

Verifying the Resilience of Neural Network Watermarking

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1 Introduction

Deep Neural Networks (DNN) seems to be everywhere nowadays both in research and in the Industry; They're used for large variety of applications, and achieving state of the art result in many fields (Computer vision, speech recognition, AI, and many more). DNNs flexibility and diversity are pushing the limits on what is possible for a computer to solve efficiently. As a result from their empiric success DNNs are now changing the way software is being designed, and broaden the role of Machine Learning trained functions in applications.

Because of the success of DNNs and because designing and training a DNN require certain expertise, time and processing power, the demand for a specifically designed DNN is growing, and Machine Learning as a Service (MLaaS) is now a thing. There are many machine learning services appearing in many forms, from data visualization and cloud computing to frameworks and semi-trained DNN's. This create some issues regarding the rights and ownership of some part of a trained network or design. Because of the relatively simple components of a DNN (Matrices, vectors and simple functions) it's quite easy to copy or use without permission.

In order to deal with such issues we need a way to authenticate a DNN. This may sound simple but the authentication needs to be robust, such that it's hard to remove. Here comes the concept of Digital Watermarking, a way of signing some Digital property such that it's hard to remove the signature, and said signature is unique. There are some proposed method of watermarking a DNN, but it's unclear how effective they are, meaning how difficult it is to remove the watermark from the DNN and what is the effect of removing the watermark.

Formal verification of DNN's is a new and promising field. We propose a novel methodology to use verification to measure and verify the robustness of a certain watermarking techniques. This method may be applicable to other verification problems that are not necessarily watermark related such as network correction, meaning we're given a network and a counter example for some desired property of the network. We're interested to find a "Fix" such that the counter example will no longer exist.

The rest of this paper is organized as follows. In Section 2 we provide the necessary background on DNN's, watermarking, and DNN verification. Next, in Section ?? we introduce our technique for casting the watermark resilience problem into a verification problem. Section 5 describes our implementation and evaluation of the approach on several watermarked DNN's for image recognition. We discuss related work in Section 6, and conclude in Section 7.

2 Background

2.1 Neural Networks

A neural network is a function that receive an input vector and returns an output vector. There are many types a architectures of neural networks.

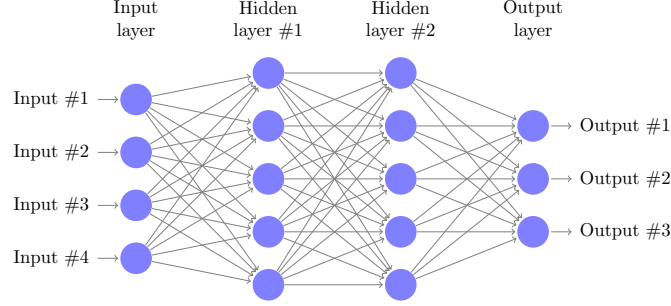


Figure 1: Example of a simple fully connected Deep Neural Network with an input layer of size 4 two hidden layers of size 5 and an output of size 3.

To keep things simple we'll only describe the way a feed forward fully connected network calculate an output from a given input. A feed forward fully connected network is constructed from a bunch of layers l_1, \dots, l_k , l_1 is the input layer and l_k is the output later, and an activation function (we're interested in the ReLU activation function). Each layer has a certain number of nodes, l_1 l_k number of nodes is determine by the size of the input and output. Between two adjacent layers l_i l_{i+1} the values of nodes in l_{i+1} is calculated by multiplying the values of l_i by a weights matrix L_i of size $n \times m$ where n and m are the number of nodes in l_{i+1} and l_i respectively. After applying the weights matrix we add a bias vector and apply the activation function, in our case the ReLU activation function, the resulting vector is of size n and it holds the values of the l_{i+1} nodes.

for a single value x

$$ReLU(x) = \max(0, x)$$

and for a vector \vec{v}

$$ReLU(\vec{v})_i = \max(0, v_i)$$

So basically a feed forward fully connected network with k layers is defined by a set of $k - 1$ matrices L_1, \dots, L_{k-1} and $k - 1$ biases vectors B_1, \dots, B_{k-1} and for a given input x an output vector y is produced like that:

$$y = B_{k-1} + L_{k-1} \cdot ReLU(\dots B_3 + L_3 \cdot ReLU(B_2 + L_2 \cdot ReLU(B_1 + L_1 \cdot x)) \dots) \quad (1)$$

For the context of this work we're only dealing with classification neural networks. For these kind of networks the output layer size usually correspond to the number of possible classifications, and each entry in the output vector correspond to the score of a specific classification.

