## Latent Class Analysis Enumeration

## IMMERSE Training Team

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#### Contents

IMMERSE Project	1
Example: Bullying in Schools	
References	21

## IMMERSE Project



The Institute of Mixture Modeling for Equity-Oriented Researchers, Scholars, and Educators (IMMERSE) is an IES funded training grant (R305B220021) to support education scholars in integrating mixture modeling into their research.

- Please visit our website to learn more.
- Visit our GitHub account to access all the IMMERSE materials.
- Follow us on Twitter!

How to reference this workshop: Institute of Mixture Modeling for Equity-Oriented Researchers, Scholars, and Educators (2023). IMMERSE Online Resources (IES No. 305B220021). Institute of Education Sciences. https://immerse-ucsb.github.io/pre-training

#### Example: Bullying in Schools

To demonstrate mixture modeling in the training program and online resource components of the IES grant we utilize the Civil Rights Data Collection (CRDC) (CRDC) data repository. The CRDC is a federally mandated school-level data collection effort that occurs every other year. This public data is currently available for selected latent class indicators across 4 years (2011, 2013, 2015, 2017) and all US states. In this example, we use the Arizona state sample. We utilize six focal indicators which constitute the latent class model in our example; three variables which report on harassment/bullying in schools based on disability, race, or sex, and three variables on full-time equivalent school staff hires (counselor, psychologist, law enforcement). This data source also includes covariates on a variety of subjects and distal outcomes reported in 2018 such as math/reading assessments and graduation rates.

#### Load packages

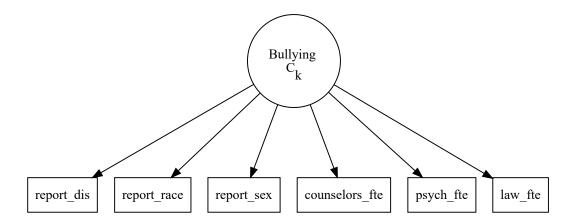
```
library(tidyverse)
library(haven)
library(glue)
library(MplusAutomation)
library(here)
library(janitor)
library(gt)
library(cowplot)
library(DiagrammeR)
here::i_am("lca_enum.Rmd")
```

#### Variable Description

	LCA indicators <sup>7</sup>	
Name	Label	Values
leaid	District Identification Code	
ncessch	School Identification Code	
report_dis	Number of students harassed or bullied on the basis of disability	0 = No reported incidents, 1 = At least one reported incident
report_race	Number of students harassed or bullied on the basis of race, color, or national origin	0 = No reported incidents, 1 = At least one reported incident
report_sex	Number of students harassed or bullied on the basis of sex	0 = No reported incidents, 1 = At least one reported incident
counselors_fte	Number of full time equivalent counselors hired as school staff	0 = No staff present, 1 = At least one staff present
report_sex	Number of full time equivalent psychologists hired as school staff	0 = No staff present, 1 = At least one staff present
counselors_fte	Number of full time equivalent law enforcement officers hired as school staff	0 = No staff present, 1 = At least one staff present
<sup>1</sup> Civil Rights Dat	a Collection (CRDC)	

## Variables have been transformed to be dichotomous indicators using the following coding strategy

Harassment and bullying count variables are recoded 1 if the school reported at least one incident of harassment (0 indicates no reported incidents). On the original scale reported by the CDRC staff variables for full time equivalent employees (FTE) are represented as 1 and part time employees are represented by values between 1 and 0. Schools with greater than one staff of the designated type are represented by values greater than 1. All values greater than zero were recorded as 1s (e.g., .5, 1,3) indicating that the school has a staff present on campus at least part time. Schools with no staff of the designated type are indicated as 0 for the dichotomous variable.



