, C. Hou, C. Zhang, W. Ge, J. Zhang, Y. Wang, C. Zhou, and G. Yang, "The sunway TaihuLight supercomputer: System and applications," *Sci. China Inf. Sci.*, vol. 59, no. 7, Jul. 2016, Art. no. 072001.

[93] N. Quesada, J. M. Arrazola, and N. Killoran, "Gaussian boson sampling using threshold detectors," *Phys. Rev. A, Gen. Phys.*, vol. 98, no. 6, Dec. 2018, Art. no. 062322.

[94] J. F. F. Bulmer, B. A. Bell, R. S. Chadwick, A. E. Jones, D. Moise, A. Rigazzi, J. Thorbecke, U.-U. Haus, T. Van Vaerenbergh, R. B. Patel, I. A. Walmsley, and A. Laing, "The boundary for quantum advantage in Gaussian boson sampling," *Sci. Adv.*, vol. 8, no. 4, pp. 1–26, Jan. 2022.

[95] A. Lamas-Linares, J. C. Howell, and D. Bouwmeester, "Stimulated emission of polarization-entangled photons," *Nature*, vol. 412, no. 6850, pp. 887–890, Aug. 2001.

[96] Y.-H. Deng et al., "Gaussian boson sampling with pseudo-photon-number-resolving detectors and quantum computational advantage," *Phys. Rev. Lett.*, vol. 131, pp. 1–26, Oct. 2023.

[97] O. Vaughan, "A platform for quantum computing," *Nature Electron.*, vol. 6, no. 5, p. 337, May 2023, doi: 10.1038/s41928-023-00974-4.

[98] (2023). *Xanadu*. [Online]. Available: https://www.xanadu.ai

[99] (2023). *IBM Quantum*. [Online]. Available: https://quantum-computing.ibm.com

[100] (2023). *Rigetti Computing*. [Online]. Available: https://www.rigetti.com

[101] A. I. Gircha, A. S. Boev, K. Avchaciov, P. O. Fedichev, and A. K. Fedorov, "Hybrid quantum-classical machine learning for generative chemistry and drug design," *Sci. Rep.*, vol. 13, no. 1, pp. 1–32, May 2023.

[102] N. Stamatopoulos, D. J. Egger, Y. Sun, C. Zoufal, R. Iten, N. Shen, and S. Woerner, "Option pricing using quantum computers," *Quantum*, vol. 4, p. 291, Aug. 2020.

[103] H.-Y. Huang, M. Broughton, J. Cotler, S. Chen, J. Li, M. Mohseni, H. Neven, R. Babbush, R. Kueng, J. Preskill, and J. R. McClean, "Quantum advantage in learning from experiments," *Science*, vol. 376, no. 6598, pp. 1182–1186, Jun. 2022.

[104] S. Ebadi et al., "Quantum optimization of maximum independent set using Rydberg atom arrays," *Science*, vol. 376, no. 6598, pp. 1209–1215, Jun. 2022.

[105] M. Machin, "Quantum-South unveils enhanced air cargo optimization solutions, harnessing the power of quantum inspired algorithms," *Quantum-South*, vol. 1, no. 1, pp. 1–28, Jul. 11, 2023.

[106] C. Portmann and R. Renner, "Security in quantum cryptography," *Rev. Mod. Phys.*, vol. 94, pp. 1–36, Nov. 2022, Art. no. 025008.

[107] J. Yin et al., "Entanglement-based secure quantum cryptography over 1,120 kilometres," *Nature*, vol. 582, no. 7813, pp. 501–505, Jun. 2020, doi: 10.1038/s41586-020-2401-y.

[108] M. Sasaki et al., "Field test of quantum key distribution in the Tokyo QKD network," *Opt. Exp.*, vol. 19, pp. 10387–10409, Sep. 2011.

[109] H. K. Lo, M. Curty, and K. Tamaki, "Secure quantum key distribution," *Nature Photon.*, vol. 8, p. 595, Jul. 2014.

[110] H.-L. Yin, T.-Y. Chen, Z.-W. Yu, H. Liu, L.-X. You, Y.-H. Zhou, S.-J. Chen, Y. Mao, M.-Q. Huang, W.-J. Zhang, H. Chen, M. J. Li, D. Nolan, F. Zhou, X. Jiang, Z. Wang, Q. Zhang, X.-B. Wang, and J.-W. Pan, "Measurement-Device-Independent quantum key distribution over a 404 km optical fiber," *Phys. Rev. Lett.*, vol. 117, no. 19, Nov. 2016, Art. no. 190501.

[111] J. Yin et al., "Satellite-based entanglement distribution over 1200 kilometers," *Science*, vol. 356, no. 6343, pp. 1140–1144, Jun. 2017.

[112] J. Yin et al., "Satellite-to-ground entanglement-based quantum key distribution," *Phys. Rev. Lett.*, vol. 119, Nov. 2017, Art. no. 200501.

[113] S.-K. Liao et al., "Space-to-Ground quantum key distribution using a small-sized payload on Tiangong-2 space lab," *Chin. Phys. Lett.*, vol. 34, no. 9, Aug. 2017, Art. no. 090302.

[114] S. K. Liao et al., "Satellite-to-ground quantum key distribution," *Nature*, vol. 549, no. 1, p. 43, 2017.

[115] A. Boaron, G. Boso, D. Rusca, C. Vulliez, C. Autebert, M. Caloz, M. Perrenoud, G. Gras, F. Bussières, M.-J. Li, D. Nolan, A. Martin, and H. Zbinden, "Secure quantum key distribution over 421 km of optical fiber," *Phys. Rev. Lett.*, vol. 121, no. 19, Nov. 2018, Art. no. 190502.

