محمدمهدی نظری ۹۹۳۱۰۴۱

①

سیگنال: $x(t) = 2t^2$

سیستم: LTI $\quad h(t) = e^{-t}u(t)$

②

الف) $2e^{j\pi/3} = 2\left(\cos(\pi/3) + j\sin(\pi/3)\right) = 2\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)$

$= \underline{1 + j\sqrt{3}}$

ب) $e^{(j\frac{5\pi}{2}+1)} = e^{j\frac{5\pi}{2}} \times e^{1} = \left(\cos(5\pi/2) + j\sin(5\pi/2)\right)e =$

$(0 + j)e = \underline{je}$

③

الف) $1 - j \longrightarrow A = \sqrt{1^2 + (-1)^2} = \sqrt{2}\ ,\ \omega = \tan^{-1}\left(\frac{-1}{1}\right) = -\pi/4$

$= \underline{\sqrt{2}\, e^{j\,-\pi/4}}$

ب) $(1-j)^2 = 1 + j^2 - 2j = 0 - 2j \longrightarrow A = \sqrt{0^2 + (-2)^2} = 2\ ,$

$\omega = \tan^{-1}(-\infty) = -\pi/2 \quad = \underline{2e^{j\,-\pi/2}}$

ب) $j(1-j) = j - j^2 = +1 + j \longrightarrow A = \sqrt{1^2 + 1^2} = \sqrt{2}\ ,$

$\omega = \tan^{-1}\left(\frac{1}{1}\right) = \pi/4 \quad = \underline{\sqrt{2}\, e^{j\pi/4}}$

ت) $\dfrac{1+j}{1-j}$

$1-j$ و $1+j$ را در مثال های الف و ب حل کردیم

پس نرم قطبی آنها را بهم تقسیم می‌کنیم

$$\frac{1+j}{1-j} = \frac{\sqrt{2}\, e^{j\pi/4}}{\sqrt{2}\, e^{j\,-\pi/4}} = \frac{\sqrt{2}}{\sqrt{2}}\, e^{j(\pi/4-(-\pi/4))} = e^{j\pi/2}$$

روش دوم: تقسیم اعداد مختلط

$$\frac{1+j}{1-j} = \frac{(1+j)(1+j)}{1^2+(-1)^2} = \frac{0+2j}{2} = j \,,$$

$$A = \sqrt{0^2+1^2} = 1 \quad , \quad \omega = \tan^{-1}(\infty) = \pi/2$$

$$= e^{j\pi/2}$$

ه) $\dfrac{\sqrt{2}+\sqrt{2}j}{1+\sqrt{3}j} \longrightarrow$ a) $\sqrt{2}+\sqrt{2}j \;,\; A = \sqrt{(\sqrt{2})^2+(\sqrt{2})^2} = 2$

$$\omega = \tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \pi/4 \qquad = 2e^{j\pi/4}$$

b) $1+\sqrt{3}j \;,\; A = \sqrt{1^2+(\sqrt{3})^2} = 2 \;,\; \omega = \tan^{-1}(\sqrt{3}) = \pi/3$

$$= 2e^{j\pi/3}$$

$$a,b) \Rightarrow \frac{2e^{j\pi/4}}{2e^{j\pi/3}} = \frac{2}{2}\, e^{j(\pi/4-\pi/3)} = e^{j\,-\pi/12}$$

(4)

a) $E_\infty = \int_{-\infty}^{+\infty} |e^{-jt}|^2 \, dt = \int_{-\infty}^{+\infty} \left| \sqrt{\sin^2(-t) + \cos^2(-t)} \right|^2 dt$

$= \int_{-\infty}^{+\infty} 1 \, dt = \underline{+\infty}$

$$P_\infty = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{+T} |e^{-jt}|^2 \, dt = \lim_{T \to \infty} \frac{[x]_{-T}^{T}}{2T}$$

$$= \lim_{T \to \infty} \frac{T - (-T)}{2T} = \lim_{T \to \infty} \frac{2T}{2T} = \lim_{T \to \infty} 1 = \underline{1}$$

b)

$$E_\infty = \sum_{n=-\infty}^{+\infty} |\sin(\tfrac{\pi}{8} n)|^2 = \sum_{n=-\infty}^{+\infty} \sin^2(\tfrac{\pi}{8} n) = \underline{+\infty}$$

$$T = \frac{2k\pi}{\pi/8} = 16k \in \mathbb{Z} \xrightarrow{k=1} N_0 = 16$$

$$P_\infty = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \sin^2(\tfrac{\pi}{8} n) = \lim_{N \to N_0/2} \frac{1}{2N} \sum_{n=-N}^{N-1} \sin^2(\tfrac{\pi}{8} n)$$

