University Of Thessaly

Mobile and Pervasive Computing

ECE515

Problem Set 2

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Exercise 1

Before solving the problem, lets remind the assumptions of Relative neighbour graph which claims that an edge (u,v) belongs to the RNG of a graph G, when for every w holds that $dist(u, v) \leq min(dist(u, w), dist(w, v))$. Keeping that in mind, we also observe that if a node has 6 neighbors who keep the same distance from the node R, and each consecutive forms an angle of $\frac{\pi}{6}$, then they all also have the same distance R, and so due to RNG they are all neighbors of the central node. If we try to place another one wherever in the perimeter or inside of the circle, then due to the RNG algorithm the center would have less than 6 neighbors.

Exercise 2

Ignoring the side areas each square corresponds to a sensor (let it be the upper left) since in each vertex 4 squares meet. First of all we must observe that $S = n \cdot r^2$ or $n = \frac{S}{r^2}$. To begin with, we preserve 2 algorithms for broadcasting with directed antennas, considering the topology of the exercise. The naive way is to use every angle mimicing the undirected antennas. Each note should then transmit 12 times considering that the angle is $\frac{\pi}{6}$. Another way is to transmit only to the four direct neighbors, so the a node should transmit only 4 times. Thus the total energy consumption is :

- $E(r) = n \cdot 12 \cdot e(\frac{\pi}{6}, r) = 12\frac{S}{r^2} \cdot e(\frac{\pi}{6}, r)$
- $E(r) = n \cdot 4 \cdot e(\frac{\pi}{6}, r) = 4\frac{S}{r^2} \cdot e(\frac{\pi}{6}, r)$

We differentiate the above and find the best r, so we get :

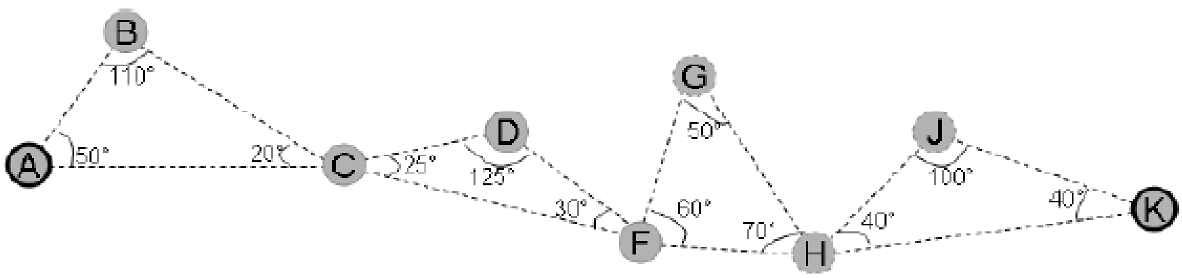
- $E(r)' = 0$ or $(12\frac{S}{r^2} \cdot e(\frac{\pi}{6}, r))' = 0$ or $r = \sqrt[a]{\frac{24 \cdot S \cdot C_1}{2 \cdot S(a-2)}}$

Note that the same value holds also for the second method.

Exercise 3

In order to solve the exercise we must make 2 important observations. Let ABC, a triangle where A, B, and C are nodes. Then the AC belongs to the Gabrel Graph if $Angle_B \leq \frac{\pi}{2}$. Moreover it holds that $AC^2 = AB^2 + BC^2 - 2 \cdot AB \cdot BC \cdot cos(Angle_B)$, so if $Angle_B \leq \frac{\pi}{2}$ then $AC^2 \leq AB^2 + BC^2$ which is exactly the same as EG when a = 2 and const = 0. Needless to say when $Angle_B > \frac{\pi}{2}$, then $AC^2 > AB^2 + BC^2$ which also holds for EG.

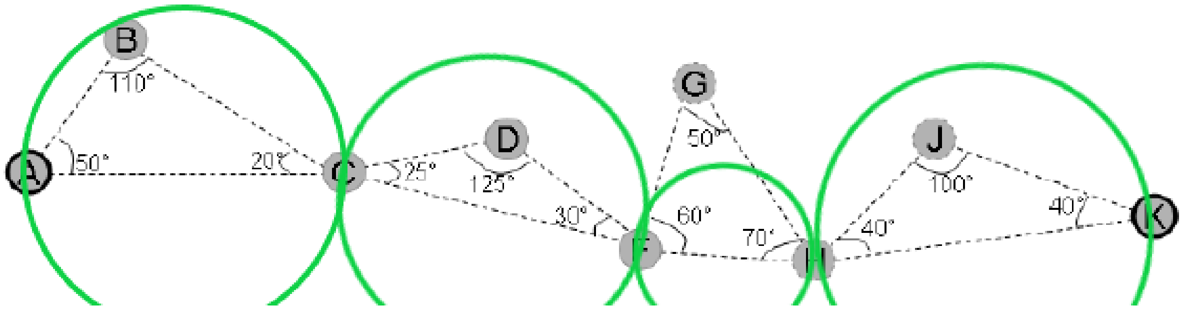
Exercise 4



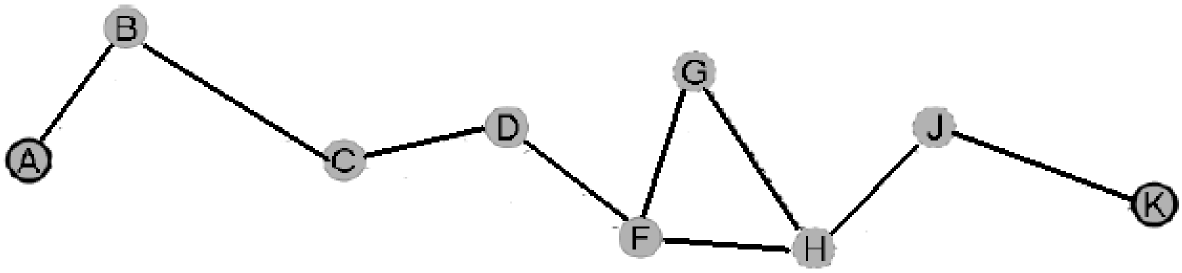
Each inscribed angle is equal to half the center angle that intersects the same arc. So given a triangle whose diameter is equal to the plane formed by the 2 points, the third point if it is on the periphery of the circle (ie if it intersects an arc of 180 degrees) will correspond to an angle of 90 degrees. Therefore, the third point involved in the formation of the triangle to be inside the circle must correspond to an angle greater than or equal to 90 degrees(obtuse angle).