[116] F. Xu, X. Ma, Q. Zhang, H.-K. Lo, and J.-W. Pan, "Secure quantum key distribution with realistic devices," *Rev. Modern Phys.*, vol. 92, no. 2, May 2020, Art. no. 025002.

[117] M. AbuGhanem, "Comprehensive characterization of three-qubit Grover search algorithm on IBM's 127-qubit superconducting quantum computers," 2024, arXiv:2406.16018.

[118] J. W. Z. Lau, K. H. Lim, H. Shrotriya, and L. C. Kwek, "NISQ computing: Where are we and where do we go?" *AAPPS Bull.*, vol. 32, no. 1, p. 27, Sep. 2022.

[119] J. Dargan, "5 crucial quantum computing applications & examples," *Quantum Insider*, vol. 1, pp. 1–16, May 2023.

[120] A. Beniza, "Industry applications of quantum computing," *IEEE Comput. Soc.*, vol. 1, no. 2, pp. 1–63, May 2022.

[121] P. Setoodeh, S. Habibi, and S. Haykin, "Deep learning," in *Nonlinear Filters: Theory and Applications*. Cambridge, MA, USA: MIT Press, 2022, pp. 113–140.

[122] R. E. Schapire and Y. Freund, "Foundations of machine learning," in *Boosting: Foundations and Algorithms*. Cambridge, MA, USA: MIT Press, 2012, pp. 23–52.

[123] J. Biamonte, P. Wittek, N. Pancotti, P. Rebentrost, N. Wiebe, and S. Lloyd, "Quantum machine learning," *Nature*, vol. 549, Aug. 2017, Art. no. 195202.

[124] M. Broughton et al., "TensorFlow quantum: A software framework for quantum machine learning," 2020, arXiv:2003.02989.

[125] C. L. Degen, F. Reinhard, and P. Cappellaro, "Quantum sensing," *Rev. Mod. Phys.*, vol. 89, Jun. 2017, Art. no. 035002.

[126] M. Benedetti, B. Coyle, M. Fiorentini, M. Lubasch, and M. Rosenkranz, "Variational inference with a quantum computer," *Phys. Rev. A, Gen. Phys.ppl.*, vol. 16, no. 4, Oct. 2021, Art. no. 044057.

[127] S. C. Kak, "Quantum neural computing," *Adv. Imag. Electron Phys.*, vol. 94, Oct. 1995, Art. no. 259313.

[128] D. Ventura, "Quantum associative memory," *Inf. Sci.*, vol. 124, nos. 1–4, pp. 273–296, May 2000.

[129] C. A. Trugenberger, "Quantum pattern recognition," *Quantum Inf. Process.*, vol. 1, no. 6, Aug. 2002, Art. no. 471493.

[130] R. A. Servedio and S. J. Gortler, "Equivalences and separations between quantum and classical learnability," *SIAM J. Comput.*, vol. 33, no. 5, pp. 1067–1092, Jan. 2004.

[131] R. Schützhold, "Pattern recognition on a quantum computer," *Phys. Rev. A*, vol. 67, no. 6, Jun. 2003, Art. no. 062311.

[132] M. Denil and N. De Freitas. (2011). *Toward the Implementation of a Quantum RBM*. [Online]. Available: https://www.cs.ubc.ca/~nando/papers/quantumrbm.pdf

[133] N. Wiebe, D. Braun, and S. Lloyd, "Quantum algorithm for data fitting," *Phys. Rev. Lett.*, vol. 109, no. 5, 2012, Art. no. 050505.

[134] G. Sentís, J. Calsamiglia, R. Muñoz-Tapia, and E. Bagan, "Quantum learning without quantum memory," *Sci. Rep.*, vol. 2, no. 1, pp. 1–69, Oct. 2012.

[135] E. Aïmeur, G. Brassard, and S. Gambs, "Quantum speed-up for unsupervised learning," *Mach. Learn.*, vol. 90, no. 2, pp. 261–287, Feb. 2013.

[136] S. Lloyd, M. Mohseni, and P. Rebentrost, "Quantum algorithms for supervised and unsupervised machine learning," 2013, arXiv:1307.0411.

[137] N. Wiebe, A. Kapoor, and K. M. Svore, "Quantum deep learning," 2014, arXiv:1412.3489.

[138] P. Wittek, *Quantum Machine Learning: What Quantum Computing Means to Data Mining*. New York, NY, USA: Academic, 2014.

[139] P. Rebentrost, M. Mohseni, and S. Lloyd, "Quantum support vector machine for big data classification," *Phys. Rev. Lett.*, vol. 113, no. 13, Sep. 2014, Art. no. 130503.

[140] M. I. Jordan and T. M. Mitchell, "Machine learning: Trends, perspectives, and prospects," *Science*, vol. 349, no. 6245, pp. 255–260, Jul. 2015.

[141] E. Akleman, "Deep learning," *Computer*, vol. 53, no. 9, pp. 1–17, Sep. 2020.

[142] S. Aaronson, "Read the fine print," *Nature Phys.*, vol. 11, p. 4, Dec. 2015, Art. no. 291293.

[143] M. Schuld, I. Sinayskiy, and F. Petruccione, "An introduction to quantum machine learning," *Contemp. Phys.*, vol. 56, no. 2, Sep. 2015, Art. no. 172185.

[144] V. Dunjko, J. M. Taylor, and H. J. Briegel, "Quantum-enhanced machine learning," *Phys. Rev. Lett.*, vol. 117, no. 13, 2016, Art. no. 130501.

