CSE4227 Digital Image Processing

Chapter 9 – **Morphological Image Processing- (Part-I)**

Dr. Kazi A Kalpoma

Professor, Department of CSE

Ahsanullah University of Science & Technology (AUST)

Contact: kalpoma@aust.edu

Google Class code: bux3jc2

Today's Contents

- **Morphological image processing: pixel shape based analysis**
  - **Structuring Element**
  - **basic morphology operations**
    - Dilation - grow image regions
    - Erosion - shrink image regions
    - Opening - structured removal of image region boundary pixels
    - Closing - structured filling in of image region boundary pixels

Chapter 9 from R.C. Gonzalez and R.E. Woods, Digital Image Processing (3rd Edition), Prentice Hall, 2008 [**Section 9.1, 9.2, 9.3**]

https://www.cs.auckland.ac.nz/courses/compsci773s1c/lectures/ImageProcessing-html/topic4.htm

Morphological Image Processing

- A tool for **extracting image components** that are useful in the representation and description of image.
- It deals with the shape (or morphology) of objects/features in an image
- Rely only on the **relative ordering of pixel values**, not on their numerical values
- Probe an image with a small shape or template called a **structuring element**.
- The language of mathematical morphology of an image is Set Theory

Morphological Image Processing

- Morphology applies certain simple rules and shapes (such as squares, circles, diamonds, cubes and spheres) to process images.

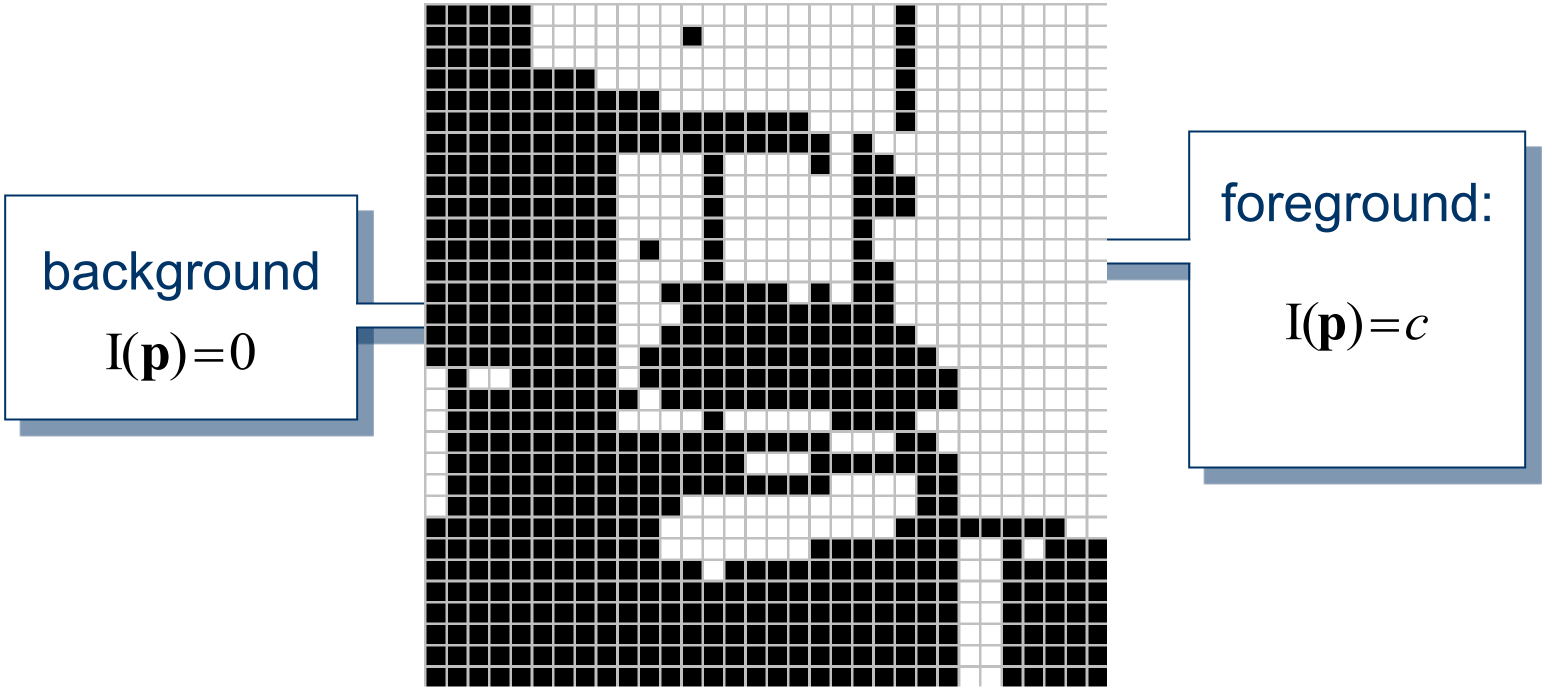
- The objective is usually to identify features of interest in images
  - as a prelude to performing high-level inspection, or
  - machining, functions.

- Two kinds :
  - Binary Morphology
  - Grey Level Morphology

- Four basic morphology operations for **binary images**:
  - **Erosion**
  - **Dilation**
  - **Openning and**
  - **Closing**

Binary Image

- Representation of individual pixels as 0 or 1, convention:
  - foreground, object = 1 (white)
  - background = 0 (black)



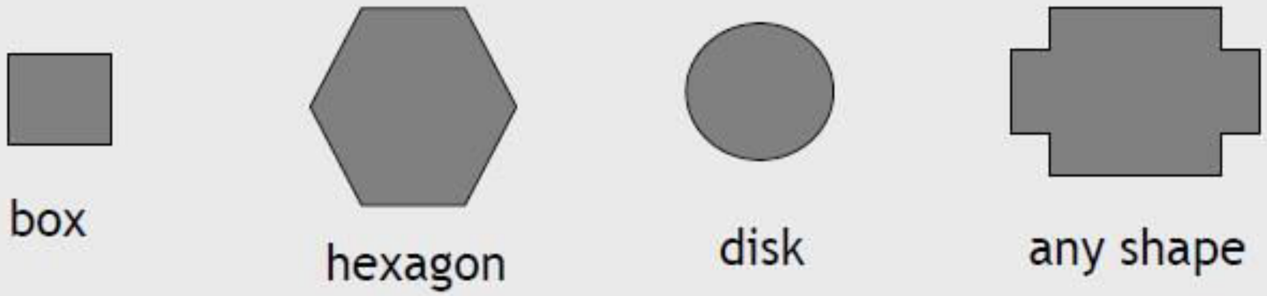
This represents a digital image. Each square is one pixel.

Structuring Element

A small image/template that helps to produce new image from the old one i.e. a small binary array.

A structuring element is a shape mask used in the basic morphological operations.

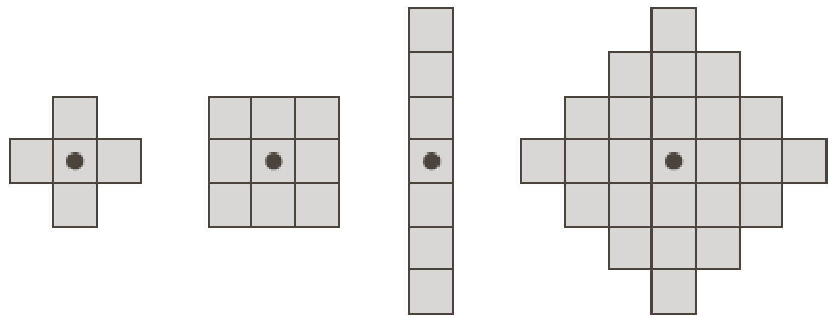
They can be any shape and size that is digitally representable, and each has an origin.