2.2 Verifying Neural Networks

DNN verification answers the following question: Given a neural network N and two predicates P, Q such that P encodes the constraint on the input to the network x and Q encodes the constraint on the output y . We ask if there exist some input x_0 and as a result the output

of the network $y_0 = N(x_0)$, such that $P(x_0) = \text{True}$ and $Q(y_0) = \text{True}$. In this work we're verifying properties of the network a bit differently. Instead of searching an input that answer certain constraints, the input is fixed and we're allowing changes to the network. So the verification question looks like this: Given a neural network N and two predicates P, Q such that P encodes the constraint on the changes to the network ε and Q encodes the constraint on the output y . We ask if there exist ε and as a result the new network $N' = N + \varepsilon$ and new output $y_0 = N'(x_0)$, such that $P(\varepsilon_0) = \text{True}$ and $Q(y_0) = \text{True}$. [2, 3]

3 Changes to the network as minimization problem

Looking at a trained neural network we're interested to find the minimal change to the network in order to "change" the prediction of a specific input. We'll look at changes only to the last layer of the network.

Given a trained network N and a set of K inputs $\{x_1, \dots, x_K\}$ we'll mark the network last layer matrix L such that L is a $m \times n$ matrix where n is the layer's number of neurons and m is the network output size. The change to the last layer will be a matrix with the same dimension as L we'll mark as ε , such that $\varepsilon_{i,j}$ is the change to the last layer matrix entry $L_{i,j}$.

3.1 Defining the problem for single input

For a certain input x we're only interested in the input to the last layer we'll mark the input to the last layer v . v is a $n \times 1$ vector. So the original network output $y = Lv$ and the changed network output is $y' = (L + \varepsilon)v$.

For a single input x we denote the original network prediction:

$$d_x := \operatorname{argmax}_{i \in [m]} \{y_i\}$$

And the changed network prediction:

$$d'_x := \operatorname{argmax}_{i \in [m]} \{y'_i\}$$

We're interested to find the minimal change to the last layer ε so that the prediction will change i.e. $d_x \neq d'_x$

We'll measure the overall change to the layer in two ways

$$\|\varepsilon\|_\infty = \max_{i,j} \{|\varepsilon_{i,j}|\}. \quad (2)$$

And

$$\|\varepsilon\|_1 = \sum_{i,j} |\varepsilon_{i,j}|. \quad (3)$$

For the ℓ_∞ norm (2) with a chosen d'_x that is different from d_x the minimization problem looks

like that:

$$\begin{aligned}
& \text{Minimize : } M \\
& \text{Subject to : } \forall i, j \quad -M \leq \varepsilon_{i,j} \leq M \\
& \quad y' = (L + \varepsilon)v \\
& \quad y'_{d_x} \leq y'_{d'_x}
\end{aligned}$$

Variables are the entries in ε, y' and M

(4)

Similarly for the ℓ_1 norm (3) the minimization problem looks like that:

$$\begin{aligned}
& \text{Minimize : } M \\
& \text{Subject to : } \forall i, j \quad -M \leq \sum_{i,j} |\varepsilon_{i,j}| \leq M \\
& \quad y' = (L + \varepsilon)v \\
& \quad y'_{d_x} \leq y'_{d'_x}
\end{aligned}$$

Variables are the entries in ε, y' and M

(5)

3.2 Defining the problem for many inputs

Our definition to a single input minimal change ε to the network last layer L can be extended to more then one input very easily by adding more constraint to the problem.

