

## Appendix A - Annotated Mplus Output

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An annotated LTA model output estimated using Mplus (version 8.6) is shown below. Mplus input and output syntax is presented within gray colored text boxes. Some of the output has been appended to highlight information most relevant to applied researchers. Annotation is directed towards providing assistance with interpretation and comprehension.

### LTA Model Input

```
TITLE: Full invariance LTA - LSAY

DATA: FILE = "Invariance_LTA_LSA.dat";

VARIABLE: NAMES =
  CASENUM COHORT FEMALE MINORITY
  AB39M AB39T AB39U AB39W AB39X
  MATHG7 MATHG10 MATHG12 OPKNOW OPKNOW2
  SIOK SIOK2 GA33A GA33H GA33I GA33K GA33L
  CPROB11 CPROB12 CPROB13 CPROB14 CMOD1
  CPROB21 CPROB22 CPROB23 CPROB24 CMOD2;

categorical =
  AB39M AB39T AB39U AB39W AB39X    # grade 7 items
  GA33A GA33H GA33I GA33K GA33L;   # grade 10 items

usevar =
  AB39M AB39T AB39U AB39W AB39X
  GA33A GA33H GA33I GA33K GA33L;

missing=all(9999);

classes = c1(4) c2(4);
# Enumeration indicated the four class solution at each time point
# c1 is the latent class variable for 7th grade
# c2 is the latent class variable for 10th grade

ANALYSIS:
  estimator = MLR;
  type = mixture;
  starts = 500 100; # Starting values may be increased to increase
                  # the likelihood of LL replication & solution being the global maximum.

MODEL:
  %overall%
```

```

# Regression of the 10th grade latent class variable, c2, on the 7th grade latent class variable, c1.
c2 on c1;

MODEL c1: # Measurement model for c1, the latent class variable for 7th grade (time point 1).

# To estimate thresholds the 7th grade items are mentioned under each class-specific statement.
%c1#1%
[AB39M$1-AB39X$1] (1-5); # (1-5) is included to label the 5 class-specific item thresholds.
#
%c1#2%
[AB39M$1-AB39X$1] (6-10); # By labeling them and using the same numbers for the first
# class under 'MODEL C2' we are constraining the item
# thresholds to be the same for each class.
%c1#3%
[AB39M$1-AB39X$1] (11-15); # This is how measurement invariance across the LCA models for each
# time point is specified.
%c1#4%
[AB39M$1-AB39X$1] (16-20); #

MODEL c2: # Measurement model for c2, the latent class variable for 10th grade (time point 2)

%c2#1%
[GA33A$1-GA33L$1] (1-5); # Here label numbers are repeated, all parameter labels
# match those above (e.g., (1-5)).
%c2#2%
[GA33A$1-GA33L$1] (6-10); # The symbol '$' is used to refer to binary or ordinal
# variable thresholds.
%c2#3%
[GA33A$1-GA33L$1] (11-15); # Because items in this example are binary, only a single
# threshold is mentioned.
%c2#4%
[GA33A$1-GA33L$1] (16-20); #

```

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## LTA Model Output

Note, due to the estimation of a series of mixture models in LTA re-ordering of the classes occurs. Although the solution is the same, the transition matrix presented in the manuscript may not match the order of the values shown in this output. This is important to check and is part of the process of conducting LTA. To avoid class switching see the Mplus manual for adding the OPTSEED statement.

SUMMARY OF ANALYSIS # The analysis sample size for this model is  $N = 3061$

Number of groups	1
Number of observations	3061
Number of dependent variables	10
Number of independent variables	0
Number of continuous latent variables	0
Number of categorical latent variables	2

## Observed dependent variables

### Binary and ordered categorical (ordinal)

AB39M	AB39T	AB39U	AB39W	AB39X	GA33A
GA33H	GA33I	GA33K	GA33L		

## Categorical latent variables

C1	C2
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## Response Probabilities

Below are the response probabilities and counts for the data. This can be used to check if your data was read in correctly. That is, category 2 reflects  $P(u_j = 1)$  from equation (1) in the main text.