[145] I. Goodfellow, Y. Bengio, and A. Courville, *Deep Learning*. Cambridge, MA, USA: MIT Press, 2016.

[146] A. Monras, G. Sentís, and P. Wittek, "Inductive supervised quantum learning," *Phys. Rev. Lett.*, vol. 118, no. 19, May 2017, Art. no. 190503.

[147] E. Farhi and H. Neven, "Classification with quantum neural networks on near term processors," 2018, arXiv:1802.06002.

[148] S. Arunachalam and R. De Wolf, "Optimal quantum sample complexity of learning algorithms," *J. Mach. Learn. Res.*, vol. 19, no. 1, pp. 2878–2879, 2018.

[149] M. Schuld and F. Petruccione, *Supervised Learning With Quantum Computers*. Cham, Switzerland: Springer, 2018.

[150] K. Mitarai, T. Yan, and K. Fujii, "Generalization of the output of a variational quantum eigensolver by parameter interpolation with a low-depth ansatz," *Phys. Rev. A, Gen. Phys.ppl.*, vol. 11, no. 4, Apr. 2019, Art. no. 044087.

[151] M. Benedetti, E. Lloyd, S. Sack, and M. Fiorentini, "Parameterized quantum circuits as machine learning models," *Quantum Sci. Technol.*, vol. 4, no. 4, Nov. 2019, Art. no. 043001.

[152] S. Y. Chen, C. H. Yang, J. Qi, P.-Y. Chen, X. Ma, and H.-S. Goan, "Variational quantum circuits for deep reinforcement learning," *IEEE Access*, vol. 8, pp. 141007–141024, 2020.

[153] B. Coyle, D. Mills, V. Danos, and E. Kashefi, "The born supremacy: Quantum advantage and training of an Ising born machine," *Npj Quantum Inf.*, vol. 6, no. 1, p. 111, Jul. 2020.

[154] M. Schuld, A. Bocharov, K. M. Svore, and N. Wiebe, "Circuit-centric quantum classifiers," *Phys. Rev. A, Gen. Phys.*, vol. 101, no. 3, Mar. 2020, Art. no. 032308.

[155] Y. Du, M.-H. Hsieh, T. Liu, and D. Tao, "Expressive power of parametrized quantum circuits," *Phys. Rev. Res.*, vol. 2, no. 3, Jul. 2020, Art. no. 033125.

[156] A. Zlokapa, H. Neven, and S. Lloyd, "A quantum algorithm for training wide and deep classical neural networks," 2021, *arXiv:2107.09200*.

[157] L. Banchi, J. Pereira, and S. Pirandola, "Generalization in quantum machine learning: A quantum information standpoint," *PRX Quantum*, vol. 2, no. 4, Nov. 2021, Art. no. 040321.

[158] H.-Y. Huang, R. Kueng, and J. Preskill, "Information-theoretic bounds on quantum advantage in machine learning," *Phys. Rev. Lett.*, vol. 126, no. 19, May 2021, Art. no. 190505.

[159] H.-Y. Huang, M. Broughton, M. Mohseni, R. Babbush, S. Boixo, H. Neven, and J. R. McClean, "Power of data in quantum machine learning," *Nature Commun.*, vol. 12, no. 1, p. 19, May 2021.

[160] Y. Liu, S. Arunachalam, and K. Temme, "A rigorous and robust quantum speed-up in supervised machine learning," *Nature Phys.*, vol. 17, no. 9, pp. 1013–1017, Sep. 2021.

[161] N. Mishra et al., "Quantum machine learning: A review and current status," *Data Manag. Analytics Innov.*, vol. 1, Dec. 2021, Art. no. 101145.

[162] R. Sweke, J.-P. Seifert, D. Hangleiter, and J. Eisert, "On the quantum versus classical learnability of discrete distributions," *Quantum*, vol. 5, p. 417, Mar. 2021.

[163] M. Schuld and N. Killoran, "Is quantum advantage the right goal for quantum machine learning?" *PRX Quantum*, vol. 3, no. 3, Jul. 2022, Art. no. 030101.

[164] U. Alvarez-Rodriguez, M. Sanz, L. Lamata, and E. Solano, "Quantum artificial life in an IBM quantum computer," *Sci. Rep.*, vol. 8, no. 1, Oct. 2018, Art. no. 14793.

[165] (2023). *Exploring Quantum Use Cases for Chemicals and Petroleum: Changing How Chemicals Are Designed and Petroleum is Refined.* [Online]. Available: https://www.ibm.com/downloads/cas/BDGQRXOZ

[166] A. Kandala, A. Mezzacapo, K. Temme, M. Takita, M. Brink, J. M. Chow, and J. M. Gambetta, "Hardware-efficient variational quantum eigensolver for small molecules and quantum magnets," *Nature*, vol. 549, no. 7671, pp. 242–246, Sep.13, 2017.

[167] (2023). *ExxonMobil and IBM to Advance Energy Sector Application of Quantum Computing*. [Online]. Available: https://news.exxonmobil.com/press-release/exxonmobil-and-ibm-advance-energy-sector-application-quantum-computing

[168] M. D'Arcangelo, D. Loco, F. team, N. Gouraud, S. Angebault, J. Sueiro, P. Monmarché, J. Forut, L.-P. Henry, L. Henriet, and J.-P. Piquemal, "Leveraging analog quantum computing with neutral atoms for solvent configuration prediction in drug discovery," 2023, *arXiv:2309.12129*.

[169] S. Greve, G. A. Kuhn, M. D. Saenz-de-Juano, A. Ghosh, F. von Meyenn, and K. Giller, "The major urinary protein gene cluster knockout mouse as a novel model for translational metabolism research," *Sci. Rep.*, vol. 12, no. 1, Aug. 2022, Art. no. 13161.

