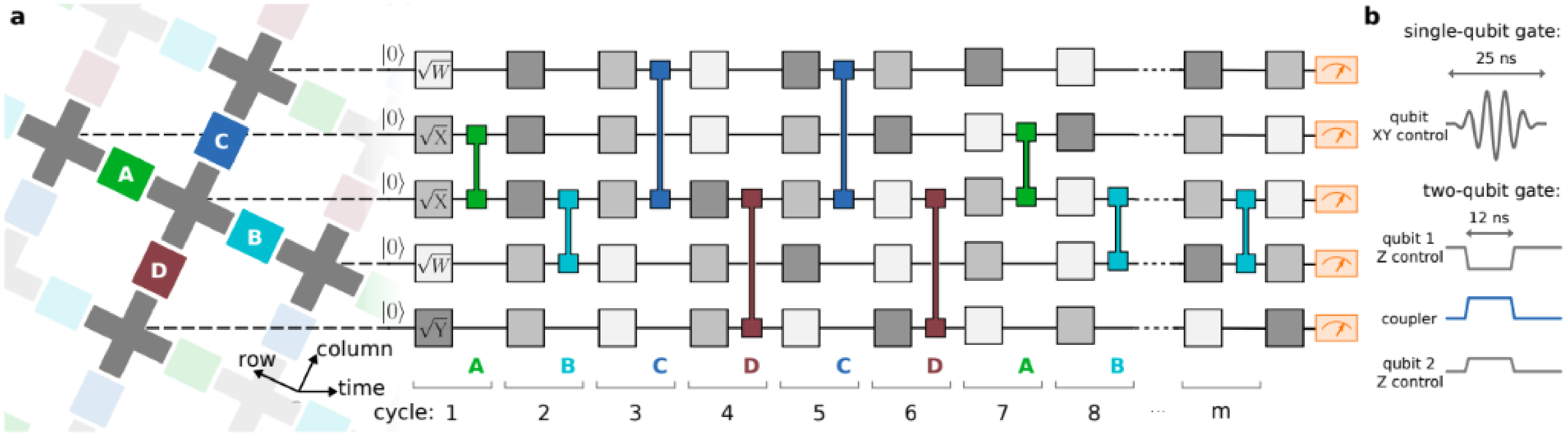
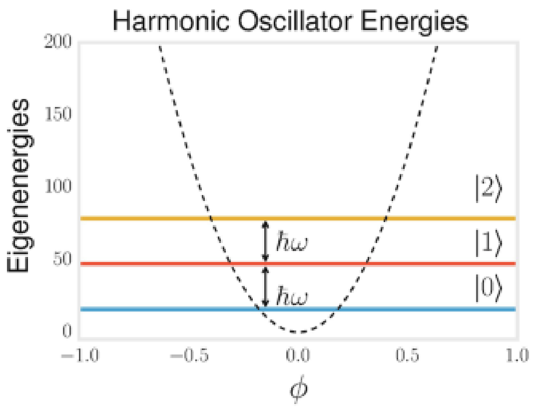
Google's quantum computer

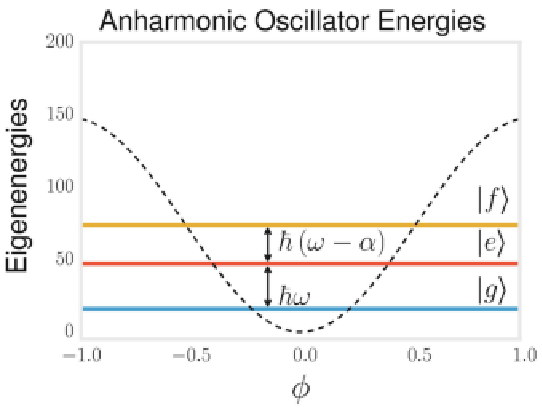


- **Transmon qubits**: superconducting qubit based on Josephson Junctions

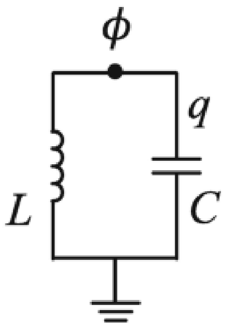
A nonlinear quantum harmonic oscillator



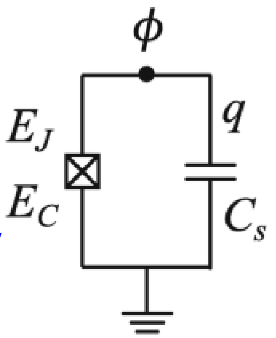
(a)



