Exercise 1.7.2

Verify the assertion of the previous paragraph.
As seen in the prompt, the assertion is

$$[\hat{A}|\hat{b}] = M[A|b]$$

Answer:

Remember we can group A and b.

$$A = \begin{bmatrix} a_{11} & \dots & a_{1m} \\ \vdots & \ddots & \vdots \\ a_{n1} & \dots & a_{nm} \end{bmatrix}, b = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}, [A|b] = \begin{bmatrix} a_{11} & \dots & a_{1m} & b_1 \\ \vdots & \ddots & \vdots & \vdots \\ a_{n1} & \dots & a_{nm} & b_n \end{bmatrix}$$

In the same way: $[\hat{A}|\hat{b}] = \begin{bmatrix} a_{11} & \dots & a_{1m} & b_1 \\ \vdots & & \vdots & \vdots \\ a_{i1} + m.a_{j1} & & a_{im} + a_{jm} & a_i + m.b_j \\ \vdots & & \vdots & \vdots \\ a_{j1} & & a_{ij} & b_j \\ \vdots & & \vdots & \vdots \\ a_{n1} & \dots & a_{nm} & b_n \end{bmatrix}$

Let start from $I.[A|b] = [A|B]$.
We know the change should be on the ith line of the matrix $[A|b]$. We known the origin for this line is the ith line of the identity matrix. If we change the line from $\begin{bmatrix} 0 \dots 0 \; 1 \; 0 \dots 0 \end{bmatrix}$ to $\begin{bmatrix} 1 \dots 1 \dots 1 \end{bmatrix}$, we known the result would be all elements of the jth column showing up on the respective $(a|b)_{ij}$. Now that it is clear, we eliminate all undesired 1s, getting: $\begin{bmatrix} 0 \dots 0 \; 1 \; 0 \dots 0 \; 1 \; 0 \dots 0 \end{bmatrix}$. All that is missing is fix the coeffiecient. Since the jth row must be multiplied by m, we make the final change: $\begin{bmatrix} 0 \dots 0 \; m \; 0 \dots 0 \; 1 \; 0 \dots 0 \end{bmatrix}$

Notice that everything that was changed is on the ith line of the identity matrix we began with. Also notice that with these changes we got exactly the matrix M.