Suppose that we have 3 nodes U, V, W .The Gabriel graph will contains a UV edge if and only if the disk with diameter UV does not contain any other node inside it.

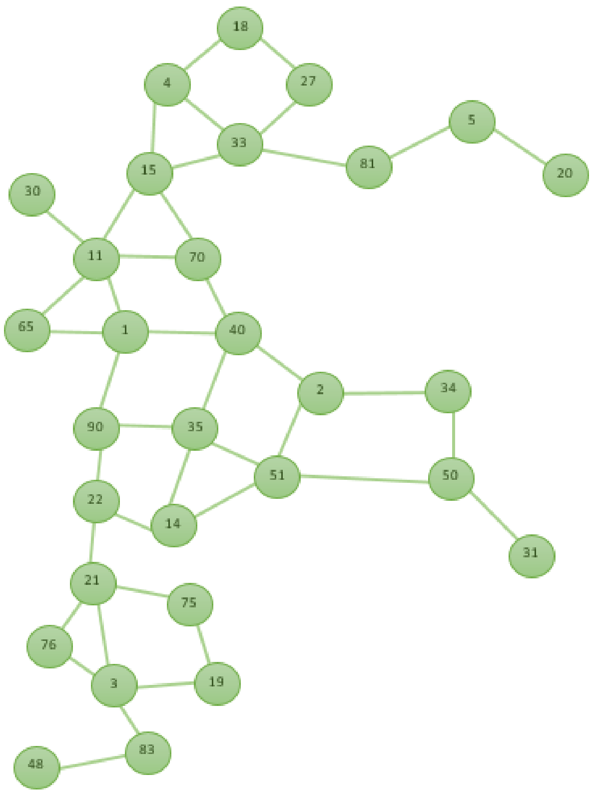
So in the given graph to find the Gabriel graph we will check the angles of the forming triangles.If the angle is obtuse then we will remove the edge which is situated across the angle. In the following image we can see and the cycles.



The Gabriel graph is the following.



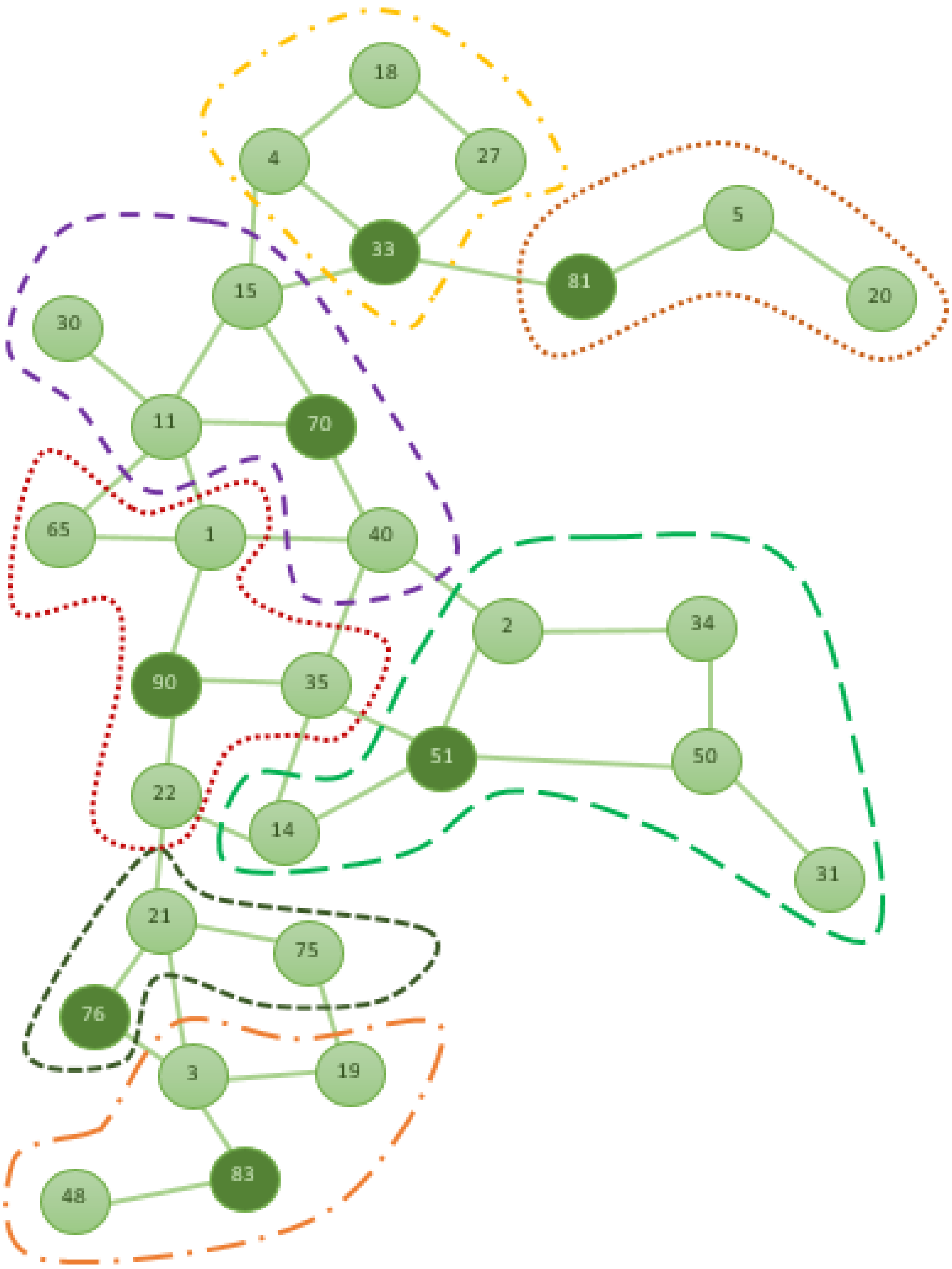
Exercise 5



We apply the max-min d-hop algorithm for d = 2.After the appropriate steps and rules we calculate the following table.

