MATRIX PROJECT
EE1390 Introduction to AI and ML

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Geometric Question

If a circle C, whose radius is 3, touches externally the circle

$$x^2 + y^2 + 2x - 4y = 4$$

at the point (2,2),then find the length of the intercept cut by this circle C on the x-axis.

Matrix Transformation of the Question

Equation of the circle which externally touches the circle C at a point (2,2) is

$$X^T X + \begin{pmatrix} 2 & -4 \end{pmatrix} X = 4$$

where X is point on the circle

$$X = \begin{pmatrix} x \\ y \end{pmatrix}$$

Find the length of intercept cut by circle C on the x-axis.

Solution in the form of matrix

Let
C1 - Centre of the circle whose equation is unknown
r - Radius of the circle whose equation is given
C - Centre of the circle whose equation is given

Solution in the form of matrix

Given

$$X^T X + \begin{pmatrix} 2 & -4 \end{pmatrix} X = 4 \tag{1}$$

Since we know that

$$(X - C)^T (X - C) = r^2$$

as

$$||X - C|| = r$$

$$X^T X - 2C^T X = r^2 - C^T C \tag{2}$$

Solution in the form of matrix

Comparing equations (1) and (2) we get

$$-2C^T = \begin{pmatrix} 2 & -4 \end{pmatrix}$$

$$C^T = \frac{-1 \times \begin{pmatrix} 2 & -4 \end{pmatrix}}{2}$$

$$C = \frac{-1 \times \begin{pmatrix} 2 & -4 \end{pmatrix}^T}{2}$$

$$C^T C = \begin{pmatrix} -1 & 2 \end{pmatrix} \begin{pmatrix} -1 \\ 2 \end{pmatrix} = 5 \quad (3)$$

$$r^2 - C^T C = 4$$

$$r^2 = C^T C + 4$$

$$r = \sqrt{C^T C + 4} \qquad (4)$$

From equations (3) and (4) we get

$$r = \sqrt{5 + 4} = 3$$

Solution in the form of matrix

Let n be the direction vector of line joining centre C and point(2,2)

$$n = \begin{pmatrix} -1 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \end{pmatrix}$$

We know that when circles touch each other externally at a point, the point and the centres of the circles lie on the same line which implies

$$C1 - C = kn$$

where k is some constant.

Solution in the form of matrix

As we know that when circles touch each other externally the distance between the centres is same as the sum of their radii.

$$|C1-C| = r1+r(since\ r = 3\ and\ radius\ of\ the\ other\ circle\ (r1)\ = 3)$$

$$(C1-C)^T(C1-C) = 6^2$$

$$(kn)^T(kn) = 36$$

$$k^2 \times n^T n = 36$$

$$k^2 = \frac{36}{n^T n}$$

$$As\ n^T n = \begin{pmatrix} -3 & 0 \end{pmatrix} \begin{pmatrix} -3 \\ 0 \end{pmatrix} = 9$$

$$k^2 = \frac{36}{9} = 4$$

$$k = \pm 2.....(a)$$

Solution in the form of matrix

Also

$$C1 = kn + C \tag{5}$$

$$(C1 - \begin{pmatrix} 2 \\ 2 \end{pmatrix})^T (C1 - \begin{pmatrix} 2 \\ 2 \end{pmatrix}) = 3^2$$

From (5)

$$(C + kn - \begin{pmatrix} 2 \\ 2 \end{pmatrix})^T (C + kn - \begin{pmatrix} 2 \\ 2 \end{pmatrix}) = 9$$

$$(\begin{pmatrix} -1 \\ 2 \end{pmatrix} + k \begin{pmatrix} -3 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \end{pmatrix})^T (\begin{pmatrix} -1 \\ 2 \end{pmatrix} + k \begin{pmatrix} -3 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \end{pmatrix}) = 9$$

$$\begin{pmatrix} -3k - 3 & 0 \end{pmatrix} \begin{pmatrix} -3k - 3 \\ 0 \end{pmatrix} = 9$$

$$(-3k - 3)^2 + 0^2 = 9$$

Solution in the form of matrix

$$9(k+1)^2 = 9$$

$$(k+1)^2 = 1$$

$$k+1 = \pm 1$$

$$k = 0, -2......(b)$$

From (a) and (b)

$$k = -2$$

$$C = \begin{pmatrix} -1 \\ 2 \end{pmatrix} + k\begin{pmatrix} -3 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} + -2\begin{pmatrix} -3 \\ 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$$

Solution in the form of matrix

We now know the centre of the circle C and its radius(r1 = 3 given). So the equation of the circle is

$$(Y - C)^T (Y - C) = 3^2$$

where Y is a point on the circle and as

$$|Y - C| = 3$$

$$Y^T Y - 2C^T Y + C^T C = 9$$

$$Y^T Y - 2\begin{pmatrix} 5 & 2 \end{pmatrix} Y + \begin{pmatrix} 5 & 2 \end{pmatrix} \begin{pmatrix} 5 \\ 2 \end{pmatrix} = 9$$

$$Y^T Y - 2\begin{pmatrix} 5 & 2 \end{pmatrix} Y + 20 = 0 \tag{6}$$

Solution of the form of matrix

To get the x-intercept we take general point on x-axis, substitute it in the circle equation and solve for points and then find distance between them.

$$\textit{Substituting } Y = \begin{pmatrix} x \\ 0 \end{pmatrix} \textit{ in } (6)$$

$$\begin{pmatrix} x & 0 \end{pmatrix} \begin{pmatrix} x \\ 0 \end{pmatrix} - 2 \begin{pmatrix} 5 & 2 \end{pmatrix} \begin{pmatrix} x \\ 0 \end{pmatrix} + 20 = 0$$

$$x^2 - 10x + 20 = 0$$

$$x = \frac{10 \pm \sqrt{10^2 - 4 \times 20}}{2}$$

$$x = \frac{10 \pm \sqrt{20}}{2}$$

$$x = 5 \pm \sqrt{5}$$

$$X - intercept = \Delta x = 2\sqrt{5}$$

Figure of the Solution