POLITECNICO
MILANO 1863

ISPL
Image and Sound Processing Lab

Musical Acoustics Module I: Modeling of Musical Instruments

HL2 - Practice Session Homework II – Electric Analogs

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Instructions

- Homeworks can be done in groups of up to 3 people

- You must upload on the HL2 beep folder the Simulink files and a report describing the work you have done.

- You will have three weeks from the day when the exercises were published.
  (23:59 – 4/12/2020)

- Each Exercise and sub-exercise corresponds to different points
  - Exercise 1 (0.15 points)
  - Exercise 2 (0.85 points)

- Complexity Coefficient: 2

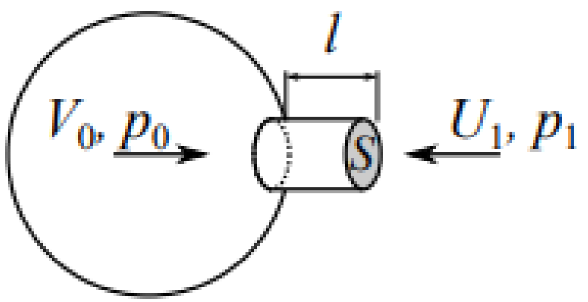
- N.B. please include your student ID in the report

Ex.1 - Single Resonator (0.15 points)

Set a simulation in Simscape and plot the frequency response

$$H(\omega) = \frac{U_1(\omega)}{p_1(\omega)},$$

using the electric analog characterized by the following parameters:



$V_0 = 0{,}1\, m^3$ $\rho = 1{,}2\ {}^{Kg}/_{m^3}$

$l = 10\ cm$ $c = 343\ {}^{m}/_{s}$

$S = 100\, m^2$

NB: model the pipe resistance with a frequency-independent resistance

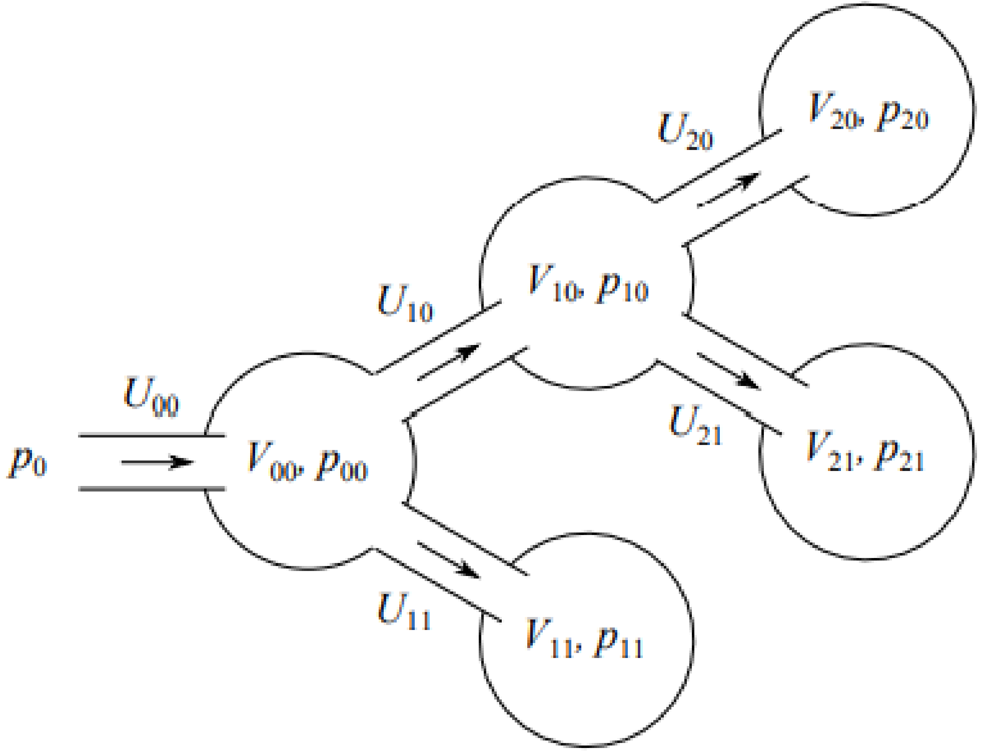
$$R = \frac{\rho \cdot c}{S}$$

Hint: Look at the Helmholtz Resonator Dimmer seen in class as a starting point

Ex.2 - Helmholtz Resonator Tree (0.85 points)

Idea

The Helmholtz resonator tree yields an intuitive way of creating physically-inspired resonating structures for synthesis. In this type of structure several resonators influence each other, and their parameters can be modified using acoustical parameters, which are intuitive even for non-technical people, although the exact resonance frequencies are unknown. Additionally, it provides the possibility of exciting the system at different points, resulting in timbre variations of the same musical instrument. This approach differs from the implementation of cascaded second order filters, since the resonators interact with each other, changing their resonance frequencies.



Ex.2 - Helmholtz Resonator Tree (0.85 points)

The acoustic impedance of a Helmholtz resonator tree can be derived in an iterative way. The acoustic impedance of a single resonator is given by

$$Z_1(\omega) = \frac{(\mathrm{j}\omega)^2 LC + \mathrm{j}\omega RC + 1}{\mathrm{j}\omega C}, \tag{5}$$

where ω is the frequency in rad/s and $\mathrm{j} = \sqrt{-1}$ is the imaginary unit. The impedance of a Helmholtz resonator tree of height two and with N leaves can be determined as

$$Z_2(\omega) = \mathrm{j}\omega L + R + \frac{1}{\mathrm{j}\omega C + \sum_{n=1}^{N} \frac{1}{Z_{1,n}}}, \tag{6}$$

where $Z_{1,n}(\omega)$ is the impedance of the n^{th} leaf, given by Eq. 5. The same procedure performed for a tree of height two can be generalized for a tree of height K, where the impedance of this tree $Z_K(\omega)$ is calculated based on the impedances of the subtrees $Z_{K-1,n}(\omega)$ connected to it, as illustrated in Fig. 2 (c). This procedure results in the general form of the Helmholtz resonator tree impedance

$$Z_K(\omega) = \frac{-\omega^2 LC + \mathrm{j}\omega RC + 1}{\mathrm{j}\omega C + \sum_{n=1}^{N} \frac{1}{Z_{K-1,n}}}. \tag{7}$$

Finally, the pressure-to-volume flow transfer function, can be obtained with the impedance $Z_K(\omega)$ as

$$H_K(\omega) = \frac{u(\omega)}{p(\omega)} = Z_K^{-1}(\omega). \tag{8}$$

This results in a complex resonating system where the resonances are given by the poles of $H_K(\omega)$ in Eq. 8.

Ex.2 - Helmholtz Resonator Tree (0.85 points)

- **Part A (0.35 points)**
  - Use the RLC circuit defined in Ex.1 and connect its replicas to build a NxK tree, where

    N – height of the tree

    K – branch division (= how many leaves for each branch)

    -> example: tree in slide 4 has N=2 and K=2
  - Use the same parameters for each component
  - Analyse the frequency response using as output the current in one of the leaves ($U_{n,k}$) and pressure p_0 as input

- **Part B (0.25 points)**
  - Try with different N and highlight what this parameter controls in the final response

- **Part C (0.25 points)**
  - Try with different K and highlight what this parameter controls in the final response