(b)



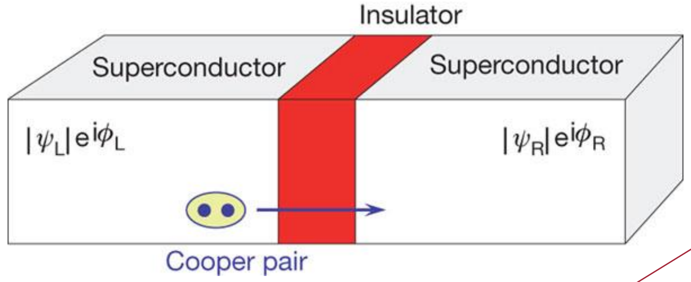
(c)



(d)

Regular inductor
+ capacitor

Josephson Junction
+ capacitor



Supercurrent: $I = I_c \sin(\phi)$

Phase difference across the junction

- Potential energy of the inductor (**not harmonic potential**)

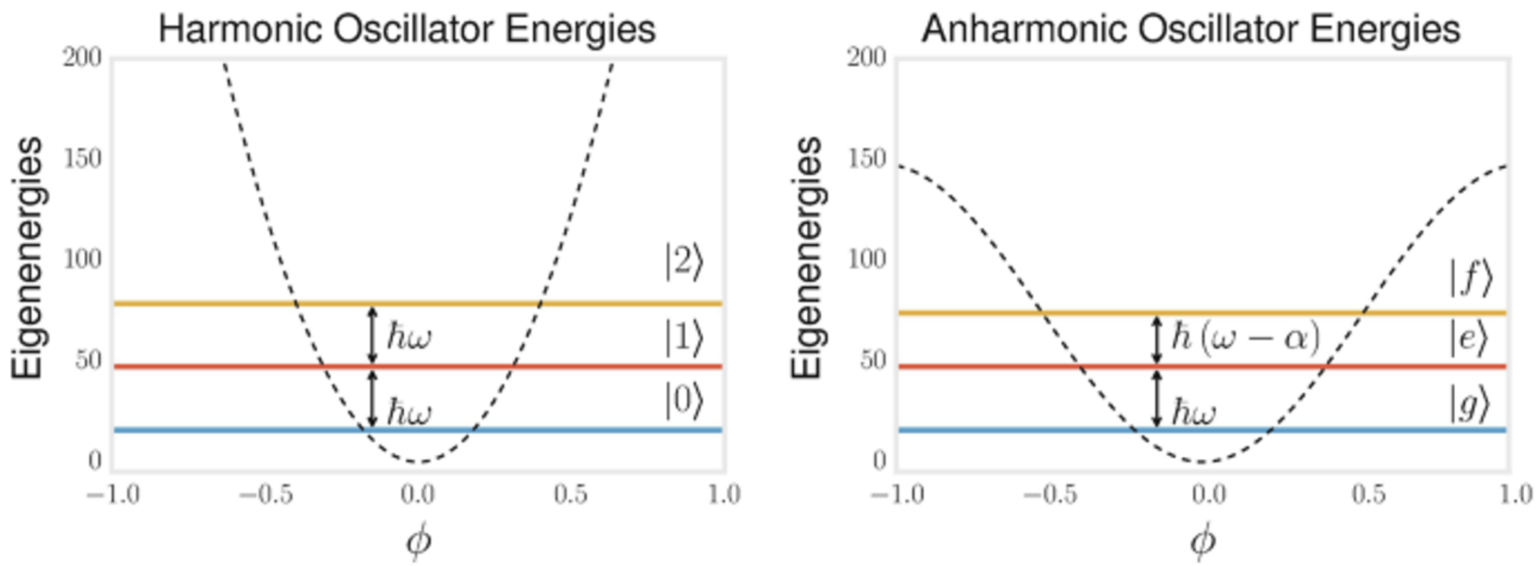
$$U = \int IV\,dt = \frac{\phi_0 I_c}{2\pi}\int \sin(\phi)\frac{d\phi}{dt}dt = -E_J\cos\phi,$$

