Uncertainty Quantification and Sensitivity Analysis Report

Fortune 500 Company

1 Model Overview

The model of interest aims to compute the critical stress σ_c in MPa for a material subjected to certain physical properties. The model can be mathematically expressed as:

$$\sigma_c = \left| \left(\frac{M\gamma}{2b} \right) \left(\sqrt{\frac{8\gamma\phi R_s}{\pi G b^2}} - \phi \right) \right| / 10^6 \tag{1}$$

where the input parameters are defined as follows:

- γ: Parameter with units [value units]
- ϕ: Parameter with units [value units]
- R_s: Parameter with units [value units]
- G: Shear modulus with units [Pa]
- M: Parameter with units [value units]

This model is crucial for understanding how variations in these parameters can impact the critical stress, which is significant in determining material performance and safety.

As can be seen in Figure 1, the grid plot of input parameters against the model output provides insights into how each parameter influences the critical stress when other parameters are fixed.

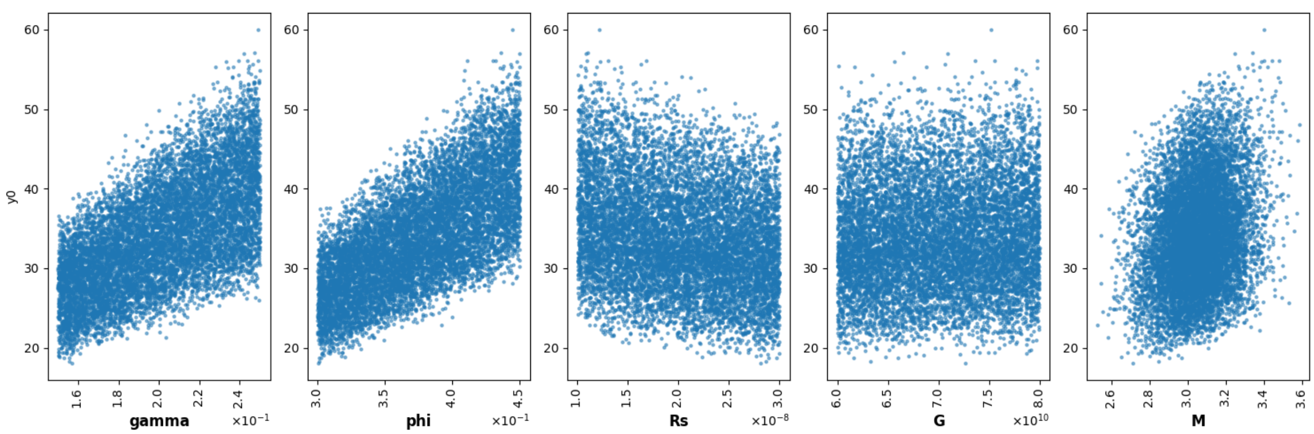

Figure 1: Grid plot of input parameters against model output.

2 Expectation Convergence Analysis

The expectation convergence analysis helps in understanding the range within which the mean value of the model output is expected to lie as the sample size increases.

As indicated in Figure 2, the mean estimate stabilizes around 34.2 MPa with lower and upper bounds converging closer as the sample size increases. This indicates reliable expectation values under the specified uncertainty.

