

Uncertainty Quantification and Sensitivity Analysis Report

1 Model Overview and Uncertain Inputs in Question

The model of interest is defined by the following equation:

$$Y = \sin(x_1) + a \cdot \sin^2(x_2) + b \cdot x_3^4 \cdot \sin(x_1) \quad (1)$$

where $a = 7$ and $b = 0.1$. The variables x_1, x_2, x_3 are uncertain inputs. The uncertain inputs and their probability distributions are detailed in Table 1:

Here $\pi \approx 3.14159$. The output Y represents a complex interaction of trigonometric and polynomial terms, likely dependent on the physical phenomena characterized by these inputs. The physical units are implied to be dimensionless for the inputs, while the output Y may also be considered dimensionless based on the operations involved. Figure 1 shows the grid plot of input parameters against the model output.

2 Expectation Convergence Analysis

The mean estimate convergence analysis indicates how the model's expected output stabilizes with increasing sample size. Table 2 summarizes the mean estimate convergence data, and Figure 2 visualizes these results.

The range of mean estimates illustrates that the model's expected output spans from approximately 3.08 to 3.54 as more samples are considered, which indicates the likely operating range under the specified uncertainty.

3 Sensitivity Analysis

3.1 Correlation Coefficients

Table 3 shows the correlation coefficients including PCC, Pearson, PRCC, Spearman, SRC, and SRRC. The PRCC coefficient for x_1 is particularly high, suggesting that x_1 has a significant and consistent impact on the output across multiple metrics.

3.2 Sobol Indices

Tables 4 and 5 provide the first-order and total-order Sobol indices respectively. These indices quantify the variance contribution by the individual inputs and their interactions.

The Sobol indices reveal that x_1 and x_2 are the most influential parameters, with x_3 having a negligible main effect but a non-trivial total effect, highlighting possible interaction effects. The total-order indices being larger than the first-order indices confirm significant interactions especially for x_3 . The mathematical formulation of the Sobol indices is given by:

$$S_i = \frac{V[E[Y | Q_i]]}{V[Y]}$$
$$S_{Ti} = 1 - \frac{V[E[Y | Q_{-i}]]}{V[Y]}$$

This underlines the comprehensive impact of each parameter inclusive of interaction effects.

Table 1: Uncertainties and associated probability distributions

Variable	Distribution	Parameters
x_1	Uniform	$[-\pi, \pi]$
x_2	Uniform	$[-\pi, \pi]$
x_3	Uniform	$[-\pi, \pi]$

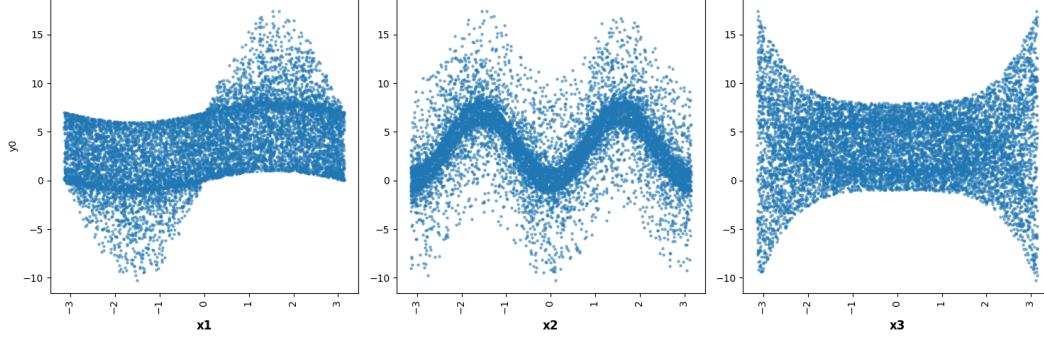


Figure 1: Grid plot of input parameters against model output.