[170] M. Motta, E. Ye, J. R. McClean, Z. Li, A. J. Minnich, R. Babbush, and G. K.-L. Chan, "Low rank representations for quantum simulation of electronic structure," *Npj Quantum Inf.*, vol. 7, no. 1, p. 17, May 2021.

[171] D. W. Berry, C. Gidney, M. Motta, J. R. McClean, and R. Babbush, "Qubitization of arbitrary basis quantum chemistry leveraging sparsity and low rank factorization," *Quantum*, vol. 3, p. 208, Dec. 2019.

[172] M. Reiher, N. Wiebe, K. M. Svore, D. Wecker, and M. Troyer, "Elucidating reaction mechanisms on quantum computers," *Proc. Nat. Acad. Sci. USA*, vol. 114, no. 29, pp. 7555–7560, Jul. 2017.

[173] V. von Burg, G. H. Low, T. Häner, D. S. Steiger, M. Reiher, M. Roetteler, and M. Troyer, "Quantum computing enhanced computational catalysis," *Phys. Rev. Res.*, vol. 3, no. 3, Jul. 2021, Art. no. 033055.

[174] J. Lee, D. W. Berry, C. Gidney, W. J. Huggins, J. R. McClean, N. Wiebe, and R. Babbush, "Even more efficient quantum computations of chemistry through tensor hypercontraction," *PRX Quantum*, vol. 2, no. 3, Jul. 2021, Art. no. 030305.

[175] D. Wecker, M. B. Hastings, N. Wiebe, B. K. Clark, C. Nayak, and M. Troyer, "Solving strongly correlated electron models on a quantum computer," *Phys. Rev. A, Gen. Phys.*, vol. 92, no. 6, Dec. 2015, Art. no. 062318.

[176] Y. Nam et al., "Ground-state energy estimation of the water molecule on a trapped-ion quantum computer," *Npj Quantum Inf.*, vol. 6, no. 1, p. 16, Apr. 2020.

[177] G. A. Q. Collaborators, "Hartree-fock on a superconducting qubit quantum computer," *Science*, vol. 369, no. 6507, pp. 1084–1089, 2020.

[178] P. J. OMalley et al., "Scalable quantum simulation of molecular energies," *Phys. Rev. X.*, vol. 6, no. 3, 2016, Art. no. 031007.

[179] J. R. McClean, J. Romero, R. Babbush, and A. Aspuru-Guzik, "The theory of variational hybrid quantum-classical algorithms," *New J. Phys.*, vol. 18, no. 2, Feb. 2016, Art. no. 023023.

[180] C.-K. Lee, J. Wei Zhong Lau, L. Shi, and L. Chuan Kwek, "Simulating energy transfer in molecular systems with digital quantum computers," 2021, *arXiv:2101.06879*.

[181] C. K. Lee, P. Patil, S. Zhang, and C. Y. Hsieh, "Neural-network variational quantum algorithm for simulating many-body dynamics," *Phys. Rev. Res.*, vol. 3, no. 2, May 2021, Art. no. 023095.

[182] Y. Cao, J. Romero, J. P. Olson, M. Degroote, P. D. Johnson, M. Kieferova, I. D. Kivlichan, T. Menke, B. Peropadre, N. P. Sawaya, S. Sim, L. Veis, and A. Aspuru-Guzik, "Quantum chemistry in the age of quantum computing," *Chem. Rev.*, vol. 119, no. 19, pp. 10856–10915, 2019.

[183] J. Olson, Y. Cao, J. Romero, P. Johnson, P.-L. Dallaire-Demers, N. Sawaya, P. Narang, I. Kivlichan, M. Wasielewski, and A. Aspuru-Guzik, "Quantum information and computation for chemistry," 2017, *arXiv:1706.05413*.

[184] P. Hauke, D. Marcos, M. Dalmonte, and P. Zoller, "Quantum simulation of a lattice schwinger model in a chain of trapped ions," *Phys. Rev. X*, vol. 3, no. 4, Nov. 2013, Art. no. 041018.

[185] E. A. Martinez, C. A. Muschik, P. Schindler, D. Nigg, A. Erhard, M. Heyl, P. Hauke, M. Dalmonte, T. Monz, P. Zoller, and R. Blatt, "Real-time dynamics of lattice gauge theories with a few-qubit quantum computer," *Nature*, vol. 534, no. 7608, pp. 516–519, Jun. 2016.

[186] C. Kokail, C. Maier, R. van Bijnen, T. Brydges, M. K. Joshi, P. Jurcevic, C. A. Muschik, P. Silvi, R. Blatt, C. F. Roos, and P. Zoller, "Self-verifying variational quantum simulation of lattice models," *Nature*, vol. 569, no. 7756, pp. 355–360, May 2019.

[187] S. Y.-C. Chen, T.-C. Wei, C. Zhang, H. Yu, and S. Yoo, "Quantum convolutional neural networks for high energy physics data analysis," *Phys. Rev. Res.*, vol. 4, no. 1, Mar. 2022, Art. no. 013231.

[188] S. L. Wu, J. Chan, W. Guan, S. Sun, A. Wang, C. Zhou, M. Livny, F. Carminati, A. D. Meglio, A. C. Y. Li, J. Lykken, P. Spentzouris, S. Y.-C. Chen, S. Yoo, and T.-C. Wei, "Application of quantum machine learning using the quantum variational classifier method to high energy physics analysis at the LHC on IBM quantum computer simulator and hardware with 10 qubits," *J. Phys. G, Nucl. Part. Phys.*, vol. 48, no. 12, Dec. 2021, Art. no. 125003.

