POLITECNICO DI MILANO

Fundamentals of Acoustics: Diffuse Field Model

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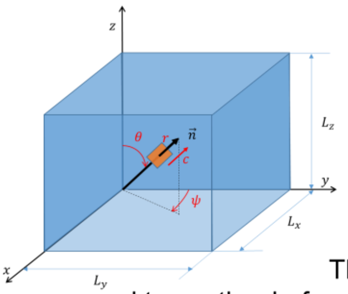
Application of the Diffuse Field Model

Diffuse Field: *Average Time between Reflections*



For a plane wave propagating in a generic direction $\vec{n}$ (identified by Ω subscript) the components of its propagation speed c are:

$$v_{p,x} = c \sin\theta \cos\psi$$
$$v_{p,y} = c \sin\theta \sin\psi$$
$$v_{p,z} = c \cos\theta$$

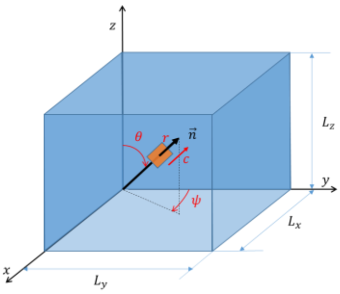
The wave front traces on each axis move from one end to another before a reflection can occurs; the related elapsed times are:

$$t_x = L_x / v_{p,x} \quad ; \quad t_y = L_y / v_{p,y} \quad ; \quad t_z = L_z / v_{p,z}$$

the reflection frequencies (number of reflections in one second) along each axis are then

$$f_x = 1/t_x = v_{p,x}/L_x = c \sin\theta \cos\psi \,/L_x$$
$$f_y = 1/t_y = v_{p,y}/L_y = c \sin\theta \sin\psi \,/L_y$$
$$f_z = 1/t_z = v_{p,z}/L_z = c \cos\theta \,/L_z$$

Diffuse Field: *Average Frequency of Reflections*



For a plane wave propagating in a generic direction $\vec{n}$ (identified by Ω subscript) the reflection frequency f_n is then

$$f_n = f_x + f_y + f_z$$

(how may times reflections occur in 1 second when traveling from one side to the other of the space)

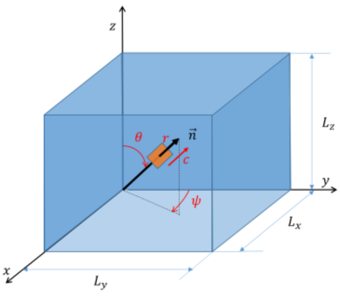
If the wave field is diffuse (*infinite number of incoherent plane equal waves traveling in all directions*) the probable mean reflection frequency for all waves through a generic point *P* is

$$f_m = \frac{1}{\Omega} \int_{\Omega=sphere} |f_n| \, d\Omega = \frac{8}{4\pi} \int_0^{\pi/2} d\psi \int_0^{\pi/2} f_n \sin\theta \, d\theta$$

i.e.

$$f_m = \frac{2c}{\pi} \int_0^{\pi/2} d\psi \int_0^{\pi/2} \left[\frac{\sin\theta \cos\psi}{L_x} + \frac{\sin\theta \sin\psi}{L_y} + \frac{\cos\theta}{L_z} \right] \sin\theta \, d\theta$$

Diffuse Field: *Reflection Mean Time*



Thus

$$f_m = \frac{2c}{\pi}\frac{\pi}{4}\left(\frac{1}{L_x}+\frac{1}{L_y}+\frac{1}{L_z}\right)$$

and for

$$V = L_x L_y L_z \quad ; \quad S = 2(S_x + S_y + S_z)$$

$$S_x = L_y L_z \quad ; \quad S_y = L_x L_z \quad ; \quad S_z = L_x L_y ;$$

$$\boxed{f_m = \frac{cS}{4V}}$$

mean frequency = mean value of a stochastic distribution

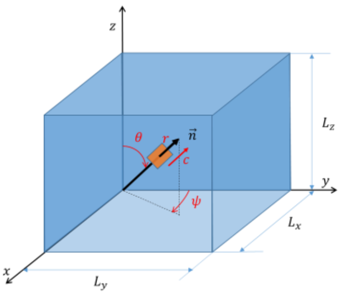
Then, the probable mean time between two reflections is

