STAC58: 2017 Assignment 3

1. Suppose $\underset{\sim}{x} = (x_1, \ldots, x_n)$ is a sample from a $N(\mu, \sigma_0^2)$ distribution and put $\underset{\sim}{r} = \underset{\sim}{x} - \bar{x}\underset{\sim}{1}$. Calculate the moment generating function of $(\bar{x}, \underset{\sim}{r})$ and from this deduce that

$$\bar{x} \sim N(\mu, \sigma_0^2/n) \quad \text{statistically independent of}$$

$$\underset{\sim}{r} \sim N_n(\underset{\sim}{0}, \sigma_0^2(\underset{\sim}{I} - \underset{\sim}{1}(\underset{\sim}{1}'\underset{\sim}{1})^{-1}\underset{\sim}{1}')).$$

Hint: calculate the joint moment generating function of $(\bar{x}, \underset{\sim}{r})$.

2. Consider the following statistical model

$\theta \backslash x$	1	2	3
a	1/3	1/3	1/3
b	1/3	2/3	0
c	0	1/3	2/3

and suppose $\Psi(\theta) = \begin{cases} 1 & \theta \in \{a, b\} \\ 0 & \theta \in \{c\} \end{cases}$ with loss function $L(\theta, a) = 1$ when $\Psi(\theta) \neq a$ and 0 otherwise.

(a) Let $d_1 : \mathcal{X} \to \Psi$ be given by $d_1(x) = 1$ when $x \in \{1, 2\}$, $d_1(x) = 0$ otherwise. Determine $R(\theta, d_1)$.

(b) Determine $R(\theta, d_2)$ when $d_2(x) = I_{\{1,3\}}(x)$.

(c) Which of d_1 and d_2 is preferred.

3. E&R 8.1.8

4. E&R 8.1.15 ($\alpha \in [0,1]$)

5. E&R 8.1.19.

6. E&R 8.1.21

7. E&R 8.1.25