	18	4	33	27	81	5	20	15	30	11	70	65	1	40	2
Floodmax	27	33	81	33	81	81	20	70	30	70	70	65	90	70	51
	33	81	81	81	81	81	81	81	70	90	70	90	90	90	70
Floodmin	33	33	81	33	81	81	81	70	70	70	70	90	90	70	51
	33	33	33	33	81	81	81	33	70	70	70	70	70	51	51
Final	33	33	CH	33	CH	81	81	70	70	70	CH	90	90	70	51

	34	90	35	22	14	51	50	31	21	75	76	3	19	83	48
Floodmax	50	90	90	90	51	51	51	50	76	75	76	83	75	83	83
	51	90	90	90	90	90	51	51	90	76	83	83	83	83	83
Floodmin	51	90	90	90	90	51	51	51	76	76	83	83	76	83	83
	51	90	51	76	51	51	51	51	76	76	76	76	76	83	83
Final	51	CH	90	90	51	CH	51	51	76	76	CH	83	83	CH	83

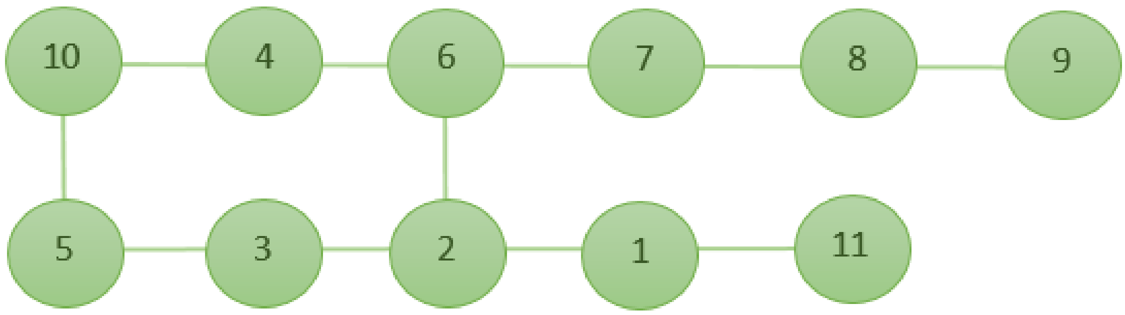


So the clusterheads are CH: 33,51,70,76,81,83,90
The cluster-members are the following :

{[33] , 4, 18, 27 }
{[51] , 2, 14, 34, 50,31 }

{[70] , 11, 15, 30, 40 }
{[76] , 21, 75 }
{[81] , 5, 20 }
{[83] ,1, 22, 35, 65 }
{[90] , 4, 18, 27 }

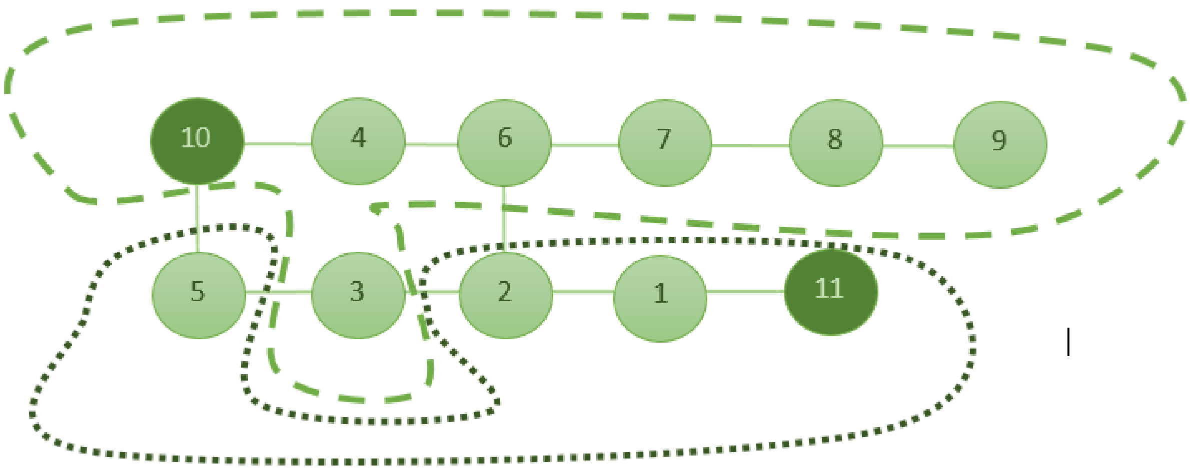
Exercise 6



We apply the max-min d-hop algorithm for d = 5. So we have 5 rounds for floodmax and 5 rounds for floodmin.In the floodmax each node listen his id and the id from his neighbor and he set as winner the biggest.In the floodmin he repeat the proccess but this time he set as winner the smallest one. In the end if a node has set in the last round of floodmin his id as winner that means that he is a clasterhead. Else if he didn't set his id as winner, he search for id pairs between floodmax and floodmin, if he has 1 pair he join to the cluster of this id, if he has 2 pairs he select the smaller id as the clusterhead of the cluster that he will join. In case that he has not found any pair he select the biggest id of floodmax to join.With these rules we calculate the following array.

	10	5	4	3	2	6	7	8	9	1	11
Floodmax	10	10	10	5	6	7	8	9	9	11	11
	10	10	10	10	11	10	9	9	9	11	11
	10	11	10	11	11	11	10	9	9	11	11
	11	11	11	11	11	11	11	10	9	11	11
	11	11	11	11	11	11	11	11	10	11	11
Floodmin	11	11	11	11	11	11	11	10	10	11	11
	11	11	11	11	11	11	10	10	10	11	11
	11	11	11	11	11	10	10	10	10	11	11
	11	11	10	11	10	10	10	10	10	11	11
	10	11	10	10	10	10	10	10	10	11	11
Final	CH	11	10	10	11	10	10	10	10	11	CH

So the clusterheads are the node 10 and the node 11.
The cluster-members are the following :
{[10] , 3, 4, 6, 7, 8, 9 }
{[11] , 1, 2, 5}

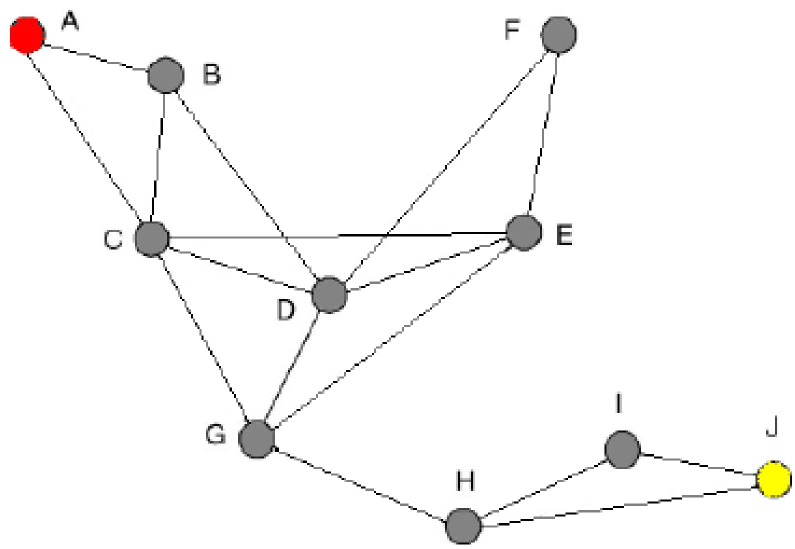


Looking at the final graph and the clusters we can infer that if the node with the id 9 did not exist then only one clusterhead would be created , because all the others nodes have 5-hop distance for each other. Additionally we can infer that the larger the d the smaller the number of clusters.

