MATRIX PROJECT
EE1390 Introduction to AI and ML

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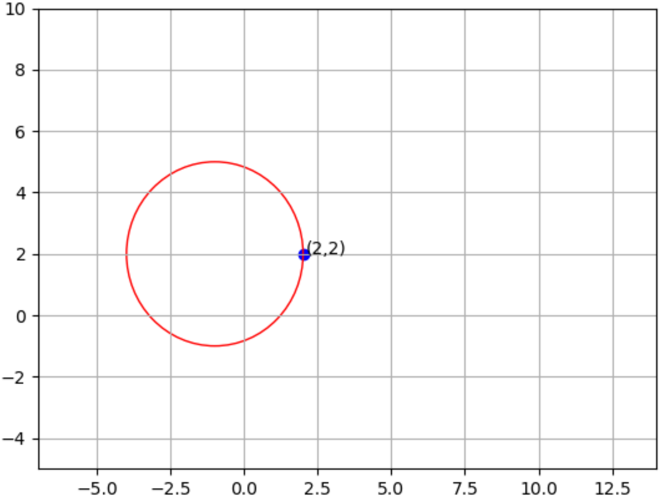
Geometric Question

If a circle **C**, whose radius is 3, touches externally the circle

$$x^2 + y^2 + 2x - 4y = 4$$

at the point (2,2),then find the length of the intercept cut by this circle **C** on the x-axis.

Figure



Matrix Transformation of the Question

Equation of the circle which externally touches the circle **C** at a point (2,2) is

$$\mathbf{X}^T\mathbf{X} + \begin{pmatrix} 2 & -4 \end{pmatrix}\mathbf{X} = 4$$

where X is point on the circle

$$\mathbf{X} = \begin{pmatrix} x \\ y \end{pmatrix}$$

Find the length of intercept cut by circle **C** on the x-axis.

Solution in the form of matrix

Let
C1 - Centre of the circle whose equation is unknown
r - Radius of the circle whose equation is given
C - Centre of the circle whose equation is given

Solution in the form of matrix

Given

$$\mathbf{X}^T\mathbf{X} + \begin{pmatrix} 2 & -4 \end{pmatrix}\mathbf{X} = 4 \tag{1}$$

Since we know that

$$(\mathbf{X} - \mathbf{C})^T(\mathbf{X} - \mathbf{C}) = r^2$$

as

$$||\mathbf{X} - \mathbf{C}|| = r$$

$$\mathbf{X}^T\mathbf{X} - 2\mathbf{C}^T\mathbf{X} = r^2 - \mathbf{C}^T\mathbf{C} \tag{2}$$

Solution in the form of matrix

Comparing equations (1) and (2) we get

$$-2\mathbf{C}^T = \begin{pmatrix} 2 & -4 \end{pmatrix}$$

$$\mathbf{C}^T = \frac{-1 \times \begin{pmatrix} 2 & -4 \end{pmatrix}}{2}$$

$$\mathbf{C} = \frac{-1 \times \begin{pmatrix} 2 & -4 \end{pmatrix}^T}{2}$$

$$\mathbf{C}^T\mathbf{C} = \begin{pmatrix} -1 & 2 \end{pmatrix} \begin{pmatrix} -1 \\ 2 \end{pmatrix} = 5 \quad (3)$$

$$r^2 - \mathbf{C}^T\mathbf{C} = 4$$

$$r^2 = \mathbf{C}^T\mathbf{C} + 4$$

$$r = \sqrt{\mathbf{C}^T\mathbf{C} + 4} \qquad (4)$$

From equations (3) and (4) we get

$$r = \sqrt{5+4} = 3$$

Solution in the form of matrix

Let **n** be the direction vector of line joining centre C and point(2,2)

$$\mathbf{n} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \end{pmatrix}$$

The line joining centres of two externally touching circles includes the point of contact.

$$\mathbf{C1} - \mathbf{C} = k\mathbf{n}$$

where k is some constant.

Solution in the form of matrix

$$\mathbf{C1} = k\mathbf{n} + \mathbf{C} \tag{5}$$

$$(\mathbf{C1} - \begin{pmatrix} 2 \\ 2 \end{pmatrix})^T(\mathbf{C1} - \begin{pmatrix} 2 \\ 2 \end{pmatrix}) = 3^2$$

From (5)

$$(\mathbf{C} + k\mathbf{n} - \begin{pmatrix} 2 \\ 2 \end{pmatrix})^T(\mathbf{C} + k\mathbf{n} - \begin{pmatrix} 2 \\ 2 \end{pmatrix}) = 9$$

$$(\begin{pmatrix} -1 \\ 2 \end{pmatrix} + k\begin{pmatrix} -3 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \end{pmatrix})^T(\begin{pmatrix} -1 \\ 2 \end{pmatrix} + k\begin{pmatrix} -3 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \end{pmatrix}) = 9$$

$$\begin{pmatrix} -3k - 3 & 0 \end{pmatrix}\begin{pmatrix} -3k - 3 \\ 0 \end{pmatrix} = 9$$

$$(-3k - 3)^2 + 0^2 = 9$$

Solution in the form of matrix

$$9(k+1)^2 = 9$$

$$(k+1)^2 = 1$$

$$k+1 = \pm 1$$

$$k = 0, -2$$

We require a non-zero value of k so

$$k = -2$$

$$\mathbf{C} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} + k \begin{pmatrix} -3 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} + -2 \begin{pmatrix} -3 \\ 0 \end{pmatrix}$$

$$\mathbf{C} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$$

Solution in the form of matrix

We now know the centre of the circle **C** and its radius($r1 = 3$ given). So the equation of the circle is

$$(\mathbf{Y} - \mathbf{C})^T(\mathbf{Y} - \mathbf{C}) = 3^2$$

where **Y** is a point on the circle and as

$$||\mathbf{Y} - \mathbf{C}|| = 3$$

$$\mathbf{Y}^T\mathbf{Y} - 2\mathbf{C}^T\mathbf{Y} + \mathbf{C}^T\mathbf{C} = 9$$

$$\mathbf{Y}^T\mathbf{Y} - 2\begin{pmatrix}5 & 2\end{pmatrix}\mathbf{Y} + \begin{pmatrix}5 & 2\end{pmatrix}\begin{pmatrix}5\\2\end{pmatrix} = 9$$

$$\mathbf{Y}^T\mathbf{Y} - 2\begin{pmatrix}5 & 2\end{pmatrix}\mathbf{Y} + 20 = 0 \tag{6}$$

Solution of the form of matrix

To get the x-intercept we take general point on x-axis, substitute it in the circle equation and solve for points and then find distance between them.

$$\textit{Substituting } \mathbf{Y} = \begin{pmatrix} x \\ 0 \end{pmatrix} \textit{ in } (6)$$

$$\begin{pmatrix} x & 0 \end{pmatrix} \begin{pmatrix} x \\ 0 \end{pmatrix} - 2 \begin{pmatrix} 5 & 2 \end{pmatrix} \begin{pmatrix} x \\ 0 \end{pmatrix} + 20 = 0$$

$$x^2 - 10x + 20 = 0$$

$$x = \frac{10 \pm \sqrt{10^2 - 4 \times 20}}{2}$$

$$x = \frac{10 \pm \sqrt{20}}{2}$$

$$x = 5 \pm \sqrt{5}$$

$$X - intercept = \Delta x = 2\sqrt{5}$$

Figure of the Solution