## UNIVARIATE PROPORTIONS AND COUNTS FOR CATEGORICAL VARIABLES

AB39M			
Category 1	0.387	1177.000	
Category 2	0.613	1865.000	
AB39T			
Category 1	0.596	1782.000	
Category 2	0.404	1206.000	
AB39U			
Category 1	0.508	1519.000	
Category 2	0.492	1473.000	
AB39W			
Category 1	0.596	1796.000	
Category 2	0.404	1216.000	
AB39X			
Category 1	0.539	1639.000	
Category 2	0.461	1404.000	
GA33A			
Category 1	0.425	956.000	
Category 2	0.575	1294.000	
GA33H			
Category 1	0.572	1278.000	
Category 2	0.428	957.000	
GA33I			
Category 1	0.491	1097.000	
Category 2	0.509	1137.000	
GA33K			
Category 1	0.580	1297.000	
Category 2	0.420	941.000	
GA33L			
Category 1	0.584	1314.000	
Category 2	0.416	936.000	

## Transition Probabilities ( $\tau$ )

The transition probability matrix describes stability (diagonal of matrix) or change (off-diagonal of matrix) for each class across the two latent class variables. Typically, these parameters are of central interest in an LTA model and are used to determine transition of observations between discrete classes across time-points.

The values in the transition matrix are derived from the multinomial logistic regression of C2 on C1 and the latent class intercepts or the class size logit estimates (e.g., [C1#1]). The value in the upper-left of the matrix is interpreted as: 56% of those in Class 1 in 7th grade remained in Class 1 in 10th grade. The value in the top row column 2 is interpreted as: 9.1% of those in Class 1 in 7th grade transitioned to Class 2 in 10th grade. Note that the multinomial logistic regression estimates are presented farther down in the output.

#### LATENT TRANSITION PROBABILITIES BASED ON THE ESTIMATED MODEL

C1 Classes (Rows) by C2 Classes (Columns)

	1	2	3	4
1	0.557	0.091	0.157	0.194
2	0.266	0.264	0.151	0.319
3	0.352	0.082	0.302	0.264
4	0.215	0.144	0.117	0.524

#### Class-specific Item Threshold ( $\omega_{rk}$ )

This section presents the class-specific item threshold parameters which are estimated in the logit scale. These are often converted to probabilities. Note that since we assume measurement invariance in this example, the thresholds for consistent patterns (e.g., C1#1) the thresholds are constrained to be the equal. Logits estimated close to the extreme ends of the logit scale are fixed by Mplus to 15 and -15 to avoid estimation complications.

#### MODEL RESULTS

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Latent Class C1#1				
Thresholds				
AB39M\$1	0.956	0.100	9.574	0.000
AB39T\$1	3.735	1.010	3.697	0.000
AB39U\$1	3.352	0.935	3.584	0.000
AB39W\$1	2.805	0.172	16.301	0.000
AB39X\$1	4.347	0.782	5.562	0.000
Latent Class C1#2				
Thresholds				
AB39M\$1	-0.724	0.102	-7.086	0.000
AB39T\$1	0.815	0.147	5.536	0.000
AB39U\$1	0.268	0.147	1.825	0.068
AB39W\$1	0.280	0.172	1.635	0.102
AB39X\$1	-0.833	0.572	-1.457	0.145
Latent Class C1#3				
Thresholds				
AB39M\$1	-0.304	0.158	-1.923	0.055
AB39T\$1	0.421	0.234	1.803	0.071

AB39U\$1	-0.890	0.776	-1.146	0.252
AB39W\$1	1.580	0.288	5.485	0.000
AB39X\$1	15.000	0.000	99.000	999.000

Latent Class C1#4

Thresholds

AB39M\$1	-2.098	0.109	19.324	0.000
AB39T\$1	-2.572	0.274	-9.383	0.000
AB39U\$1	-2.824	0.225	12.574	0.000
AB39W\$1	-2.215	0.150	14.768	0.000
AB39X\$1	-2.566	0.184	13.964	0.000

