POLITECNICO
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MSc. Music and Acoustic Engineering

Musical Acoustics - A.Y. 2020/2021

H5 - Synthesis of the guitar sound

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1 Bridge impedance

The bridge impedance is computed as the ratio between the pressure on the top plate (the force applied to the top plate divided by the effective top plate area) and the volume velocity of the plate itself. The guitar body is modelled as a two-mass system, neglecting the back plate and the ribs, and each element is assumed to be lumped, a reasonable assumption considering that we are interested in the frequencies below 500 Hz. The elements in question are the top plate, the air cavity and the sound hole, and their impedances are:

$$Z_p = \mathrm{j}\omega M_p + R_p + \frac{1}{\mathrm{j}\omega C_p}, \qquad \text{where } M_p = \frac{m_p}{A_p^2},\ C_p = \frac{A_p^2}{K_p}$$

$$Z_v = \frac{1}{\mathrm{j}\omega C_v} + R_v, \qquad \text{where } C_v = \frac{V}{\rho c^2}$$

$$Z_h = \mathrm{j}\omega M_h + R_h \qquad \text{where } M_h = \frac{m_h}{A_h^2}.$$

The system can be thought of as composed of two coupled oscillators: one is the top plate, while the other is the cavity/sound hole resonator. The resulting equivalent circuit can be seen in Fig. 1.

The input impedance of this circuit is the series of Z_p with the parallel of Z_v and Z_h

$$Z = Z_p + \frac{Z_v Z_h}{Z_v + Z_h}.$$

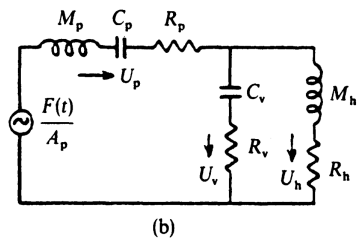

Figure 1: Two-mass model of the guitar body.

The magnitude and phase of Z as functions of the frequency are shown in Fig. 2. The magnitude graph shows two points of resonance with one anti-resonance in between, as expected. The antiresonance corresponds to the Helmholtz resonance frequency f_h of the cavity/sound hole subsystem (the A_0 mode of the cavity). The highest resonance occurs at the resonance frequency f_p of the system without the sound hole. This tells us that the effects of the losses due to the resistances are very small, since these frequencies are computed for the conservative case.

C_p	$1.48 \times 10^{-9}\,\mathrm{N/m^5}$
M_p	$236.4\,\mathrm{kg\,m^{-2}}$
R_p	$32.0\,\mathrm{Ns/m^5}$
C_v	$1.22 \times 10^{-7}\,\mathrm{N/m^5}$
R_v	0
M_h	$13.05\,\mathrm{kg\,m^{-2}}$
R_h	$30.0\,\mathrm{Ns/m^5}$

Table 1: Values of the compliances, inertances and resistances of the system.

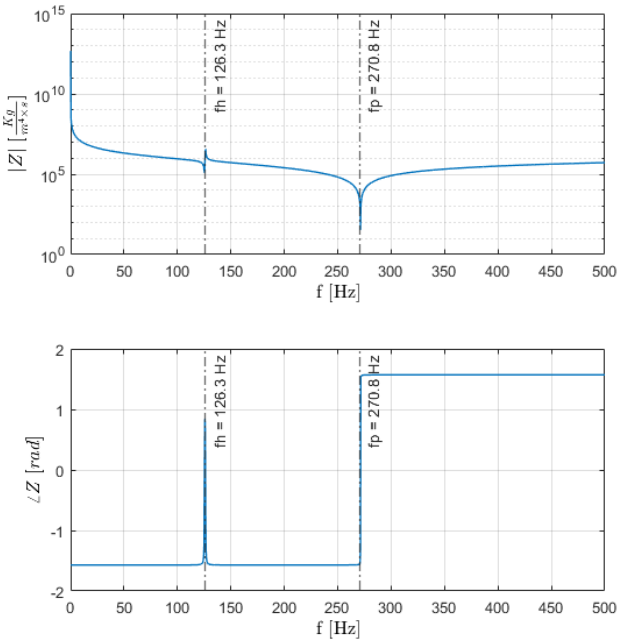

Figure 2: Bridge impedance as a function of the frequency. The values of f_h and f_p are highlighted.

2 Transfer function from the plucking point to the bridge

A guitar string can be modelled as two transmission lines in opposite directions with two reflection filters at the end points (see Fig. 3), with the input being given at a certain fraction β of the length. This model can be implemented easily with digital waveguides, as is done in most synthesis applications. However, if the model is an LTI system, the various components can be commuted to obtain a single-delay loop (SDL), resulting in an increase in efficiency, which is a desirable feature in real time synthesis[**?**]. When we use acceleration as a wave variable, the transfer function of the system is:

$$H_{EB}(s) = \frac{1}{2}\left[1 + H_{E2R1}(s)\right]\frac{H_{E1R1}(s)}{1 - H_{loop}(s)}Z(s)\frac{1}{s}\left[1 - R_b(s)\right] =$$
$$= H_E(s)H_{E1R1}(s)S(s)H_B(s)$$

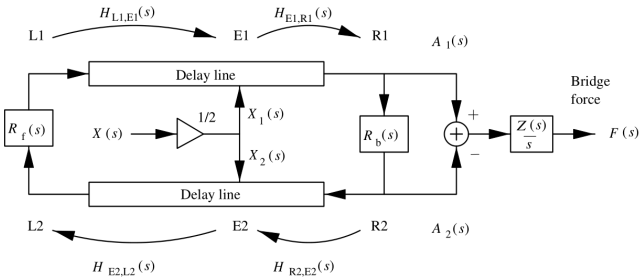

Figure 3: Model of the bidirectional waveguide model coupled with the bridge impedance $Z(s)$.