Exercise 7

On max-min d-hop cluster formation each node propagates node ids for 2d rounds to elect clusterheads.Since no node is more than d hops from its clusterhead the convertcast will be O(d) rounds of messages.Therefore, the time complexity of the heuristic is 0(2d + d) rounds = O(d) rounds. More specific 2d is for the communication and the exchange of messages and d for the final announcement if someone is CH or a clustermember.This is the complexity for each node. For all the network(graph) if the number of nodes is n then the complexity is O(n*(2d+d)) = O(d).The communication comlexity is the time complexity, so O(d).

Exercise 8



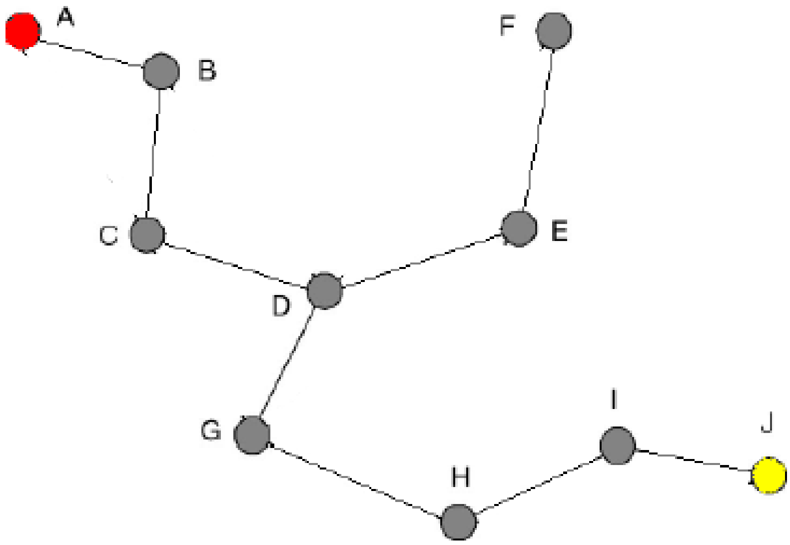
On this exerise we have to use only the perimeter mode.GPSR forwards perimeter-mode packets using a simple planar graph traversal. In essence, when a packet enters perimeter mode at node x bound for node D, GPSR forwards it on progressively closer faces of the planar graph, each of which is crossed by the line xD. A planar graph has two types of faces. Interior faces are the closed polygonal regions bounded by the graph's edges. The exterior face is the one unbounded face outside the outer boundary of the graph. On each face, the traversal uses the right-hand rule to reach an edge that crosses line xD. At that edge, the traversal moves to the adjacent face crossed by xD. So every time that the right-hand rule is used we maybe meet some links that is not useful, but we should select this one that lead us on the face that we want to reach on our destination. For this exercise, we use the second algorithm that is provided in Routing with Guaranteed Delivery in Ad Hoc Wireless Networks paper. To be more specific, we travece a face using the right hand rule until we reach a node z which edge intersects xD. Then we use the right hand rule using the line zD, and start travesing the face. We should mention that this problem is similar to the worst case example (Figure 3) in the corresponding paper.

Perimeter mode needs a planar graph. Our graph is not planar because we have crossing links. So we found the RNG graph.

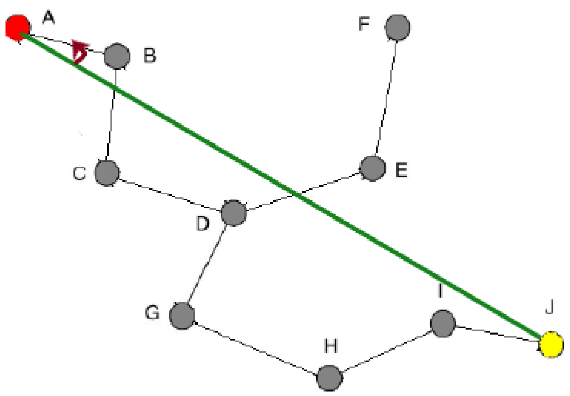
Given a collection of vertices with known positions, the RNG is defined as follows:

An edge (u, v) exists between vertices u and v if the distance between them, d(u, v), is less than or equal to the distance between every other vertex w, and whichever of u and v is farther from w.

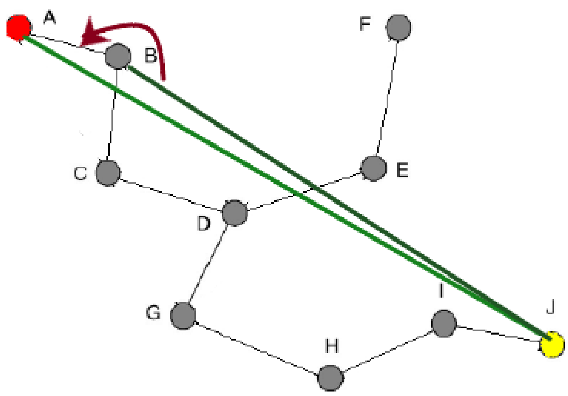
So the rng graph is the following:



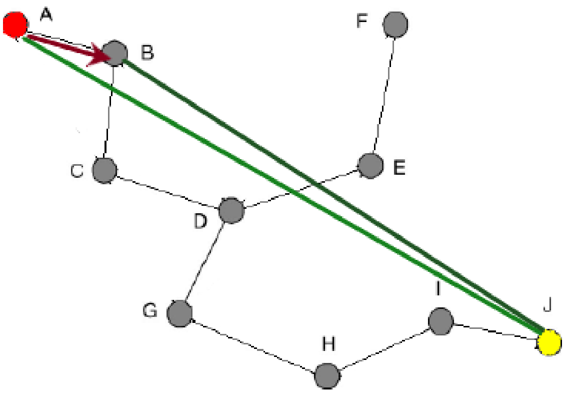
On the next pictures we can see the steps that algorithm do to reach on the final destination.