Given inputs $\{x_1, \dots, x_k\}$ and their respective values:

$$\begin{aligned}
\{v_1, \dots, v_k\} & \quad \text{:Inputs to the last layer} \\
\{d_1, \dots, d_k\} & \quad \text{:Decisions}
\end{aligned}$$

Such that

$$\forall 1 \leq j \leq k \quad d_j = \operatorname{argmax}_{i \in [m]} \{(Lv_j)_i\}$$

With chosen new desired decisions $\{d'_1, \dots, d'_k\}$ Such that

$$\forall 1 \leq j \leq k \quad d'_j \neq d_j$$

our minimization problem for the ℓ_∞ norm looks like this:

$$\begin{aligned}
& \text{Minimize : } M \\
& \text{Subject to : } \forall i, j \quad -M \leq \varepsilon_{i,j} \leq M \\
& \quad \forall j \quad y'_j = (L + \varepsilon)v_j \\
& \quad \forall j \quad (y'_j)_{d_j} \leq (y'_j)_{d'_j}
\end{aligned}$$

Variables are the entries in $\varepsilon, y'_1, \dots, y'_k$ and M

(6)

Similarly for the ℓ_1 norm the minimization problem looks like that:

Minimize : M

Subject to : $\forall i, j \quad -M \leq \sum_{i,j} |\varepsilon_{i,j}| \leq M$

$$\forall j \quad y'_j = (L + \varepsilon)v_j$$

$$\forall j \quad (y'_j)_{d_j} \leq (y'_j)_{d'_j}$$

Variables are the entries in $\varepsilon, y'_1, \dots, y'_k$ and M

(7)

Guy: Overall, this is good. Some stuff needs to be moved to other sections according to the paper layout.

4 Methods

As seen in the previous section we have in our hands a minimization problem. One minimization is according to ℓ_∞ norm and the other is according to ℓ_a norm. For the ℓ_∞ norm when choosing new predictions all the constraint of the minimization problems (4) (6) are linear. There for we choose to solve the ℓ_∞ problem using the Gurobi linear programming solver. Gurobi has a well known LP solver that is capable of solving relatively large scale problems. On the other hand ℓ_1 norm minimization problems (5) (7) have non linear constraint so a linear programming solver is not enough. Instead we used Marabou which is a verification tool for neural networks that support piecewise-linear activation functions, Marabou can supply a feasible solution to a set of piecewise-linear constraints so combined with the interval halving method we can find the minimal change for the ℓ_1 case as well.

5 Evaluation

We tested both approaches on a simple neural network trained on the MNIST data set, the network have one hidden layer with 150 nodes. The network was watermarked with 100 images of noise, examples in Figure 2a.

5.1 Removing a single watermark

The first test we did was to find the minimal change of a single input as defined (4) (5) for every water mark (noise) image. For every decision possible beside the original decision we found the minimal change and choose the overall minimum. It turns out that the minimal change was always the second best score in the original prediction output. For example an watermark image w with an original output y .

$$d_w = \underset{i \in \{0, \dots, 9\}}{\operatorname{argmax}} \{y_i\}$$

d_w is the original tagging of w .

$$d'_w = \underset{i \in \{0, \dots, 9\} \setminus \{d_w\}}{\operatorname{argmax}} \{y_i\}$$

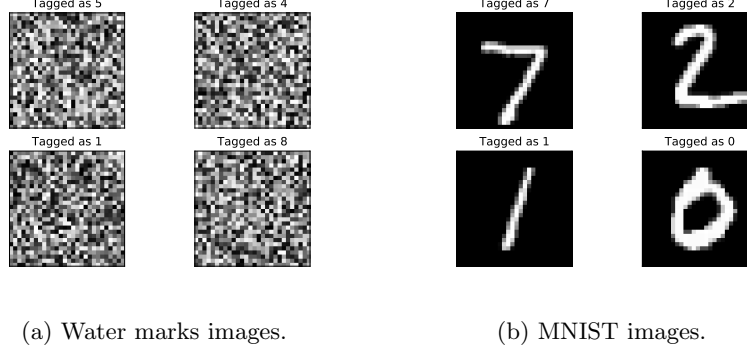


Figure 2: Input images examples.

| Number of watermarks | Avrg change | Min change | Max change | Avrg acc | Min acc | Max acc | Norm |
|----------------------|-------------|------------|------------|----------|---------|---------|----------|
| 0 | 0 | 0 | 0 | 0.97 | 0.97 | 0.97 | |
| 1 | 0.34 | 0.3 | 0.38 | 0.87 | 0.86 | 0.88 | Infinity |
| 1 | 27.95 | 21.56 | 35.91 | 0.94 | 0.56 | 0.97 | One |

Table 1: Minimal changes and Accuracy

d'_w will be the new tagging of w .