Latent Class C2#1

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Thresholds				
GA33A\$1	0.956	0.100	9.574	0.000
GA33H\$1	3.735	1.010	3.697	0.000
GA33I\$1	3.352	0.935	3.584	0.000
GA33K\$1	2.805	0.172	16.301	0.000
GA33L\$1	4.347	0.782	5.562	0.000

Latent Class C2#2

Thresholds				
GA33A\$1	-0.724	0.102	-7.086	0.000
GA33H\$1	0.815	0.147	5.536	0.000
GA33I\$1	0.268	0.147	1.825	0.068
GA33K\$1	0.280	0.172	1.635	0.102
GA33L\$1	-0.833	0.572	-1.457	0.145

Latent Class C2#3

Thresholds				
GA33A\$1	-0.304	0.158	-1.923	0.055
GA33H\$1	0.421	0.234	1.803	0.071
GA33I\$1	-0.890	0.776	-1.146	0.252
GA33K\$1	1.580	0.288	5.485	0.000
GA33L\$1	15.000	0.000	999.000	999.000

Latent Class C2#4

Thresholds				
GA33A\$1	-2.098	0.109	-19.324	0.000
GA33H\$1	-2.572	0.274	-9.383	0.000
GA33I\$1	-2.824	0.225	-12.574	0.000
GA33K\$1	-2.215	0.150	-14.768	0.000
GA33L\$1	-2.566	0.184	-13.964	0.000

# OUTPUT APPENDED

## Multinomial Logistic Regression & Logit Intercept Parameters

The following output are the parameters of the latent class variable model expressed as logits. The first section of parameters are the multinomial logistic regression or structural paths between the latent variables. These parameters are used to compute the transition probability matrix shown below (b11...b13). The section labeled 'Means' presents the logit intercepts for the latent class variable which are used to compute class size counts & proportions for the estimated model. These intercept parameters are used to compute the transition probability matrix shown in table below (a1...a3).

		C2 = 1	C2 = 2	C2 = 3	C2 = 4
Time 1	C1 = 1	a1+b11	a2+b21	a3+b31	0
	C1 = 2	a1+b12	a2+b22	a3+b32	0
	C1 = 3	a1+b13	a2+b23	a3+b33	0
	C1 = 4	a1	a2	a3	0

### Categorical Latent Variables

*# NOTE: Symbols (%) and labels (!) added to output which are not found in Mplus syntax.*

			Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
%C2#1%	ON					
%C1#1%		!b11!	1.946	0.194	10.019	0.000
%C1#2%		!b12!	0.708	0.234	3.023	0.003
%C1#3%		!b13!	1.180	0.280	4.219	0.000
%C2#2%	ON					
%C1#1%		!b21!	0.532	0.375	1.417	0.156
%C1#2%		!b22!	1.099	0.301	3.646	0.000
%C1#3%		!b23!	0.124	1.088	0.114	0.909
%C2#3%	ON					
%C1#1%		!b31!	1.289	0.329	3.917	0.000
%C1#2%		!b32!	0.750	0.357	2.100	0.036
%C1#3%		!b33!	1.632	0.535	3.049	0.002
Means						
%C1#1%			0.079	0.117	0.676	0.499
%C1#2%			0.101	0.153	0.660	0.509
%C1#3%			-0.615	0.464	-1.327	0.185
%C2#1%		!a1!	-0.891	0.157	-5.674	0.000
%C2#2%		!a2!	-1.288	0.267	-4.815	0.000
%C2#3%		!a3!	-1.498	0.407	-3.685	0.000

## Conditional Item Probabilities

Conditional item probabilities are derived from the threshold parameters using equation (2) in the paper. These are the values that are used when graphing the posterior probability plots as in Figure 2 (bottom). Note that since we assume measurement invariance in this analysis, the conditional item probabilities are constrained to be equal across time when the classes are equal.