- Energy from the capacitor

$$T = \frac{1}{2}CV^2 = \frac{1}{2}C\left(\frac{\phi_0}{2\pi}\dot{\phi}\right)^2$$

Anharmonic oscillator: energy spacing not equal, hence qubits can be introduced

A nonlinear quantum harmonic oscillator



Anharmonic oscillator: energy spacing not equal, hence qubits can be introduced

With microwave, we can couple |0> and |1>, and safely ignore the higher levels (of course, there will be some excitation leakage!)

Second quantization

Second quantization: treat the relative phase and its derivative as conjugate variables (similar to position and momentum)

$$\hat{\phi} = \kappa \frac{\hat{a}^\dagger + \hat{a}}{\sqrt{2}}, \qquad \dot{\hat{\phi}} = \kappa^{-1} \frac{\hat{a}^\dagger - \hat{a}}{\sqrt{2}i}.$$

Without anharmonic terms (approximate the potential term with quadratic function), the Josephson LC circuit's Hamiltonian

$$\hat{H}_0 = \omega_0 \left(\hat{a}^\dagger \hat{a} + \frac{1}{2} \right)$$

The lowest order anharmonic term

$$\hat{H}_1 = \gamma \hat{\phi}^4 = \frac{\gamma \kappa^4}{4} (\hat{a} + \hat{a}^\dagger)^4$$

Interaction between qubits

Qubits are connected via a capacitor

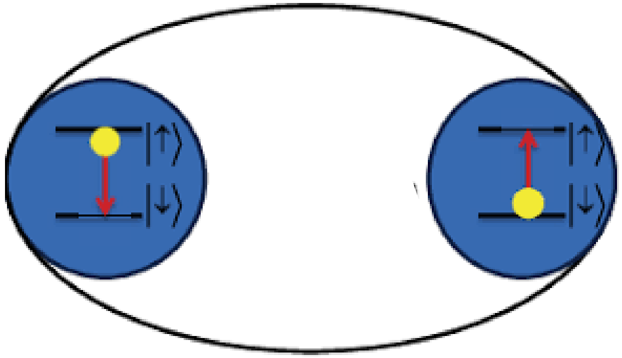
The capacitive coupling between the two Josephson LC circuits

$$\hat{H}_2 = -\alpha'(t)\dot{\phi}_1\dot{\phi}_2 = -\frac{\alpha'(t)}{2\kappa_1\kappa_2}(\hat{a}_1^\dagger - \hat{a}_1)(\hat{a}_2^\dagger - \hat{a}_2)$$

It leads to **an iSWAP interaction – exchange of excitation between two qubits**

|10> (1st qubit in excited state, 2nd qubit in ground state)

|01> (1st qubit in ground state, 2nd qubit in excited state)



Microwave pulse to control individual qubit

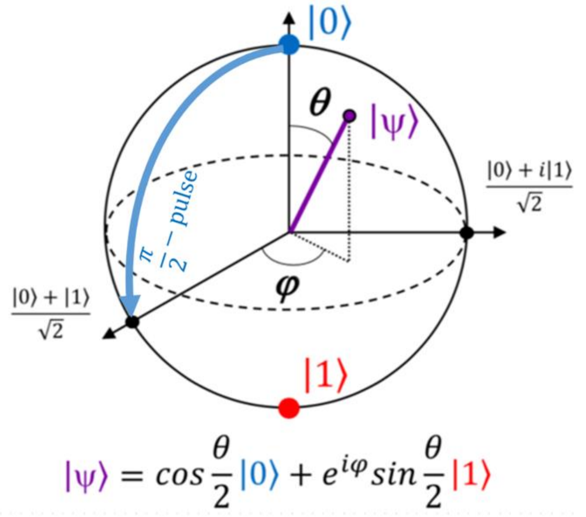
The transmon qubit resonant frequency (energy difference between |0> and |1>) ~ microwave frequency