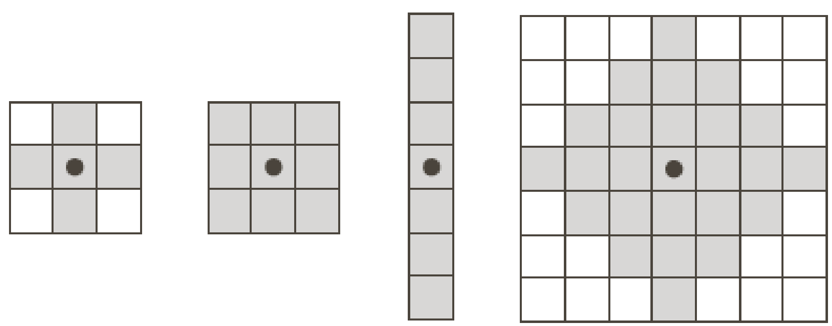
box(length,width) disk(diameter)

Structuring Element

A structuring element is a small image – used as a moving window



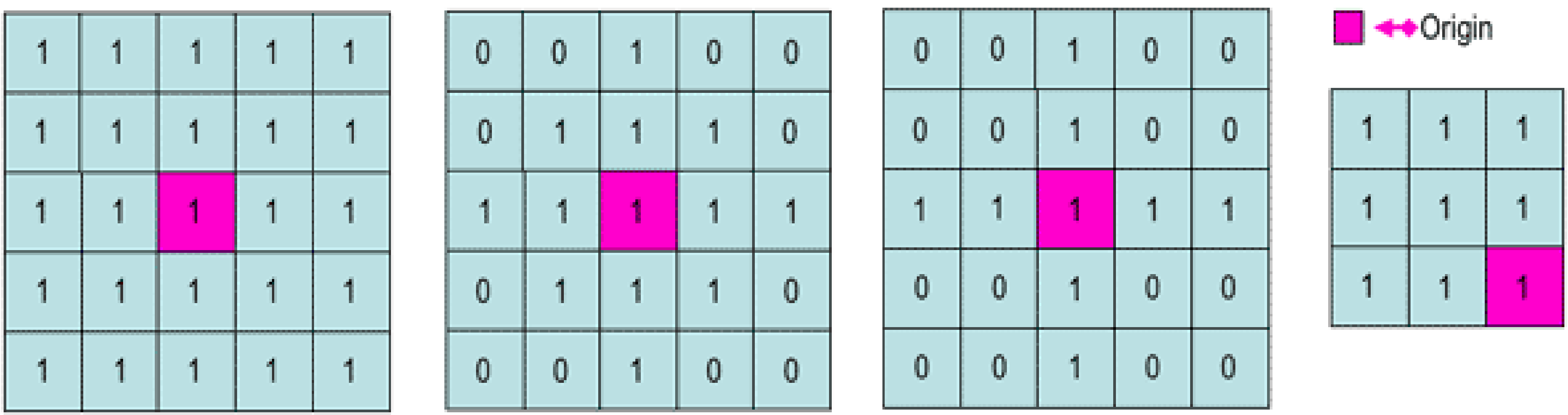
Example Structuring Elements



Structuring Elements converted to rectangular arrays

Structuring Element

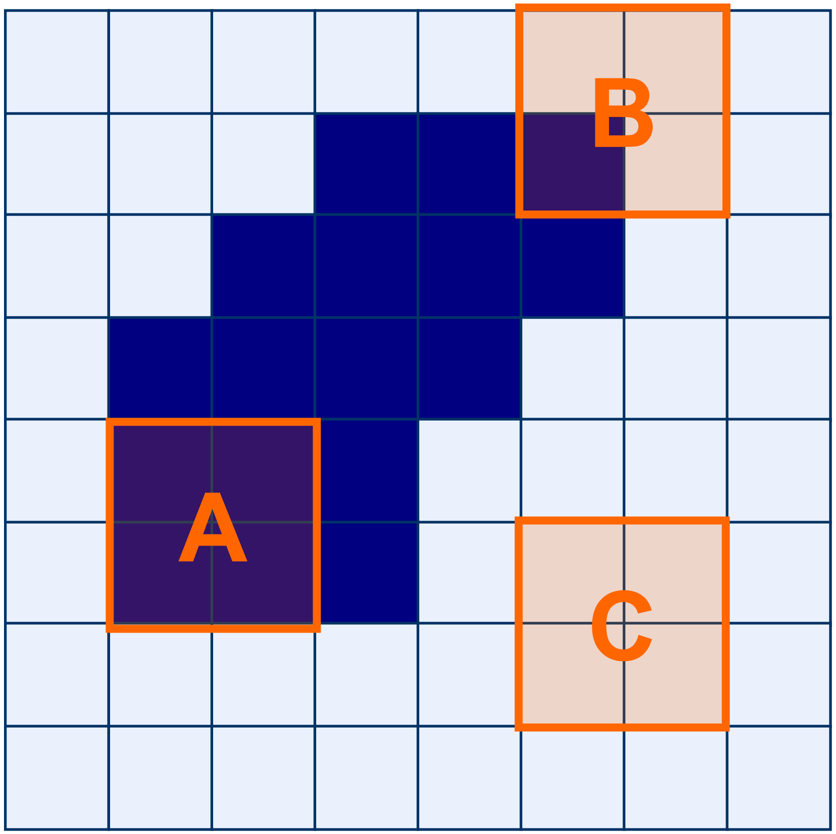
- For simplicity we will use rectangular structuring elements with their origin point.
- Origin point can be at the middle pixel or at any position.

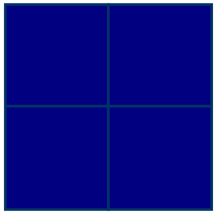


Square 5x5 element | Diamond-shaped 5x5 element | Cross-shaped 5x5 element | Square 3x3 element

❖ By default origin point is at the center

Structuring Elements: Hits & Fits





Fit: All "***ON***" *pixels* in the structuring element cover "***ON***" *pixels* in the image

Hit: Any "***ON***" *pixel* in the structuring element covers an "***ON***" *pixel* in the image

All morphological processing operations are based on these simple ideas

Fitting and hitting of a binary image with structuring elements s_1 and s_2.