### Item Probabilities for 7th Grade ( $C_1$ )

#### RESULTS IN PROBABILITY SCALE

*# The conditional item probabilities are equal across C1 and C2 (invariance assumed).*

#### Results for Class-specific Model Parts of C1

##### Latent Class C1#1

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
AB39M				
Category 1	0.722	0.020	36.062	0.000
Category 2	0.278	0.020	13.859	0.000
AB39T				
Category 1	0.977	0.023	42.453	0.000
Category 2	0.023	0.023	1.013	0.311
AB39U				
Category 1	0.966	0.031	31.601	0.000
Category 2	0.034	0.031	1.107	0.268
AB39W				
Category 1	0.943	0.009	101.838	0.000
Category 2	0.057	0.009	6.164	0.000
AB39X				
Category 1	0.987	0.010	100.102	0.000
Category 2	0.013	0.010	1.296	0.195

##### Latent Class C1#2

AB39M				
Category 1	0.327	0.022	14.536	0.000
Category 2	0.673	0.022	29.978	0.000
AB39T				
Category 1	0.693	0.031	22.143	0.000
Category 2	0.307	0.031	9.805	0.000
AB39U				
Category 1	0.567	0.036	15.731	0.000
Category 2	0.433	0.036	12.037	0.000
AB39W				
Category 1	0.570	0.042	13.549	0.000
Category 2	0.430	0.042	10.235	0.000
AB39X				
Category 1	0.303	0.121	2.510	0.012
Category 2	0.697	0.121	5.773	0.000

##### Latent Class C1#3

AB39M				
Category 1	0.425	0.039	11.001	0.000
Category 2	0.575	0.039	14.905	0.000
AB39T				
Category 1	0.604	0.056	10.804	0.000
Category 2	0.396	0.056	7.091	0.000
AB39U				
Category 1	0.291	0.160	1.818	0.069
Category 2	0.709	0.160	4.425	0.000
AB39W				
Category 1	0.829	0.041	20.325	0.000
Category 2	0.171	0.041	4.187	0.000
AB39X				
Category 1	1.000	0.000	0.000	1.000
Category 2	0.000	0.000	0.000	1.000
Latent Class C1#4				
AB39M				
Category 1	0.109	0.011	10.344	0.000
Category 2	0.891	0.011	84.260	0.000
AB39T				
Category 1	0.071	0.018	3.927	0.000
Category 2	0.929	0.018	51.415	0.000
AB39U				
Category 1	0.056	0.012	4.716	0.000
Category 2	0.944	0.012	79.473	0.000
AB39W				
Category 1	0.098	0.013	7.394	0.000
Category 2	0.902	0.013	67.757	0.000
AB39X				
Category 1	0.071	0.012	5.859	0.000
Category 2	0.929	0.012	76.274	0.000

## Item Probabilities for 10th Grade ( $C_2$ )

Results for Class-specific Model Parts of C2

Latent Class C2#1

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
GA33A				
Category 1	0.722	0.020	36.062	0.000
Category 2	0.278	0.020	13.859	0.000
GA33H				
Category 1	0.977	0.023	42.453	0.000
Category 2	0.023	0.023	1.013	0.311
GA33I				
Category 1	0.966	0.031	31.601	0.000
Category 2	0.034	0.031	1.107	0.268
GA33K				



Category 1	0.943	0.009	101.838	0.000
Category 2	0.057	0.009	6.164	0.000
GA33L				
Category 1	0.987	0.010	100.102	0.000
Category 2	0.013	0.010	1.296	0.195
Latent Class C2#2				
GA33A				
Category 1	0.327	0.022	14.536	0.000
Category 2	0.673	0.022	29.978	0.000
GA33H				
Category 1	0.693	0.031	22.143	0.000
Category 2	0.307	0.031	9.805	0.000
GA33I				
Category 1	0.567	0.036	15.731	0.000
Category 2	0.433	0.036	12.037	0.000
GA33K				
Category 1	0.570	0.042	13.549	0.000
Category 2	0.430	0.042	10.235	0.000
GA33L				
Category 1	0.303	0.121	2.510	0.012
Category 2	0.697	0.121	5.773	0.000
Latent Class C2#3				
GA33A				
Category 1	0.425	0.039	11.001	0.000
Category 2	0.575	0.039	14.905	0.000
GA33H				
Category 1	0.604	0.056	10.804	0.000
Category 2	0.396	0.056	7.091	0.000
GA33I				
Category 1	0.291	0.160	1.818	0.069
Category 2	0.709	0.160	4.425	0.000
GA33K				
Category 1	0.829	0.041	20.325	0.000
Category 2	0.171	0.041	4.187	0.000
GA33L				
Category 1	1.000	0.000	0.000	1.000
Category 2	0.000	0.000	0.000	1.000
Latent Class C2#4				
GA33A				
Category 1	0.109	0.011	10.344	0.000
Category 2	0.891	0.011	84.260	0.000
GA33H				
Category 1	0.071	0.018	3.927	0.000
Category 2	0.929	0.018	51.415	0.000
GA33I				
Category 1	0.056	0.012	4.716	0.000
Category 2	0.944	0.012	79.473	0.000

