Independent Study Proposal
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Online Ramsey Games played on Planar Graphs

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The Online Ramsey Game is played alternately between two players, called Builder and Painter, on a graph that is not complete. In their turns, the Builder creates an edge and the Painter paints it. Builder's objective is to create a monochromatic subgraph that is isomorphic to a target graph and Painter's objective is to avoid it as long as possible. The Ramsey Theorem, from which the game is named after, guarantees that if there are enough vertices and the Builder is able to choose edges freely, then he/she always wins, regardless of the target.

Some of the interesting questions come from trying to parameterize the number of moves needed by the Builder and the amount of colors for him/her to win. Other problems arise from limiting rules added to the ones listed above. Consider a class of graphs $\mathcal{G}$, a graph G in $\mathcal{G}$ and a target graph H. It is interesting to know whether there exists a sequence of moves starting on G such that all board states $(G_1, \ldots, G_n)$ are inside $\mathcal{G}$ and H is isomorphic to a subgraph of G_n. A second question is to define a strategy for the Builder to win for given $\mathcal{G}, G$ and H.

This is an independent study proposal. The following text provides some more information about the area of interest, outlines a specific problem, provides the method and schedule for the study and lists intended deliverables. It ends by providing a suggestion of method to evaluate the work done. The sponsor of this Independent Study is Dr. Stanisław P. Radziszowski. The credits of this Independent Study will count towards Advanced Electives.

Context

The planar graphs form an important class of graphs and the result of considering the Online Ramsey Games with the class $\mathcal{G} = \{G \mid G \text{ is planar}\}$ is a focus of study as well. In the past few years many facts were proved. For example, Grytczuk, Haluszczak and Kierstead proved that every forest in the class of forests (a subclass of planar graphs) is a win for the Builder [1]. In the same text, the authors conjectured that "The class of graphs unavoidable on planar graphs is exactly the class of outerplanar graphs".

Ten years later in 2014, the conjecture was proved wrong by Sarka Petrickova [2]. She showed that there is an entire class of planar graphs, that are not outerplanar, which is unavoidable. In the following year, Hojin Choi showed in a master thesis [3, 4] that the Painter wins the game with target K_3 on the class of K_4-minor-free graphs (another subclass of the planar graphs). There are still many conjectures open and new results come up almost every other year.

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Just like with the Ramsey Numbers counter-part, progress is made both in the algebraic part of the theory and in the computational part. In 2018, Przemysław Gordinowicz and Paweł Prałat[5] showed a new method for calculating Online Ramsey Numbers that increased performance for a special case by a factor of $2 \cdot 10^6$. That is not specific to the planar version but serves to show that this is another area under active development.

For this Independent Study project, the topic of interest is taking advantage of the fact the graphs discussed can be embedded in a plane by generating visual representations of Builder and Painter games and strategies. At times, these strategies are used in proofs for theorems, which will be the focus of the visualizations generated.

The idea is not only making them more accessible, but also help finding patterns between different proofs for different theorems. Additionally, visualization is a valid method of identifying problematic strategies and allowing human interaction, which are important for testing purposes. A software that generates such visualization would be a great tool for future contributions to the field.

Steps

From the many possibilities, the Independent Study project will start by experimenting with a visualization tool and generating visual representations or animations. As stated previously this is an important contribution, but this is also a vital point for the project to be considered a class for the IGM department. Around half of the academic semester will be allocated to studying and developing this area.

Hopefully such a tool already exists and it is a matter of finding it. In that case more time can be allocated to implementing Builder and Painter strategies. The second part of the semester will be used to study the current status and developments of the field.

At this time it is hard to gather all the information on this topic. It is still only found on papers and there is not a place that gathers and lists the information in a concise manner with a more standardized language, like a book or a chapter in one. A very important part of this study will be to write a report with the current status of Online Ramsey Games played on planar graphs.

Finally, nearing the end of the semester, work will be done towards a final report. The report will discuss the results of the work done during the semester. It will explain the difficulties faced and possible adaptations made to the schedule.

Based on the text found at the proposal form, this study has a total dedication time of at least 135 hours. The table below details how these 135h willbe split among the proposed activities.

[1] One using https://www.boost.org/doc/libs/1_49_0/libs/graph/doc/boyer_myrvold.html, maybe

Weeks	Min. Hours	Activities
1	9	• Find and test visualizations tools for planar graphs • *Decide what tool to use and if a new one is necessary[1]
2 - 6	50	• Develop a program that simulates a builder-painter game for planar graphs. It draws the planar embedding of the graphs generated during play.
7 - 8	20	• Use a visualization tool to generate a proof that every forest in unavoidable in the class of forests • Show that if all trees with n vertices are unavoidable, then so are all trees with $n+1$ vertices • This would be done by following the inductive proof provided on Online Ramsey Theory [1]
9 - 12	41	• Gather all papers that discuss Online Ramsey Games played on Planar Graphs • Write a report that tells the status and the important milestones in this area of study
13 - 14	15	• Write a report of the work done during the semester • Outline main challenges faced both during the implementation of the proof visualization and the writing of the report

* If a new tool is not necessary: the activities for weeks 2 - 6 become animating other proofs (like the 7-8 week period).

Deliverables and Evaluation

There are four anticipated deliverables. The first is a software for generating drawings of plane graphs. The second is an animation of the proof that every forest is unavoidable for Online Ramsey Games played on forests. The third and fourth are the report on status of Online Ramsey Games played on planar graphs and the final report. Their due dates follow the schedule presented in the previous section.

1. 10/01/2021 Software to draw plane graphs and simulate Builder and Painter actions
2. 10/15/2021 Animation showing every forest is unavoidable
3. 11/15/2021 Report on the status of Online Ramsey Games played on Planar Graphs

4. 12/01/2021 Final Report

The full evaluation also takes into account the expected weekly meetings and full log of activities. Although this is not the result of the work, it may reflect the time and effort put into the project better than the set of deliverables. In this sense, note that the schedule presented in the previous section might go through some adjustments if needed. These changes will be approved by the sponsoring faculty.

Bibliography

[1] J. A. Grytczuk, M. Haluszczak, and H. A. Kierstead. On-line Ramsey Theory. *Electronic Journal of Combinatorics 11*, 2004.

[2] Sarka Petrickova. Online Ramsey Theory for Planar Graphs. *Electronic Journal of Combinatorics 21*, 2014.

[3] Hojin Choi. *On online Ramsey theory*. PhD thesis, Department of Mathematical Sciences, KAIST, Daejeon, Republic of Korea, 2015.

[4] Hojin Choi, Ilkyoo Choi, Jisu Jeong, and Sang-il Oum. Online Ramsey theory for a triangle on F-free graphs. *Journal of Graph Theory*, 2019.

[5] Przemysław Gordinowicz and Paweł Prałat. Small On-line Ramsey Numbers — A New Approach. 2018.