So after we apply change to the last layer of the network the new output y' is such that $\argmax_{i \in \{0, \dots, 9\}} \{y'_i\} = d'_w$

For each watermark image we have found the minimal ε . This can give us some measure of how difficult it is to remove a single watermark image. And there may be some correlation in the minimal change distribution that is not related to the norm as seen in this figure 3. Beside removing a watermark we're interested in the effect the change we introduced has on the network goal. By applying the change to the original network and evaluating on the MNIST dataset we measured the resulting accuracy of each change. As it turns out we get widely different results from both removal methods (norms) when measuring the accuracy of the changed network. As seen in this table 1 the ℓ_1 gives better accuracy result on average but can have a bad accuracy result, compared to the ℓ_∞ after removing a single watermark that has a consistent result (there minimal accuracy and the maximal accuracy are quite close) but the average is lower than the ℓ_1 method.

These results beg the question what is the nature of the change and how it differs between the methods? Turns out that when minimizing the change according to ℓ_∞ (4) the linear programming solver assigns all the entries of ε a small value. This translates to a change to every weight in the last layer which can explain the uniformity of the accuracy test. On the other hand when minimizing the change according to ℓ_1 (5) the optimal change is to change only a single entry in ε by a seemingly large value. So only one weight in the last layer is changed. See figure 4

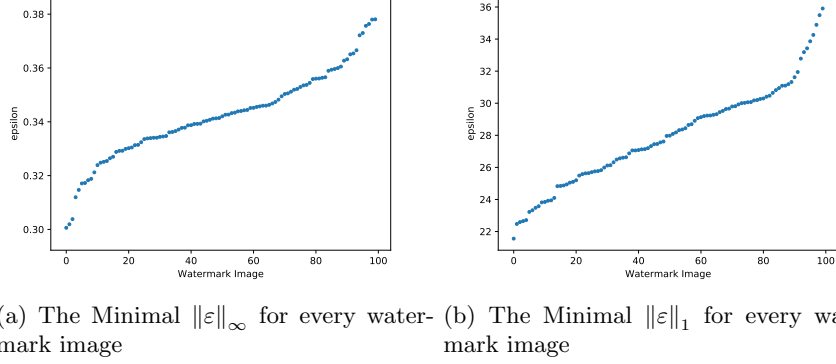


Figure 3: Notice the scale of the graphs, the ℓ_∞ values are much smaller than the ℓ_1 values. As expected

The result of removing a single watermark shows us that the minimal change according to ℓ_∞ is changing the network in a broad way and according to the ℓ_1 norm the change is local, but then not all watermarks are "equal", some are harder to remove and some are easy (as seen in the accuracy test 1).

5.2 Removing Multiple watermark

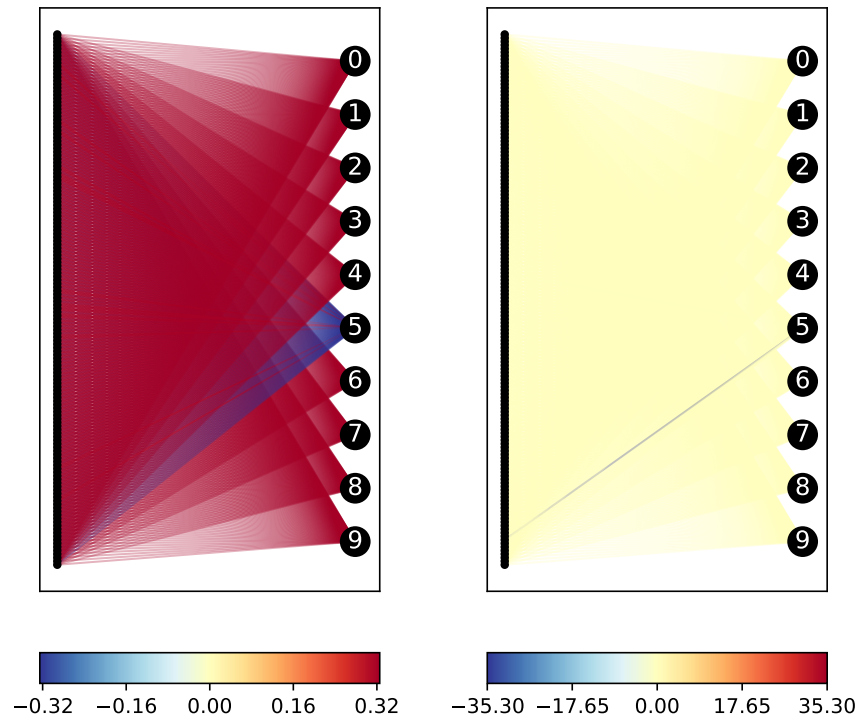
As described above the removal of watermarks can be generalized to many watermarks (6) (7). We tested the same MNIST digits classifying network that is signed with 100 watermark. While running the problems on multiple watermarks the linear problem that we solved using the Gurobi solver had no issues with scalability and we were able to find the change that fits to as many watermarks as we want, on the other hand the non-linear problem had some issues with scalability so we were able to find the change that fits to up to 5 watermarks.