B

C

A

$$
\begin{matrix}
0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&1&0&0&0&0&0&0&0\\
0&0&1&1&1&1&1&0&0&0&0&0\\
0&1&1&1&1&1&1&1&0&0&0&0\\
0&1&1&1&1&1&1&1&0&0&0&0\\
0&0&1&1&1&1&1&1&0&0&0&0\\
0&0&1&1&1&1&1&1&1&0&0&0\\
0&0&1&1&1&1&1&1&1&1&1&0\\
0&0&0&0&0&1&1&1&1&1&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0
\end{matrix}
$$

$$s_1 = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$s_2 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

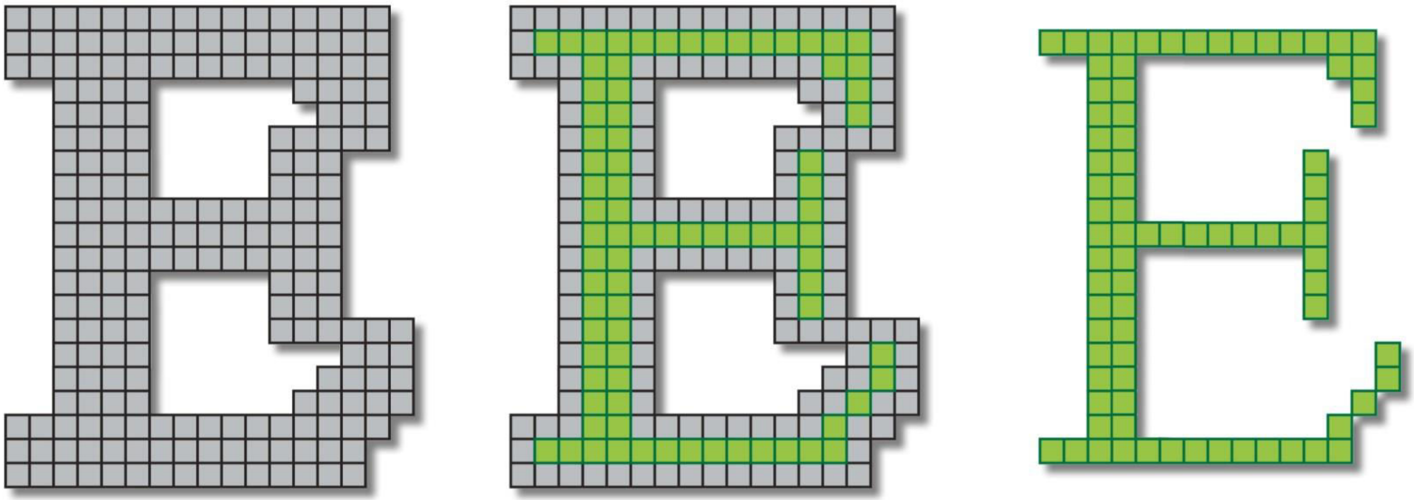
		A	B	C
fit	s_1	yes	no	no
	s_2	yes	yes	no
hit	s_1	yes	yes	yes
	s_2	yes	yes	no

Fundamental Operations

- Fundamentally morphological image processing is very like spatial filtering
- The structuring element is moved across every pixel in the original image to give a pixel in a new processed image
- The value of this new pixel depends on the operation performed

There are two basic morphological operations: **Erosion** and **Dilation**

Erosion

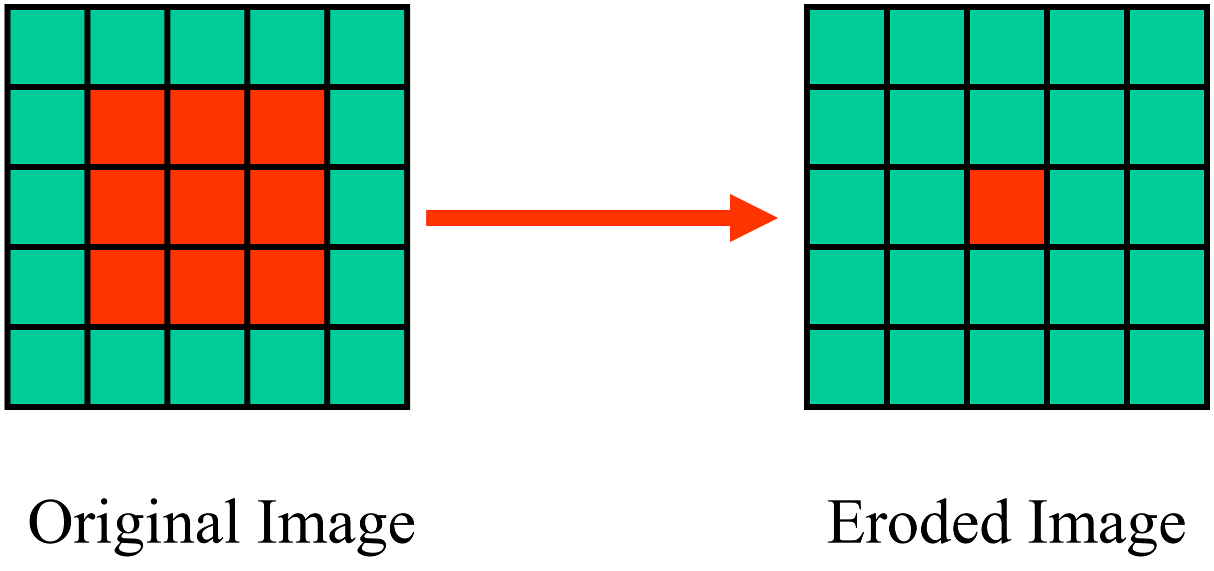


Shrink the object

Shrinks foreground, enlarges Background

Erosion

- Erosion is used for shrinking of element A by using element B.
- One of the simplest uses of erosion is for eliminating irrelevant details from a binary image.



Erosion

Definition 1:

Fit: All "***ON***" *pixels* in the structuring element cover "***ON***" *pixels* in the image

- Does the structuring element **fit the set?**

$$A \ominus B = \{z \mid (B)_z \subseteq A\}$$

- Erosion of a set A by structuring element B:
  - Set of all points z, such that B translated by z is contained in A.

If YES, output pixel g(x,y) will be foreground (i.e. 1)

Erosion

Definition 2:

Does the structuring element **fit the set?**

Erosion of image *f* by structuring element *s* is given by $f \ominus s$

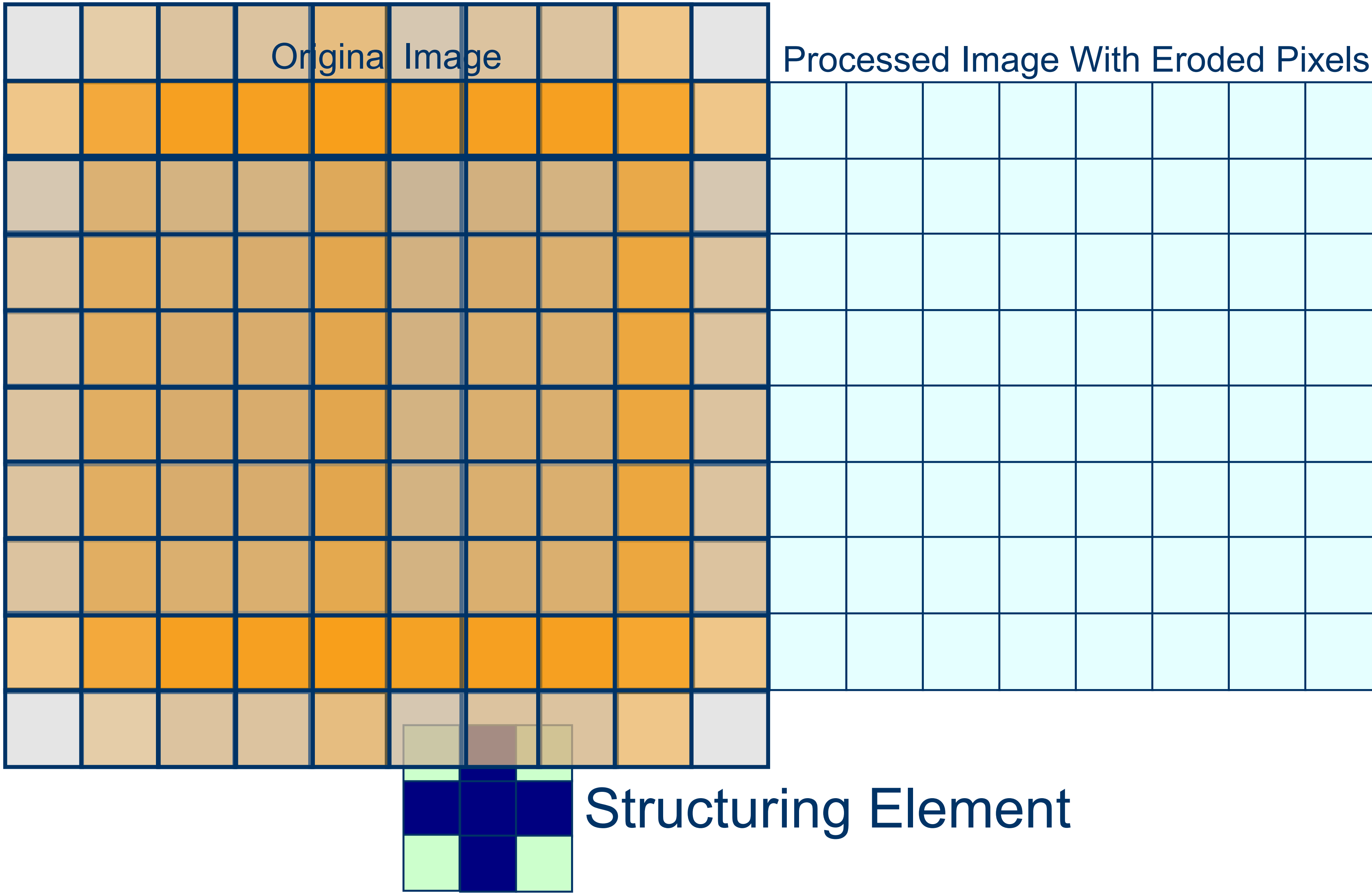
The structuring element s is positioned with its origin at *(x, y)* and the new pixel value is determined using the rule:

$$g(x,y) = \begin{cases} 1 \text{ if } s \text{ fits } f \\ 0 \text{ otherwise} \end{cases}$$

Erosion – How to compute

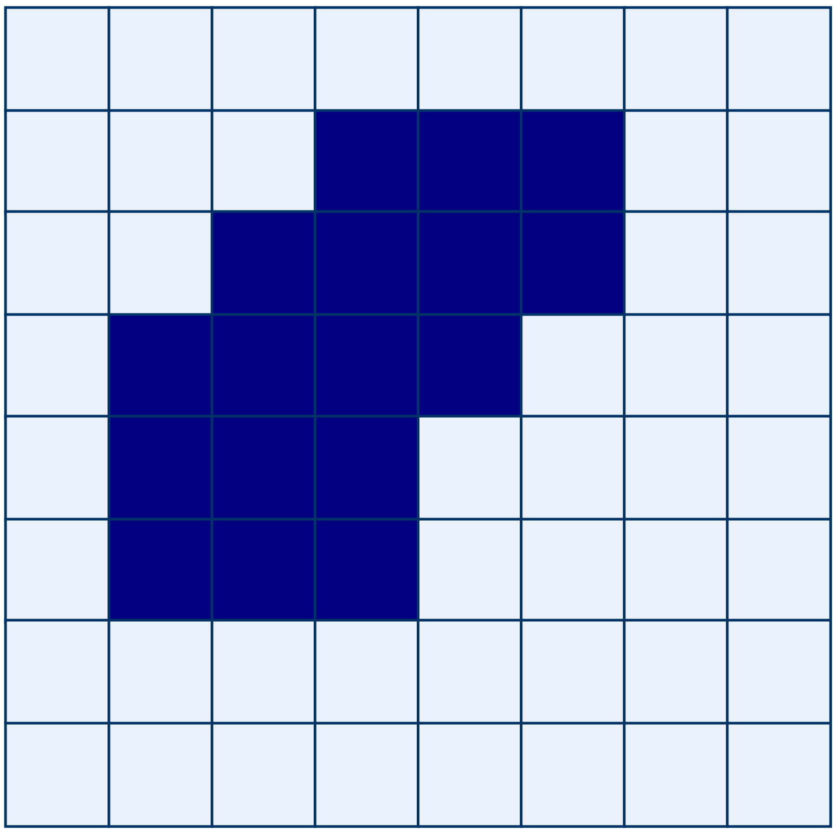
- For each foreground pixel (which we will call the *input pixel*)
  - Superimpose the structuring element on top of the input image so that the origin of the structuring element coincides with the input pixel position.
  - If *for every* pixel in the structuring element, the corresponding pixel in the image underneath is a foreground pixel, then the input pixel is left as it is.
  - If any of the corresponding pixels in the image are background, however, the input pixel is also set to background value.