[189] M. C. Banuls, R. Blatt, J. Catani, A. Celi, J. I. Cirac, M. Dalmonte, L. Fallani, K. Jansen, M. Lewenstein, S. Montangero, C. A. Muschik, B. Reznik, E. Rico, L. Tagliacozzo, K. Van Acoleyen, F. Verstraete, U.-J. Wiese, M. Wingate, J. Zakrzewski, and P. Zoller, "Simulating lattice gauge theories within quantum technologies," *Eur. Phys. J. D*, vol. 74, no. 8, p. 142, 2020.

[190] N. Klco, A. Roggero, and M. J. Savage, "Standard model physics and the digital quantum revolution: Thoughts about the interface," *Rep. Prog. Phys.*, vol. 85, no. 6, Jun. 2022, Art. no. 064301.

[191] C. Mc Keever and M. Lubasch, "Classically optimized Hamiltonian simulation," *Phys. Rev. Res.*, vol. 5, no. 2, Jun. 2023, Art. no. 023146.

[192] M. Suzuki, "Fractal decomposition of exponential operators with applications to many-body theories and Monte Carlo simulations," *Phys. Lett. A*, vol. 146, no. 6, pp. 319–323, Jun. 1990.

[193] M. Suzuki, "General theory of fractal path integrals with applications to many-body theories and statistical physics," *J. Math. Phys.*, vol. 32, no. 2, pp. 400–407, Feb. 1991.

[194] M. Benedetti, M. Fiorentini, and M. Lubasch, "Hardware-efficient variational quantum algorithms for time evolution," *Phys. Rev. Res.*, vol. 3, no. 3, Jul. 2021, Art. no. 033083.

[195] J.-B. You, D. E. Koh, J. F. Kong, W.-J. Ding, C. E. Png, and L. Wu, "Exploring variational quantum eigensolver ansatzes for the long-range XY model," 2021, *arXiv:2109.00288*.

[196] F. Y. Leong, D. E. Koh, W.-B. Ewe, and J. F. Kong, "Variational quantum simulation of partial differential equations: Applications in colloidal transport," *Int. J. Numer. Methods Heat Fluid Flow*, vol. 33, no. 11, pp. 3669–3690, Oct. 2023.

[197] W.-B. Ewe, D. E. Koh, S. T. Goh, H.-S. Chu, and C. E. Png, "Variational quantum-based simulation of waveguide modes," *IEEE Trans. Microwave Theory Techn.*, vol. 70, no. 5, pp. 2517–2525, May 2022.

[198] G. Wang, D. E. Koh, P. D. Johnson, and Y. Cao, "Minimizing estimation runtime on noisy quantum computers," *PRX Quantum*, vol. 2, no. 1, Mar. 2021, Art. no. 010346.

[199] D. E. Koh, G. Wang, P. D. Johnson, and Y. Cao, "Foundations for Bayesian inference with engineered likelihood functions for robust amplitude estimation," *J. Math. Phys.*, vol. 63, no. 5, May 2022, Art. no. 052202.

[200] I. C. Cloet et al., "Opportunities for nuclear physics quantum information science," 2019, *arXiv:1903.05453*.

[201] D.-B. Zhang, H. Xing, H. Yan, E. Wang, and S.-L. Zhu, "Selected topics of quantum computing for nuclear physics," *Chin. Phys. B*, vol. 30, no. 2, Feb. 2021, Art. no. 020306.

[202] C. P. Koch, "Controlling open quantum systems: Tools, achievements, and limitations," *J. Phys., Condens. Matter*, vol. 28, no. 21, Jun. 2016, Art. no. 213001.

[203] P. K. Barkoutsos, F. Gkritsis, P. J. Ollitrault, I. O. Sokolov, S. Woerner, and I. Tavernelli, "Quantum algorithm for alchemical optimization in material design," *Chem. Sci.*, vol. 12, no. 12, pp. 4345–4352, Apr. 2021.

[204] I. M. Georgescu, S. Ashhab, and F. Nori, "Quantum simulation," *Rev. Mod. Phys.*, vol. 86, no. 1, p. 153, 2014.

[205] F. Tacchino, A. Chiesa, S. Carretta, and D. Gerace, "Quantum computers as universal quantum simulators: State-of-the-art and perspectives," *Adv. Quantum Technol.*, vol. 3, no. 3, Mar. 2020, Art. no. 1900052.

[206] K. Meichanetzidis. (2020). *Quantum Natural Language Processing*. [Online]. Available: https://medium.com/cambridge-quantum-computing/quantum-natural-language-processing-748d6f27b31d

[207] *Cambridge Quantum Releases Worlds First Quantum Natural Language Processing Toolkit and Library*, HPCwire, San Diego, CA, USA, 2021.

[208] D. Kartsaklis, I. Fan, R. Yeung, A. Pearson, R. Lorenz, A. Toumi, G. de Felice, K. Meichanetzidis, S. Clark, and B. Coecke, "Lambeq: An efficient high-level Python library for quantum NLP," 2021, *arXiv:2110.04236*.

[209] lambeq. *Github*. Accessed: May 7, 2024. [Online]. Available: https://github.com/CQCL/lambeq

[210] B. Coecke, G. de Felice, K. Meichanetzidis, and A. Toumi, "How to make qubits speak," 2021, *arXiv:2107.06776*.

[211] K. Meichanetzidis, A. Toumi, G. de Felice, and B. Coecke, "Grammar-aware sentence classification on quantum computers," *Quantum Mach. Intell.*, vol. 5, no. 1, p. 10, Jun. 2023.