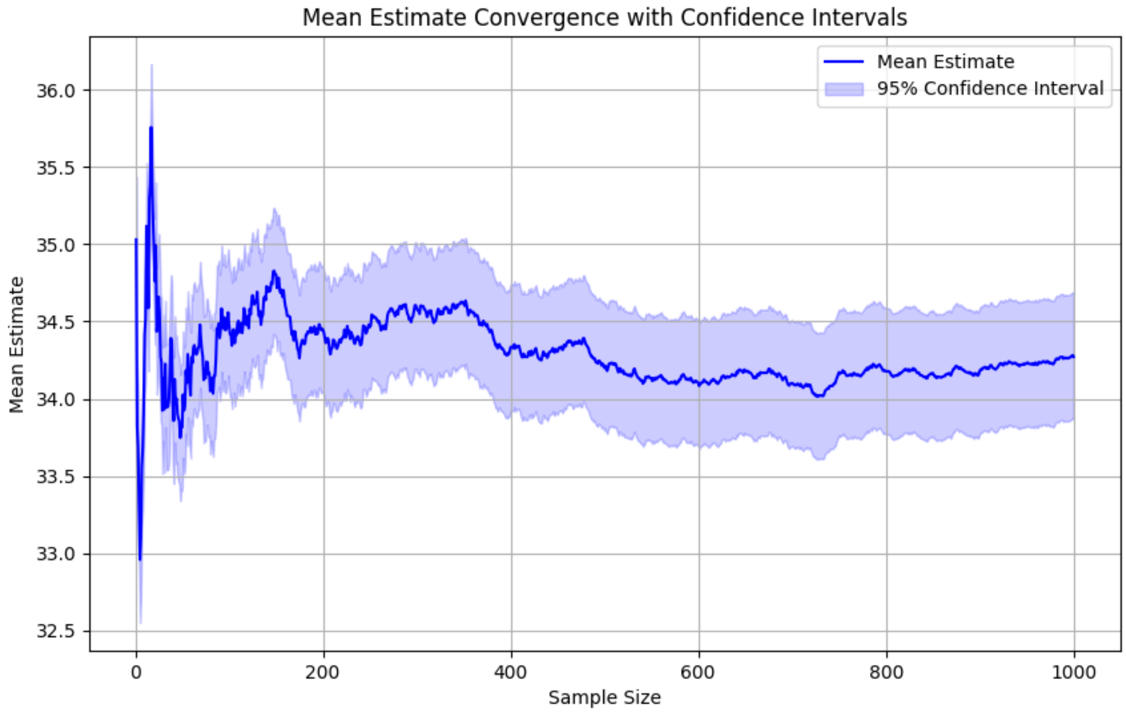

Figure 2: Mean estimate convergence plot.

3 Sensitivity Analysis

3.1 Correlation Coefficients

The correlation coefficients provide initial insights into the relationships between input variables and the model's output. The table below summarizes the different types of coefficients used:

From Figure 3, we observe that parameters γ and ϕ exhibit high PRCC values, reflecting strong monotonic relationships with σ_c. Interestingly, Rs has a negative PRCC, indicating an inverse relationship. This pattern is also consistent with Spearman and SRRC, validating the non-linear and monotonic nature of their influence.

3.2 Sobol Indices

The Sobol' indices provide an extensive variance-based decomposition of model sensitivity. The first-order and total-order Sobol indices are summarized below.

From Figure 4, it is evident that ϕ has the highest first-order Sobol index, indicating its dominant impact on the variance of σ_c. γ follows closely, underscoring its significant contribution. The total-order indices further reinforce these observations, depicting how ϕ and γ dominate the sensitivity profile with both their individual and interactive effects.

The R_s, though having a lesser impact, shows some interaction effects in its total index but with broader bounds, pointing towards some non-linear dependencies. Parameters G and M demonstrate minimal influence, correlating well with their low correlation coefficients.

4 Key Findings

The analysis reveals the following key points:

- Among the input variables, ϕ and γ are the most critical, influencing the output strongly both individually and interactively.

Coefficient	Mathematics	Description
PCC (Pearson)	$\rho_{U,V} = \frac{\mathrm{Cov}(U,V)}{\sigma_U \sigma_V}$	Measures the strength of a linear relationship between two variables. Can be positive or negative. Values range from -1 to 1, where 1 indicates a perfect positive linear relationship and -1 indicates a perfect negative linear relationship. Zero does not necessarily imply independence. If PCC for a given model input is 0.5, it means there is a moderate positive linear relationship with the output.
PRCC (Partial Rank Correlation Coefficient)	$\widehat{\rho}^S_{U,V}$ (on ranked data)	Computes the Pearson correlation coefficient on ranked input and output variables. Useful for identifying monotonic relationships when linearity is not present. Values range from -1 to 1. If PRCC for a given model input is 0.5, it means there is a moderate positive monotonic relationship with the output.
SRC (Standard Regression Coefficient)	$\widehat{\mathrm{SRC}}_i = \widehat{a}_i \frac{\widehat{\sigma}_i}{\widehat{\sigma}}$	Measures the influence of input variables on output using multiple linear regression. Useful for linear relationships. The closer to 1, the greater the impact on the variance of Y. Negative values are possible. If SRC for a given model input is 0.5, it means the input has a moderate positive influence on the output variance.
SRRC (Standard Rank Regression Coefficient)	$\widehat{\mathrm{SRC}}_i$ (on ranked data)	Computes SRC on ranked input and output variables. Useful for monotonic relationships where linearity is not present. Values range from -1 to 1. If SRRC for a given model input is 0.5, it means the input has a moderate positive monotonic influence on the output.
Spearman	$\rho^S_{U,V} = \rho_{F_U(U),F_V(V)}$	Measures the strength of a monotonic relationship between two variables using ranks. Equivalent to Pearson's on ranked data. Values range from -1 to 1, indicating perfect monotonic relationships. Can be positive or negative. If Spearman for a given model input is 0.5, it means there is a moderate positive monotonic relationship with the output.
Variables: $\mathrm{Cov}(U,V)$: covariance between U and V; σ_U, σ_V: standard deviations of U and V; $\widehat{a}_i$: estimated regression coefficients; $\widehat{\sigma}_i, \widehat{\sigma}$: sample standard deviations; F_U, F_V: cumulative distribution functions; U, V: random variables; ρ: correlation coefficient.		

