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MSc. Music and Acoustic Engineering

Musical Acoustics - A.Y. 2020/2021

# Homework 3 - Sound Radiation from Plates

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November 14, 2020

# 1 Adjusting the radiation cutoff by choosing the plate thickness

Assuming an infinite plate, we can study the sound radiation by imposing that the air's velocity field and the plate velocity are equal on the plate's surface. This yields the following expression for the component of the acoustic wave vector normal to the surface:

$$k_z = \omega\sqrt{\frac{1}{c^2} - \frac{1}{v_p^2}}$$

where $c$ is the sound velocity in air and $v_p$ is the velocity of the bending waves in the plate. Since bending waves in a plate are dispersive ($v_p \propto \omega^{\frac{1}{2}}$), the argument of the square root increases with frequency, which means there'll be a cutoff frequency below which $k_z$ becomes imaginary. In this case the solution for the acoustic field takes the form of a vanishing wave in the z direction, which doesn't carry any energy away from the plate's surface. Therefore, below the cutoff frequency there will not be any sound radiation.

In particular, the cutoff frequency is the one for which $v_p = c$. The velocity of the bending waves in a thin plate is:

$$v_p(\omega) = \sqrt{\frac{\omega h c_L}{\sqrt{12}}}$$

where $c_L$ is the corresponding velocity of the quasi-longitudinal waves. For a wooden plate the longitudinal velocity depends on the direction of the wave; indicating with $x$ and $y$ the longitudinal and radial axes of the material respectively:

$$c_x = \sqrt{E_x/\rho(1-\nu_{xy}\nu_{yx})} \qquad c_y = \sqrt{E_y/\rho(1-\nu_{yx}\nu_{xy})}$$

where $\nu_{xy}$ and $\nu_{yx}$ are the Poisson's ratios relative to the two axes, while $E_x$ and $E_y$ are the Young's moduli.

The values of these parameters for Sitka spruce can be seen in Tab. 1, together with the resulting velocities. Since the longitudinal velocity is the highest of the two, waves that propagate in the $x$ direction will be cutoff at a lower frequency than those that propate in the $y$ direction. The cuttoff frequency of the plate, therefore, is the one at which the velocity of the bending waves in the $x$ direction matches the speed of sound in the air. We can tune this frequency by varying the plate's thickness $h$: if we want a cutoff at 1.2 kHz, we will need to use a plate of thickness $h = 9.5\,\mathrm{mm}$.

| $E_x$ [GPa] | $E_y$ [GPa] | $\nu_{xy}$ | $\nu_{yx}$ | $c_x$ [$\mathrm{m\,s^{-1}}$] | $c_y$ [$\mathrm{m\,s^{-1}}$] |
|---|---|---|---|---|---|
| 10.80 | 1.112 | 0.372 | 0.040 | $5.73 \times 10^3$ | $1.60 \times 10^3$ |

Table 1: Mechanical parameters of Sitka spruce, from [1].

## 2 Frequency dependence of the directivity

The wave vector of the radiated wave lies on the $xz$ plane. The angle it forms with the plate's surface can be recovered by writing:

$$\tan\theta = \frac{k_z}{k_x} = \sqrt{\frac{v_p^2}{c^2} - 1}$$

Much like before, the presence of $v_p$ in this expression implies a dependence on frequency of the direction of propagation. In Fig. 1 we report a plot of the angle $\theta$ as a function of the frequency $f$.

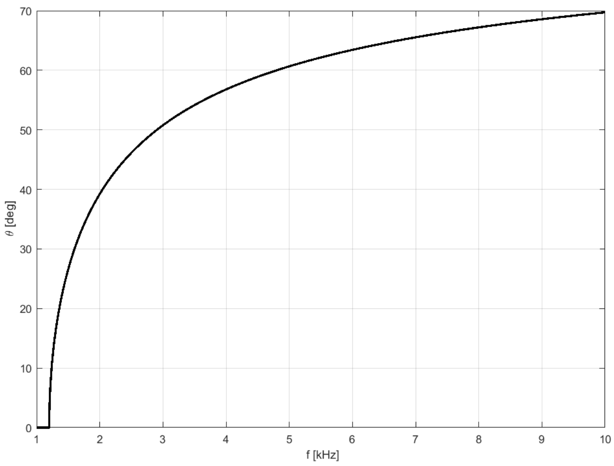


Figure 1: Plot of $\theta$ as a function of $f$.

## 3 Effects of the finite extension of real plates

Of course all the discussion presented up until now stands on the assumption that the plate we are considering is infinite in its extension. This is clearly a limiting case, but it can at least give us a picture of the expected radiating behavior of a real "large" plate, i. e. one in which the linear dimensions are all significantly greater than the wavelengths of interest. In a real plate, typically, the radiation spectrum will exhibit a roll-off below the theoretical cut-off frequency, as opposed to the discontinuity we have in the case of the infinite plate.

## References

[1] Forest Products Laboratory. *Wood Handbook - Wood as an Engineering Material.* Madison, Wisconsin: United States Department of Agriculture, 2010.