$$h(t) = \begin{cases} 1 & , \quad 0 < t < 2 \\ 0 & , \quad \text{ow} \end{cases}$$

$$H(j\omega) = \int_{-\infty}^{+\infty} h(t) e^{-j\omega t} dt =$$

$$+ \int_{0}^{2} e^{-j\omega t} dt$$

$$= \frac{e^{-j\omega t}}{-j\omega} \Big|_{0}^{2}$$

$$= \left(\frac{e^{-j\omega(2)}}{-j\omega} + \frac{1}{j\omega} \right)$$

$X_1(j\omega) = 2\pi \delta(\omega - \omega_0) \longleftarrow$ تبدیل فوریه سیگنال $x_1(t)$ با ω_0

$X_2(j\omega) = \pi(\delta(\omega-\omega_0) + \delta(\omega+\omega_0)) \longleftarrow$ $x_2(t)$ " "

$Y_1(j\omega) = X_1(j\omega) . H(j\omega) \longleftarrow$ پاسخ فرکانسی سیستم به $x_1(t)$

$Y_2(j\omega) = X_2(j\omega) . H(j\omega) \longleftarrow$ $x_2(t)$ " " "

روش دوم : با جابجایی زمانی

$$h(t+1) = \begin{cases} 1 & , \quad |t|<1 \\ 0 & , \quad |t|>1 \end{cases} = h_1(t)$$

$$H_1(j\omega) = \frac{2\sin(\omega)}{\omega} = e^{-j\omega(-1)} H(j\omega)$$

$$\rightarrow H(j\omega) = e^{-j\omega} \times \frac{2\sin(\omega)}{\omega}$$

$$= e^{-j\omega} \times \frac{e^{j\omega} - e^{-j\omega}}{j\omega} = \frac{1 - e^{-j\omega 2}}{j\omega}$$

بقیه مراحل مشابه است.

$$Y_1(j\omega) = 2\pi\,\delta(\omega-\omega_0)\left(\frac{1-e^{-j\omega 2}}{j\omega}\right)$$

$$= \begin{cases} \dfrac{2\pi - 2\pi e^{-j\omega 2}}{j\omega} & \omega = \omega_0 \\ 0 & \omega \neq \omega_0 \end{cases}$$

$$= \underbrace{\frac{2\pi}{j\omega_0}}_{\pi\,\mathrm{sgn}(t)} - \underbrace{\frac{2\pi e^{-j\omega_0 2}}{j\omega_0}}_{\pi\,\mathrm{sgn}(t-2)}$$

$$\rightarrow \quad y_1(t) = F^{-1}\{Y_1(j\omega)\} = \pi\left(\mathrm{sgn}(t) - \mathrm{sgn}(t-2)\right)$$

$$Y_2(j\omega) = \pi\left(\delta(\omega-\omega_0) - \delta(\omega+\omega_0)\right)\left(\frac{1-e^{-j\omega 2}}{j\omega}\right)$$

$$= \begin{cases} +\dfrac{\pi}{j\omega} - \dfrac{\pi e^{-j\omega 2}}{j\omega} & , \quad \omega = \omega_0 \\[2ex] -\dfrac{\pi}{j\omega} + \dfrac{\pi e^{-j\omega 2}}{j\omega} & , \quad \omega = -\omega_0 \\[2ex] 0 & , \quad \omega \neq \omega_0 \ \& -\omega_0 \end{cases}$$

با هم برابرند

$$y_2(t) = F^{-1}\{Y_2(j\omega)\} = \frac{\pi\,\mathrm{sgn}(t)}{2} - \frac{\pi\,\mathrm{sgn}(t-2)}{2}$$

۲

$$H(j\omega) = \int_{-\infty}^{+\infty} h(t)\, e^{-j\omega t}\, dt = \int_{-\infty}^{+\infty} ۳e^{-۱۰t}\, u(t)\, e^{-j\omega t}\, dt$$

$$= ۳\int_{۰}^{+\infty} e^{(-۱۰-j\omega)t}\, dt = ۳\, \frac{e^{(-۱۰-j\omega)t}}{-۱۰-j\omega}\Bigg|_{۰}^{+\infty}$$

$$= ۳\left(\lim_{t\to+\infty} \frac{e^{(-۱۰-j\omega)t}}{-۱۰-j\omega} - \frac{۱}{-۱۰-j\omega}\right) = \frac{-۳}{-۱۰-j\omega}$$

$$X(j\omega) \xrightarrow{\text{طبق روابط}} \begin{matrix} \omega_۰ = ۵ \\ \text{ضریب} = ۳ \end{matrix} = ۳\pi\left(\delta(\omega-۵) + \delta(\omega+۵)\right)$$

$$Y(j\omega) = X(j\omega) \times H(j\omega)$$

$$Y(j\omega) = ۳\pi\big(\delta(\omega-۵) - \delta(\omega+۵)\big)\left(\frac{-۳}{-۱۰-j\omega}\right) =$$

$$\begin{cases} \dfrac{-۹\pi}{-۱۰-j\omega}, & \omega = ۵ \quad \xleftrightarrow{F^{-1}} \quad +۹\pi e^{-۱۰t} u(t) \\[2ex] \dfrac{-۹\pi}{+۱۰+j\omega}, & \omega = -۵ \quad \times \end{cases}$$

