



**POLITECNICO**  
MILANO 1863

MSC. MUSIC AND ACOUSTIC ENGINEERING

MUSICAL ACOUSTICS - A.Y. 2020/2021

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## **H7 - Eigenfrequency Study of a Marimba Bar**

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# 1 Constructing the mesh

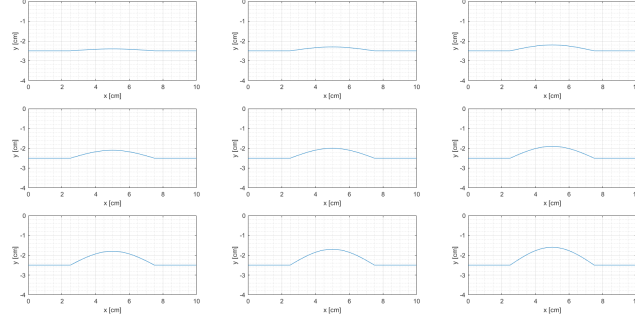


Figure 1: Profiles of the lower surface of a marimba bar with the depth of the arching varying from 1 to 9 mm.

Our aim in this homework is to study the dependency of the modes of a marimba bar on the depth  $a$  of the arching. Fig. 1 shows how we modelled the profiles: the cutaway is sinusoidal in shape and varies from 1 to 9 mm in depth in steps of 1 mm. The model for the bar was implemented in Comsol with a length of 10 cm, a height of 2.5 cm and a width of 2.5 cm. The depth  $a$  was defined as a global parameter in order to perform a parametric study of the eigenfrequencies. The values of the Young moduli, shear moduli and Poisson coefficients were taken from [1], considering Honduran mahogany as our material of choice. Ideally, the most sought after instruments of this kind are made of rosewood; however, for this material, it is harder to find measurements of all of the nine relevant parameters that are needed to completely characterize it. Fig. 2 shows the profiles of the generated meshes.

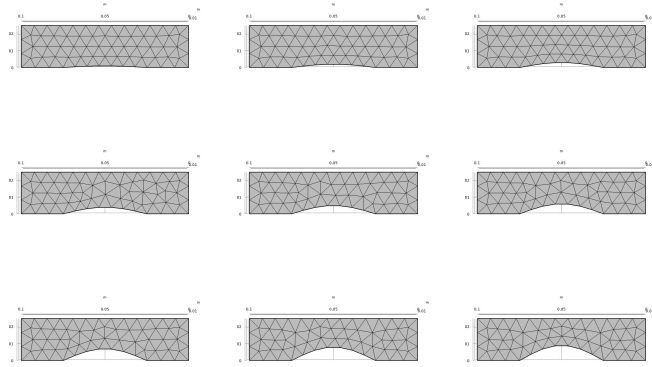


Figure 2: Profiles of the Comsol mesh for all values of  $a$  used in the parametric study.

## 2 Eigenfrequencies

As mentioned in the previous section, a parametric eigenfrequency study was performed, with the values we already reported for  $a$  and with free boundary conditions. The first five eigenfrequencies obtained from this study for each value of  $a$  are reported in Tab. 1. They are also reported as functions of  $a$  in Fig. 3.

| $a$ [cm]    | 0.1     | 0.2     | 0.3     | 0.4     | 0.5     | 0.6     | 0.7     | 0.8     | 0.9     |
|-------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| $f_1$ [kHz] | 5.9071  | 5.7717  | 5.6294  | 5.4788  | 5.3202  | 5.1534  | 4.9770  | 4.7969  | 4.6076  |
| $f_2$ [kHz] | 8.1063  | 8.0827  | 7.8760  | 7.5803  | 7.2655  | 6.9319  | 6.5852  | 6.2287  | 5.8557  |
| $f_3$ [kHz] | 8.4015  | 8.1523  | 8.0464  | 8.0000  | 7.9430  | 7.8755  | 7.7968  | 7.7086  | 7.6096  |
| $f_4$ [kHz] | 12.2164 | 12.3037 | 12.3886 | 12.4762 | 12.5592 | 12.6413 | 12.7214 | 12.7876 | 12.5428 |
| $f_5$ [kHz] | 14.0013 | 13.8828 | 13.7447 | 13.5885 | 13.4160 | 13.2263 | 13.0134 | 12.7992 | 12.8797 |

Table 1: First five eigenfrequencies of the bar for each value of  $a$ .

