# STAC42: 2015 Assignment 4

1. Suppose that $X_1, X_2, \ldots$ are i.i.d $N(0,1)$ and put $S_n = \sum_{i=1}^{n} X_i$

(a) Determine the mean and covariance function of $\{S_n : n = 1, 2, \ldots\}$

(b) Prove that $\{S_n : n=1,2,\ldots\}$ is a Gaussian process.

2. For the gambler's ruin problem prove that

$$q_k = P(S_n = N \text{ for some } n \text{ where } S_1, \ldots, S_{n-1} \neq 0 \mid S_0 = k) = \begin{cases} \dfrac{1-(q/p)^k}{1-(q/p)^N}, & p \neq q \\ \dfrac{k}{N}, & p = q \end{cases}$$

3. Suppose that in an election the winner receives $\alpha$ votes and the loser receives $\beta$ votes. If the voters vote one after another then prove that if all possible sequences of voting patterns are equiprobable then the probability that the winner is always ahead is $\dfrac{\alpha-\beta}{\alpha+\beta}$.

4. 3.11.18

5. 5.1.2

6. 5.7.3

7. 5.8.6