STAC62F: 2015 Assignment 1

1. If $\mathcal{F}$ is a σ-field on a set Ω then prove (i) $\Omega \in \mathcal{F}$ (ii) $A_1, \ldots, A_n \in \mathcal{F}$ implies $\bigcup_{i=1}^{n} A_i \in \mathcal{F}$ and $\bigcap_{i=1}^{n} \in \mathcal{F}$ (iii) $A_1, A_2, \ldots \in \mathcal{F}$ implies $\bigcap_{i=1}^{\infty} A_i \in \mathcal{F}$.

2. If $\{\mathcal{F}_i : i \in I\}$ is a collection of σ-fields on Ω, then prove that $\bigcap_{i \in I} \mathcal{F}_i$ is a σ-field on Ω.

3. For probability model $(\Omega, \mathcal{F}, P)$ prove (i) $P(\phi) = 0$ (ii) if $A_1, \ldots, A_n \in \mathcal{F}$ are mutually disjoint then $P(\bigcup_{i=1}^{n} A_i) = \sum_{i=1}^{n} P(A_i)$ (iii) for $A \in \mathcal{F}$ then $P(A^c) = 1 - P(A)$ (iv) for $A, B \in \mathcal{F}$ with $A \subseteq B$ then $P(A) \leq P(B)$ (iv) for $A_1, \ldots, A_n \in \mathcal{F}$ $P(\bigcup_{i=1}^{n} A_i) = \sum_{i=1}^{n} P(A_i) - \sum_{i<j} P(A_i \cap A_j) + \sum_{i<j<k} P(A_i \cap A_j \cap A_k) - \cdots + (-1)^{n+1} P(A_1 \cap A_2 \cap \cdots \cap A_k)$.

4. If $B \in \mathcal{F}$ with $P(B) > 0$ then prove that $P(\cdot \mid B) : \mathcal{F} \to [0,1]$ given by $P(A \mid B) = P(A \cap B)/P(B)$ is a probability measure on $\mathcal{F}$.

5. If $A, B \in \mathcal{F}$ are statistically independent, then prove that each element of $\{\phi, A, A^c, \Omega\}$ is statistically independent of each element of $\{\phi, B, B^c, \Omega\}$.

6. Suppose $\Omega = \{1, 2, 3\}$, $\mathcal{F} = 2^{\Omega}$, $P(\{1\}) = \frac{1}{2}$, $P(\{2\}) = \frac{1}{4}$, $P(\{3\}) = 1/4$ and $X : \Omega \to R$ is given by $X(1) = 0$, $X(2) = 0$, $X(3) = 1$. Prove that X is a random variable and determine P_X on $\mathcal{B}^1$.

7. 1.4.6

8. 1.5.8

9. 1.8.14

10. 1.8.29