



**IME-USP**

Universidade de São Paulo - Instituto de Matemática e Estatística  
Bachelor's Programme in Computer Science

# HOT GAMES

Temperature, advantage and numbers

by

Matheus Tararam de Laurentys

**Abstract** (The Abstract is a short summary of what your thesis is about. It accurately reflects the content of the thesis providing information about the research problem, research aims, methods and procedures, results and implications. It is a short section. Abstracts give readers the opportunity to quickly see the main contents of the paper and enable them to decide whether the paper is of particular interest to their needs. This section will be one of the last sections that you write. No subheadings are used in an abstract.)

MAC0499

Undergraduate Thesis

(Month) (Year)

Supervisor: José Coelho Pina

# Acknowledgements

It is usual, but not compulsory, to thank those who have been of particular help to you in completing the thesis.

# Contents

<b>1</b>	<b>Introduction</b>	<b>6</b>
<b>2</b>	<b>Definitions</b>	<b>7</b>
2.1	Numbers . . . . .	8
2.2	Games . . . . .	8
2.2.1	Combinatorial Games . . . . .	8
2.3	Rulesets . . . . .	8
2.3.1	Red, Green and Blue Hacklebush . . . . .	8
<b>3</b>	<b>Theory</b>	<b>9</b>
3.1	Previous Research . . . . .	9
3.2	Theoretical Approach . . . . .	9
<b>4</b>	<b>Data</b>	<b>10</b>
4.1	Source Material . . . . .	10
<b>5</b>	<b>Methods</b>	<b>11</b>
5.1	The Approach [or Model] . . . . .	11
<b>6</b>	<b>Empirical Analysis</b>	<b>12</b>
6.1	Results . . . . .	12
6.2	Discussion . . . . .	13
<b>7</b>	<b>Conclusion</b>	<b>14</b>
7.1	Research Aims . . . . .	14
7.2	Research Objectives . . . . .	14
7.3	Practical Implications . . . . .	14
7.4	Future Research . . . . .	14
7.5	Chapter Summary . . . . .	14
	<b>References</b>	<b>14</b>
<b>A</b>	<b>(Appendix A title)</b>	<b>15</b>
<b>B</b>	<b>(Appendix B title)</b>	<b>16</b>

# List of Figures

6.1	Dunning–Kruger effect ( <a href="#">?</a> , <a href="#">?</a> ) . . . . .	13
-----	---	----

# List of Tables

6.1 The economic argument ([?](#), [?](#)) . . . . . 13

# 1

## Introduction

(The Introduction chapter should contain background information as appropriate, plus definitions of all special and general terms. Your topic should be: clearly stated and defined; have a clear overall purpose; and have clear, relevant and coherent aims and objectives. It is also informative to give a brief description of the contents of the remaining chapters of the thesis. This alerts the reader and prepares them for the rest of the thesis.)

## 2

# Definitions

In order to study mathematical plays and answer the many questions they raise a new mathematical field of study was developed and many new terms were created. The phrase "mathematical play" is in itself a new term, for instance. While the most common term is "Combinatorial Games", the canonical reference for this field *Winning Ways for you mathematical plays* 1981, uses the former, not the later. As more is said about the topic, more meaning the term "mathematical play" is going to acquire.

The name "Combinatorial Game" might bring to light some information. It, at least, means that this field will deal with games, as in, an instance of a Game Theory problem, and, more specifically, a subset of those games. It also brings to light that the use of counting, finite structures and, most likely, graph representations will be heavily used (combinatorics). However, a definition of the object of interest becomes possible with the name mathematical play.

To play something mathematical could be understood as to engage in an activity in which the better use of mathematical ability, such as counting and logic, would result in advantage over its poor use. However it could be detailed further to an activity in which mathematical ability is the single defining factor. The later might make more sense because there are games, like poker, that do require some counting ability; however, luck and reading behavior skill are much more valuable to a successful game and this is something the definition would be better off forbidding.

**Chance moves**, like throwing a dice or flipping a card, are not fit for mathematical plays. Even with their removal, however, there are possibilities that would not me comfortably called mathematical plays. The nature of a mathematical plays is that both players can engage the same activity and generate advantages out of "good play". For instance, it would be hard to agree that two people play rock-paper-scissors are battling a mathematical fight (even though there are no chance moves).

It is very important that all players have **complete information** of the position. Games like rock-paper-scissors, in which players take action simultaneously, block complete information. Therefore, players must **move alternately**. The last concerning factor in discerning mathematical from non-mathematical plays during this analysis is the number of players.