Table 2: Mean estimate convergence data

Sample Size	Mean Estimate	Lower Bound	Upper Bound
1	-0.754	-0.986	-0.521
501	3.311	3.078	3.544

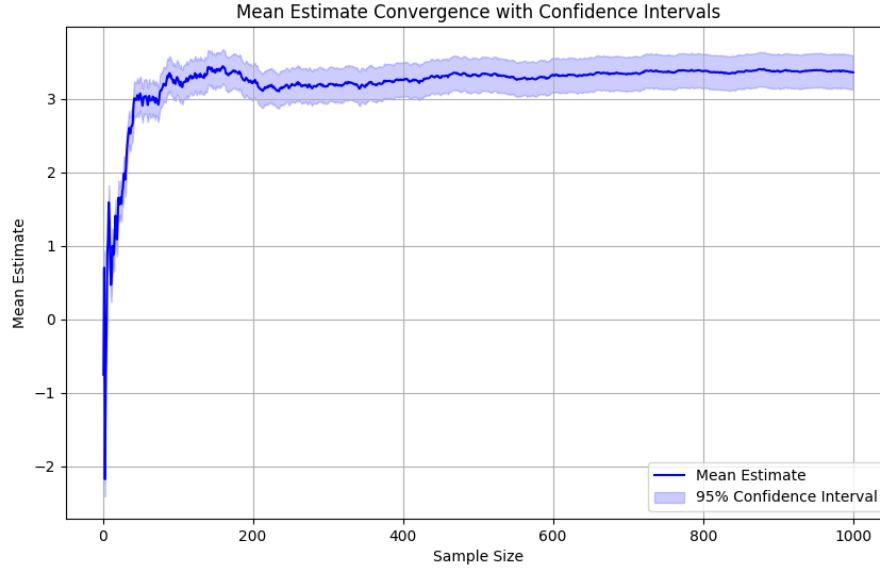


Figure 2: Mean estimate convergence plot.

Table 3: Correlation coefficients

Variable	PCC	Pearson	PRCC	Spearman	SRC	SRRC
x_1	0.435	0.435	0.435	0.435	0.435	0.435
x_2	0.010	0.008	0.013	0.011	0.009	0.011
x_3	0.006	0.003	-0.002	-0.004	0.005	-0.002

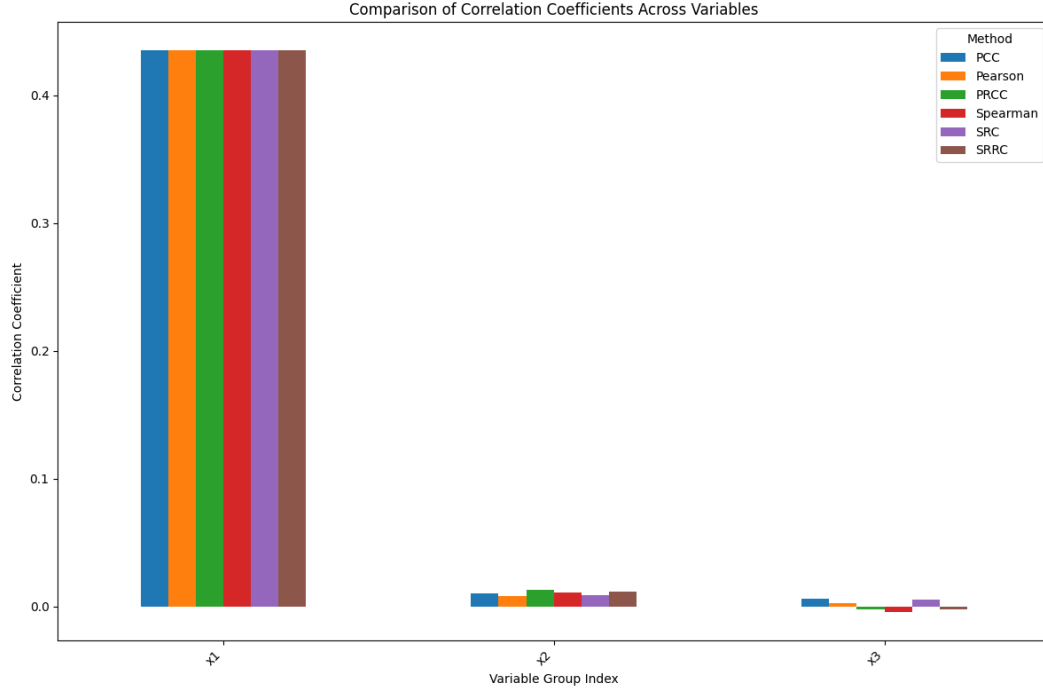


Figure 3: Correlation coefficients plot.