Applying a microwave pulse of different duration can rotate the qubit quantum state, e.g.,

- π pulse
- $\pi/2$ pulse

The higher levels are off-resonant (but still there can be a small excitation)

$$\hat{H}_3 = \sum_{j=1,2} i\left(\hat{a}_j - \hat{a}_j^\dagger\right) f_j(t)\cos(\omega_j t + \varphi_j)\ ,$$



Final Bosonic Hamiltonian

After a rotating-wave approximation

$$\hat{H}_{\mathrm{RWA}} = \frac{\eta}{2}\sum_{j=1}^{2}\hat{n}_j(\hat{n}_j - 1) + g(t)(\hat{a}_2^\dagger\hat{a}_1 + \hat{a}_1^\dagger\hat{a}_2)$$
$$+ \sum_{j=1}^{2}\delta_j(t)\hat{n}_j + \sum_{j=1}^{2} i f_j(t)\left(\hat{a}_j e^{i\varphi_j(t)} - \hat{a}_j^\dagger e^{-i\varphi_j(t)}\right)$$

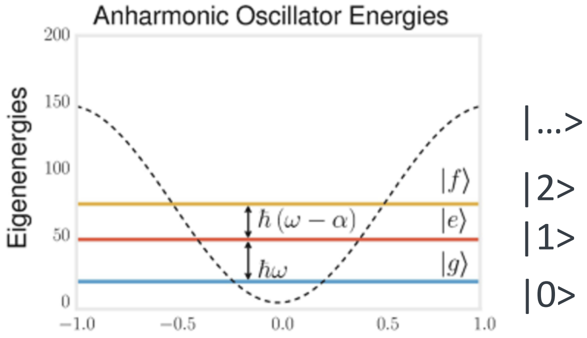
Typical parameters

	η	$g(t)$	$\delta_j(t)$	$f_j(t)$	$\varphi_j(t)$
amplitude	200 MHz	[-20, 20] MHz	[-20, 20] MHz	[-20, 20] MHz	[0, 2π]
error amplitude	$\pm$ 1 MHz	$\pm$ 1 MHz	$\pm$1 MHz	$\pm$ 1 MHz	

η: anharmonicity of the Josephson junction. It induces a large energy gap separating the qubit subspace from higher energy subspaces.

The computational subspace is spanned by the two lowest energy levels of each bosonic mode:

$$\mathcal{H}_2 = \mathrm{Span}\{|0\rangle_j, |1_j\rangle\}$$



If there is no leakage to higher levels …

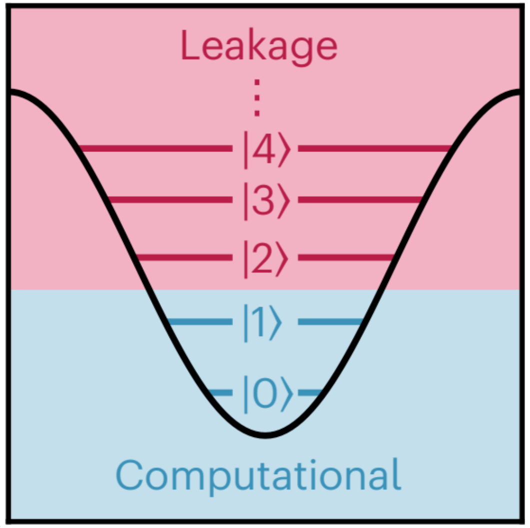
Each bosonic field excitation can be |0>, |1>, |2>, …

If limited to the lowest two levels, or projecting the Bosonic Hamiltonian to the qubit basis

$$\hat{H}_{\mathrm{RWA}} = \frac{g(t)}{2}(\sigma_1^x \sigma_2^x + \sigma_1^y \sigma_2^y) + \sum_{j=1}^{2} \left[\frac{\delta_j(t)}{2}\sigma_j^z - f_j(t)\left(\sin\varphi_j(t)\sigma_j^x + \cos\phi_j(t)\sigma_j^y\right)\right].$$

Due to the finite gap to the higher excitations, the higher levels |2>, |3>, … will also be excited → Leakage error.