When each player has more than one opponent a greater goal (than gaining advantage) arises. When playing with over two people it is frequent that the best move is not the one that leaves you in a better position but one that prevents

any of the opponents from gaining advantage over you. While that can be very mathematical, there is a clear distinction between sticking to two player games and allowing any number of players (notice that you can consider soccer as a two player game - even though there are multiple agents in a team). In order to focus on the mathematical ability to make the best move, the option to allow only **two players** is the most interesting.

The only remaining criteria of this definition (as established in [**TODO: WW**]), that is related to the term play, and not the term mathematical, is preventing an infinite game. The rules of the game must guarantee that from any starting position, **TODO: play should always end because a player will not have moves available**. If following "normal play" convention, a player that cannot move is lost. It is correct to assume normal play, unless specified otherwise, in this field of study.

## 2.1 Numbers

## 2.2 Games

### 2.2.1 Combinatorial Games

In this text, when the word game refers to an instance of a game theory problem, it will be of an instance of a combinatorial game. You can think of a combinatorial game as a two people mathematical play. It usually comes in the form of a board or drawing game. Here is a formal definition: a combinatorial game meets the criteria below:

- **Two Players** (called Left and Right).
- Clearly defined **ruleset** that:
  - specifies the **moves** available from one position to its **options**.
  - establish an **ending condition** such that play will always end because a player is unable to move.
- Left and Right move **alternately**, as a whole.
- both players have **complete information**.
- there are **no chance moves**.

Many known games, chess (checkmate, stalemate and draw by repetition), poker and ludo (hidden information), do not fit all the criteria and, therefore, are not combinatorial games.

## 2.3 Rulesets

### 2.3.1 Red, Green and Blue Hacklebush

A game of Hacklebush consists of a initial position



# 3

## Theory

This chapter should outline, compare and discuss key ideas, explanations, concepts, models and theories. You should present these ideas in a systematic, well-structured and logical sequence. It is expected that you use prominent and up-to-date books and articles. All literature should be referenced, not just for quotations, but also for ideas and information/knowledge drawn from the works of others. Refer to the [Teaching and Learning platform](#) for guidance on how to incorporate references into your text

### 3.1 Previous Research

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Duis ut ipsum nec orci interdum sollicitudin ut eu nunc. Pellentesque ultricies eros in justo sagittis, eget blandit velit aliquet. Aenean ac lectus nibh. Quisque ac est pellentesque, ullamcorper sem sit amet, pharetra quam. Morbi ullamcorper placerat diam, sed tincidunt odio.

### 3.2 Theoretical Approach

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Duis ut ipsum nec orci interdum sollicitudin ut eu nunc. Pellentesque ultricies eros in justo sagittis, eget blandit velit aliquet. Aenean ac lectus nibh. Quisque ac est pellentesque, ullamcorper sem sit amet, pharetra quam. Morbi ullamcorper placerat diam, sed tincidunt odio.

# 4

## Data

This Chapter should demonstrate that you have conducted a thorough and critical investigation of relevant sources. Apart from a presentation of the sources of your data, this chapter allows you to critically discuss the data (whatever these data are, ‘quantitative’ or ‘qualitative’, primary or secondary), which is proof of good research. You can even do good research with poor data but you must demonstrate that you are aware of the data quality and accordingly are careful in your interpretations. Essentially, there are three aspects to consider:

1. Reliability, which, for example, could depend on whether they are estimates or more direct evidence;
2. Representativity, which is about how typical the data are; for example, you may have arguments why the very few cases are typical or you may carry out statistical tests;
3. Validity, which is about the relevance of the data for your case. Strictly speaking, sometimes no valid data are available but one may argue that there are other data which could be used as ‘proxies’.)

### 4.1 Source Material

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Duis ut ipsum nec orci interdum sollicitudin ut eu nunc. Pellentesque ultricies eros in justo sagittis, eget blandit velit aliquet. Aenean ac lectus nibh. Quisque ac est pellentesque, ullamcorper sem sit amet, pharetra quam. Morbi ullamcorper placerat diam, sed tincidunt odio.

# 5

## Methods

In this Chapter, you present in more concrete terms the method(s) you are going to apply. And as always in research, it is good to demonstrate awareness of the weaknesses or limitations of the method you use. It makes no difference if you work with interviews, econometric models, or a comprehensive analysis of data from various sources. Transparency should be the guideline: make it possible for your readers to follow, or even repeat, your analysis!

### 5.1 The Approach [or Model]

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Duis ut ipsum nec orci interdum sollicitudin ut eu nunc. Pellentesque ultricies eros in justo sagittis, eget blandit velit aliquet. Aenean ac lectus nibh. Quisque ac est pellentesque, ullamcorper sem sit amet, pharetra quam. Morbi ullamcorper placerat diam, sed tincidunt odio.