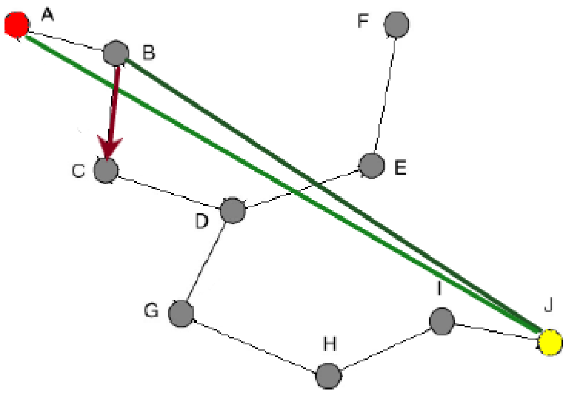
A->B



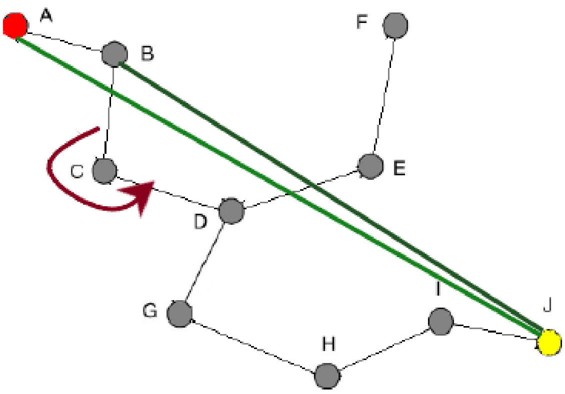
B->A



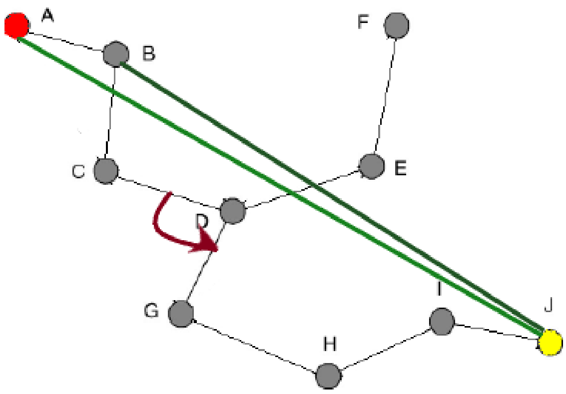
A->B



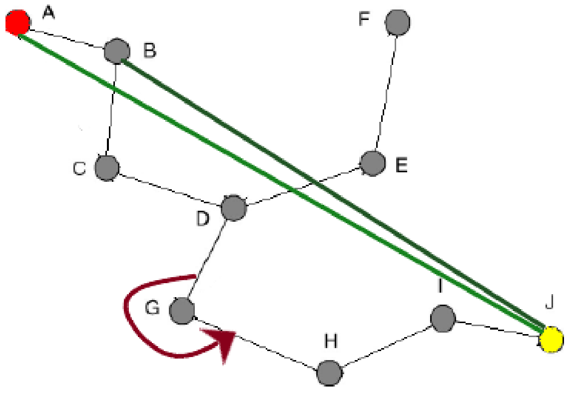
B->C



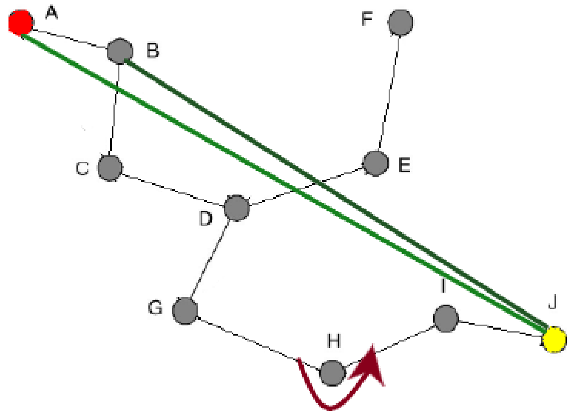
C->D



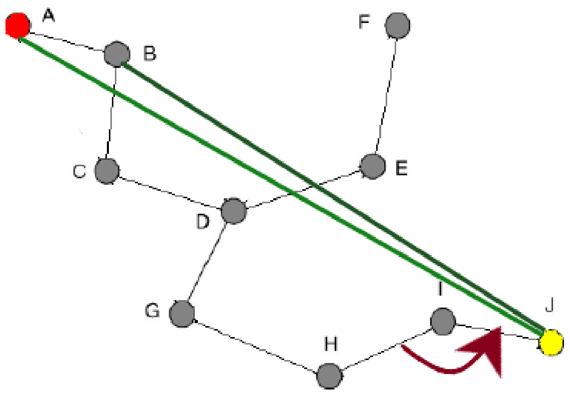
D->G



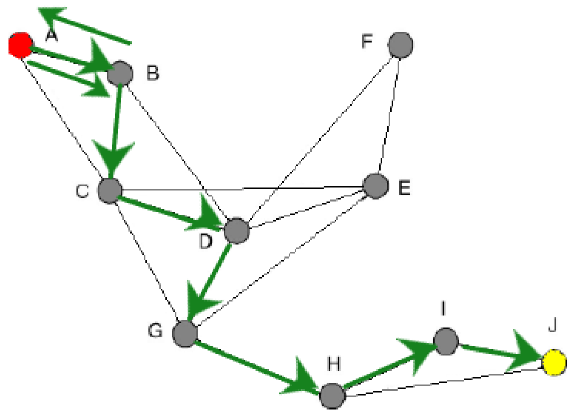
G->H



H->I



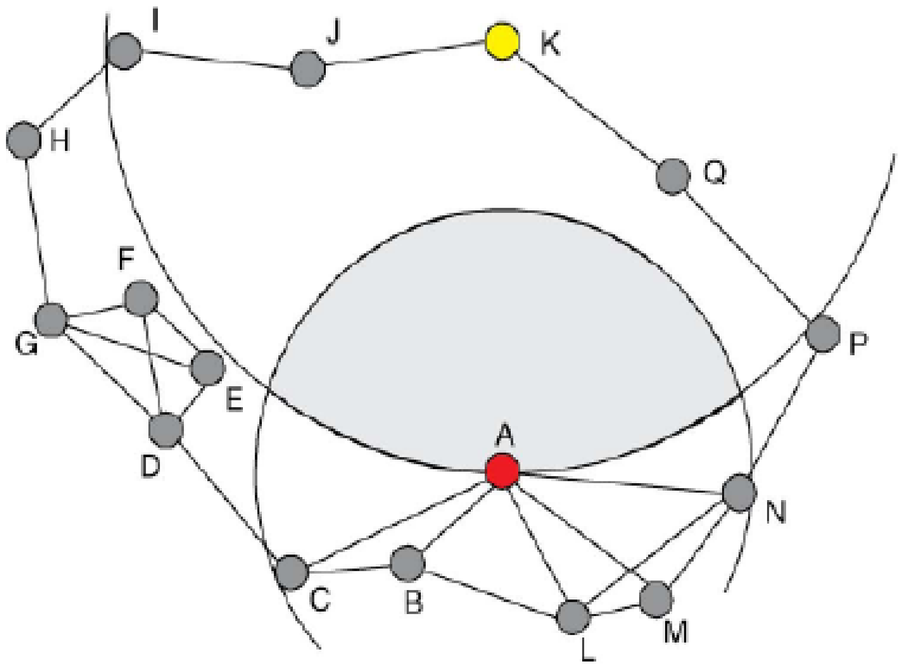
I->J



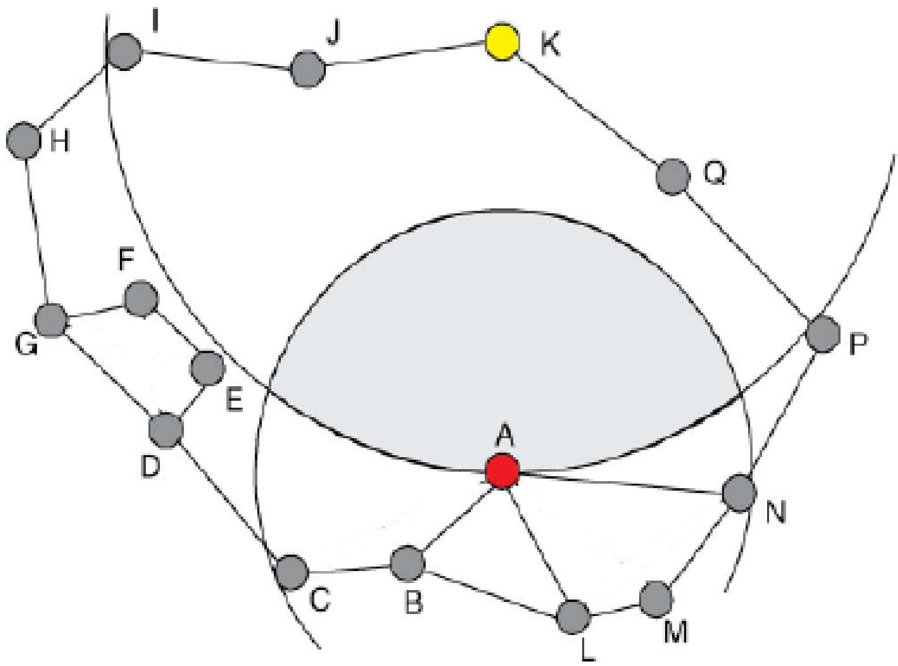
final path

So the the total path to reach the packet from A to J only with perimeter mode is : A->B->A->B->C->D->G->H->I->J.

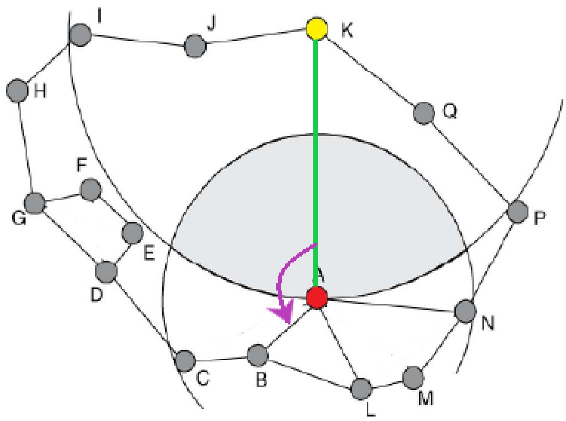
Exercise 9



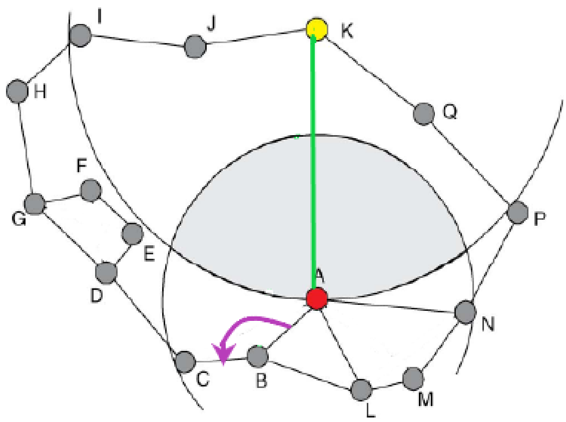