The leakage error bound can be computed (complicated)



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ARTICLE OPEN

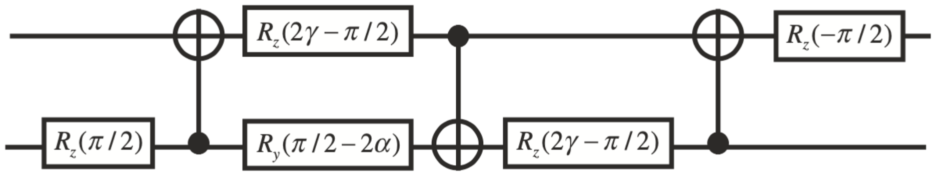
Universal quantum control through deep reinforcement learning

Murphy Yuezhen Niu[1,2], Sergio Boixo[2], Vadim N. Smelyanskiy[2] and Hartmut Neven[2]

Emerging reinforcement learning techniques using deep neural networks have shown great promise in control optimization. They harness non-local regularities of noisy control trajectories and facilitate transfer learning between tasks. To leverage these powerful capabilities for quantum control optimization, we propose a new control framework to simultaneously optimize the speed and fidelity of quantum computation against both leakage and stochastic control errors. For a broad family of two-qubit unitary gates that are important for quantum simulation of many-electron systems, we improve the control robustness by adding control noise into training environments for reinforcement learning agents trained with trusted-region-policy-optimization. The agent control solutions demonstrate a two-order-of-magnitude reduction in average-gate-error over baseline stochastic-gradient-descent solutions and up to a one-order-of-magnitude reduction in gate time from optimal gate synthesis counterparts. These significant improvements in both fidelity and runtime are achieved by combining new physical understandings and state-of-the-art machine learning techniques. Our results open a venue for wider applications in quantum simulation, quantum chemistry and quantum supremacy tests using near-term quantum devices.

npj Quantum Information (2019)5:33 ; https://doi.org/10.1038/s41534-019-0141-3

A quantum circuit with two transmon qubits only



Bosonic Hamiltonian

$$\hat{H}_{\mathrm{RWA}}(t) = \frac{\eta}{2}\sum_{j=1}^{2}\hat{n}_j(\hat{n}_j - 1) + g(t)(\hat{a}_2^\dagger\hat{a}_1 + \hat{a}_1^\dagger\hat{a}_2) + \sum_{j=1}^{2}\delta_j(t)\hat{n}_j + \sum_{j=1}^{2} i f_j(t)\left(\hat{a}_j e^{-i\varphi_j(t)} - \hat{a}_j^\dagger e^{i\varphi_j(t)}\right),$$

The corresponding unitary evolution

$$U(T) = \mathbb{T}\left[\exp\left(-i\int_0^T \hat{H}_{\mathrm{RWA}}(t)dt\right)\right]$$

Question?

Can it be used to simulate an arbitrary target unitary evolution denoted by U_{target} ?

$$\mathcal{N}(\alpha,\alpha,\gamma) = \exp[i(\alpha\sigma_1^x\sigma_2^x + \alpha\sigma_1^y\sigma_2^y + \gamma\sigma_1^z\sigma_2^z)],$$

Gate fidelity

$$F[U(T)] = (1/16)\left|\mathrm{Tr}(U^\dagger(T)U_{\mathrm{target}})\right|^2$$

equals to 1 if they are the same, up to a global phase

Universal control cost Function Optimization (UFO)