Table 4: First-order Sobol indices

Inputs	Sobol Index	Upper Bound	Lower Bound
x_1	0.334	0.271	0.410
x_2	0.429	0.339	0.507
x_3	-0.025	-0.122	0.071

Table 5: Total-order Sobol indices

Inputs	Sobol Index	Upper Bound	Lower Bound
x_1	0.539	0.411	0.687
x_2	0.440	0.295	0.575
x_3	0.262	0.164	0.357

4 Key Findings

From the correlation and Sobol analysis, it is evident that:

- x_1 and x_2 are the primary influencers of the model output.
- x_3 has minimal direct effect but important interaction effects.
- The high PRCC value for x_1 aligns with its significant Sobol index.

5 Conclusion

Further analysis can focus on deeper interactions involving x_3 and its influence in combination with other parameters. Additional sampling may refine the confidence bounds of the estimates.

6 Summary and Insights for Decision Making

Key parameters like x_1 and x_2 should be the focus of management strategies. Negative correlations, albeit small, suggest certain balancing effects in the model dynamics. Utilizing sensitivity insights can help in targeted interventions.

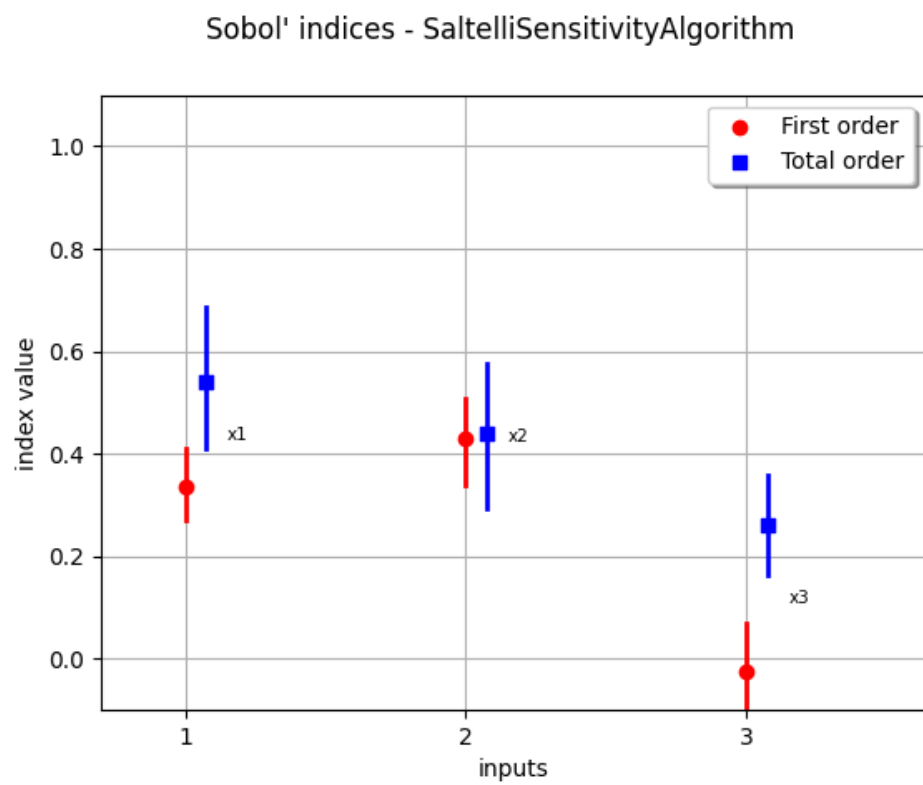


Figure 4: Sobol indices plot.