#### Prepare Data

```
df_bully <- read_csv(here("data", "crdc_lca_data.csv")) %>%
  clean_names() %>%
  dplyr::select(report_dis, report_race, report_sex, counselors_fte, psych_fte, law_fte)
```

#### **Descriptive Statistics**

```
# Set up data to find proportions of binary indicators
ds <- df_bully %>%
  pivot_longer(c(report_dis, report_race, report_sex, counselors_fte, psych_fte, law_fte), names_to = "
# Create table of variables and counts, then find proportions and round to 3 decimal places
prop_df <- ds %>%
  count(variable, value) %>%
  group_by(variable) %>%
  mutate(prop = n / sum(n)) %>%
  ungroup() %>%
  mutate(prop = round(prop, 3))
\# Make it a gt() table
prop_table <- prop_df %>%
  gt(groupname_col = "variable", rowname_col = "value") %>%
  tab_stubhead(label = md("*Values*")) %>%
  tab header(
    md(
      "Variable Proportions"
    )
  ) %>%
  cols label(
   variable = md("*Variable*"),
   value = md("*Value*"),
   n = md("*N*"),
    prop = md("*Proportion*")
  )
prop_table
```

#### Variable Proportions

Values	N	Proportion
counselo	ors_fte	
0	1081	0.533
1	919	0.453

NA	27	0.013					
law_fte	law_fte						
0	1749	0.863					
1	251	0.124					
NA	27	0.013					
psych_f	îte						
0	1050	0.518					
1	947	0.467					
NA	30	0.015					
report_	report_dis						
0	1915	0.945					
1	85	0.042					
NA	27	0.013					
report_	report_race						
0	1794	0.885					
1	206	0.102					
NA	27	0.013					
report_sex							
0	1660	0.819					
1	340	0.168					
NA	27	0.013					

```
# Save as img
gtsave(prop_table, here("figures", "prop_table.png"))
```

#### Quick Introduction to MplusAutomation

#### WHAT?

- MplusAutomation is an R package
- $\bullet~$  It "wraps around" the  ${\tt Mplus}$  program
- Requires both R & Mplus software
- Requires learning some basics of 2 programming languages
- Car metaphor: R/Rstudio is the steering wheel or dashboard & Mplus is the engine

#### WHY?

- $\bullet$  MplusAutomation can provide clearly organized work procedures in which every research decision can be documented in a single place
- $\bullet\,$  Increase reproducibility, organization, efficiency, and transparency

#### HOW?

• The interface for MplusAutomation is entirely within R-Studio. You do not need to open Mplus

• The code presented will be very repetitive by design

Below is a template for mplusObject() & mplusModeler() functions. Use this template to run statistical models with Mplus.

```
m_template <- mplusObject(</pre>
  TITLE =
   "",
  VARIABLE =
    "",
  ANALYSIS =
    "",
  PLOT =
   Шη,
  OUTPUT =
    ш,
  usevariables = colnames(),
  rdata = )
m_template_fit <- mplusModeler(m_template,</pre>
                   dataout=here("", ".dat"),
                   modelout=here("", ".inp"),
                   check=TRUE, run = TRUE, hashfilename = FALSE)
```

#### Enumeration

This code uses the mplusObject function in the MplusAutomation package and saves all model runs in the enum folder.

```
lca_6 <- lapply(1:6, function(k) {
    lca_enum <- mplusObject(

    TITLE = glue("{k}-Class"),

    VARIABLE = glue(
        "categorical = report_dis-law_fte;
        usevar = report_dis-law_fte;
        classes = c({k}); "),

ANALYSIS =
    "estimator = mlr;
    type = mixture;
    starts = 200 100;
    processors = 10;",

OUTPUT = "sampstat residual tech11 tech14;",</pre>
```

**IMPORTANT**: Before moving forward, make sure to open each output document to ensure models were estimated normally. In this example, the last two models (5- and 6-class models) did not produce reliable output and are excluded.

