POLITECNICO
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MSc. Music and Acoustic Engineering

Musical Acoustics - A.Y. 2020/2021

H4 - Impedance maxima of a compound horn

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1 Impedance maxima of the pipe

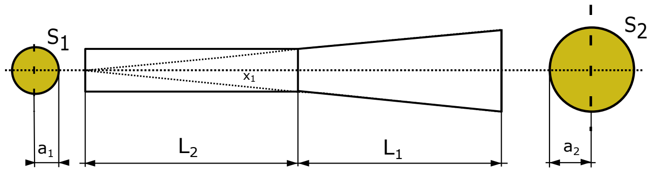

Figure 1: Scheme of the compound horn system. Beside each end a drawing of its cross section can be seen.

In order to compute the acoustic input impedance of a pipe, one needs to account not only for the air contained in the pipe as an ideal fluid but also for two phenomena occurring around it: the radiation load at the end and the wall losses along its length. The latter is relevant for narrow pipes in particular, but in this case, since the ratio between the pipe's length and the radius of its cross section is $L_1/a_1 = 10$, it is clear that we are not dealing with this condition. Moreover, a common way to characterize the influence of the viscous drag is the ratio of the pipe radius with the boundary layer thickness r_v. If this parameter is greater than 10, it is usually considered safe to neglect the wall losses. Since $r_v \propto \sqrt{f}$, this might not be true at low frequencies; however, in our case, we can see that r_v increases very rapidly from zero and at 1 Hz is already ~ 30. We assume, therefore, that it is safe for us to ignore such losses in the computation of the impedance.

Concerning the radiation, instead, we know that its influence on an open-ended pipe is usually not negligible. At low frequency, the strategy that is commonly used to address the study of radiation is to add an end correction Δ^{open} . At first order in ka the radiation impedance of an open end is equal to the input impedance of an open pipe of length Δ^{open}. We take the value $\Delta^{open} = 0.61a$; this holds well under the assumption $ka \ll 1$. However, for us $ka = 1$ at ~ 1 kHz, which is well inside the range of frequencies we are interested in. At higher frequencies it is still possible to use this model[1], but the end correction decreases with ka. This means that the values for the frequencies of the impedance maxima we will compute will be slightly underestimated in the higher range.

The input impedance of the pipe is therefore, in our model:

$$Z_{in} = jZ_0 \tan kL \qquad \text{, with } L = L_2 + \Delta^{open}$$

The corresponding maxima are at

$$f_n = \frac{nc}{2L}, \qquad n \in \mathbb{N}$$

[1]As long as the wavefronts are planar, i. e. for $ka < 3.83$.