STAC62F: 2016 Assignment ④

① For the gambler's ruin problem prove

$$P(S_n = N \text{ some } n \text{ and } S_1, \dots, S_{n-1} \neq 0 \mid S_0 = k)$$

$$= \begin{cases} \dfrac{1-\left(\frac{q}{p}\right)^k}{1-\left(\frac{q}{p}\right)^N} & p \neq q \\ \dfrac{k}{N} & p = q \end{cases}.$$

② Suppose that $X_1, X_2, \dots$ are i.i.d. Poisson(λ).

(a) Determine $m_{X_i}(t)$.

(b) Using (a) determine $E(X_i)$, $Var(X_i)$.

(c) If $Y = X_1 + \dots + X_n$ then determine the distribution of Y using moment generating functions.

(d) In general for a r.v. taking values in $\mathbb{N}_0$ and with mgf m_X and pgf r_X show that $r_X(t) = m_X(\log t)$ whenever $\log t$ is such that $m_X(\log t)$ exists. Explain how this is an inversion result for the mgf.

③ Suppose that $X_1, \dots, X_n \overset{i.i.d.}{\sim} N(\mu, \sigma^2)$ and $Y_i = X_i^3$ for $i = 1, \dots, n$. Indicate how you would approximate the probability $P(\bar{Y} \leq k)$ when n is large.