University of Toronto Scarborough

Department of Computer & Mathematical Sciences

STAD57H3 Time Series Analysis

Midterm Examination
November 2, 2016

Duration: 110 minutes

Examination aids allowed: Non-programmable Scientific Calculator, formula sheet (provided by instructor)

Last Name: _____
First Name: _____
Student #: _____

Instructions:

- Read the questions carefully and answer only what is being asked.
- Answer all questions directly on the examination paper; use the last pages if you need more space, and provide clear pointers to your work.
- Show your intermediate work, and write clearly and legibly.

Question:	1	2	3	4	Total
Points:	20	22	35	23	100
Score:					

1. Let $W_t \sim \text{WN}(0, \sigma_W^2)$ follow White Noise, and $Z_t = Z_{t-1} + W_t$ be a Random Walk process. Define the processes:

   - $X_t = t^2 + Z_t$
   - $Y_t = (-1)^t + Z_t$

   (a) [10 points] Show that the *2nd order differences* of X_t, i.e. $\nabla^2 X_t$, are weakly stationary.

   (b) [10 points] Show that the *differences at lag 2* of Y_t, i.e. $\nabla_2 Y_t$, are weakly stationary.

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2. Consider the AR(2) model $X_t = \phi_1 X_{t-1} + -.25 X_{t-2} + W_t$, where $W_t \sim \text{WN}(0, \sigma_W^2)$.
   (a) [10 points] Find the range of values of ϕ_1 for which the model is stationary.
   (b) [12 points] Find the ACF of the model for $\phi_1 = 1$.

(contd...)

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3. Consider the AR(1) model $X_t = \phi X_{t-1} + W_t$, where $W_t \sim \text{WN}(0, \sigma_W^2)$. Assume you know the parameter ϕ of the model.

   (a) [5 points] Find the 1-step-ahead Best Linear Predictor (BLP) X_{n+1}^n, for predicting X_{n+1} based on $X_1, \ldots, X_n$; i.e. write the formula for X_{n+1}^n in terms of ϕ and $X_1, \ldots, X_n$.

   (b) [10 points] Verify that the 2-step-ahead BLP is $X_{n+2}^n = \phi^2 X_n$.

   (c) [10 points] Find the Mean Square Prediction Error (MSPE) of the 1- & 2-step-head predictors; i.e. write the formula for $\mathbb{E}\left[(X_{n+1} - X_{n+1}^n)^2\right]$ and $\mathbb{E}\left[(X_{n+2} - X_{n+2}^n)^2\right]$ in terms of ϕ and σ_W^2.

   (d) [10 points] Find the correlation $\text{Corr}[(X_{n+1} - X_{n+1}^n), (X_{n+2} - X_{n+2}^n)]$ of the 1- & 2-step-ahead *prediction errors*, expressed in terms of ϕ and σ_W^2.

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4. Consider the MA(1) model $X_t = W_t + \theta W_{t-1}$, where $W_t \sim \text{WN}(0, \sigma_W^2)$.

   (a) [5 points] Find the autocovariance function of the model.

   (b) [18 points] Find the PACF $\phi_{h,h}$ for $h = 1, 2, 3$ of the model, expressed in terms of θ and σ_W^2.

(contd...)

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Extra Space (use if needed and clearly indicate which questions you are answering)

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