$$= \lim_{N \to 8} \frac{1}{2N} \sum_{n=-N}^{N-1} \sin^2(\tfrac{\pi}{8} n) = \frac{1}{16} \times \left(\sum_{n=-8}^{7} \sin^2(\tfrac{\pi}{8} n) \right) \xrightarrow{\text{اتحاد طلایی}}$$

$$= \frac{1}{16} \times \left(\sum_{n=-8}^{7} \frac{1 - \cos(\tfrac{\pi}{4} n)}{2} \right) =$$

$$= \frac{1}{16} \times \left(\sum_{n=-8}^{7} \frac{1}{2} - \frac{1}{2} \sum_{n=-8}^{7} \cos\left(\frac{\pi}{4} n\right) \right) =$$

$$\frac{1}{16} \left(16 \times \frac{1}{2} - \frac{1}{2} \left(1 + \frac{\sqrt{2}}{2} + 0 - \frac{\sqrt{2}}{2} - 1 - \frac{\sqrt{2}}{2} + 0 + \frac{\sqrt{2}}{2} + 1 + \frac{\sqrt{2}}{2} + 0 - \frac{\sqrt{2}}{2} - 1 - \frac{\sqrt{2}}{2} + 0 + \frac{\sqrt{2}}{2} \right) \right)$$

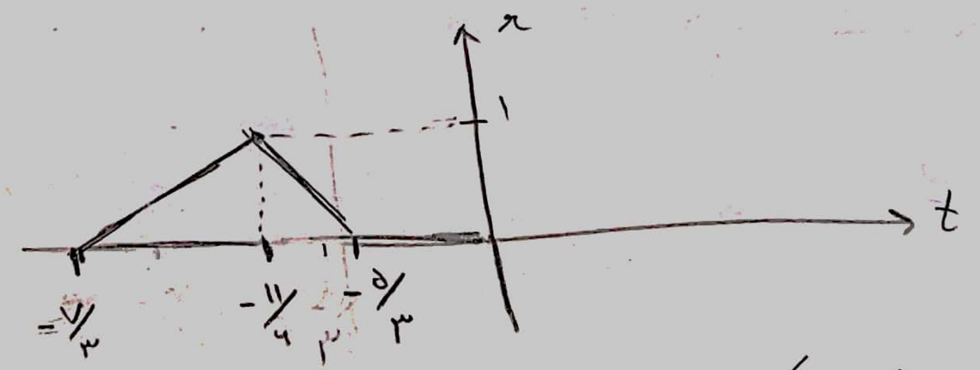
$$\frac{1}{16} (8 - 0) = \underline{\frac{1}{2}}$$

۵

$$t = -1 \rightarrow -\frac{t}{3} - 2 = -\frac{5}{3} \rightarrow x\left(-\frac{5}{3}\right) = 0$$

$$t = -\frac{1}{2} \rightarrow -\frac{t}{3} - 2 = -\frac{11}{6} \rightarrow x\left(-\frac{11}{6}\right) = 1$$

$$t = 1 \rightarrow -\frac{t}{3} - 2 = -\frac{7}{3} \rightarrow x\left(-\frac{7}{3}\right) = 0$$

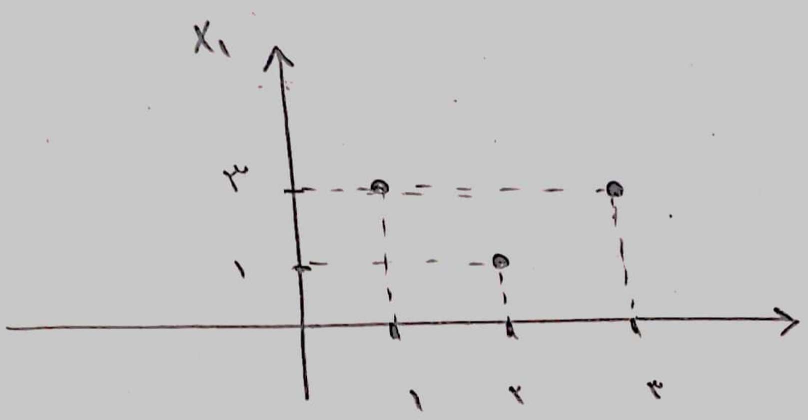


روش دوم: ابتدا t ما را در $-\frac{1}{3}$ ضرب کرده سپس ۲ واحد به چپ انتقال می‌دهم.

$2n-4=-2 \longrightarrow n=1 \longrightarrow x_1[1]=3$

$2n-4=0 \longrightarrow n=2 \longrightarrow x_1[2]=1$

$2n-4=2 \longrightarrow n=3 \longrightarrow x_1[3]=3$



روش دوم: ابتدا ~~x~~ ها را ۴ واحد به راست برده و سپس تقسیم بر ۲ می کنیم