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MSc. Music and Acoustic Engineering

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H6 - Design of a Recorder Flute

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1 Resonator

1.1 Fixing the shape of the bore

The resonances of the instrument occur at the zeros of the series $Z_m + Z_p$ of the impedances of the mouth and the resonator pipe. The pipe is a finite cone tapering towards the foot; we can write its input impedance neglecting the radiation load:

$$Z_p = \frac{\mathrm{j}\rho c}{S_1} \frac{\sin kL \sin k\theta_1}{\sin k(L+\theta_1)}$$

where $L = 0.45$ m is the length of the bore, S_1 is the bore cross section at the mouth, x_1 is the distance between the apex of the cone and S_1, and $k\theta_1 = \arctan kx_1$. The mouth impedance can be written as a pure inertance term $\mathrm{j}\omega M$, where $M = \rho l/S_m$ is determined by fixing arbitrarily the surface area and the depth of the mouth opening. Here we choose $S = 0.65$ cm^2 and $l = 3$ mm. The resonance condition becomes:

$$\frac{\rho}{S_1} \sin kL \sin k\theta_1 + kM \sin k(L+\theta_1) = 0.$$

Fixing $k = 2\pi f_0/c$, with $f_0 = 392$ Hz, and solving this for x_1, we find $x_1 = 99.9$ cm, which corresponds to a diameter at the resonator head $d_1 = 26.2$ mm. The corresponding values at the foot are $x_2 = 54.9$ cm and $d_2 = 14.4$ mm.

2 Flue channel and mouth

2.1 Channel thickness

Given the pressure difference Δp between the player's mouth and the flue channel entrance, the flow velocity U_j in the flue channel can be easily recovered from the Bernoulli equation:

$$\frac{1}{2}\rho_0 \left(v_2^2 - v_1^2\right) = p_2 - p_1 \quad \Rightarrow \quad U_j = \sqrt{\frac{2\Delta p}{\rho_0}}$$

where $v_2 = U_j$ is the jet velocity in the channel, $v_1 = 0$ is the air velocity in the player's mouth (assumed negligible) and p_1 and p_2 are the corresponding pressures.

Now, we know that the amplification of the jet perturbation is strongly dependent on the frequency of the acoustic field, and that it is strongest for frequencies around $0.3U_j/h$. This allows us to choose h according to the desired spectral characteristics of the instrument: we can "tune" the channel thickness to maximize the perturbation amplification at a target spectral centroid f_c, choosing $h = 0.3U_j/f_c$.

In our particular case we can compute:

$$\Delta p = 62\,\mathrm{Pa},\ f_c = 2\,\mathrm{kHz},\ \rho_0 = 1.2\,\mathrm{kg\,m^{-3}} \qquad \longrightarrow \qquad \boxed{h = 1.5\,\mathrm{mm}}$$

The structure of the jet can be characterized using the Reynolds number:

$$\mathrm{Re} = \frac{U_j h}{\nu} = 1033.3$$

where $\nu = 1.5 \cdot 10^{-5}\ \mathrm{m}^2\,\mathrm{s}^{-1}$ is the kinematic viscosity of the air.

2.2 Boundary layer effects

Of course the result for the flow velocity we found above is only valid at the center of the flow. Indeed, even if the fluid can be safely assumed to be frictionless in open space, the boundary conditions in a channel make the viscosity effects significant near the walls. There will be a thin layer extending from the boundary into the fluid where the velocity increases rapidly from zero to the value U_j: this is the so-called *boundary layer*. The thickness of the boundary layer increases along the length of the channel: at position x along the channel it is:

$$\delta(x) \approx \sqrt{\frac{\nu x}{U_j}}.$$

This means that for a channel length of 20 mm we get $\delta = 0.54\,\mathrm{mm}$. Notice that $\delta/h = 0.36 < \frac{1}{2}$, so we can expect the Bernoulli approximation to hold reasonably well in the center of the channel.