Table 1: Summary of Sensitivity Analysis Coefficients

Inputs	Sobol Index	Upper Bound	Lower Bound
γ	0.382657	0.451473	0.298333
ϕ	0.470565	0.528412	0.392007
R_s	0.0499193	0.108297	-0.0347545
G	-0.0127661	0.0426575	-0.0926921
M	0.0338171	0.0891464	-0.0451639

Table 2: First-order Sobol Indices

- The Rs parameter has a noticeable inverse relationship with the output, suggesting certain scenarios where higher Rs values might decrease σ_c.
- Variables G and M have minimal impact, as indicated by their Sobol indices and correlation coefficients.
- The convergence of the mean estimate implies operational reliability near the 34.2 MPa mark, under the given uncertainty.

5 Conclusion

The sensitivity analysis demonstrates that focusing on the precise control of ϕ and γ is crucial for managing the critical stress σ_c. The analyses corroborate well across both correlation coefficients and Sobol indices, reinforcing the robustness of the findings. It is suggested to further refine the model by exploring interaction effects between the less influential parameters to validate assumptions more thoroughly.

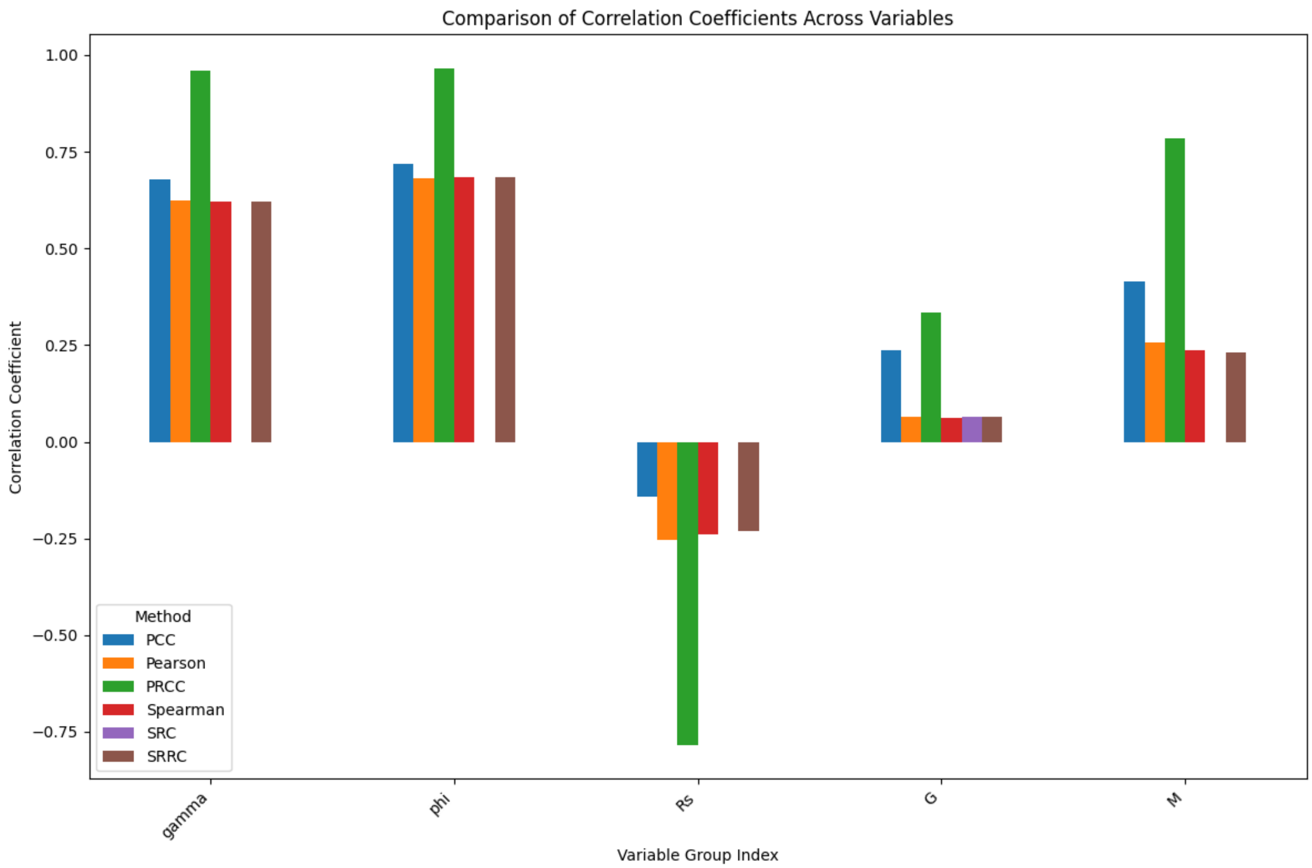

Figure 3: Correlation Coefficients plot.