[212] R. Lorenz, A. Pearson, K. Meichanetzidis, D. Kartsaklis, and B. Coecke, "QNLP in practice: Running compositional models of meaning on a quantum computer," *J. Artif. Intell. Res.*, vol. 76, pp. 1305–1342, Apr. 2023.

[213] J. Russell, *Multiverse, Pasqal, and Credit Agricole Tout Progress Using Quantum Computing in FS*. San Diego, CA, USA: HPCwire, 2023.

[214] (2024). *Pasqal*. [Online]. Available: https://www.pasqal.com

[215] (2023). *Multiverse Computing*. [Online]. Available: https://multiverse computing.com

[216] (2023). *Credit Agricole Corporate and Investment Bank*. [Online]. Available: https://www.credit-agricole.fr

[217] R. G. Patel, C.-W. Hsing, S. Sahin, S. Palmer, S. S. Jahromi, S. Sharma, T. Dominguez, K. Tziritas, C. Michel, V. Porte, M. Abid, S. Aubert, P. Castellani, S. Mugel, and R. Orus, "Quantum-inspired tensor neural networks for option pricing," 2022, *arXiv:2212.14076*.

[218] L. Leclerc et al., "Financial risk management on a neutral atom quantum processor," 2022, *arXiv:2212.03223*.

[219] R. Patel, C.-W. Hsing, S. Sahin, S. S. Jahromi, S. Palmer, S. Sharma, C. Michel, V. Porte, M. Abid, S. Aubert, P. Castellani, C.-G. Lee, S. Mugel, and R. Orus, "Quantum-inspired tensor neural networks for partial differential equations," 2022, *arXiv:2208.02235*.

[220] K. Plekhanov, M. Rosenkranz, M. Fiorentini, and M. Lubasch, "Variational quantum amplitude estimation," *Quantum*, vol. 6, p. 670, Mar. 2022.

[221] M. Sipser, *Introduction to the Theory of Computation*, 3rd ed., Boston, MA, USA: Course Technology, 2013.

[222] E. Farhi, J. Goldstone, and S. Gutmann, "A quantum approximate optimization algorithm," 2014, *arXiv:1411.4028*.

[223] E. Farhi, J. Goldstone, S. Gutmann, and M. Sipser, "Quantum computation by adiabatic evolution," 2000, *arXiv: quant-ph/0001106*.

[224] T. Albash and D. A. Lidar, "Adiabatic quantum computation," *Rev. Mod. Phys.*, vol. 90, Oct. 2018, Art. no. 015002.

[225] A. Lucas, "Ising formulations of many NP problems," *Frontiers Phys.*, vol. 2, p. 5, Nov. 2014.

[226] D. Wecker, M. B. Hastings, and M. Troyer, "Training a quantum optimizer," *Phys. Rev. A, Gen. Phys.*, vol. 94, no. 2, Aug. 2016, Art. no. 022309.

[227] F. Barahona, "On the computational complexity of Ising spin glass models," *J. Phys. A, Math. Gen.*, vol. 15, no. 10, pp. 3241–3253, Oct. 1982.

[228] E. Farhi, J. Goldstone, S. Gutmann, J. Lapan, A. Lundgren, and D. Preda, "A quantum adiabatic evolution algorithm applied to random instances of an NP-complete problem," *Science*, vol. 292, no. 5516, pp. 472–475, Apr. 2001.

[229] V. Bapst, L. Foini, F. Krzakala, G. Semerjian, and F. Zamponi, "The quantum adiabatic algorithm applied to random optimization problems: The quantum spin glass perspective," *Phys. Rep.*, vol. 523, no. 3, pp. 127–205, Feb. 2013.

[230] E. Farhi, D. Gosset, I. Hen, A. W. Sandvik, P. Shor, A. P. Young, and F. Zamponi, "Performance of the quantum adiabatic algorithm on random instances of two optimization problems on regular hypergraphs," *Phys. Rev. A, Gen. Phys.*, vol. 86, no. 5, Nov. 2012, Art. no. 052334.

[231] A. P. Young, S. Knysh, and V. N. Smelyanskiy, "First-order phase transition in the quantum adiabatic algorithm," *Phys. Rev. Lett.*, vol. 104, no. 2, Jan. 2010, Art. no. 020502.

[232] S. Knysh, "Zero-temperature quantum annealing bottlenecks in the spin-glass phase," *Nature Commun.*, vol. 7, no. 1, Aug. 2016, Art. no. 12370.

[233] R. Herrman, P. C. Lotshaw, J. Ostrowski, T. S. Humble, and G. Siopsis, "Multi-angle quantum approximate optimization algorithm," *Sci. Rep.*, vol. 12, no. 1, Apr. 2022, Art. no. 6781, doi: 10.1038/s41598-022-10555-8.

[234] M. P. Harrigan et al., "Quantum approximate optimization of non-planar graph problems on a planar superconducting processor," *Nature Phys.*, vol. 17, Nov. 2021, Art. no. 332336.

[235] T. M. Graham et al., "Demonstration of multi-qubit entanglement and algorithms on a programmable neutral atom quantum computer," 2021, *arXiv:2112.14589*.

[236] G. Pagano, A. Bapat, P. Becker, K. S. Collins, A. De, P. W. Hess, H. B. Kaplan, A. Kyprianidis, W. L. Tan, C. Baldwin, L. T. Brady, A. Deshpande, F. Liu, S. Jordan, A. V. Gorshkov, and C. Monroe, "Quantum approximate optimization of the long-range Ising model with a trapped-ion quantum simulator," *Proc. Nat. Acad. Sci. USA*, vol. 117, no. 41, pp. 25396–25401, Oct. 2020.

