STAC62 : 2016 Assignment 2

1. Suppose that $\Omega = \{1,2,3,4,5\}$ and $\mathcal{F} = \mathcal{F}(\mathcal{a})$, $\mathcal{a} = \{\phi, \Omega, \{1,2\}, \{3,4\}, \{5\}\}$, $P(\{1,2\}) = P(\{3,4\}) = \frac{1}{4}$, $P(\{5\}) = \frac{1}{2}$

(a) Let $X : \Omega \to R$ be given by $X(1) = 1$, $X(2) = 1$, $X(3) = 2$, $X(4) = 3$, $X(5) = 3$. Is X a random variable? Why or why not? If X is a random variable determine P_X.

(b) Repeat (a) with $Y : \Omega \to R$ given by $Y(1) = Y(2) = 0$, $Y(3) = Y(4) = 1$, $Y(5) = 2$.

2. For an random vector $\underset{\sim}{X} : (\Omega, \mathcal{F}) \to (R^k, B^k)$ prove that $P_{\underset{\sim}{X}}$ is a probability measure on (R^k, B^k).

3. Suppose that $\Omega = \{1,2,3,4\}$, $\mathcal{F} = 2^\Omega$ and define $\underset{\sim}{X} : \Omega \to R^2$ by $X_1(i) = i$ and $X_2(1) = X_2(2) = 1$, $X_2(3) = X_2(4) = 2$.

(a) Prove $\underset{\sim}{X}$ a random vector.

(b) If P is given by the uniform distribution on Ω then determine $P_{\underset{\sim}{X}}$.

(c) Let $Y(\omega) = X_1(\omega) + X_2(\omega)$. Is Y a random variable and, if it is, determine P_Y where P is uniform on Ω.

4. Suppose that $F(x) = \begin{cases} 0 & x \le 0 \\ 1 - e^{-x} & x > 0 \end{cases}$. Then show that F satisfies the three properties necessary for a cdf ((i) $\Delta_{a,b} F \geq 0 \ \forall a,b$ (ii) $\lim_{x\to-\infty} F(x) = 0$, $\lim_{x\to\infty} F(x) = 1$ (iii) F is right continuous).

5. Suppose that $\underset{\sim}{X} \sim N_k(\underset{\sim}{0}, I)$. Then prove that $\underset{\sim}{Y} = (X_1, \ldots, X_{k-1}) \sim N_{k-1}(\underset{\sim}{0}, I)$. Explain why this determines a Gaussian process on $\mathbb{R}^T$ for any time domain T.

6. Suppose that the joint density $f_{X,Y}$ is given by $f_{X,Y}(x,y) = Cye^{-xy}$ for $(x,y) \in (0,1)^2$.
(a) Determine C so that $f_{X,Y}$ is a density
(b) Compute $P(1/2 < X < 1,\ 1/2 < Y < 1)$.
(c) Compute f_X and f_Y.
(d) Are X and Y independent r.v.'s? Why or why not?

7. Suppose $(X_1, X_2, X_3) \sim \text{multinomial}(n, \theta_1, \theta_2, \theta_3)$
(a) Determine the distribution of X_1.
(b) Determine the distribution of $X_2 | X_1 = x_1$.