Inputs	Sobol Index	Upper Bound	Lower Bound
γ	0.360596	0.466441	0.239452
ϕ	0.489098	0.597767	0.372993
R_s	0.0845983	0.176282	-0.0181134
G	0.00993587	0.0947672	-0.0912478
M	0.0757934	0.155485	-0.0185773

Table 3: Total-order Sobol Indices

6 Summary and Insights for Decision Making

Understanding that ϕ and γ are critical influencers allows for targeted interventions in material properties to ensure optimal stress performance. The negative correlation with Rs points towards potential material modifications to manage critical stress scenarios effectively. Utilizing sensitivity analysis provides a strong basis for informed decision-making and prioritizing efforts on parameters with substantial impacts.

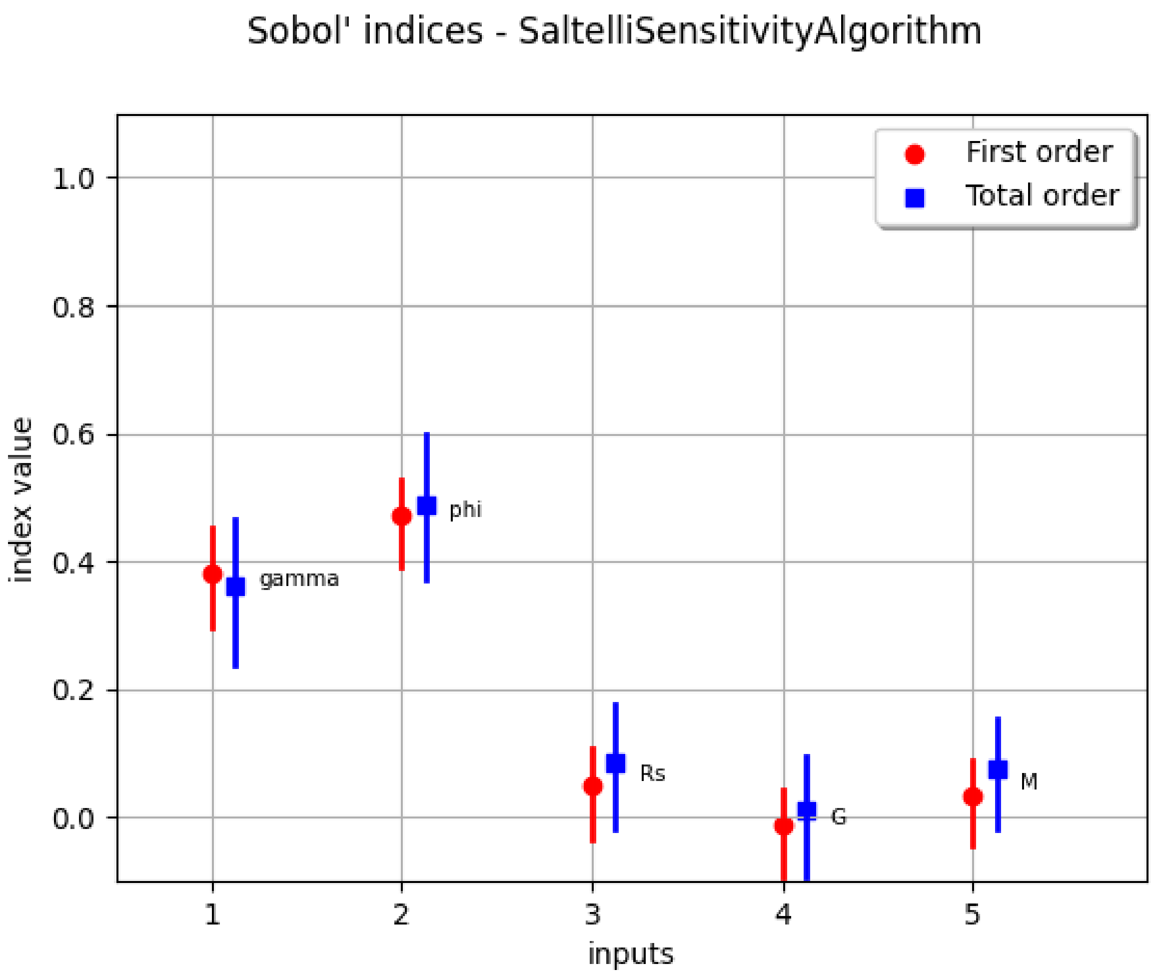

Figure 4: Sobol Indices plot.