Example for 2D Erosion

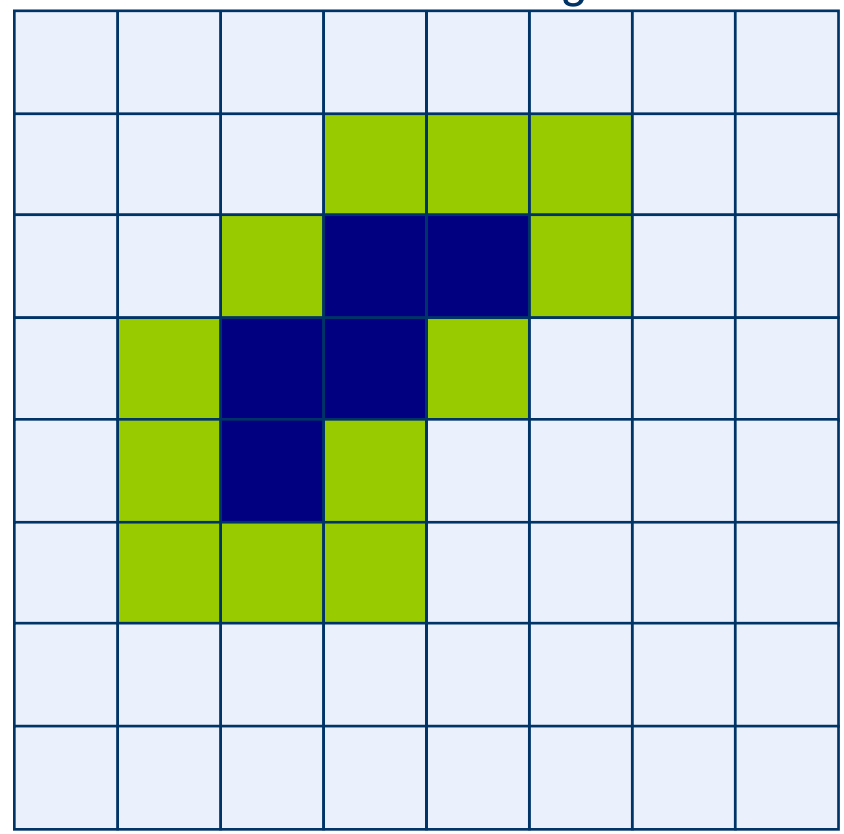


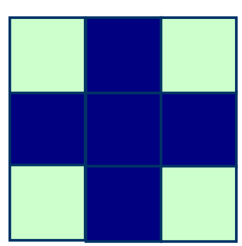
Erosion: Example 1

Original Image



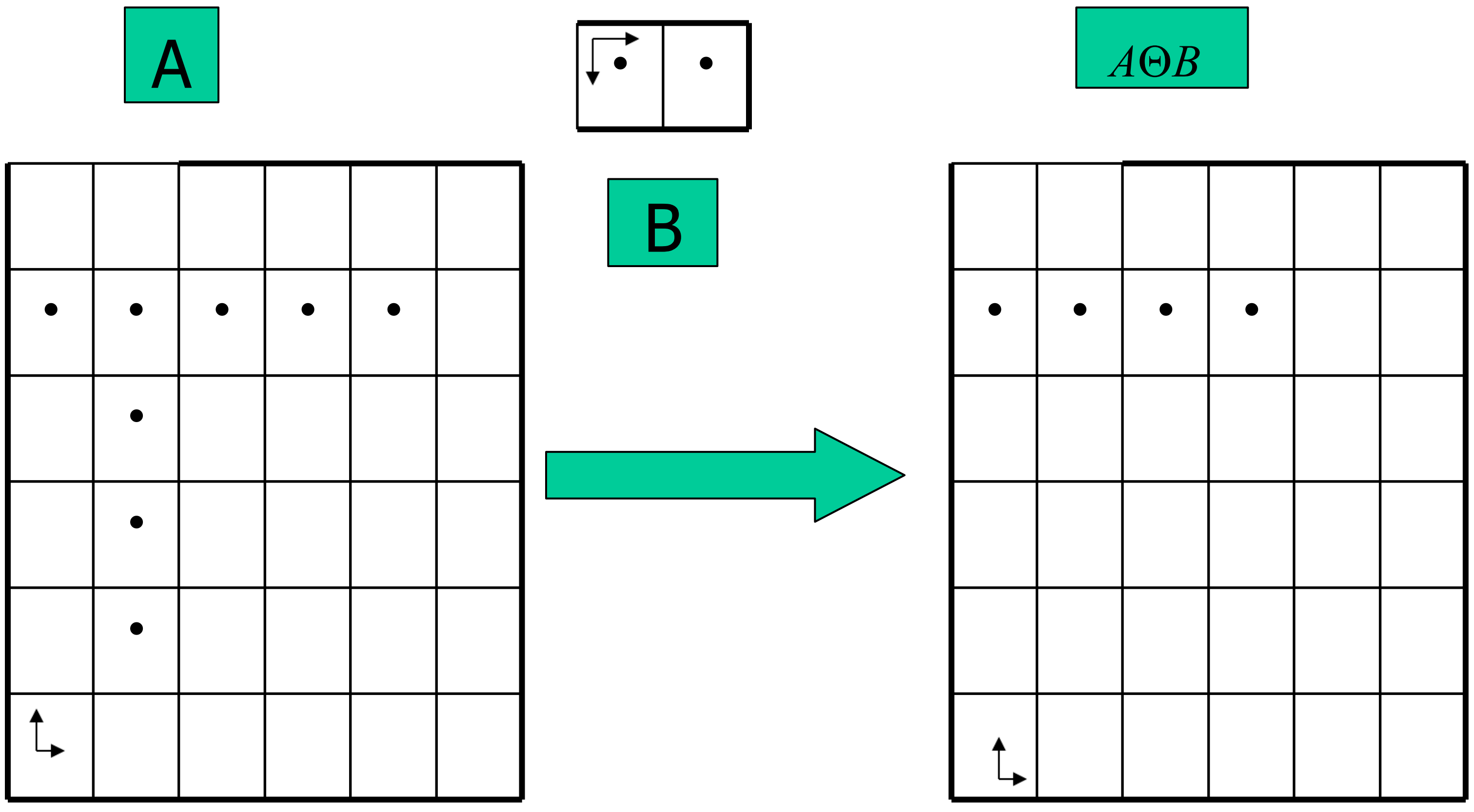
Processed Image



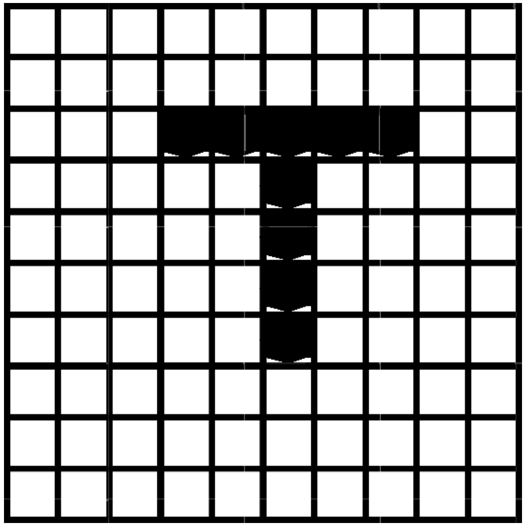


Structuring Element

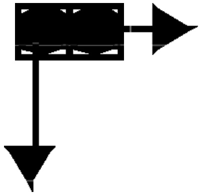
Erosion explained pixel by pixel



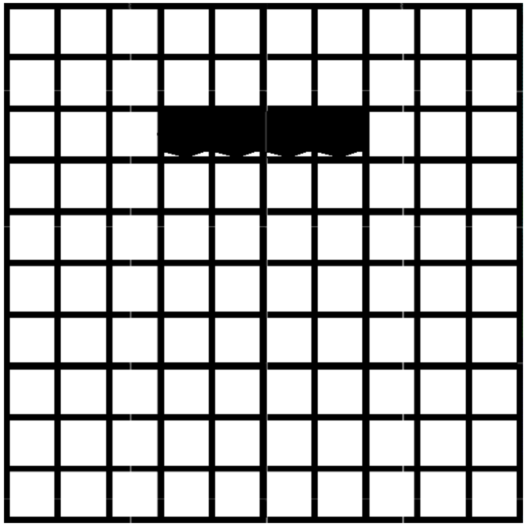
Structuring Element in Erosion Example



Image

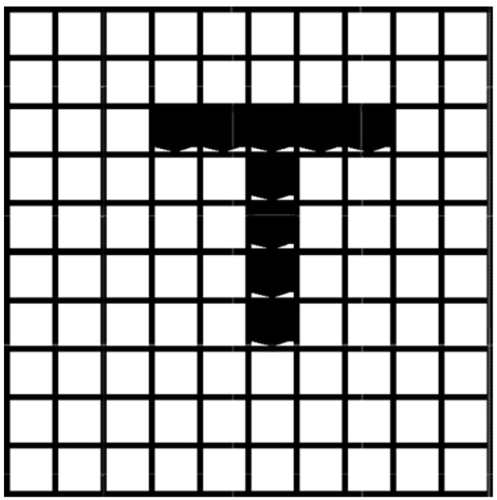


Structuring Element

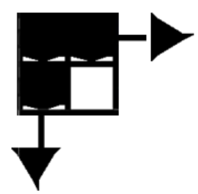