GA33K				
Category 1	0.098	0.013	7.394	0.000
Category 2	0.902	0.013	67.757	0.000
GA33L				
Category 1	0.071	0.012	5.859	0.000
Category 2	0.929	0.012	76.274	0.000
# OUTPUT APPENDED				

## Model Estimated Class Counts & Proportions

In this section the first column lists the latent variables C1 and C2. The second Column lists the latent classes  $k$  that compose each latent variable. The third column is the model estimated class counts which are distinct from modal assignment counts as indicated by the non-integer estimates. Column four are the model estimated class proportions, note that each latent variable's respective proportions sum to one (i.e., For C1 the first 4 rows).

FINAL CLASS COUNTS AND PROPORTIONS FOR EACH LATENT CLASS VARIABLE  
BASED ON THE ESTIMATED MODEL

Latent Class	Variable	Class		
C1	1	888.43665	0.29024	
	2	908.18719	0.29670	
	3	443.55807	0.14491	
	4	820.81805	0.26815	
C2	1	1069.36035	0.34935	
	2	475.95325	0.15549	
	3	506.74429	0.16555	
	4	1008.94208	0.32961	

## Conditional Transition Probabilities

Below are the estimated latent transition probabilities. Without covariates these are the same estimates found in the transition matrix. However, when covariates are included TECH15 will include probabilities conditioned by level of the covariate. This is useful for presenting covariate results.

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ESTIMATED CONDITIONAL PROBABILITIES FOR THE LATENT CLASS VARIABLES

P(C1=1)=0.290

P(C1=2)=0.297

P(C1=3)=0.145

P(C1=4)=0.268

P(C2=1|C1=1)=0.557

P(C2=2|C1=1)=0.091

P(C2=3|C1=1)=0.157

P(C2=4|C1=1)=0.194

$P(C2=1|C1=2)=0.266$   
 $P(C2=2|C1=2)=0.264$   
 $P(C2=3|C1=2)=0.151$   
 $P(C2=4|C1=2)=0.319$

$P(C2=1|C1=3)=0.352$   
 $P(C2=2|C1=3)=0.082$   
 $P(C2=3|C1=3)=0.302$   
 $P(C2=4|C1=3)=0.264$

$P(C2=1|C1=4)=0.215$   
 $P(C2=2|C1=4)=0.144$   
 $P(C2=3|C1=4)=0.117$   
 $P(C2=4|C1=4)=0.524$

#### ESTIMATED MARGINAL PROBABILITIES FOR THE LATENT CLASS VARIABLES

$P(C2=1)=0.349$   
 $P(C2=2)=0.155$   
 $P(C2=3)=0.166$   
 $P(C2=4)=0.330$

### Relative & Absolute Class Size

This Mplus model includes a total of 16 estimated response patterns, that is all combinations of the 4 classes from C1 by the 4 classes from C2, as shown in the output below. Class counts and proportions are shown in the first and second columns respectively. This section provides the joint distribution of C1 and C2. Note that the proportions in the last column sum to one.

