Exercise 3.2.1

(a) Show that if Q is orthogonal,then Q^{-1} is orthogonal. (b) Show that if Q_1 and Q_2 are othogonal, then Q_1Q_2 is orthogonal.

Answer

a) By definition:

$$Q^tQ = I$$

is orthogonal.
Inverting both sides:

$$(Q^t)^{-1}Q^{-1} = I^{-1}$$

Using the orthogonal matrices property $Q^t = Q^{-1}$:

$$(Q^{-1})^tQ^{-1} = I^{-1}$$

$$(Q^{-1})^tQ^{-1} = I$$

Therefore, Q^{-1} is orthogonal, from the same definition.

b)Show that $(Q_1Q_2)^t(Q_1Q_2) = I$

$$\begin{gathered} Q_1^tQ_1 = I \\ Q_1^tQ_1Q_2 = Q_2 \\ Q_2^tQ_1^tQ_1Q_2 = Q_2^tQ_2 \\ (Q_2^tQ_1^t)Q_1Q_2 = (Q_2^tQ_2) \\ (Q_1Q_2)^tQ_1Q_2 = I \end{gathered}$$