#### Table of Fit

First, extract data:

```
#
output_bully <- readModels(here("enum"), filefilter = "bully", quiet = TRUE)</pre>
enum_extract <- LatexSummaryTable(</pre>
  output_bully,
  keepCols = c(
    "Title",
    "Parameters",
    "LL",
    "BIC".
    "aBIC",
    "BLRT PValue",
    "T11 VLMR PValue",
    "Observations"
 ),
 sortBy = "Title"
) %>%
  slice_head(n=4) # Select first four models (Class 1 through 4)
allFit <- enum_extract %>%
  mutate(CAIC = -2 * LL + Parameters * (log(Observations) + 1)) %>%
  mutate(AWE = -2 * LL + 2 * Parameters * (log(Observations) + 1.5)) %>%
  mutate(SIC = -.5 * BIC) \%
  mutate(expSIC = exp(SIC - max(SIC))) %>%
  mutate(BF = exp(SIC - lead(SIC))) %>%
  mutate(cmPk = expSIC / sum(expSIC)) %>%
  dplyr::select(1:5, 9:10, 6:7, 13, 14) %>%
  arrange(Parameters)
```

Then, create table:

```
fit table1 <- allFit %>%
  gt() %>%
  tab_header(title = md("**Model Fit Summary Table**")) %>%
  cols label(
   Title = "Classes",
   Parameters = md("Par"),
   LL = md("*LL*"),
   T11_VLMR_PValue = "VLMR",
   BLRT_PValue = "BLRT",
   BF = md("BF"),
   cmPk = md("*cmPk*")
 ) %>%
 tab footnote(
   footnote = md(
      "*Note.* Par = Parameters; *LL* = model log likelihood;
BIC = Bayesian information criterion;
aBIC = sample size adjusted BIC; CAIC = consistent Akaike information criterion;
AWE = approximate weight of evidence criterion;
BLRT = bootstrapped likelihood ratio test p-value;
VLMR = Vuong-Lo-Mendell-Rubin adjusted likelihood ratio test p-value;
*cmPk* = approximate correct model probability."
   ),
locations = cells_title()
 tab_options(column_labels.font.weight = "bold") %>%
 fmt_number(c(3:7),
             decimals = 2) %>%
  sub_missing(1:11,
              missing_text = "--") %>%
  fmt(
   c(8:9, 11),
   fns = function(x)
      ifelse(x < 0.001, "<.001",
             scales::number(x, accuracy = .01))
  ) %>%
  fmt(
   10,
   fns = function (x)
      ifelse(x > 100, ">100",
             scales::number(x, accuracy = .01))
  ) %>%
  tab_style(
   style = list(
      cell_text(weight = "bold")
      ),
   locations = list(cells_body(
    columns = BIC,
    row = BIC == min(BIC[c(1:4)]) # Change this to the number of classes you are evaluating
   ),
   cells body(
    columns = aBIC,
    row = aBIC == min(aBIC[1:4])
```

```
cells_body(
    columns = CAIC,
    row = CAIC == min(CAIC[1:4])
    ),
    cells_body(
    columns = AWE,
    row = AWE == min(AWE[1:4])
    ),
    cells_body(
     columns = cmPk,
    row = cmPk == max(cmPk[1:4])
    ),
    cells_body(
     columns = BF,
    row = BF > 10),
    cells_body(
     columns = T11_VLMR_PValue,
    row = ifelse(T11_VLMR_PValue < .001 & lead(T11_VLMR_PValue) > .05, T11_VLMR_PValue < .001, NA)),
    cells_body(
     columns = BLRT_PValue,
     row = ifelse(BLRT_PValue < .001 & lead(BLRT_PValue) > .05, BLRT_PValue < .001, NA))
  )
)
fit_table1
```

#### Model Fit Summary Table<sup>1</sup>

Classes	Par	LL	BIC	aBIC	CAIC	AWE	BLRT	VLMR	BF	cmPk
1-Class	6	-5,443.41	10,932.50	10,913.44	10,938.50	10, 996.19	_	_	0.00	<.001
2-Class	13	-5,194.14	10,487.26	10,445.96	10,500.26	10,625.24	<.001	<.001	0.00	<.001
3-Class	20	-5,122.48	10,397.24	10,333.70	10,417.24	10,609.53	<.001	<.001	> 100	1.00
4-Class	27	-5,111.76	10,429.10	10,343.32	10,456.10	10,715.69	<.001	0.01	_	<.001

<sup>1</sup> Note. Par = Parameters; LL = model log likelihood; BIC = Bayesian information criterion; aBIC = sample size adjusted BIC; CAIC = consistent Akaike information criterion; AWE = approximate weight of evidence criterion; BLRT = bootstrapped likelihood ratio test p-value; VLMR = Vuong-Lo-Mendell-Rubin adjusted likelihood ratio test p-value; cmPk = approximate correct model probability.