On GPSR algorithm when a node meet a void area the algorithm turn into perimeter mode to surpass the void area. On graph we can see that there are crossing links so before go to perimeter mode we have to found the Gabriel graph (GG). The Gabriel graph is the following.



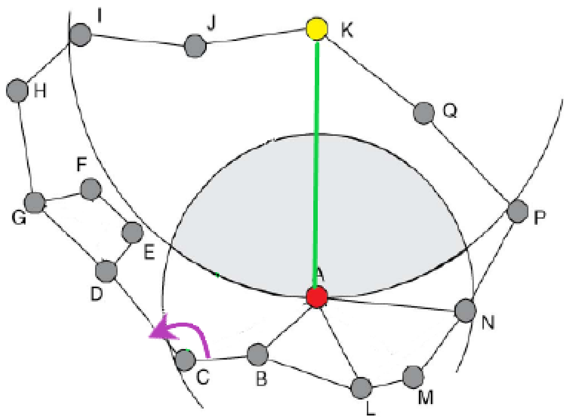
We have to note that every time that algorithm turns into greedy mode use the hole graph and not only the planar graph. TH planar graph that we found with GG is only for the perimeter mode to avoid loop situation, but the edges which remove from graph are not disconnected so when the algorithm go back to greedy mode is totally fine to use them



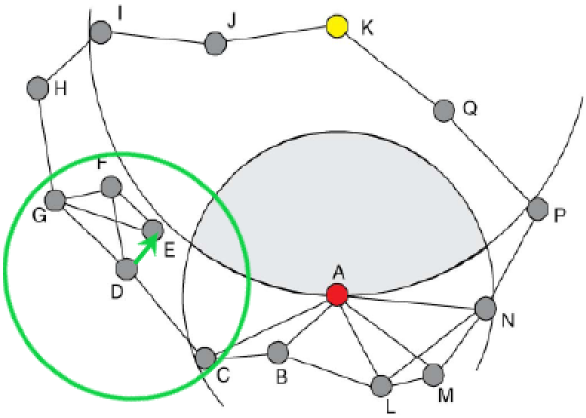
1st step is to draw a line from A to K and with the rule of right hand starting from the AK we can find the next node which is the node B