$$\boxed{t_m = \frac{1}{f_m} = \frac{4V}{cS}}$$

and the *free mean path*

$$\boxed{l_m = c t_m = 4V/S}$$

Acoustic Energy Balance of a Room



The integral acoustic energy balance on a confined space of volume $V = L_x L_y L_z$ is

$$\frac{dW}{dt} = \Phi_W + \Sigma_W$$

where

- W acoustic energy, [J]
- Φ_W acoustic energy "flow" through the boundary, [W]
- Σ_W acoustic energy source inside V, [W]

For

$W = \int_V w dV = \langle \overline{w} \rangle_V V$ volume-averaged acoustic energy density times V

$\Phi_W = -\overline{\dot{W}}_{abs}$ absorbed sound power by the boundary

$\Sigma_W = \overline{\dot{W}}_S$ internal sound sources power

Acoustic Energy Balance of a Room

The integral acoustic energy balance becomes

$$V\frac{d\langle \overline{w}\rangle_V}{dt} = \bar{\dot{W}}_S - \bar{\dot{W}}_{abs}$$

Assuming to have a quasi-diffuse sound field, compound of a **direct** sound field produced by a sound source (**d**) and a **reverberant** sound field due to infinite reflections on the bounding surfaces (**r**), (ideally diffuse), then

$$\langle \overline{w}\rangle_V = \langle \overline{w}_d\rangle_V + \langle \overline{w}_r\rangle_V = \langle \overline{w}_d\rangle_V + \overline{w}_r$$

and

$$\bar{\dot{W}}_{abs} = \bar{\dot{W}}_{abs,d} + \bar{\dot{W}}_{abs,r}$$

with

$\bar{\dot{W}}_{abs,d}$ absorbed sound power – direct sound from the source

$\bar{\dot{W}}_{abs,r}$ absorbed sound power – from reverberant field ONLY

$$\boxed{V\frac{d}{dt}\left[\langle \overline{w}_d\rangle_V + \overline{w}_r\right] = \bar{\dot{W}}_S - \bar{\dot{W}}_{abs,d} - \bar{\dot{W}}_{abs,r}}$$

Acoustic Energy Balance of a Room

Defining the room average *apparent* absorption coefficient as

$$\alpha_{av} \equiv \sum_{i=1}^{N}\alpha_i S_i \Big/ \sum_{i=1}^{N} S_i = \frac{1}{S}\sum_{i=1}^{N}\alpha_i S_i$$

indicating the reverberant field power incident on the bounding surface with $\bar{\dot{W}}_r$, it is

$$\bar{\dot{W}}_{abs,d} = \alpha_{av}\bar{\dot{W}}_S$$
$$\bar{\dot{W}}_{abs,r} = \alpha_{av}\bar{\dot{W}}_r$$

it is

$$\boxed{V\frac{d}{dt}\left[\langle \overline{w}_d\rangle_V + \overline{w}_r\right] = (1-\alpha_{av})\bar{\dot{W}}_S - \alpha_{av}\bar{\dot{W}}_r}$$

Introducing the statistical follow up of the diffuse field hypothesis:

- $t_m = 4V/cS$ mean travel time of volume V

$$\boxed{\bar{\dot{W}}_r = V\,\overline{w}_r/t_m = \overline{w}_r cS/4}$$

Acoustic Energy Balance of a Room

Under **diffuse field hypothesis**

$$V\frac{d}{dt}[\langle\bar{w}_d\rangle_V + \bar{w}_r] = (1-\alpha_{av})\bar{\dot{W}}_S - \frac{c\alpha_{av}S}{4}\bar{w}_r \qquad 1)$$

or, using the energy density of the whole acoustic field (d+r), $\langle\bar{w}\rangle_V$

$$V\frac{d\langle\bar{w}\rangle_V}{dt} = \bar{\dot{W}}_S - \frac{c\alpha_{av}S}{4}\langle\bar{w}\rangle_V \qquad 2)$$