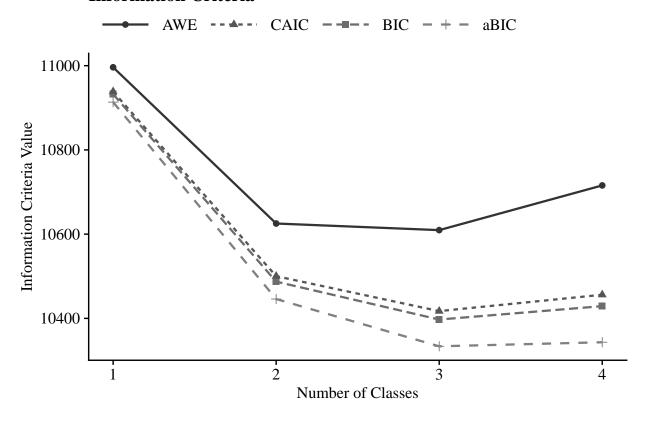
Save table:

```
gtsave(fit_table1, here("figures", "fit_table1.png"))
```

Information Criteria Plot

```
allFit %>%
  dplyr::select(2:7) %>%
  rowid_to_column() %>%
  pivot_longer(`BIC`:`AWE`,
              names_to = "Index",
               values_to = "ic_value") %>%
  mutate(Index = factor(Index,
                        levels = c ("AWE", "CAIC", "BIC", "aBIC"))) %>%
  ggplot(aes(
   x = rowid,
   y = ic_value,
   color = Index,
    shape = Index,
    group = Index,
   lty = Index
  )) +
  geom_point(size = 2.0) + geom_line(size = .8) +
  scale_x_continuous(breaks = 1:nrow(allFit)) +
  scale_colour_grey(end = .5) +
  theme_cowplot() +
  labs(x = "Number of Classes", y = "Information Criteria Value", title = "Information Criteria") +
  theme(
    text = element_text(family = "serif", size = 12),
    legend.text = element_text(family="serif", size=12),
    legend.key.width = unit(3, "line"),
   legend.title = element_blank(),
    legend.position = "top"
```

## **Information Criteria**



Save figure:

```
ggsave(here("figures", "info_criteria.png"), dpi=300, height=5, width=7, units="in")
```

#### **Compare Class Solutions**

Compare probability plots for K = 1:6 class solutions

```
model_results <- data.frame()

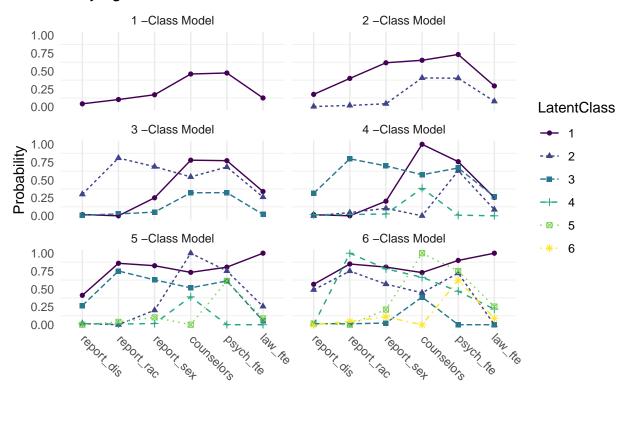
for (i in 1:length(output_bully)) {
   temp <- output_bully[[i]] parameters probability.scale %>%
      mutate(model = paste(i,"-Class Model"))

   model_results <- rbind(model_results, temp)
}

rm(temp)</pre>
```

```
compare_plot <-</pre>
  model_results %>%
  filter(category == 2) %>%
  dplyr::select(est, model, LatentClass, param) %>%
  mutate(param = as.factor(str_to_lower(param)))
compare_plot$param <- fct_inorder(compare_plot$param)</pre>
ggplot(
  compare_plot,
  aes(
    x = param,
    y = est,
    color = LatentClass,
    shape = LatentClass,
    group = LatentClass,
    lty = LatentClass
) +
  geom_point() +
  geom_line() +
  scale_colour_viridis_d() +
 facet_wrap( ~ model, ncol = 2) +
  labs(title = "Bullying Items",
       x = " ", y = "Probability") +
  theme_minimal() +
  theme(panel.grid.major.y = element_blank(),
                          axis.text.x = element_text(angle = -45, hjust = -.1))
```

