MSc. Music and Acoustic Engineering

Musical Acoustics - A.Y. 2020/2021

H7 - Eigenfrequency Study of a Marimba Bar

Authors' IDs:
10743504, 10751919,

January 21, 2021

1 Constructing the mesh

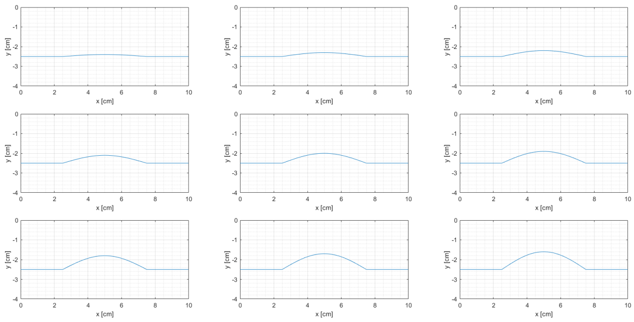

Figure 1: Profiles of the lower surface of a marimba bar with the depth of the arching varying from 1 to 9 mm.

Our aim in this homework is to study the dependency of the modes of a marimba bar on the depth a of the arching. Fig. 1 shows how we modelled the profiles: the cutaway is sinusoidal in shape and varies from 1 to 9 mm in depth in steps of 1 mm. The model for the bar was implemented in Comsol with a length of 10 cm, a height of 2.5 cm and a width of 2.5 cm. The depth a was defined as a global parameter in order to perform a parametric study of the eigenfrequencies. The values of the Young moduli, shear moduli and Poisson coefficients were taken from [1], considering Honduran mahogany as our material of choice. Ideally, the most sought after instruments of this kind are made of rosewood; however, for this material, it is harder to find measurements of all of the nine relevant parameters that are needed to completely characterize it. Fig. 2 shows the profiles of the generated meshes.

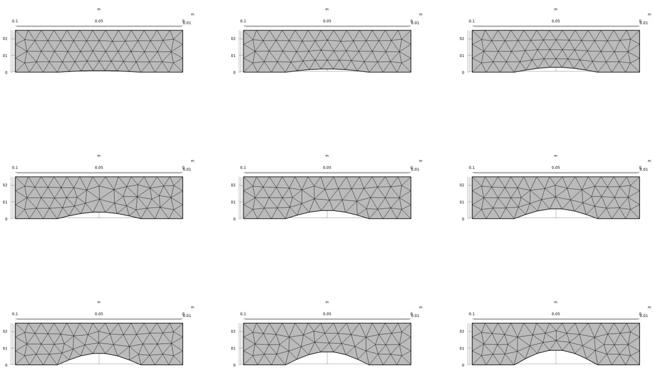

Figure 2: Profiles of the Comsol mesh for all values of a used in the parametric study.

2 Eigenfrequencies

As mentioned in the previous section, a parametric eigenfrequency study was performed, with the values we already reported for a and with free boundary conditions. The first five eigenfrequencies obtained from this study for each value of a are reported in Tab. 1. They are also reported as functions of a in Fig. 3.

a [cm]	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
f_1[kHz]	5.9071	5.7717	5.6294	5.4788	5.3202	5.1534	4.9770	4.7969	4.6076
f_2[kHz]	8.1063	8.0827	7.8760	7.5803	7.2655	6.9319	6.5852	6.2287	5.8557
f_3[kHz]	8.4015	8.1523	8.0464	8.0000	7.9430	7.8755	7.7968	7.7086	7.6096
f_4[kHz]	12.2164	12.3037	12.3886	12.4762	12.5592	12.6413	12.7214	12.7876	12.5428
f_5[kHz]	14.0013	13.8828	13.7447	13.5885	13.4160	13.2263	13.0134	12.7992	12.8797

Table 1: First five eigenfrequencies of the bar for each value of a.