Result

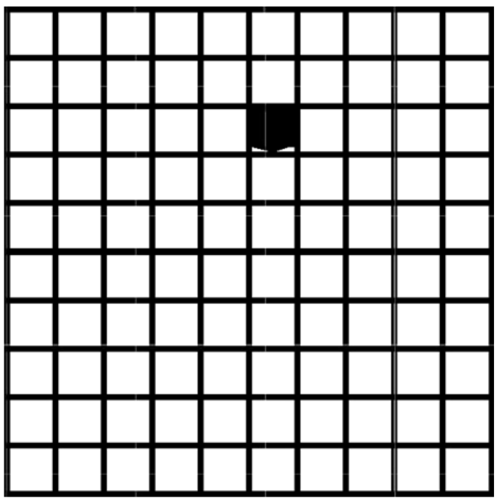
Structuring Element in Erosion Example



Image

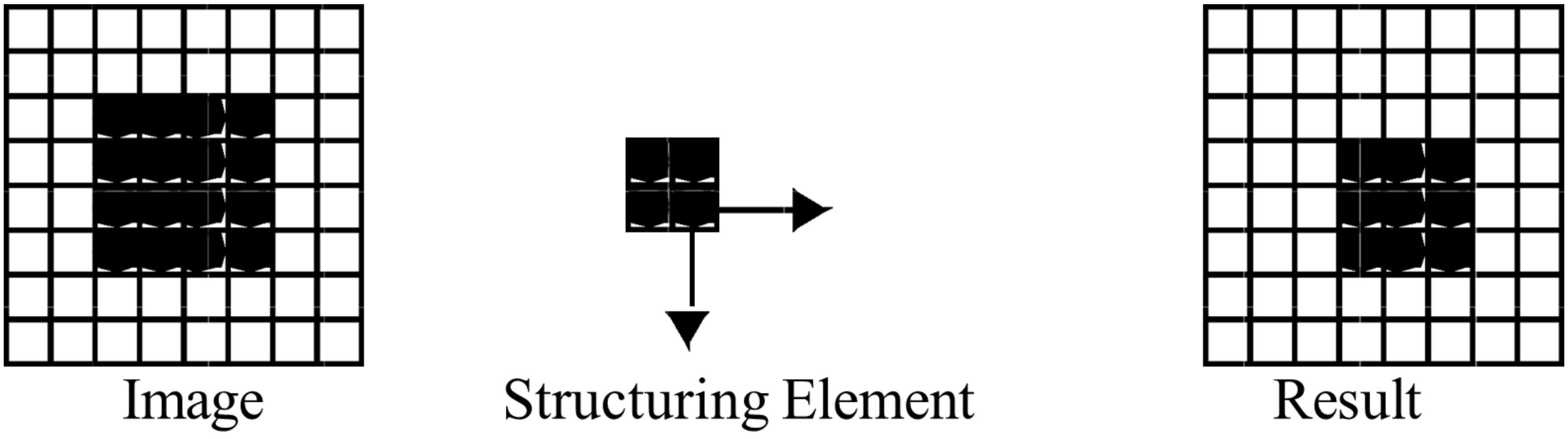


Structuring Element

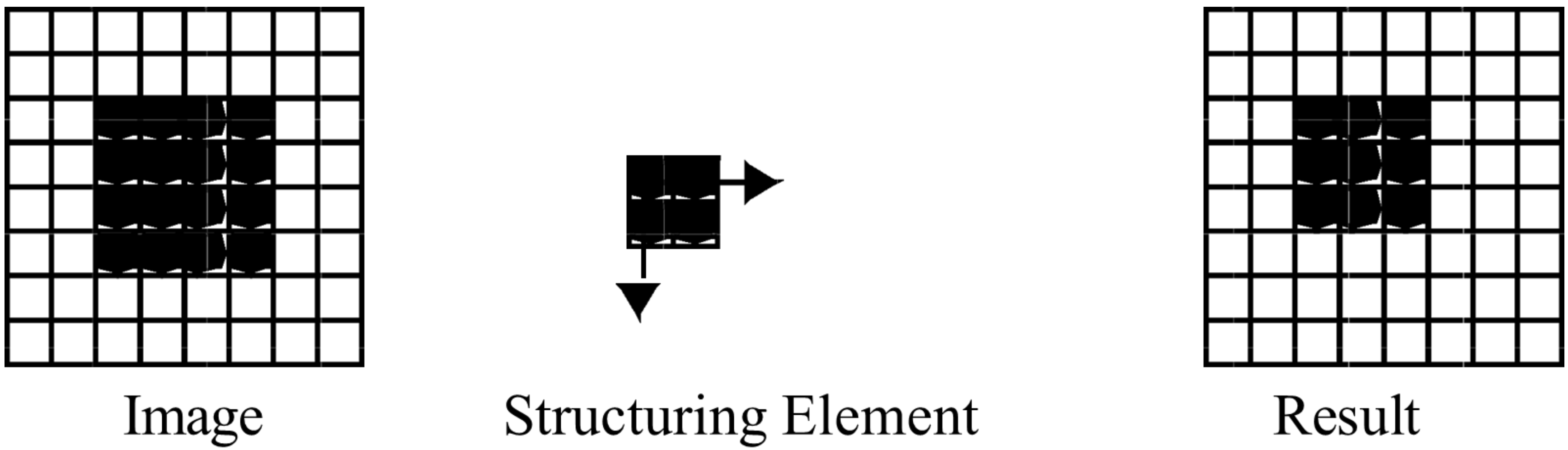


Result

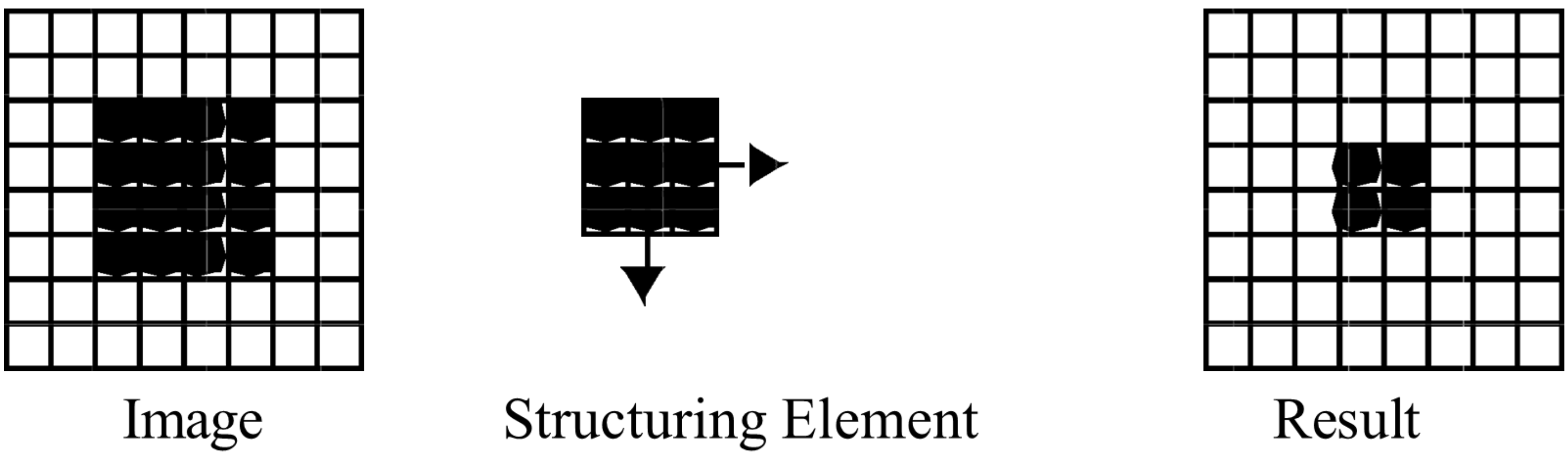
Structuring Element in Erosion Example



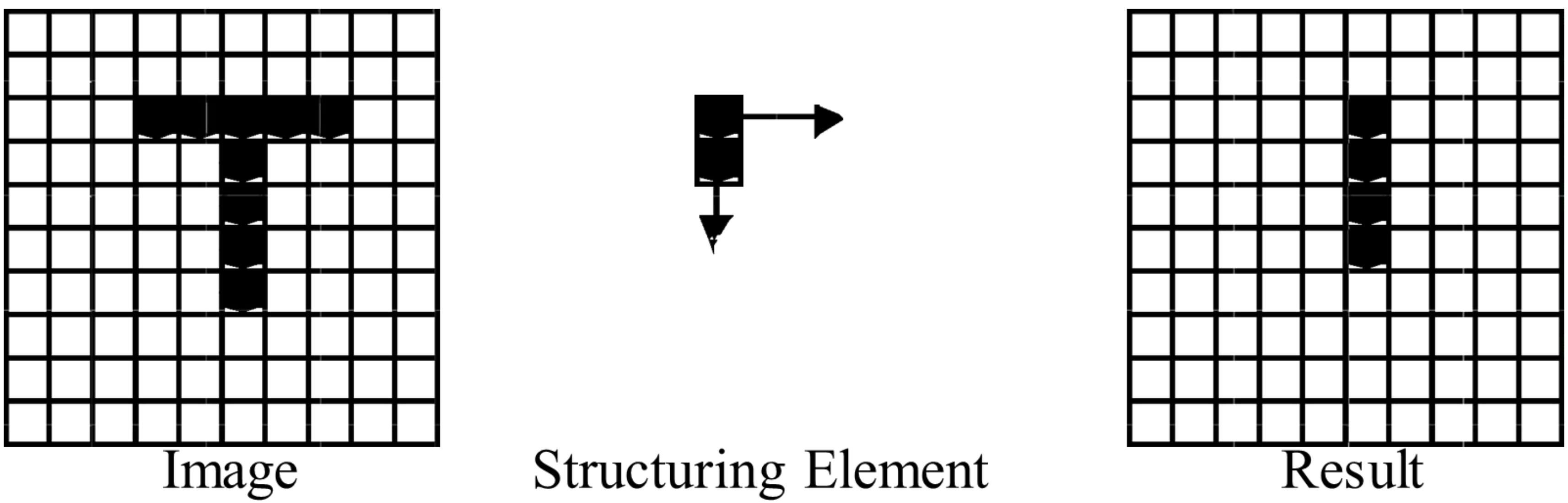
Structuring Element in Erosion Example



Structuring Element in Erosion Example



Structuring Element in Erosion Example



Erosion: Example 2

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	0	0	0	0	1	1	1	0	0
0	0	1	1	1	1	0	0	0	0	1	1	1	1	0	0
0	0	0	1	1	0	0	0	0	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0
0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0
0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0
0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0
0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0
0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0
0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Erosion with a structuring element of size 3x3