## **Bullying Items**



Save figure:

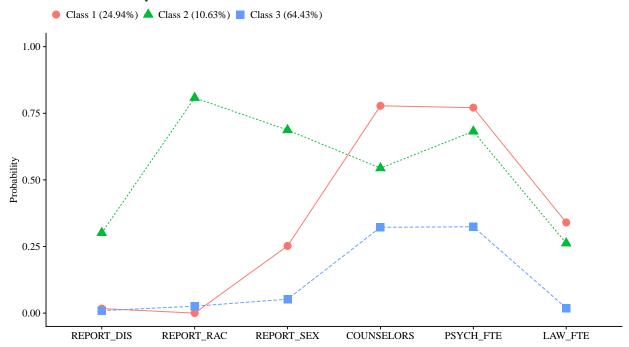
```
ggsave(here("figures", "compare_kclass_plot.png"), dpi=300, height=5, width=7, units="in")
```

#### 3-Class Probability Plot

Use the plot\_lca function provided in the folder to plot the item probability plot. This function requires one argument: - model\_name: The name of the Mplus readModels object (e.g., output\_bully\$c3\_bully.out)

```
source("plot_lca.txt")
plot_lca(model_name = output_bully$c3_bully.out)
```

#### 3-Class Probability Plot



Save figure:

```
ggsave(here("figures", "C3_bully_LCA_Plot.png"), dpi="retina", height=5, width=7, units="in")
```

#### Observed Response Patterns

Save response frequencies for the 3-class model from the previous lab with response is \_\_\_\_.dat under SAVEDATA.

```
patterns <- mplusObject(

TITLE = "C3 LCA - Save response patterns",

VARIABLE =
   "categorical = report_dis-law_fte;
   usevar = report_dis-law_fte;
   classes = c(3);",

ANALYSIS =
   "estimator = mlr;
   type = mixture;
   starts = 0;
   processors = 10;</pre>
```

 $\begin{tabular}{ll} ## < simple Error in bivar Fit Data [mPos, ] <- c(vars, values): number of items to replace is not a multiple of items of items to replace is not a multiple of items of$ 

Note: You may see an error that says <simpleError in bivarFitData[mPos, ] <- c(vars, values): number of items to replace is not a multiple of replacement length>, the developers are aware of this and are working to fix it.

Read in observed response pattern data and relabel the columns

## <simpleError in bivarFitData[mPos, ] <- c(vars, values): number of items to replace is not a multipl

```
# Add the names back to the dataset
colnames(patterns) <- c("Frequency", names)</pre>
```

Create a table with the top 5 unconditional response pattern, then top of conditional response pattern for each modal class assignment

```
# Order responses by highest frequency
order_highest <- patterns %>%
   arrange(desc(Frequency))

# Loop `patterns` data to list top 5 conditional response patterns for each class
loop_cond <- lapply(1:max(patterns$C), function(k) {</pre>
```

```
order_cond <- patterns %>%
  filter(C == k) %>%
  arrange(desc(Frequency)) %>%
  head(5)
  })

# Convert loop into data frame
table_data <- as.data.frame(bind_rows(loop_cond))

# Combine unconditional and conditional responses patterns
response_patterns <- rbind(order_highest[1:5,], table_data)</pre>
```