[237] T. F. Rønnow, Z. Wang, J. Job, S. Boixo, S. V. Isakov, D. Wecker, J. M. Martinis, D. A. Lidar, and M. Troyer, "Defining and detecting quantum speedup," *Science*, vol. 345, no. 6195, pp. 420–424, Jul. 2014.

[238] H. G. Katzgraber, F. Hamze, Z. Zhu, A. J. Ochoa, and H. Munoz-Bauza, "Seeking quantum speedup through spin glasses: The good, the bad, and the ugly," *Phys. Rev. X*, vol. 5, no. 3, Sep. 2015, Art. no. 031026.

[239] C.-N. Chou, P. J. Love, J. Singh Sandhu, and J. Shi, "Limitations of local quantum algorithms on random Max-k-XOR and beyond," 2021, *arXiv:2108.06049*.

[240] E. Farhi, D. Gamarnik, and S. Gutmann, "The quantum approximate optimization algorithm needs to see the whole graph: A typical case," 2020, *arXiv:2004.09002*.

[241] P. Festa, P. M. Pardalos, M. G. C. Resende, and C. C. Ribeiro, "Randomized heuristics for the max-cut problem," *Optim. Methods Softw.*, vol. 17, no. 6, pp. 1033–1058, Jan. 2002.

[242] J. S. Otterbach et al., "Unsupervised machine learning on a hybrid quantum computer," 2017, *arXiv:1712.05771*.

[243] L. Zhou, S.-T. Wang, S. Choi, H. Pichler, and M. D. Lukin, "Quantum approximate optimization algorithm: Performance, mechanism, and implementation on near-term devices," *Phys. Rev. X*, vol. 10, no. 2, Jun. 2020, Art. no. 021067.

[244] S. Bravyi, A. Kliesch, R. Koenig, and E. Tang, "Obstacles to variational quantum optimization from symmetry protection," *Phys. Rev. Lett.*, vol. 125, no. 26, Dec. 2020, Art. no. 260505.

[245] M. B. Hastings, "Classical and quantum bounded depth approximation algorithms," 2019, *arXiv:1905.07047*.

[246] G. E. Crooks, "Performance of the quantum approximate optimization algorithm on the maximum cut problem," 2018, *arXiv:1811.08419*.

[247] D. Amaro, C. Modica, M. Rosenkranz, M. Fiorentini, M. Benedetti, and M. Lubasch, "Filtering variational quantum algorithms for combinatorial optimization," *Quantum Sci. Technol.*, vol. 7, no. 1, Jan. 2022, Art. no. 015021.

[248] (2023). *Honeywell Quantum Solutions*. Accessed: Oct. 7, 2023. [Online]. Available: https://www.honeywell.com/us/en

[249] V. Vijendran, A. Das, D. Enshan Koh, S. M. Assad, and P. Koy Lam, "An expressive ansatz for low-depth quantum approximate optimisation," 2023, *arXiv:2302.04479*.

[250] P. Dupuy de la Grand'rive and J.-F. Hullo, "Knapsack problem variants of QAOA for battery revenue optimisation," 2019, *arXiv:1908.02210*.

[251] A. Robert, P. K. Barkoutsos, S. Woerner, and I. Tavernelli, "Resource-efficient quantum algorithm for protein folding," *Npj Quantum Inf.*, vol. 7, no. 1, p. 15, Feb. 2021.

[252] M. Fingerhuth, T. Babej, and C. Ing, "A quantum alternating operator ansatz with hard and soft constraints for lattice protein folding," 2018, *arXiv:1810.13411*.

[253] T. Babej, C. Ing, and M. Fingerhuth, "Coarse-grained lattice protein folding on a quantum annealer," 2018, *arXiv:1811.00713*.

[254] S. A. Metwalli, F. Le Gall, and R. Van Meter, "Finding small and large $k$-clique instances on a quantum computer," *IEEE Trans. Quantum Eng.*, vol. 1, pp. 1–11, 2020.

[255] Y.-H. Oh, H. Mohammadbagherpoor, P. Dreher, A. Singh, X. Yu, and A. J. Rindos, "Solving multi-coloring combinatorial optimization problems using hybrid quantum algorithms," 2019, *arXiv:1911.00595*.

[256] (2023). *Amazon Braket*. Accessed: Oct. 7, 2023. [Online]. Available: https://aws.amazon.com/braket

[257] M. Machin, "Improving cargo loading plans by using a hybrid approach with quantum annealers to optimize cargo in the bellyhold of passenger aircraft," *Quantum-South*, vol. 1, pp. 1–28, Oct. 17, 2022.

[258] (2023). *IAG Cargo*. [Online]. Available: https://www.iagcargo.com

[259] M. Machin, "Automatic ULD selection and ordering," *Quantum-South*, vol. 1, pp. 1–19, Jul. 18, 2023.

[260] M. Machin, "Quantum-South identifies alternatives to boost Amerijet Internationals cargo load factor by up to 30% with cutting-edge solution," Quantum-South, Miami, FL, USA, Jul. 2023. [Online]. Available: https://quantum-south.com/airlinescargo/quantum-south-identifies-alternatives-to-boost-amerijet-internationals-cargo-load-factor-by-up-to-30-with-cutting-edge-solution/

[261] A. W. Harrow, A. Hassidim, and S. Lloyd, "Quantum algorithm for linear systems of equations," *Phys. Rev. Lett.*, vol. 103, no. 15, Oct. 2009, Art. no. 150502.

[262] A. M. Childs, R. Kothari, and R. D. Somma, "Quantum algorithm for systems of linear equations with exponentially improved dependence on precision," *SIAM J. Comput.*, vol. 46, no. 6, pp. 1920–1950, Jan. 2017.