6 Related Work

7 Conclusion and Future Work

References

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- [2] G. Katz, C. Barrett, D. Dill, K. Julian, and M. Kochenderfer. Reluplex: An Efficient SMT Solver for Verifying Deep Neural Networks. In *Proc. 29th Int. Conf. on Computer Aided Verification (CAV)*, pages 97–117, 2017.
- [3] G. Katz, D. Huang, D. Ibeling, K. Julian, C. Lazarus, R. Lim, P. Shah, S. Thakoor, H. Wu, A. Zeljić, D. Dill, M. Kochenderfer, and C. Barrett. The Marabou Framework for Verification and Analysis of Deep Neural Networks. In *Proc. 31st Int. Conf. on Computer Aided Verification (CAV)*, 2019.



(a) The Minimal change according to ℓ_∞ for a single input (b) The Minimal change according to ℓ_1 for a single input

Figure 4: Examples of the change to the last layer. Positive change is colored red and negative change is colored blue. There are 150 nodes on the left.

| Number of watermarks | Avrg change | Min change | Max change | Avrg acc | Min acc | Max acc |
|----------------------|-------------|------------|------------|----------------------|----------------------|----------------------|
| 0 | 0 | 0 | 0 | 0.97 | 0.97 | 0.97 |
| 1 | 0.34 | 0.3 | 0.38 | 0.87 | 0.86 | 0.88 |
| 2 | 0.43 | 0.3 | 1.83 | 0.79 | 0.76 | 0.88 |
| 3 | 0.53 | 0.33 | 1.79 | 0.71 | 0.66 | 0.88 |
| 4 | 0.68 | 0.33 | 1.79 | 0.64 | 0.57 | 0.88 |
| 5 | 0.79 | 0.33 | 1.87 | 0.59 | 0.47 | 0.8 |
| 6 | 0.89 | 0.35 | 1.83 | 0.53 | 0.38 | 0.78 |
| 7 | 1.05 | 0.34 | 1.91 | 0.48 | 0.28 | 0.78 |
| 25 | 1.86 | 1.45 | 2.09 | $9.49 \cdot 10^{-2}$ | $2.7 \cdot 10^{-3}$ | 0.41 |
| 50 | 2.05 | 1.9 | 2.15 | $4.89 \cdot 10^{-2}$ | $2.3 \cdot 10^{-3}$ | 0.2 |
| 75 | 2.13 | 2.03 | 2.17 | $5.95 \cdot 10^{-2}$ | $2.8 \cdot 10^{-3}$ | 0.18 |
| 100 | 2.18 | 2.18 | 2.18 | $5.11 \cdot 10^{-2}$ | $5.11 \cdot 10^{-2}$ | $5.11 \cdot 10^{-2}$ |

(a) Change and accuracy when solving for minimal ℓ_∞ change.

| Number of watermarks | Avrg change | Min change | Max change | Avrg acc | Min acc | Max acc |
|----------------------|-------------|------------|------------|----------|---------|---------|
| 0 | 0 | 0 | 0 | 0.97 | 0.97 | 0.97 |
| 1 | 27.95 | 21.56 | 35.91 | 0.94 | 0.56 | 0.97 |
| 2 | 51.12 | 22.65 | 145.75 | 0.9 | 0.56 | 0.97 |
| 3 | 73.52 | 25.83 | 172.67 | 0.86 | 0.53 | 0.97 |
| 4 | 98.39 | 25.83 | 205.22 | 0.86 | 0.53 | 0.97 |

(b) Change and accuracy when solving for minimal ℓ_1 change.

Table 2: Minimal changes and Accuracy for multiple watermarks