Finally, use {gt} to make a nicely formatted table

```
resp_table <- response_patterns %>%
  gt() %>%
    tab_header(
    title = "Observed Response Patterns",
    subtitle = html("Response patterns, estimated frequencies, estimated posterior class probabilities
    tab_source_note(
    source_note = md("Data Source: **Civil Rights Data Collection (CRDC)**")) %>%
    cols_label(
      Frequency = html("<i>f</i><sub>r</sub>"),
    REPORT_D = "Harrassment: Disability",
    REPORT_R = "Harrassment: Race",
    REPORT_S = "Harrassment: Sex",
    COUNSELO = "Staff: Counselor",
    PSYCH_FT = "Staff: Psychologist",
    LAW_FTE = "Staff: Law Enforcement",
    CPROB1 = html("P < sub > < i > k < / i > < / sub > = 1"),
    CPROB2 = html("P < sub > < i > k < / i > < / sub > = 2"),
    CPROB3 = html("P < sub > < i > k < / i > < / sub > = 3"),
    C = md("*k*")) %>%
  tab_row_group(
    label = "Unconditional response patterns",
    rows = 1:5) %>%
  tab_row_group(
    label = md("*k* = 1 Conditional response patterns"),
    rows = 6:10) %>% #EDIT THESE VALUES BASED ON THE LAST COLUMN
  tab_row_group(
    label = md("*k* = 2 Conditional response patterns"),
    rows = 11:15) %>% #EDIT THESE VALUES BASED ON THE LAST COLUMN
  tab_row_group(
    label = md("*k* = 3 Conditional response patterns"),
    rows = 16:20) %>% #EDIT THESE VALUES BASED ON THE LAST COLUMN
    row_group_order(
      groups = c("Unconditional response patterns",
                 md("*k* = 1 Conditional response patterns"),
                 md("*k* = 2 Conditional response patterns"),
                 md("*k* = 3 Conditional response patterns"))) %>%
    tab_footnote(
    footnote = html(
      "<i>Note.</i> <i>f</i><sub>r</sub> = response pattern frequency; P<sub><i>k</i></sub> = posterior
```

```
)
) %>%

cols_align(align = "center") %>%

opt_align_table_header(align = "left") %>%

gt::tab_options(table.font.names = "Times New Roman")

resp_table
```

### Observed Response Patterns

Response patterns, estimated frequencies, estimated

<i>f</i> <sub>r</sub>	Harrassment: Disability	Harrassment: Race	Harrassment: Sex	Staff: Counselor	Staff:
Unconditional response pat	terns				
525	0	0	0	0	
299	0	0	0	0	
293	0	0	0	1	
251	0	0	0	1	
75	0	0	0	1	
k = 1 Conditional response	patterns				
251	0	0	0	1	
75	0	0	0	1	
72	0	0	1	1	
38	0	0	1	0	
34	0	0	0	0	
k = 2 Conditional response	patterns				
24	0	1	0	0	
20	0	1	1	0	
19	0	1	1	1	
18	0	1	1	1	
12	0	1	1	1	
k = 3 Conditional response	patterns				
525	0	0	0	0	
299	0	0	0	0	
293	0	0	0	1	
36	0	0	1	0	
27	0	0	0	NA	

<i>Note.</i> <i>f</i><<isb>r</sub> = response pattern frequency; P<sub><i>k</i></isb> = posterior class probabilities

Data Source: Civil Rights Data Collection (CRDC)