# 6

## Empirical Analysis

This chapter covers three areas: analysis of the data; discussion of the results of the analysis; and how your findings relate to the literature. The analysis of the data can be discussed here but the details of any analysis, such as statistical calculations, should be shown in the appendices. You should present any discussion clearly and logically and it should be relevant to your research questions/hypotheses or aims and objectives. Insert any tables or figures that you decide are important in a relevant part of the text not in the appendices, and discuss them fully. Make sure that you relate the findings of your primary research to your literature review. You can do this by comparison: discussing similarities and particularly differences. If you think your findings have confirmed some literature findings, say so and say why. If you think your findings are at variance with the literature, say so and say why.

### 6.1 Results

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Duis ut ipsum nec orci interdum sollicitudin ut eu nunc. Pellentesque ultricies eros in justo sagittis, eget blandit velit aliquet. Aenean ac lectus nibh. Quisque ac est pellentesque, ullamcorper sem sit amet, pharetra quam. Morbi ullamcorper placerat diam, sed tincidunt odio.

When placing tables ([table 6.1](#)) within the body of the text, the citation is placed above the table.

Table 6.1: The economic argument (?, ?)

CRAZY PHENOMENON	IF IT WORKED, COMPANIES WOULD BE USING IT TO MAKE A KILLING IN...	ARE THEY?
REMOTE VIEWING	OIL PROSPECTING	
DOWSING		
AURAS	HEALTH CARE COST REDUCTION	
HOMEOPATHY		
REMOTE PRAYER		
ASTROLOGY	FINANCIAL/BUSINESS PLANNING	
TAROT		
CRYSTAL ENERGY	REGULAR ENERGY	
CURSES, HEXES	THE MILITARY	
RELATIVITY	GPS DEVICES	✓
QUANTUM ELECTRODYNAMICS	SEMICONDUCTOR CIRCUIT DESIGN	✓

EVENUALLY, ARGUING THAT THESE THINGS WORK  
MEANS ARGUING THAT MODERN CAPITALISM  
ISN'T *THAT* RUTHLESSLY PROFIT-FOCUSED.

## 6.2 Discussion

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Duis ut ipsum nec orci interdum sollicitudin ut eu nunc. Pellentesque ultricies eros in justo sagittis, eget blandit velit aliquet. Aenean ac lectus nibh. Quisque ac est pellentesque, ullamcorper sem sit amet, pharetra quam. Morbi ullamcorper placerat diam, sed tincidunt odio.

When placing figures (illustrations, pictures, graphs, diagrams, charts, maps etc.) within the body of the text, the citation is placed below the figure (figure 6.1)

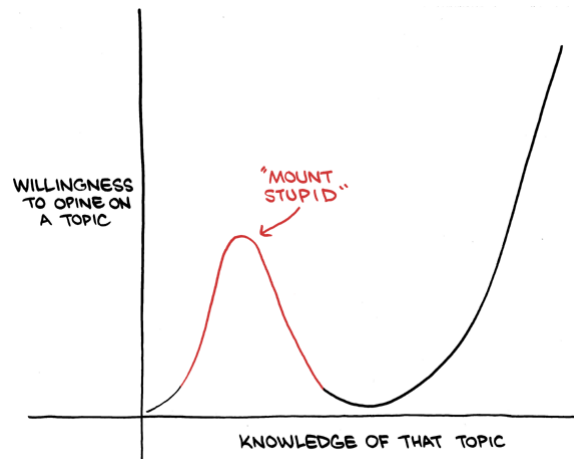


Figure 6.1: Dunning-Kruger effect (?, ?)

# 7

## Conclusion

State the main conclusions of your study. State explicitly how and to what extent you have fulfilled your aims and objectives/answered your research questions/proved your hypotheses (whichever is appropriate). Your conclusions should follow logically from your findings and not contain any new material.

### 7.1 Research Aims

### 7.2 Research Objectives

### 7.3 Practical Implications

### 7.4 Future Research

### 7.5 Chapter Summary

Refer to LUSEM's Harvard referencing guidelines in the Teaching and Learning platform. [Lusem.lu.se/asks](https://lusem.lu.se/asks)

# Appendix A

## (Appendix A title)

The final sections of your thesis are the appendices. Each appendix should be lettered (A, B, etc.,) and should consist of detailed information that is interesting but not essential to the main thrust of your findings section.

The appendices should be in the order that they are referred to in the main text. For instance, if Appendix A refers to something on page 25 and Appendix B refers to something on page 15, the appendices need to be re-lettered. This inconsistency occurs when text is moved around or inserted.)

# Appendix B

## (Appendix B title)