- **Steady-state solutions**
  - From 1)

$$\bar{w}_r = \frac{4\bar{\dot{W}}_S}{c\alpha_{av}S/(1-\alpha_{av})} = \frac{4\bar{\dot{W}}_S}{cR} \qquad with \quad R = \frac{\alpha_{av}S}{1-\alpha_{av}}$$

Room constant

  - From 2)

$$\langle\bar{w}\rangle_V = \frac{4\bar{\dot{W}}_S}{c\alpha_{av}S} = \frac{4\bar{\dot{W}}_S}{cA} \qquad with \quad A = \alpha_{av}S$$

equivalent sound absorption area

Room Acoustic Energy Density

Under **diffuse field hypothesis** and **Steady-State** condition

Reverberant field ONLY energy density

$$\bar{w}_r = \frac{4\bar{\dot{W}}_S}{cR} \qquad with \quad R = \frac{\alpha_{av}S}{1-\alpha_{av}}$$

Room constant

Whole direct + reverberant field energy density

$$\langle\bar{w}\rangle_V = \frac{4\bar{\dot{W}}_S}{cA} \qquad with \quad A = \alpha_{av}S$$

equivalent sound absorption area

If a **point source** of sound power $\bar{\dot{W}}_S$ and directivity Q is considered, the direct field energy density can be expressed through the *isotropic spherical far field energy density* as

$$\bar{w}_d(r,\theta,\psi) = Q(\theta,\psi)\frac{\bar{\dot{W}}_S}{c4\pi r^2}$$

Direct field energy density

and the whole energy density spatial distribution can be seen as the sum of the direct field energy density distribution and the uniform reverberant (diffuse) field distribution.

Room Acoustic Energy Density Distribution

Under **diffuse field hypothesis** and **Steady-State** condition, the energy density space distribution in a close reverberant space is given by:

$$\bar{w}_{SS}(r,\theta,\psi) = \bar{w}_d(r,\theta,\psi) + \bar{w}_r$$

Thus

$$\boxed{\bar{w}_{SS}(r,\theta,\psi) =} Q(\theta,\psi)\frac{\bar{\dot{W}}_S}{c4\pi r^2} + \frac{4\bar{\dot{W}}_S}{cR} = \boxed{\frac{\bar{\dot{W}}_S}{c}\left[\frac{Q(\theta,\psi)}{c4\pi r^2} + \frac{4}{R}\right]}$$

- **Sound Decay Solutions from Steady-State condition**

At time $t = t_0 - t_m$ the sound source is switched off ➔ $\bar{\dot{W}}_S = 0$

At time $t = t_0$ the energy density due to direct sound field $\langle \bar{w}_d \rangle_V$ is null and the reverberant field energy density $\bar{w}_r$ starts to decay according to

equivalent sound absorption area

$$\boxed{\frac{d\bar{w}_r(t)}{dt} = -\frac{cA}{4V}\bar{w}_r(\mathrm{t})}$$

Sabine's Continuity Hypothesis

Sabine's Reverberation Time $T_{60,\omega}$

From

$$\frac{d\bar{w}_r(t)}{dt} = -\frac{cA}{4V}\bar{w}_r(t)$$

Sabine's Continuity Hypothesis

integrating

$$\Rightarrow \quad \bar{w}_r(t) = \bar{w}_r(t_0)e^{-\frac{cA}{4V}(t-t_0)}$$

In terms of levels:

$$10\log_{10}\frac{\bar{w}_r(t)}{\bar{w}_r(t_0)} = 10\log_{10}\frac{\Delta p^2_{RMS}(t)}{\Delta p^2_{RMS}(t_0)} = L_p(t) - L_P(t_0) = 10\log_{10} e^{-\frac{cA}{4V}(t-t_0)}$$

fixing

$$T_{60,\omega} = t - t_0 \quad : \quad L_{p,\omega}(t) - L_{p,\omega}(t_0) = -60\ dB$$

it follows

$$-60 = 10\log_{10} e^{-\frac{cA_\omega}{4V}T_{60,\omega}} = \frac{10}{\ln 10}\ln e^{-\frac{cA_\omega}{4V}T_{60,\omega}} = -\frac{10}{\ln 10}\frac{cA_\omega}{4V}T_{60,\omega}$$

Sabine's Reverberation Time

$$\boxed{T_{60,\omega} = \frac{24\ln 10}{c}\frac{V}{A} = 0.16\frac{V}{A}}$$

Eyring's Reverberation Time $T_{60,\omega}$

Sabine's Reverberation Time formula, $T_{60,\omega} = 0.16\,V/A$, fails for poorly reverberant or very large rooms; specifically

$$\lim_{\alpha_{av}\to 1} T_{60,\omega} = 0.16\,V/S > 0$$

while it should be zero (*anechoic chamber*).

