STAC62 : 2015 Assignment 2

1. Suppose that $(U_1, U_2) \sim \text{Uniform}([0,1]\times[0,1])$.
(i) Determine the cdf $F_{(U_1,U_2)} : \mathbb{R}^2 \to [0,1]$;
(ii) Determine the density $f_{(U_1,U_2)}$ from $F_{(U_1,U_2)}$.
(iii) Suppose $(X,Y) = (U_1+U_2, U_1-U_2)$. Determine $F_{(X,Y)}$ and $f_{(X,Y)}$.

2. Suppose that $T : \Omega \to \mathcal{J}$. Establish the following properties for the inverse T^{-1}
(i) $T^{-1}A^c = (T^{-1}A)^c$ for $A \subseteq \mathcal{J}$
(ii) $T^{-1}\bigcup_{i\in I} A_i = \bigcup_{i\in I} T^{-1}A_i$ for $A_i \subseteq \mathcal{J}$, $i \in I$.
(iii) $T^{-1}\bigcap_{i\in I} A_i = \bigcap_{i\in I} T^{-1}A_i$.

3. Suppose it is specified that $X_1 \sim N(0,1)$, $X_2 \sim N(0,1)$ while $(X_1,X_2) \sim N_2\left(\begin{pmatrix}0\\0\end{pmatrix}, \begin{pmatrix}1 & 1/2\\ 1/2 & 1\end{pmatrix}\right)$. Does this specify a stochastic process with state space $S = \mathbb{R}$, time domain $T = \{1,2\}$? Justify your answer.

4. Suppose that $\underset{\sim}{X} \sim N_k(\underset{\sim}{\mu}, \Sigma)$ where $\Sigma = BB'$ with $B \in \mathbb{R}^{k\times k}$ invertible. Prove that $\underset{\sim}{Z} = B^{-1}(\underset{\sim}{X} - \underset{\sim}{\mu}) \sim N_k(\underset{\sim}{0}, I)$.

5. Suppose $\underset{\sim}{X} \in \mathbb{R}^k$ and $\underset{\sim}{X} \sim \text{multinomial}(n, p_1, \ldots, p_k)$. Let $\underset{\sim}{Y} = (X_1, \ldots, X_{l-1}, X_l + \cdots + X_k)'$. Prove that $\underset{\sim}{Y} \sim \text{multinomial}(n, p_1, \ldots, p_{l-1}, p_l + \cdots + p_k)$.

6. Suppose $\underset{\sim}{X}$ and $\underset{\sim}{Y}$ are as in 6. Prove that $(X_l, \ldots, X_k) \mid \underset{\sim}{Y} = \underset{\sim}{y}$
$\sim \text{multinomial}\left(n - x_1 - \cdots - x_{l-1}, \frac{p_l}{1-p_1-\cdots-p_{l-1}}, \ldots, \frac{p_k}{1-p_1-\cdots-p_{l-1}}\right)$

7. 2, 5, 6

8. 2, 7, 9

9. 2, 7, 16

10. 2, 7, 20