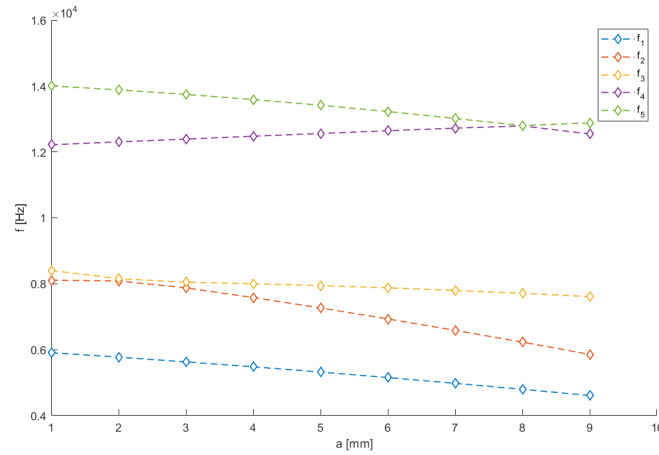


Figure 3: The five lowest eigenfrequencies plotted as functions of  $a$ .

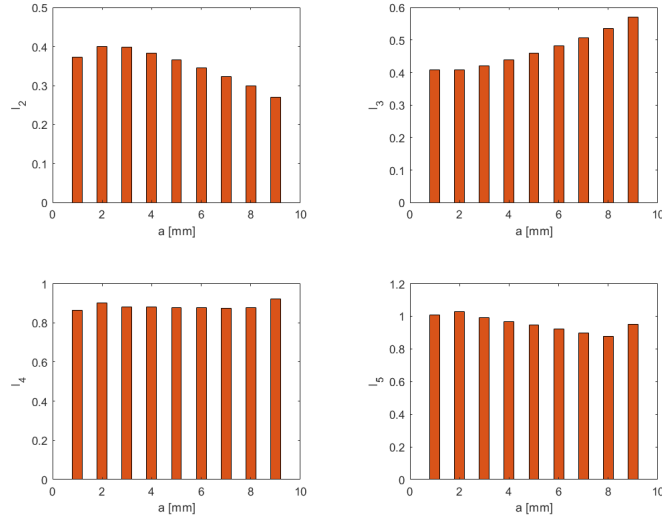
## 3 Inharmonicity

Lastly, we compute the inharmonicity of the sets of eigenfrequencies obtained before. The inharmonicity is a descriptor defined as:

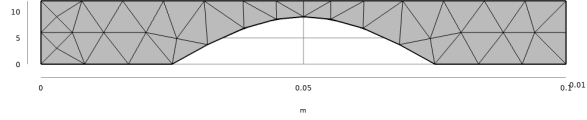
$$I = \sum_{n=2}^N \left| \frac{f_n}{f_{n-1}} - m_n \right|,$$

$$m_n = \arg \min_m |f_n - m f_{n-1}|, \quad m \in \mathbb{N}^+,$$

where in our case we take all value of  $N$  from 2 to 5. The values of  $I$  we computed are reported in Fig. 4.

Figure 4: Inharmonicity  $I_N$  as a function of  $a$ .

## 4 A better shape of the bar

Figure 5: Profile of the alternate mesh (with  $a = 9$  mm).

Looking at the values we obtained from the eigenfrequency study, a problem seems to arise. While we expect the values of the ratios between the frequencies of the first two bending modes is  $\sim 4$ , the actual ratio is much lower. We tried therefore to alter the shape of the bar to see if we could obtain better results. We ran a parametric eigenfrequency study like before, but with an height of 1.2 cm. The new shape of the bar can be seen in Fig. 5, while the results are reported in Tab. 2 and Fig. ??.

| $a$ [cm]    | 0.1     | 0.2     | 0.3     | 0.4     | 0.5     | 0.6     | 0.7     | 0.8     | 0.9     |
|-------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| $f_1$ [kHz] | 4.9342  | 4.5558  | 4.1467  | 3.7147  | 3.2757  | 2.8217  | 2.3750  | 1.8859  | 1.3926  |
| $f_2$ [kHz] | 11.7320 | 11.3950 | 10.9690 | 10.4580 | 9.8794  | 9.2027  | 8.4219  | 7.5093  | 6.4632  |
| $f_3$ [kHz] | 19.2460 | 19.0400 | 18.7580 | 18.3670 | 17.8910 | 17.2240 | 16.357  | 15.1960 | 13.7910 |
| $f_4$ [kHz] | 26.6820 | 26.5320 | 26.3570 | 26.1740 | 26.0210 | 25.8050 | 25.6420 | 24.4050 | 23.6590 |
| $f_5$ [kHz] | 34.0560 | 33.8520 | 33.5890 | 33.3040 | 33.0770 | 32.7960 | 32.4080 | 31.8430 | 31.0930 |

Table 2: Frequencies of the first five **bending** modes of the thinner bar. Notice how, given the relatively high frequency of the fundamental, we are reaching way beyond the range of audible frequencies.