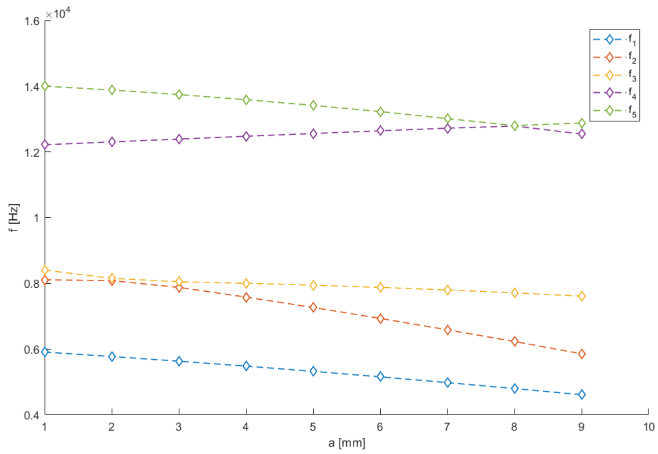

Figure 3: The five lowest eigenfrequencies plotted as functions of a.

3 Inharmonicity

Lastly, we compute the inharmonicity of the sets of eigenfrequencies obtained before. The inharmonicity is a descriptor defined as:

$$I = \sum_{n=2}^{N} \left| \frac{f_n}{f_{n-1}} - m_n \right| ,$$
$$m_n = \arg\min_m |f_n - m f_{n-1}| \, , \; m \in \mathbb{N}^+ ,$$

where in our case we take all value of N from 2 to 5. The values of I we computed are reported in Fig. 4.

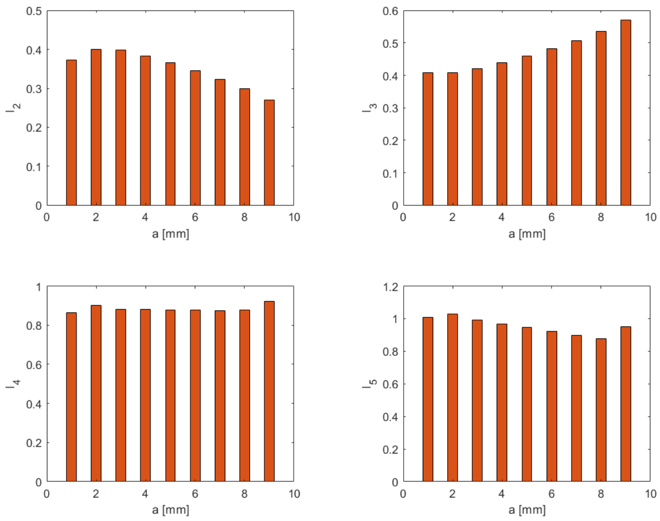

Figure 4: Inharmonicity I_N as a function of a.

4 A better shape of the bar

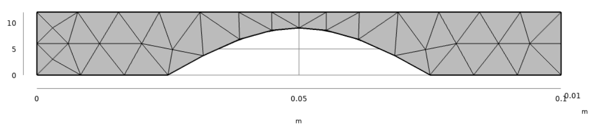

Figure 5: Profile of the alternate mesh (with $a = 9$ mm).

Looking at the values we obtained from the eigenfrequency study, a problem seems to arise. While we expect the values of the ratios between the frequencies of the first two bending modes[1] is ~ 4, the actual ratio is much lower. We tried therefore to alter the shape of the bar to see if we could obtain better results. We ran a parametric eigenfrequency study like before, but with an height of 1.2 cm. The new shape of the bar can be seen in Fig. 5, while the results are reported in Tab. 2 and Fig. 6.

In Tab. 3 we report the ratios between the frequencies of the first two bending modes for both bars. Notice how, while for the original shape we never get much past 2, for the thinner bar we get the desired value at $a = 8$ mm.

[1] Focusing on bending modes makes sense as they contribute the most to the radiation from the bar. Torsional modes might contribute too if the bar isn't struck on its symmetry axis, but this is usually avoided as they aren't tuned to the bending modes.