#### FINAL CLASS COUNTS AND PROPORTIONS FOR THE LATENT CLASS PATTERNS BASED ON THE ESTIMATED MODEL

Latent Class  
Pattern

1	1	495.29300	0.16181
1	2	80.91665	0.02643
1	3	139.85871	0.04569
1	4	172.36826	0.05631
2	1	241.41055	0.07887
2	2	239.90708	0.07838
2	3	137.09390	0.04479
2	4	289.77570	0.09467
3	1	156.27037	0.05105
3	2	36.54762	0.01194
3	3	133.74095	0.04369
3	4	116.99915	0.03822
4	1	176.38644	0.05762
4	2	118.58190	0.03874
4	3	96.05074	0.03138
4	4	429.79898	0.14041

## Joint Probabilities

This section presents the joint probability patterns by latent class. Given there are 4 classes in 7th grade and 4 classes in 10th grade, there are 16 possible patterns. This set of marginals presents the prevalence of each of these 16 patterns. These patterns can be used to see the prevalence of specific transition patterns. For example, the pattern of being in class 1 at time 1 and class 1 at time 2 has 16% of the sample, whereas the pattern of being in class 3 in 7th grade and class 2 in 10th grade only has 1.1% of the sample.

### FINAL CLASS COUNTS AND PROPORTIONS FOR THE LATENT CLASS PATTERNS BASED ON ESTIMATED POSTERIOR PROBABILITIES

Latent Class Pattern			
1	1	495.29413	0.16181
1	2	80.91538	0.02643
1	3	139.86096	0.04569
1	4	172.37002	0.05631
2	1	241.41062	0.07887
2	2	239.90595	0.07838
2	3	137.09562	0.04479
2	4	289.77310	0.09467
3	1	156.27062	0.05105
3	2	36.54607	0.01194
3	3	133.73765	0.04369
3	4	117.00055	0.03822
4	1	176.38637	0.05762
4	2	118.57912	0.03874
4	3	96.05312	0.03138
4	4	429.80072	0.14041

## Marginal Probabilities Based on Estimated Posterior Probabilities ( $\pi_k$ )

This estimate of the relative size of the classes are the **most commonly reported**. Note that these estimates account for the non-perfect assignment to classes, thus shown see non-integer values for the class counts (column 3). This estimate is based on summing the columns for each posterior probability.

### FINAL CLASS COUNTS AND PROPORTIONS FOR EACH LATENT CLASS VARIABLE BASED ON ESTIMATED POSTERIOR PROBABILITIES

Latent Class Variable	Class		
C1	1	888.44048	0.29025
	2	908.18528	0.29670
	3	443.55490	0.14491
	4	820.81934	0.26815
C2	1	1069.36174	0.34935
	2	475.94652	0.15549
	3	506.74735	0.16555
	4	1008.94438	0.32961

## Modal Assignment (Most Likely Class Membership)

Below are the class counts and proportions if you assign individuals into classes based on their modal class assignment (e.g., the class with the highest posterior probability of membership). These are not commonly used for describing the classes. Note, if you use the ML 3-step, these are used but adjusted for non-perfect assignment to class.

### FINAL CLASS COUNTS AND PROPORTIONS FOR THE LATENT CLASSES BASED ON THEIR MOST LIKELY LATENT CLASS PATTERN

#### Class Counts and Proportions

##### Latent Class Pattern

1	1	648	0.21170
1	2	49	0.01601
1	3	126	0.04116
1	4	164	0.05358
2	1	197	0.06436
2	2	109	0.03561
2	3	93	0.03038
2	4	358	0.11696
3	1	267	0.08723
3	2	16	0.00523
3	3	88	0.02875
3	4	99	0.03234
4	1	141	0.04606
4	2	67	0.02189
4	3	78	0.02548
4	4	561	0.18327

## Marginal Distribution

Marginal distribution of the classes based on modal class assignment. These are not commonly reported as they do not account for the classification error present in the model. When reporting class distributions the model estimated posterior probabilities are most commonly reported.

### FINAL CLASS COUNTS AND PROPORTIONS FOR EACH LATENT CLASS VARIABLE BASED ON THEIR MOST LIKELY LATENT CLASS PATTERN

##### Latent Class Variable      Class

C1	1	987	0.32244
	2	757	0.24730
	3	470	0.15354
	4	847	0.27671
C2	1	1253	0.40934
	2	241	0.07873
	3	385	0.12578
	4	1182	0.38615

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