This is due to that the Sabine's hypothesis of continuous decay is not correct:

- statistically a sound wave travels for t_m before to hit a surface and loose some energy: it is a step-wise process.

At $t = t_0$ $\quad \bar{w}_r(t) = \bar{w}_r(t_0) = \bar{w}_{r,SS} = 4\bar{\bar{W}}_S/cR$

$t = t_0 + t_m$ $\quad \bar{w}_r(t) = \bar{w}_{r,SS}(1-\alpha_{av})$

$t = t_0 + 2t_m$ $\quad \bar{w}_r(t) = \bar{w}_{r,SS}(1-\alpha_{av})^2$

…… $\quad$ ….. ……..

$t = t_0 + nt_m$ $\quad \bar{w}_r(t) = \bar{w}_{r,SS}(1-\alpha_{av})^n$

Eyring's Reverberation Time $T_{60,\omega}$

Thus for $t = t_0 + nt_m$

$$\bar{w}_r(t) = \bar{w}_r(t_0)(1-\alpha_{av})^n$$

In terms of levels:

$$10\log_{10}\frac{\bar{w}_r(t)}{\bar{w}_r(t_0)} = 10\log_{10}\frac{\Delta p^2_{RMS}(t)}{\Delta p^2_{RMS}(t_0)} = L_p(t) - L_P(t_0) = 10\log_{10}(1-\alpha_{av})^n$$

Fixing

$$T_{60,\omega} = t - t_0 \;:\; L_{p,\omega}(t) - L_{p,\omega}(t_0) = -60\ dB$$

it follows, for $n = (t-t_0)/t_m = T_{60,\omega}/t_m$ and $t_m = 4V/cS$

$$-60 = 10\log_{10}(1-\alpha_{av})^n = \frac{10}{\ln 10}n\ln(1-\alpha_{av}) = \frac{10}{\ln 10}\frac{cS}{4V}\ln(1-\alpha_{av})\,T_{60,\omega}$$

Eyring's Reverberation Time

$$\boxed{T_{60,\omega}} = \frac{24\ln 10}{c}\frac{V}{-S\ln(1-\alpha_{av,\omega})} = \boxed{0.16\frac{V}{S\ln|1-\alpha_{av,\omega}|}}$$

Eyring's versus Sabine's Reverberation Time

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Eyring's Reverberation Time

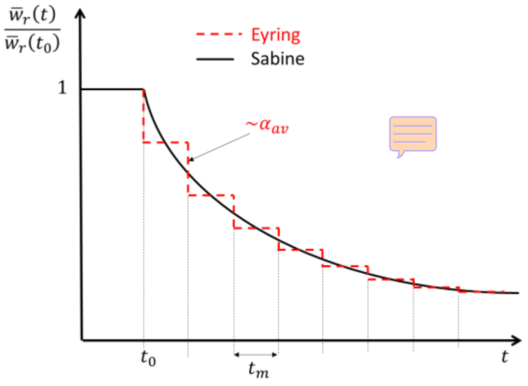
$$T_{60,\omega} = 0.16\frac{V}{S\ln|1-\alpha_{av,\omega}|}$$

$$\lim_{\alpha_{av}\to 1} T_{60,\omega} = 0$$

Sabine's Reverberation Time

$$T_{60,\omega} = 0.16\frac{V}{A_\omega}$$

$$\lim_{\alpha_{av}\to 1} T_{60,\omega} = 0.16\,V/S > 0$$



The real energy density decay is a "*staircase*" or "*step*" function. The step depth is the mean time between two reflections t_m, the step height is proportional to the average absorption coefficient α_{av}.

When t_m and α_{av} are very small (small, highly reflecting room) the step function is very close to the continuous decay curve: $T_{60,Sabine} \approx T_{60,Eyring}$