MAC0300 - Lista 1

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PROMPT: Exercises 4.1, 5.1, 5.8 from IEEE (Overton) book.
4.1 Give the single precision float representation of the following numbers: $2, 30, 31, 32, 33, 23/4, (23/4)\times 2^{100}, (23/4)\times 2^{-100}, (23/4)\times 2^{-135}, 1/5, 1024/5, (1/10)\times 2^{-140}$

$(2) \longrightarrow \boxed{0}\boxed{10000000}\boxed{00000000000000000000000}$

$(30) \longrightarrow \boxed{0}\boxed{10000011}\boxed{11100000000000000000000}$

$(31) \longrightarrow \boxed{0}\boxed{10000011}\boxed{11110000000000000000000}$

$(32) \longrightarrow \boxed{0}\boxed{10000100}\boxed{00000000000000000000000}$

$(33) \longrightarrow \boxed{0}\boxed{10000100}\boxed{00001000000000000000000}$

$(23/4) \longrightarrow \boxed{0}\boxed{10000001}\boxed{01110000000000000000000}$

$((23/4)\times 2^{100}) \longrightarrow \boxed{0}\boxed{11100101}\boxed{01110000000000000000000}$

$((23/4)\times 2^{-100}) \longrightarrow \boxed{0}\boxed{00011101}\boxed{01110000000000000000000}$

$((23/4)\times 2^{-135}) = (1.4375\times 2^{-7})\times 2^{-126} \longrightarrow \boxed{0}\boxed{00000000}\boxed{00000010111000000000000}$

$(1/5 = 1/8 \times 8/5) \longrightarrow \boxed{0}\boxed{01111100}\boxed{10011001100110011001101}$

$(1024/5) \longrightarrow \boxed{0}\boxed{10000110}\boxed{10011001100110011001101}$

$((1/10)\times 2^{100}) \longrightarrow \boxed{0}\boxed{11011111}\boxed{10011001100110011001101}$

5.1 Give the rounded values of 1/10, using each of the rounding modes? What are they for $(1+2^{-25})$ and 2^{130}.

(1/10):
Round Down: $\boxed{0}\boxed{01111011}\boxed{10011001100110011001110}$
Round Up: $\boxed{0}\boxed{01111011}\boxed{10011001100110011001101}$
Round Towards Zero: $\boxed{0}\boxed{01111011}\boxed{10011001100110011001110}$
Round to Nearest: $\boxed{0}\boxed{01111011}\boxed{10011001100110011001101}$

$(1+2^{-25})$
Round Down: $\boxed{0}\boxed{01111111}\boxed{00000000000000000000000}$
Round Up: $\boxed{0}\boxed{01111111}\boxed{00000000000000000000001}$
Round Towards Zero: $\boxed{0}\boxed{01111111}\boxed{00000000000000000000000}$
Round to Nearest: $\boxed{0}\boxed{01111111}\boxed{00000000000000000000000}$

(2^{130})

Round Down:	0	11111110	11111111111111111111111
Round Up:	0	11111111	00000000000000000000000
Round Towards Zero:	0	11111110	11111111111111111111111
Round to Nearest:	0	11111111	00000000000000000000000

5.8 Do bounds (5.10) and (5.11) hold when $|x| < N_{min}$?

No, the relative rounding error bounds do not hold, but as seen in exercise 5.7, the absolute rounding error bound holds.

For example, let $0 < x < (2^{-23} \times 2^{-126})$. In this case, either round(x) $= 0$, or round(x) $= 2^{-23} \times 2^{-126}$. In either case it is easy to verify that the rounding error is greater than ϵ since the round to nearest error can be up to $2^{-24} \times 2^{-126}$ and, in this case, the relerr is $\frac{1}{2}$.