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HL3 – Modeling Techniques

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1 Comments on the FD implementation

The MATLAB code `homework_piano.m` is able to perform a backward finite difference (FD) scheme which is used to simulate a piano string struck with a felt hammer. In the first part of the following report the numerical values chosen are illustrated while the second part describes briefly the actual FD algorithm.

1.1 Simulation data

The string under consideration is tuned to C2 ($f_0 = 52.8221\,\text{Hz}$). We chose a sampling frequency of $f_s = 176.4\,\text{kHz}$ as suggested in [1] and it's well above the Nyquist frequency so we don't expect aliasing in the time domain. Other dimensions that are not suggested in the assignment, such as string mass, hammer mass, string length etc. have been taken from [1] and can be found listed in Tab. 1 . The string has a linear mass $\mu = M_s/L$ and the tension applied on it at rest has been computed as $T_0 = 4L^2 f_0^2 \mu$ which determines that the speed of sound along the string is $c = \sqrt{T_0/\mu}$. The spatial sampling has been chosen under the limit imposed by the stability condition [2]

$$N_{max} = \sqrt{\frac{-1 + \sqrt{1 + 16\epsilon\gamma^2}}{8\epsilon}}$$

where $\gamma = f_s/2f_0$. The number of spatial steps has been set as $N = N_{max} - 1 = 537$ and the relative spatial resolution is $X = 0.0036\,\text{m}$.

string length	L	$1.92\,\text{m}$
string mass	M_s	$35 \times 10^{-3}\,\text{kg}$
string stiffness parameter	ϵ	7.5×10^{-6}
string stiffness coefficient	κ	7.5×10^{-6}
relative striking position	a	0.12
viscous damping coefficient	b_h	$1 \times 10^{-4}\,\text{s}^{-1}$
hammer mass	M_h	$4.9 \times 10^{-3}\,\text{kg}$
hammer stiffness	K	$4 \times 10^{8}\,\text{N}\,\text{m}^{-1}$
stiffness exponent	p	2.3
hinge normalized impedance	ζ_l	$10^{20}\,\text{s}^3\text{m}^2/\text{kg}^2$
bridge normalized impedance	ζ_b	$10^{3}\,\text{s}^3\text{m}^2/\text{kg}^2$

Table 1: Values considered in the simulation.

1.2 FD algorithm

The definition of temporal and spatial coordinate is the very first step of the FD implementation

```
t = linspace(0, duration, tSamples); %temporal vector
x = linspace( 1 , L , L/X  ); %spatial coordinate vector
```

followed by the initialization of the string displacement $y(n, m)$, the hammer displacement (with respect to the contact point) $\eta(n)$, the hammer force $F(n)$ and the Hanning window $g(m)$, where $n \in t$ and $m \in x$.

```
y = zeros(length(t), length(x)); %string displacement vector
eta = zeros(1 , length(t)); %hammer displacement vector
Fh = zeros(1 , length(t)); %hammer force vector
g = zeros(length(x), 1 ); %Hanning window vector
```

At this point the Hanning window must be defined centred in the striking point m_0 and with a width of $2w$

```
m0 = ceil(x0/X); % hammer striking coordinate
wh = 0.1; %[m] half width of the hammer
w = ceil(wh/X); %sample of the hammer's width
gloc = hann(2*w); %local hanning window
g(m0-w:m0-w+length(gloc)-1)=gloc;
```

and used to filter the force that is acting on the string $F(n,m) = F(n)g(m)$. At time $t = 0$ ($n = 1$) the string and the hammer are supposed to be still $y(1,m) = \eta(1) = 0$ and therefore the force $F(1) = 0$, since it is defined by the non-linear relation $F(n) = K|\eta(n) - y(n, m0)|^p$. The hammer displacement and the force at time $t = dt$ ($n = 2$) is given by the velocity at time $t = 0$, which is $Vh_0 = 2.5\,\mathrm{m\,s^{-1}}$

```
eta(2) = Vh0*ts;
Fh(2) = K*(abs(eta(2) - y(2, m0)))^p;
```

where $y(2, m_0)$ is assumed to be zero. After having defined the initial conditions, the backward FD loop can start by computing the displacement at time sample $n + 1$ by knowing the displacement at n and $n - 1$

References

[1] Charalampos Saitis, Sarah Orr, and Maarten Van Walstijn. "Physical modeling of the piano: An investigation into the effect of string stiffness on the hammer string interaction." In: *The Journal of the Acoustical Society of America* 125 (May 2009), p. 2684. DOI: 10.1121/1.4784255.

[2] Antoine Chaigne and Anders Askenfelt. "Numerical simulations of piano strings. I. A physical model for a struck string using finite difference methods". In: *Journal of The Acoustical Society of America - J ACOUST SOC AMER* 95 (Feb. 1994), pp. 1112–1118. DOI: 10.1121/1.408459.