Homework 2: Soundboard modeling and string coupling

It is given a rectangular aluminium plate of dimensions 1m x1.4 m with clamped, free and supported edges. The plate is characterized by the following characteristics

- thickness: 3 mm.
- E=69 x10^9 N/m^2
- Poisson ratio: 0.35.
- Density 2650 kg/m^3

Question 1: find the modal frequencies of the lowest five modes under the assumption that the magnitude of the displacement of the vibration is much smaller than the thickness.
Question 2: Under the same assumptions of question 1, compute the frequency separation between adjacent modes for all the three boundary conditions, and make considerations on the modal density.
Question 3: Plot the frequency of the lowest five modes as a function of time when the initial amplitude of the vibration is 0.4 mm and all the three modes are characterized by a decay time of 1ms.

In the following, consider that the plate is used as the soundboard of an instrument, and the plate in this use is simply supported.
A doublet of identical strings is attached to the soundboard. The two strings are deputed to the production of the C_3 tone (f_0 = 130.8 Hz). The length of the strings is 1.007 m. The weight per unit length of the strings is 12 g/m. One of the strings is tuned to produce f_0 (i.e. $\omega_1 = 2\pi f_0$), while the second is detuned and $\omega_1 = 2\pi f_0(1+2\varepsilon)$.

Question 4: find the location(s) on the soundboard (x_{min}, y_{min}) where the impedance is an absolute minimum for the considered frequency f_0.
Question 5: Compute the eigenfrequencies of the two strings when they are mounted on the soundboard, through a rigid bridge, at the locations (x_{min}, y_{min}) computed above.
Question 6: compute the decay time for the two eigenfrequencies.

Difficulty coefficient for this homework: 3.0