STA C68: 2017 Assignment (1)

(1) Suppose that $\Omega = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $X: \Omega \to \mathbb{R}$ is given by $X(1) = X(5) = X(10) = 0$, $X(2) = X(4) = X(8) = 1$, $X(3) = X(6) = X(7) = X(9) = -1$.

(a) Determine f_X.

(b) Determine μ_X and σ_X^2.

(c) Suppose $Y: \Omega \to \mathbb{R}$ is given by $Y(\omega) = 0$ when ω is even and $Y(\omega) = 1$ when ω is odd. Are X and Y related? If X and Y are related display the relationship.

(2.) Prove that for finite population Ω with real-valued measurement X that $\mu_X = \sum_x x f_X(x) = \sum_{\omega \in \Omega} X(\omega) / \#(\Omega)$ and establish a similar result for σ_X^2.

(3.) Suppose that Ω is finite and X and Y are measurements on Ω. Then prove that X and Y are unrelated iff $f_{X,Y}(x,y) = f_X(x) f_Y(y)$ for all x, y.

(4.) Suppose that $x = (x_1, \ldots, x_n) \overset{iid}{\sim} N(\mu, \sigma^2)$ where $\mu \in \mathbb{R}$, $\sigma^2 > 0$ are unknown.

(a) Using R generate a sample of $n = 10$ from a $N(5, 2)$ distribution.

(b) Generate a sample of 10^3 from the distribution of the skewness statistic based on $n = 10$ and use this to check that the sample you obtained in (a) is from

a normal distribution. Provide a plot of the density histogram based on the sample and the shapiro statistic.

(c) Generate a sample of $n=10$ from the exponential (1) distribution. Using the sample of 10^3 obtained in (b) check whether or not the normal model makes sense for this new data.

(5) For the 3 prisoners problem prove that the conditional probability that prisoner 1 will not be executed is $p/(1+p)$ when the jailer randomly names prisoner 2 with probability p when this option is available.