[263] A. Ambainis, "Variable time amplitude amplification and quantum algorithms for linear algebra problems," in *Proc. 29th Symp. Theor. Aspects Comput. Sci.*, vol. 14, 2023, pp. 636–647.

[264] C. Bravo-Prieto, R. LaRose, M. Cerezo, Y. Subasi, L. Cincio, and P. J. Coles, "Variational quantum linear solver," 2019, *arXiv:1909.05820*.

[265] J. Wen, X. Kong, S. Wei, B. Wang, T. Xin, and G. Long, "Experimental realization of quantum algorithms for a linear system inspired by adiabatic quantum computing," *Phys. Rev. A, Gen. Phys.*, vol. 99, no. 1, Jan. 2019.

[266] X. Xu, J. Sun, S. Endo, Y. Li, S. C. Benjamin, and X. Yuan, "Variational algorithms for linear algebra," *Sci. Bull.*, vol. 66, no. 21, pp. 2181–2188, Nov. 2021.

[267] H.-Y. Huang, K. Bharti, and P. Rebentrost, "Near-term quantum algorithms for linear systems of equations with regression loss functions," *New J. Phys.*, vol. 23, no. 11, Nov. 2021, Art. no. 113021.

[268] A. Ü. Keskin, "Nonlinear ordinary differential equations," in *Ordinary Differential Equations for Engineers*. Cham, Switzerland: Springer, 2019, pp. 631–696.

[269] M. Lubasch, J. Joo, P. Moinier, M. Kiffner, and D. Jaksch, "Variational quantum algorithms for nonlinear problems," *Phys. Rev. A, Gen. Phys.*, vol. 101, no. 1, Jan. 2020, Art. no. 010301.

[270] O. Kyriienko, A. E. Paine, and V. E. Elfving, "Solving nonlinear differential equations with differentiable quantum circuits," *Phys. Rev. A, Gen. Phys.*, vol. 103, no. 5, May 2021, Art. no. 052416.

[271] F. Y. Leong, W.-B. Ewe, and D. E. Koh, "Variational quantum evolution equation solver," 2022, *arXiv:2204.02912*.

[272] F. Gaitan, "Finding flows of a Navier–Stokes fluid through quantum computing," *Npj Quantum Inf.*, vol. 6, no. 1, p. 16, Jul. 2020.

[273] P. W. Shor, "Polynomial-time algorithms for prime factorization and discrete logarithms on a quantum computer," *SIAM Rev.*, vol. 41, no. 2, pp. 303–332, Jan. 1999.

[274] C. J. Burges, *Factoring as Optimization*, document MSR-TR-200, 2000.

[275] N. Xu, J. Zhu, D. Lu, X. Zhou, X. Peng, and J. Du, "Quantum factorization of 143 on a dipolar-coupling nuclear magnetic resonance system," *Phys. Rev. Lett.*, vol. 108, no. 13, Mar. 2012, Art. no. 130501.

[276] N. S. Dattani and N. Bryans, "Quantum factorization of 56153 with only 4 qubits," 2014, *arXiv:1411.6758*.

[277] A. H. Karamlou, W. A. Simon, A. Katabarwa, T. L. Scholten, B. Peropadre, and Y. Cao, "Analyzing the performance of variational quantum factoring on a superconducting quantum processor," *Npj Quantum Inf.*, vol. 7, no. 1, pp. 1–16, Oct. 2021.

[278] T. Haug and K. Bharti, "Generalized quantum assisted simulator," *Quantum Sci. Technol.*, vol. 7, no. 4, Oct. 2022, Art. no. 045019.

**MUHAMMAD ABUGHANEM** received the bachelor's degree in science and education from the Faculty of Education, Department of Mathematics, Alexandria University's in 2008, the B.Sc. degree (Hons.) in mathematics from the Faculty of Science, Department of Mathematics and Computer Science in 2012, and the M.Sc. degree in applied mathematics with a specialization in quantum computation from the Faculty of Science, Ain Shams University, Cairo, Egypt, in 2019, where he is currently pursuing the Ph.D. degree.

With an extensive teaching experience spanning over a decade, he has imparted knowledge at prestigious academic institutions including Alexandria University, AASTMT, and Zewail City of Science and Technology. Originally from Rosetta, a historic city situated 239 mi west of Gaza, Palestine. His research interests include philosophy, comparative education, applied computing, quantum technology, and the pursuit of fault-tolerant quantum computers.

**HICHEM ELEUCH** received the Ph.D. degree in quantum physics from Kastler Brossel Laboratory, École Normale Supérieure de Paris and Université Pierre-et-Marie-Curie, Sorbonne University, France.

He worked at and visited several prestigious universities and research institutes with Texas A&M University, Max Planck Institute for the Physics of Complex Systems, Princeton University, McGill University, Auckland University, and University of Montreal. He is a Full Professor of physics with the University of Carthage, Tunis, Tunisia. Recently, he joined the University of Sharjah (UoS) as a Full Professor. He is a fellow with African Academy of Sciences and a member with Sigma Xi, The Scientific Research Honor Society, USA. He is also a member with the Mohammed Bin Rashid Academy of Scientists, United Arab Emirates. His academic contributions extend to more than 280 articles in Scopus and *Web of Science*-indexed journals, along with 14 U.S. patents. He gave more than 50 invited talks. He has refereed articles for more than 50 journals of physics and mathematics. His research interests include quantum optics, quantum computing, mathematical physics, and complex systems. He has been awarded several fellowships with the Fulbright Foundation, Max Planck Society, and the International Center of Theoretical Physics, Trieste, Italy.

● ● ●