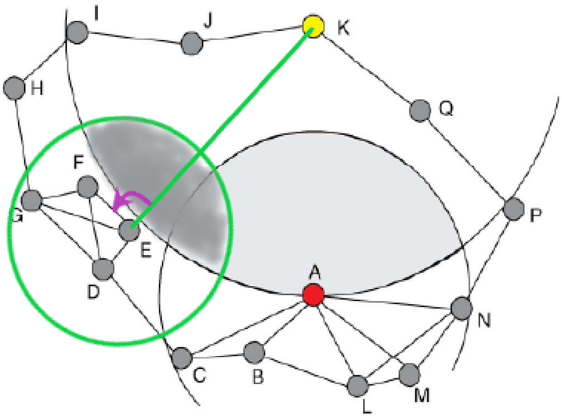
We continue with the same process.The following node is C



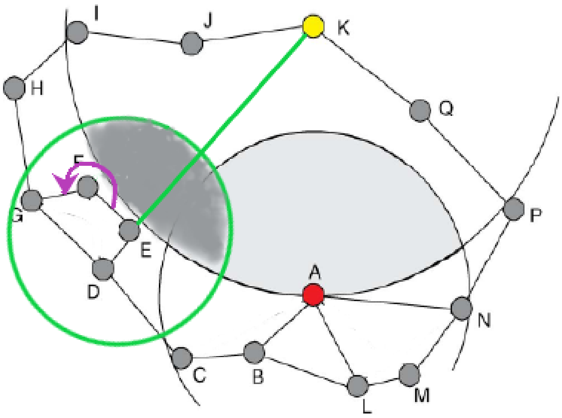
Last move to surpass the void area. The next node is D.



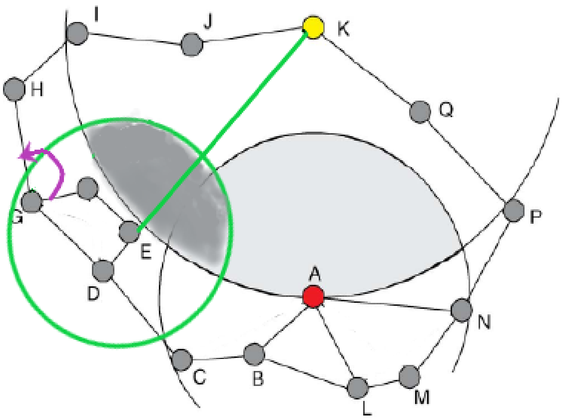
Now the algorithm go back to the greedy mode and we draw a cycle around the area of D node. We see that the node which is more close to K is the E so this is the next selected node.



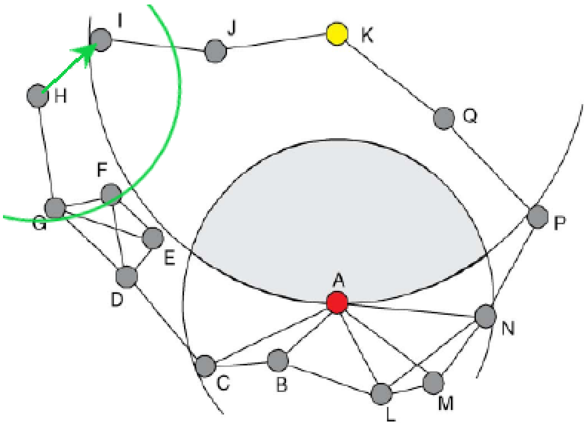
In this step another void area is showing up so the perimeter mode is again on. We draw the EK line and counterclockwise we select the F as the next node.



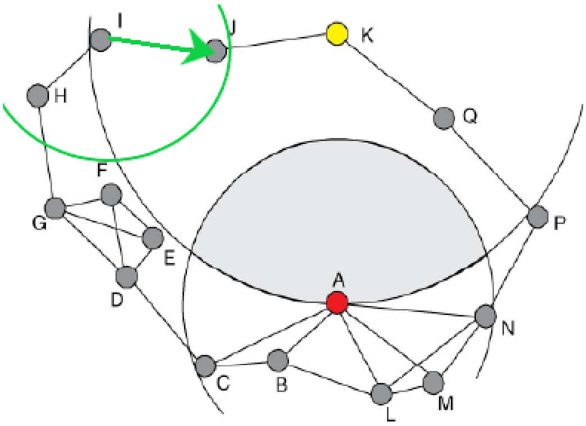
With the same way the following node is G.



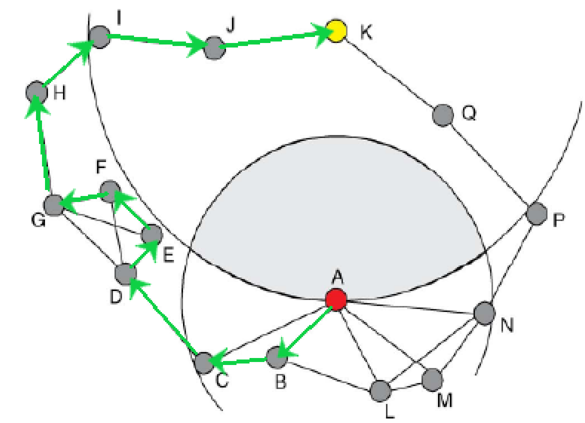
With similar we found the last node to surpass and this void area. The node is the H.



The algorithm go back to the greedy mode. We can see that the node which is more close to our destination is the I.



