

Beta has to be the spline formed by four polynomials that respects the following rules:

$$P_0(2) = 0 \text{ -(1)}$$

$$P'_0(2) = 0 \text{ -(2)}$$

$$P''_0(2) = 0 \text{ -(3)}$$

$$P_1(0) = 1 \text{ -(4)}$$

$$P'_1(0) = 0 \text{ -(5)}$$

$$P_1(1) = P_0(1) \text{ -(6)}$$

$$P'_1(1) = P'_0(1) \text{ -(7)}$$

$$P''_1(1) = P''_0(1) \text{ -(8)}$$

And symetric:

$$P_3 = -P_0 \text{ -(9)}$$

$$P_2 = -P_1 \text{ -(10)}$$

All polynomials $P_i = a_i * t^3 + b_i * t^2 + c_i * t + d_i$

Given (1), (2) and (3), $P_0 = (2 - t)^3 * a_0$ Given 4, $d_1 = 1$

Given 5, $c_1 = 0$

Given 6, 7 and 8:

$$\begin{cases} a_1 * 1^3 + b_1 * 1^2 + 1 = (2 - 1)^3 * a_0 \\ 3 * a_1 * 1^2 + 2 * b_1 * 1 = -3 * (2 - 1)^2 * a_0 \\ 6 * a_1 * 1 + 2 * b_1 = 6 * (2 - 1) * a_0 \end{cases}$$

$$\begin{cases} a_1 + b_1 - a_0 = -1 \\ 3 * a_1 + 2 * b_1 + 3 * a_0 = 0 \\ 6 * a_1 + 2 * b_1 - 6 * a_0 = 0 \end{cases}$$

$$a_1 = 3/4$$

$$b_1 = -3/2$$

$$a_0 = 1/4$$

Given (9),

$$P_0 = (2 - t)^3 * 1/4$$

$$P_3 = (2 + t)^3 * 1/4$$

Given (10),

$$P_1 = 3t^3/4 - 3t^2/2 + 1$$

$$P_2 = -3t^3/4 - 3t^2/2 + 1$$

Also needed:

$$P''_1 = 9t/2 - 3$$

$$P''_0 = 3(2 - t)/2$$