Save table:

```
gtsave(resp_table, here("figures","resp_table.png"))
```

#### Classification Diagnostics

Use Mplus to calculate k-class confidence intervals (Note: Change the synax to make your chosen k-class model):

```
classification <- mplusObject(</pre>
  TITLE = "C3 LCA - Calculated k-Class 95% CI",
  VARIABLE =
    "categorical = report_dis-law_fte;
  usevar = report_dis-law_fte;
  classes = c(3);",
  ANALYSIS =
    "estimator = ml;
   type = mixture;
   starts = 0;
    processors = 10;
    stseed = 802779;
    bootstrap = 1000;",
 MODEL =
  !CHANGE THIS SECTION TO YOUR CHOSEN k-CLASS MODEL
  %OVERALL%
  [C#1](c1);
  [C#2](C2);
 Model Constraint:
 New(p1 p2 p3);
 p1 = \exp(c1)/(1+\exp(c1)+\exp(c2));
  p2 = \exp(c2)/(1+\exp(c1)+\exp(c2));
 p3 = 1/(1+exp(c1)+exp(c2));,
  OUTPUT = "cinterval(bcbootstrap)",
  usevariables = colnames(df_bully),
 rdata = df_bully)
classification_fit <- mplusModeler(classification,</pre>
                dataout=here("mplus", "bully.dat"),
                modelout=here("mplus", "class.inp") ,
                check=TRUE, run = TRUE, hashfilename = FALSE)
```

Read in the 3-class model:

```
# Read in the 3-class model and extract information needed
output_bully <- readModels(here("mplus", "class.out"))</pre>
# Entropy
entropy <- c(output_bully$summaries$Entropy, rep(NA, output_bully$summaries$NLatentClasses-1))
# 95% k-Class and k-class 95% Confidence Intervals
k_ci <- output_bully$parameters$ci.unstandardized %>%
  filter(paramHeader == "New.Additional.Parameters") %>%
  unite(CI, c(low2.5,up2.5), sep=", ", remove = TRUE) %>%
  mutate(CI = paste0("[", CI, "]")) %>%
  rename(kclass=est) %>%
  dplyr::select(kclass, CI)
# AvePPk = Average Latent Class Probabilities for Most Likely Latent Class Membership (Row) by Latent C
avePPk <- tibble(avePPk = diag(output_bully$class_counts$avgProbs.mostLikely))</pre>
# mcaPk = modal class assignment proportion
mcaPk <- round(output_bully$class_counts$mostLikely,3) %>%
 mutate(model = paste0("Class ", class)) %>%
 add column(avePPk, k ci) %>%
 rename(mcaPk = proportion) %>%
  dplyr::select(model, kclass, CI, mcaPk, avePPk)
# OCCk = odds of correct classification
OCCk <- mcaPk %>%
 mutate(OCCk = round((avePPk/(1-avePPk))/(kclass/(1-kclass)),3))
# Put everything together
class_table <- data.frame(OCCk, entropy)</pre>
```

Now, use {gt} to make a nicely formatted table

```
class_table <- class_table %>%
  gt() %>%
    tab_header(
    title = "Model Classification Diagnostics for the 3-Class Solution") %>%
    cols_label(
      model = md("*k*-Class"),
     kclass = md("*k*-Class Proportions"),
      CI = "95\% CI",
      mcaPk = html("McaP<sub>k</sub>"),
      avePPk = md("AvePP<sub>k</sub>"),
      OCCk = md("OCC < sub > k < / sub > "),
      entropy = "Entropy") %>%
    sub_missing(7,
              missing_text = "") %>%
    tab_footnote(
    footnote = html(
      "<i>Note.</i> McaP<sub>k</sub> = Modal class assignment proportion; AvePP<sub>k</sub> = Average p
  ) %>%
  cols_align(align = "center") %>%
```

```
opt_align_table_header(align = "left") %>%
  gt::tab_options(table.font.names = "Times New Roman")
class_table
```

#### Model Classification Diagnostics for the 3-Class Solution

k-Class	k-Class Proportions	95% CI	McaP <sub>k</sub>	AvePPk	OCCk	Entropy
Class 1	0.106	[0.083, 0.136]	0.095	0.904	79.420	0.635
Class 2	0.249	[0.166, 0.329]	0.282	0.675	6.264	
Class 3	0.644	[0.561,  0.731]	0.623	0.893	4.614	

<i>Note.</i> McaP<sub>k</sub> = Modal class assignment proportion; AvePP<sub>k</sub> = Average posterior class probabilities; OCC<sub>k</sub> = Odds of correct classification;

Save table:

```
gtsave(class_table, here("figures","class_table.png"))
```

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