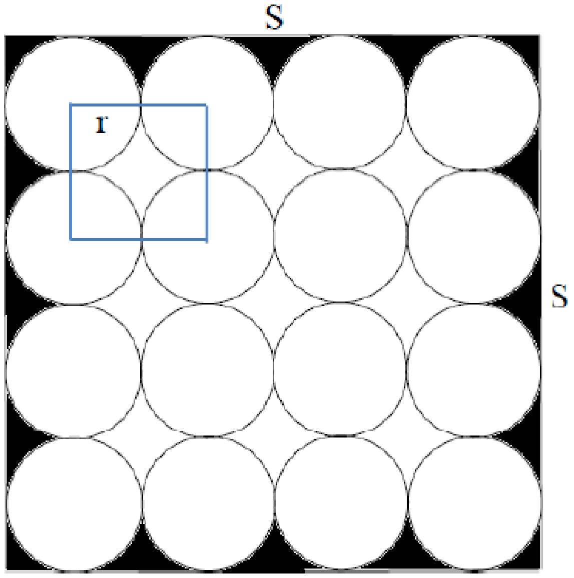
With the same way of GPSR algorithm the J is selected. J is the final node before reach to our destination

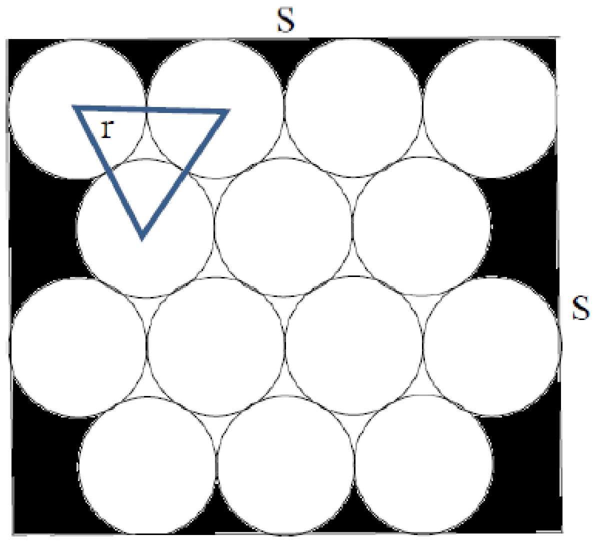


This is the final path for the packet to reach his final destination starting from A. The algorithm switching between perimeter and greedy mode to avoid the void areas.

More specific the path is A->B->C->D->E->F->G->H->I->J->K

Exercise 10





Considering that the appearance of events is uniform , the probability of loosing an event is equals to the total area of the space that remains uncovered from the sensors.More specific is equal with the total area of the void space between circular discs.
Square:
The area of the space model when we use the square mode is $(2 \cdot r)^2 - 4 \cdot \frac{\pi \cdot r^2}{4}$. We calculate the area of the cycle which is $\pi \cdot r^2$ and the area of the square which is $2r^2$. From the area of the square we remove 4 times the area of the circular sector corresponding to an angle of 90 degrees. So $4 \cdot \pi \cdot r - \pi \cdot r^2$. Each row has $\frac{S}{2 \cdot r} - 1$ void areas and each column has also $\frac{S}{2 \cdot r} - 1$. So the total area of the voids is $(\frac{S}{2 \cdot r} - 1) \cdot (\frac{S}{2 \cdot r} - 1) \cdot (4 - \pi) \cdot r^2 = (\frac{S}{2 \cdot r} - 1)^2 \cdot (4 - \pi) \cdot r^2$.
Triangle:
for the triangle method we need the area of an equilateral triangle with side equal to 2r. To calculate the area of this triangle we need the height which with the pythagorean theorem is calculated as $height^2 = (2 \cdot r)^2 - r^2 \Rightarrow height = \sqrt{3} \cdot r$. So the area of the triangle is $\frac{base \cdot height}{2} = \frac{2 \cdot r \cdot \sqrt{3} \cdot r}{2} = \sqrt{3} \cdot r^2$. To find the

area of the void we should remove 3 times the area of a circular sector corresponding to an angle of 60 degrees. So the area of the void is equal to $\sqrt{3} \cdot r^2 - 3 \cdot \frac{\pi \cdot r^2}{6} = \sqrt{3} \cdot r^2 - \frac{\pi \cdot r^2}{2}$. We can see that in S there are $\frac{S}{2r} - 1$ rows with void areas and in each of them there are $\frac{S}{2r} + 1$ voids. So the total area of voids is equal to $(\frac{S}{2r} - 1) \cdot (\frac{S}{2r} + 1) \cdot (\sqrt{3} \cdot r^2) - \frac{\pi \cdot r^2}{2} = ((\frac{S}{2r})^2 - 1^2) \cdot (r^2 \cdot \frac{2\sqrt{3} - \pi}{2})$.

Having calculated the area of voids in each method we need to find the probability of not perceiving the birth of an event with the square packing and the corresponding probability with hexagonal packing. Excluding black areas , because no event can show up there, we calculate the rest area and then we find the probability of each case.

Square packing probability :

We use the mathematical relation S = 8*r

$$\frac{(4-\pi)\cdot(r^2 - S\cdot r + \frac{S^2}{4})}{16\cdot\pi\cdot r^2 + (4-\pi)\cdot(r^2 - S\cdot r + \frac{S^2}{4})} = \frac{(4-\pi)\cdot(r^2 - 8\cdot r^2 + \frac{64\cdot r^2}{4})}{16\cdot\pi\cdot r^2 + (4-\pi)\cdot(r^2 - 8\cdot r^2 + \frac{64\cdot r^2}{4})} = \frac{(4-\pi)\cdot(9\cdot r^2)}{16\cdot\pi\cdot r^2 + (4-\pi)\cdot(9\cdot r^2)} =$$

$$\frac{7{,}74}{58} = 0{,}01334 = 13{,}34\%$$

Hexagonal packing probability:

We use the mathematical relation S = 8*r

$$\frac{(\frac{S}{2\cdot r}^2 - 1)\cdot(r^2 \cdot \frac{2\cdot\sqrt{3}-\pi}{2})}{14\cdot\pi\cdot r^2 + (\frac{S}{2\cdot r}^2 - 1)\cdot(r^2 \cdot \frac{2\cdot\sqrt{3}-\pi}{2})} = \frac{(\frac{S^2}{4\cdot r^2} - 1)\cdot(r^2 \cdot \frac{2\cdot\sqrt{3}-\pi}{2})}{14\cdot\pi\cdot r^2 + (\frac{S^2}{4\cdot r^2} - 1)\cdot(r^2 \cdot \frac{2\cdot\sqrt{3}-\pi}{2})} = \frac{(\frac{64\cdot r^2}{4\cdot r^2} - 1)\cdot(r^2 \cdot \frac{2\cdot\sqrt{3}-\pi}{2})}{14\cdot\pi\cdot r^2 + (\frac{64\cdot r^2}{4\cdot r^2} - 1)\cdot(r^2 \cdot \frac{2\cdot\sqrt{3}-\pi}{2})} =$$

$$\frac{2{,}418\cdot r^2}{43{,}98\cdot r^2 + 2{,}418\cdot r^2} = 0{,}052 = 5{,}2\%$$

So we can infer that the hexagonal packing has smaller probability to lose an event.