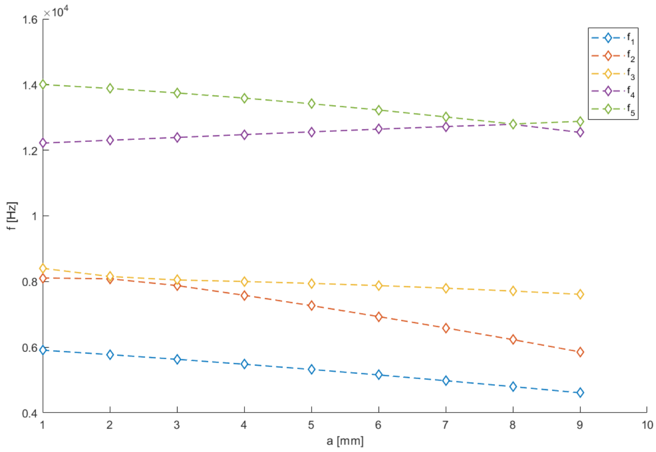

Figure 6: Frequencies of the five lowest **bending** modes of the thin bar plotted as functions of a.

a [cm]	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
f_1[kHz]	4.9342	4.5558	4.1467	3.7147	3.2757	2.8217	2.3750	1.8859	1.3926
f_2[kHz]	11.7320	11.3950	10.9690	10.4580	9.8794	9.2027	8.4219	7.5093	6.4632
f_3[kHz]	19.2460	19.0400	18.7580	18.3670	17.8910	17.2240	16.357	15.1960	13.7910
f_4[kHz]	26.6820	26.5320	26.3570	26.1740	26.0210	25.8050	25.6420	24.4050	23.6590
f_5[kHz]	34.0560	33.8520	33.5890	33.3040	33.0770	32.7960	32.4080	31.8430	31.0930

Table 2: Frequencies of the first five **bending** modes of the thinner bar. Notice how, given the relatively high frequency of the fundamental, we are reaching far beyond the range of audible frequencies.

We can now compute the inharmonicities of the bending modes of the thin bar. The results can be seen in Fig. 7. It must be remarked, however, that only the first two graphs (I_2 and I_3) give us a measure of the perceived harmonicity of the sound, as the last two frequencies are well into ultrasound territory. Once again, $a = 8$ mm seems to be the sweet spot, with the related inharmonicities being the smallest.

a [cm]	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
f_2/f_1	1.84	1.88	1.93	1.97	2.02	2.08	2.14	2.20	2.27
f_2/f_1 - thin bar	2.38	2.50	2.66	2.82	3.02	3.26	3.55	3.98	4.64

Table 3: Ratios of the frequencies of the first two bending modes for both bars.

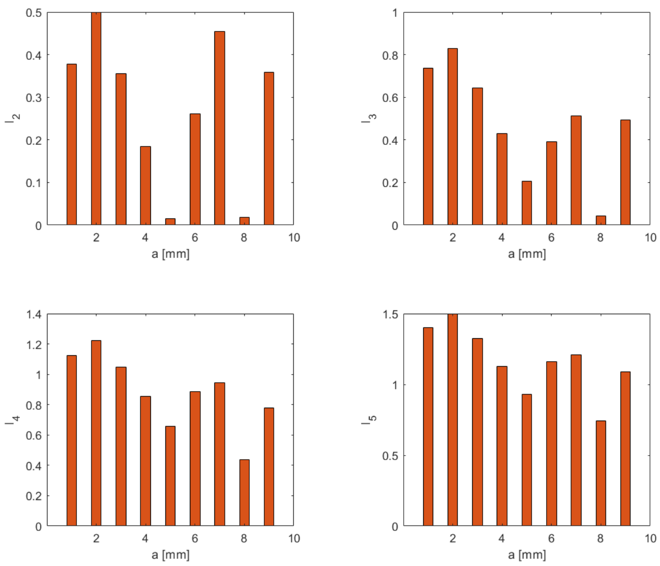

Figure 7: Inharmonicity I_N as a function of a, for the thin bar.