Minimize the cost function

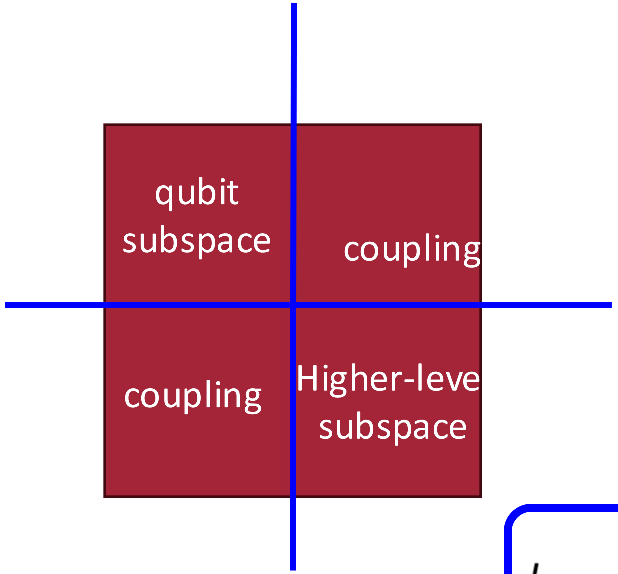
$$C(\chi,\beta,\gamma,\kappa) = \boxed{\chi[1 - F[U(T)]]} + \boxed{\beta L_{\text{tot}}} + \boxed{\mu \sum_{t\in\{0,T\}} [g^2(t) + f^2(t)]} + \boxed{\kappa T}$$

