Exercise 1.8.1

After $k-1$ steps of the Gaussian elimination process the coefficient matrix has been transformed to the form

$$B = a \begin{bmatrix} B_{11} & B_{12} \\ 0 & B_{22} \end{bmatrix}$$

where B_{11} is $(k-1)$ by $(k-1)$ and upper triangular.

Prove that B is singular if the first column of B_{22} is zero. (*Remark:* The fact that B_{11} is upper triangular is of no consequence).

Answer

From definition: If B non-singular, $detB \neq 0$

Calculate determinant of B using the coefficient matrix form and applying Laplace along the first column always until reaching line column k. Note that doing the same on column k gives *0* times the rest of determinant.

$$detB = \prod_{i}^{k-1} a_{ii} \times 0$$

$$detB = 0$$

Remember that since B is not non-singular, B is singular.