$\omega_0 = ۵$ مال $(۳\cos ۵t)$

$$y(t) = F^{-1}\{Y(j\omega)\} = ۹\pi e^{-۱۰t} u(t)$$

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③

a)

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt = \int_{-\infty}^{+\infty} \delta(t)\, e^{-j\omega t}\, dt =$$

$$e^{-j\omega(0)} = 1$$

b)

$$X(j\omega) = \int_{-\infty}^{+\infty} e^{-\delta t}\, u(t)\, e^{-j\omega t}\, dt = \int_{0}^{+\infty} e^{(-\delta - j\omega)t}\, dt =$$

$$\left.\frac{e^{(-\delta-j\omega)t}}{-\delta-j\omega}\right|_{0}^{+\infty} = \lim_{t\to+\infty} \frac{e^{(-\delta-j\omega)t}}{-\delta-j\omega} - \frac{1}{-\delta-j\omega}$$

$$= \frac{1}{+\delta+j\omega}$$

c)

$$x(t) = e^{-\delta|t|} = \begin{cases} e^{-\delta t}, & t \geq 0 \\ e^{\delta t}, & t < 0 \end{cases}$$

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt = \int_{-\infty}^{0} e^{\delta t} e^{-j\omega t}\, dt + \underbrace{\int_{0}^{+\infty} e^{-\delta t} e^{-j\omega t}\, dt}_{\text{در قسمت b محاسبه شد}}$$

$$= \int_{-\infty}^{0} e^{(\sigma - j\omega)t}\, dt + \frac{1}{-\sigma - j\omega} = \left. \frac{e^{(\sigma - j\omega)t}}{\sigma - j\omega} \right|_{-\infty}^{0} + \frac{1}{-\sigma - j\omega}$$

$$= \frac{1}{\sigma - j\omega} - \lim_{t \to -\infty} \frac{e^{(\sigma - j\omega)t}}{\sigma - j\omega} + \frac{1}{\sigma + j\omega} =$$

$$\frac{1}{\sigma - j\omega} + 0 + \frac{1}{\sigma + j\omega} = \frac{\sigma + j\omega + \sigma - j\omega}{\sigma^2 - j^2\omega^2} = \frac{2\sigma}{\sigma^2 + \omega^2}$$

د) ابتدا از تابع مشتق بگیریم به شکل تابع

$$\frac{dx(t)}{dt} = \begin{cases} 1 , & |t| < 1 \\ 0 , & |t| > 1 \end{cases}$$

در می آید ـ طبق خواص تبدیل فوریه

$$j\omega X(j\omega) = F\left\{ \frac{dx(t)}{dt} \right\} = \frac{2\sin(\omega)}{\omega}$$

$$\rightarrow X(j\omega) = \frac{1}{j\omega} \times \frac{2\sin(\omega)}{\omega}$$

۳) فرض برای همه قسمت‌ها

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt$$

a)

$$F\{x(t-t_0)\} = \int_{-\infty}^{+\infty} x(t-t_0)\, e^{-j\omega t}\, dt$$

$$\rightarrow \quad t - t_0 = k \;,\quad dt = dk \;,\quad t = t_0 + k$$

$$= \int_{-\infty}^{+\infty} x(k)\, e^{-j\omega(t_0+k)}\, dk = e^{-j\omega t_0} \int_{-\infty}^{+\infty} x(k)\, e^{-j\omega k}\, dk$$

$$= e^{-j\omega t_0}\, X(j\omega)$$

b)

$$F\{x^*(t)\} = \int_{-\infty}^{+\infty} x^*(t)\, e^{-j\omega t}\, dt = \left(\int_{-\infty}^{+\infty} x(t)\, e^{j\omega t}\, dt\right)^*$$

$$= \left(\int_{-\infty}^{+\infty} x(t)\, e^{-(-j)\omega t}\, dt\right)^* = \left(X(-j\omega)\right)^* = X^*(-j\omega)$$

c)

$$F\{x(at)\} = \int_{-\infty}^{+\infty} x(at)\, e^{-j\omega t}\, dt$$

$$\rightarrow \quad at = k \;,\quad t = \frac{k}{a} \;,\quad dt = \frac{dk}{a}$$

$$= \int_{-\infty}^{+\infty} x(k)\, e^{-j\omega \frac{k}{a}} \frac{1}{a}\, dk = \frac{1}{a} \int_{-\infty}^{+\infty} x(k)\, e^{-\left(\frac{j\omega}{a}\right)k}\, dk$$

$$= \frac{1}{a} X\left(\frac{j\omega}{a}\right)$$

د) طبق فرض

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(\omega)\, e^{j\omega t}\, d\omega$$

$$\rightarrow 2\pi x(t) = \int_{-\infty}^{+\infty} X(\omega)\, e^{j\omega t}\, d\omega$$

$$F \xrightarrow[\omega \leftrightarrow t]{t \leftrightarrow -t} \quad 2\pi x(-t) = \int_{-\infty}^{+\infty} X(\omega)\, e^{-j\omega t}\, d\omega$$

$$\rightarrow 2\pi x(-\omega) = \int_{-\infty}^{+\infty} X(t)\, e^{-j\omega t}\, dt$$

$$\rightarrow X(t) \xleftrightarrow{F} 2\pi x(-\omega)$$