Enhance gate fidelity

Suppress leakage error

Reducing the violation of boundary constraints

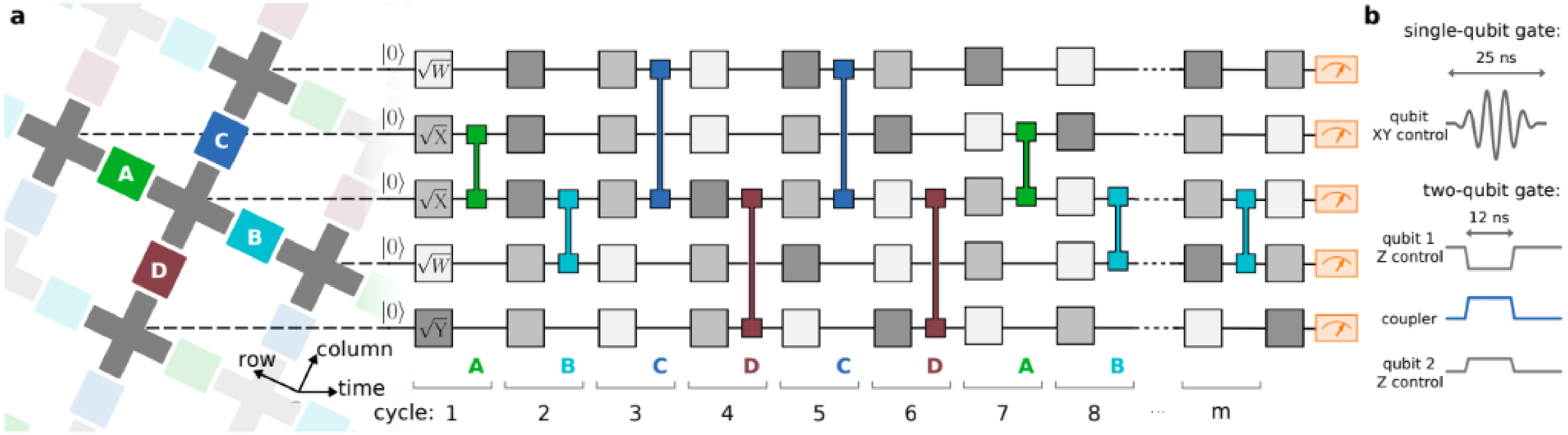
Minimize total runtime



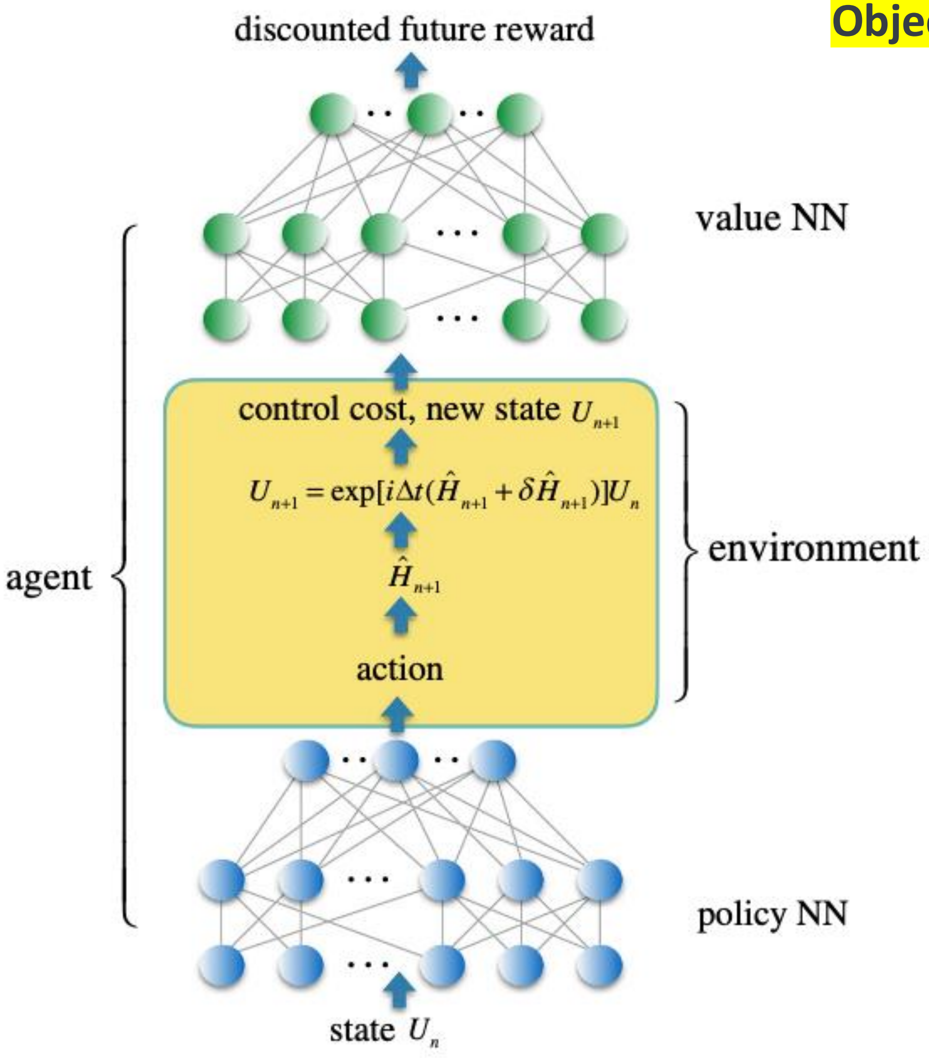
$$\boxed{L_{tot} = \frac{\left\|\hat{\mathbb{H}}_{\text{od}}(0)\right\|}{\Delta(0)} + \frac{\left\|\hat{\mathbb{H}}_{\text{od}}(T)\right\|}{\Delta(T)} + \int_0^T \frac{1}{\Delta^2(t)}\left\|\frac{d^2\hat{\mathbb{H}}_{\text{od}}(t)}{dt^2}\right\| dt,}$$

Boundary constraints

To Facilitate convenient gate concatenations: microwave pulses (manipulating individual qubit) and the g-pulse (coupling between qubits) should vanish at both boundaries



RL Architecture for Quantum Gate Optimization



Objective: optimize the policy to reduce the cost function

Agent

- **Policy neural network**: takes the current unitary gate U_n as input and outputs a control action (Hamiltonian H_{n+1})
- **Value neural network**: predicts the expected future reward from the updated state U_{n+1}

Environment:

- Takes the action and simulates the systems evolution (with additional noise)
- **Returns** the cost function

Integerating Trust Region into Policy Optimization (TRPO)

- Avoids large, unstable updates to the policy
- Ensures safe, smooth improvements in each training step
- Especially important in high-dimensional control like quantum systems

Line search
(like gradient ascent)

Trust region

How?

When updating the policy NN, add a KL-divergence constraint:

$$\text{maximize expected return subject to } D_{KL}(\pi_{\text{new}} \| \pi_{\text{old}}) \leq \delta$$

It ensures that new Hamiltonian proposals don't deviate too far from the previous ones, helps with robust convergence under noise