1	1	1
1	1	1
1	1	1

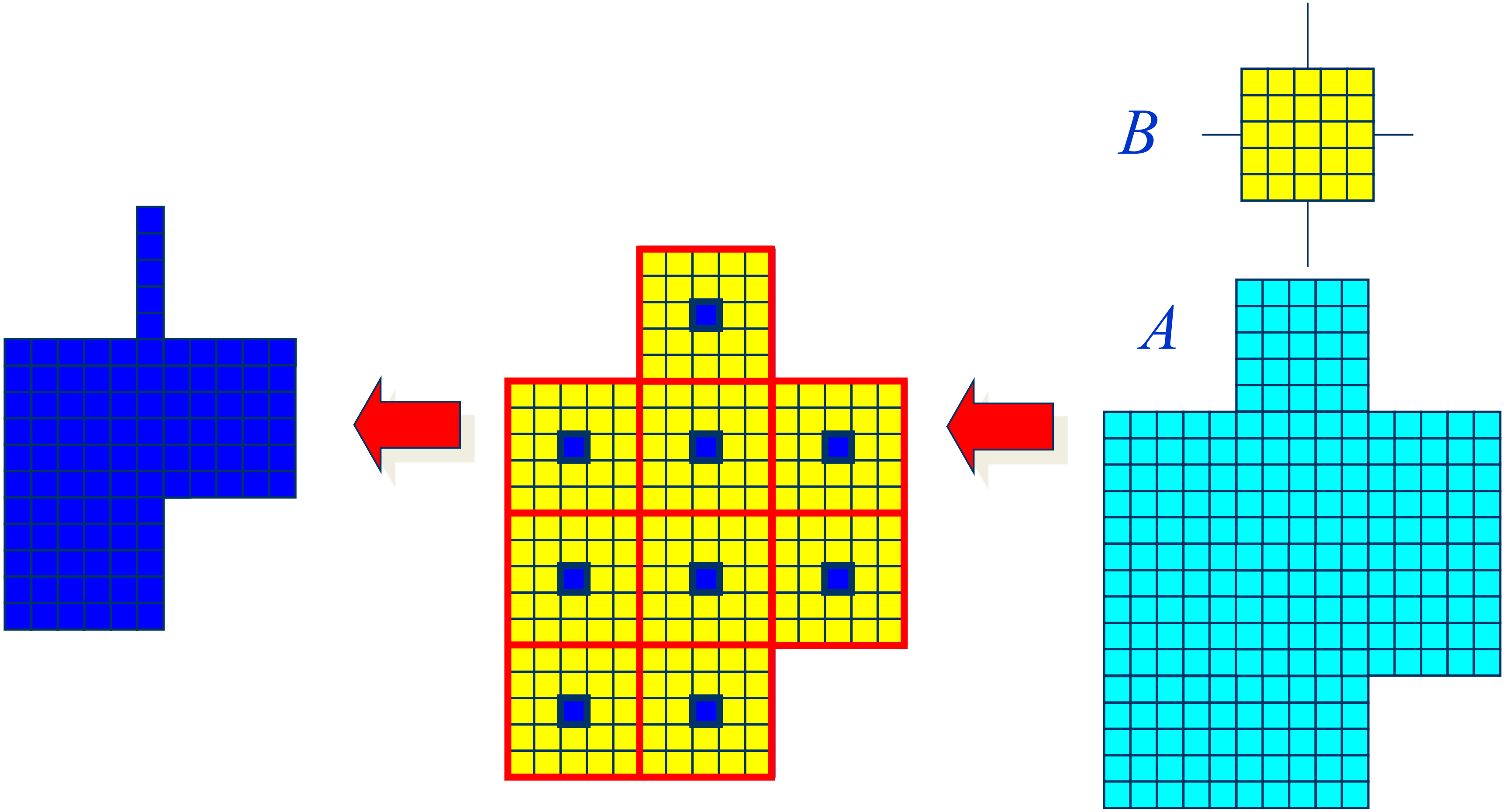
Set of coordinate points =

{ (-1, -1), (0, -1), (1, -1),

(-1, 0), (0, 0), (1, 0),

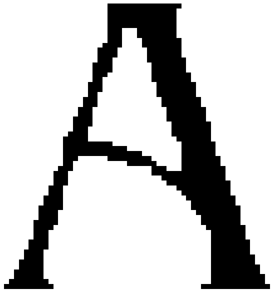
(-1, 1), (0, 1), (1, 1) }

Erosion: Example 3



Erosion: Example 4

Original image



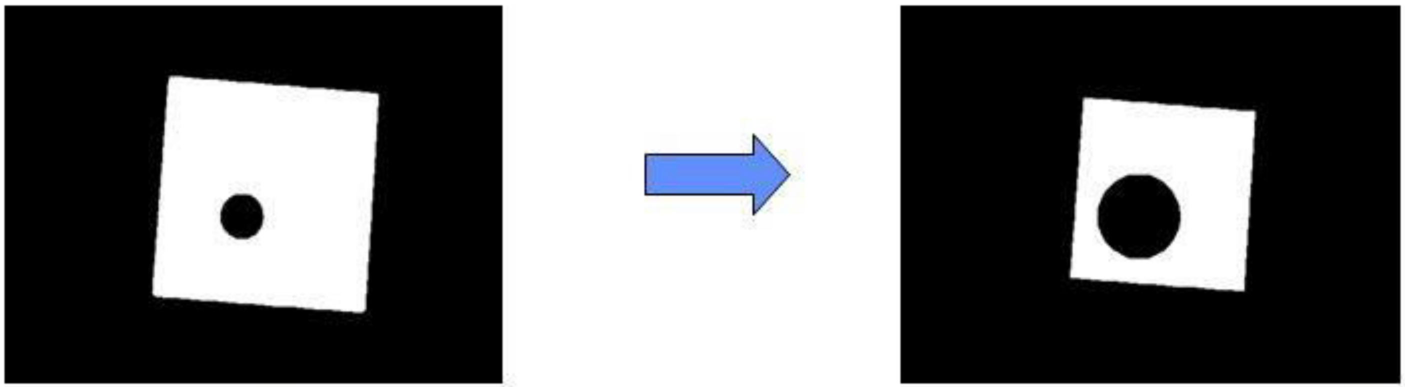
Erosion by 3*3 square structuring element

Erosion by 5*5 square structuring element

Note: In these examples a 1 refers to a black pixel!

Erosion: Example 5

⇨ Example: Binary erosion



Original thresholded image

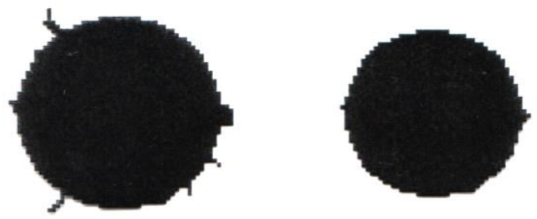
Result of erosion four times with a disk shaped structuring element of 11 pixels in diameter

- It shows that the hole in the middle of the image increases in size as the border shrinks.
- Erosion using a disk shaped structuring element will tend to round concave boundaries, but will preserve the shape of convex boundaries.

Erosion: Example 6

Erosion can split apart joined objects

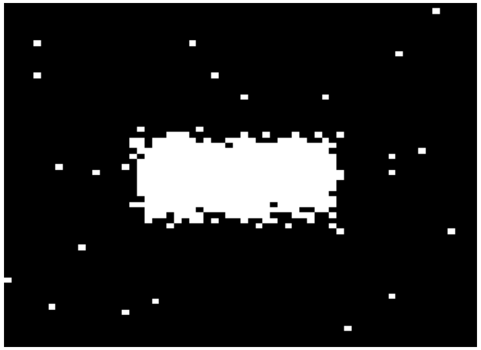
Erosion can strip away extrusions



Watch out: Erosion shrinks objects

Typical use of Erosion

- Removes isolated noisy pixels.
- Smoothes object boundary(removes spiky edges).
- Removes the outer layer of object pixels:



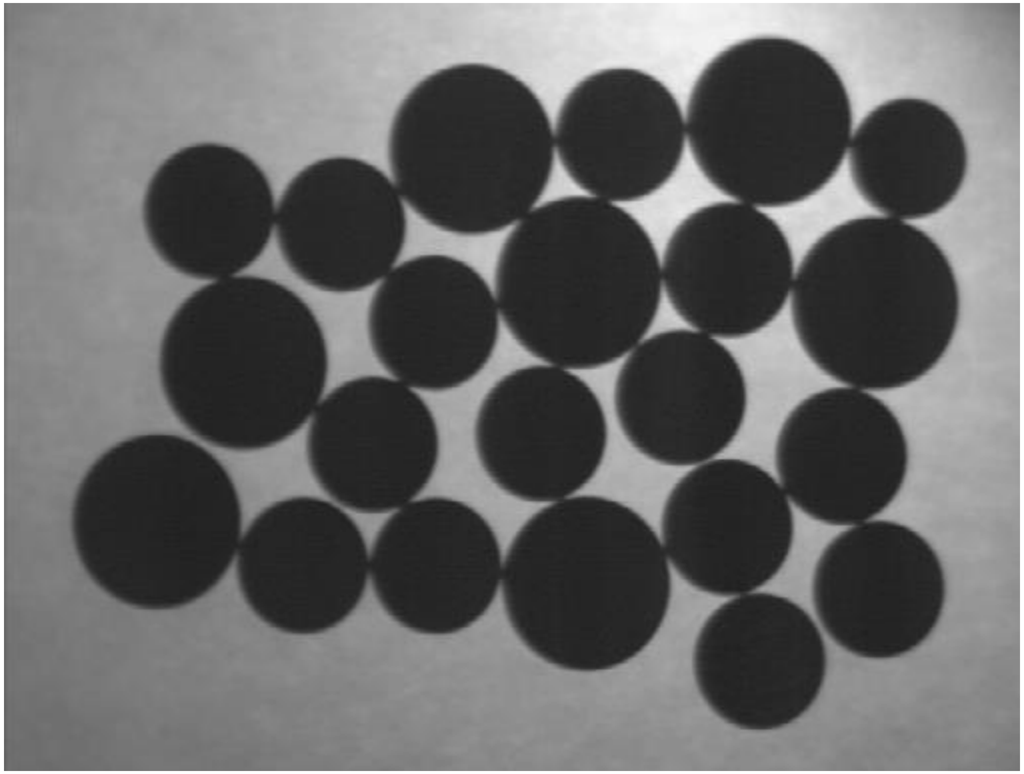
- Object becomes slightly smaller.
- Sets contour pixels of object to background value

Erosion

- Effects
  - Shrinks the size of foreground (1-valued) objects
  - Smoothes object boundaries
  - Removes small objects
- Rule for Erosion
  In a binary image, if any of the pixel (in the neighborhood defined by structuring element) is 0, then output is 0

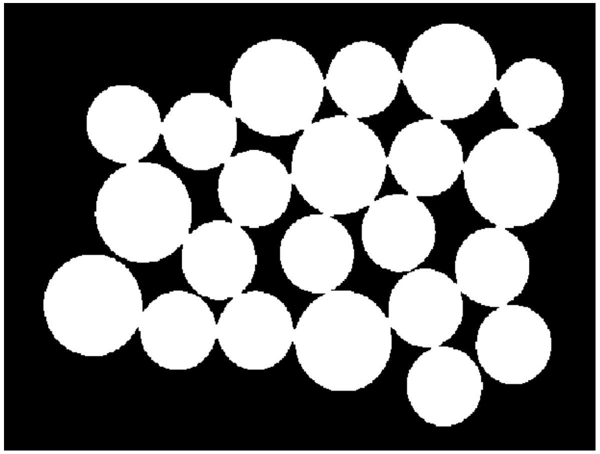
Exercise

- Count the number of coins in the given image performing Erosion operation.

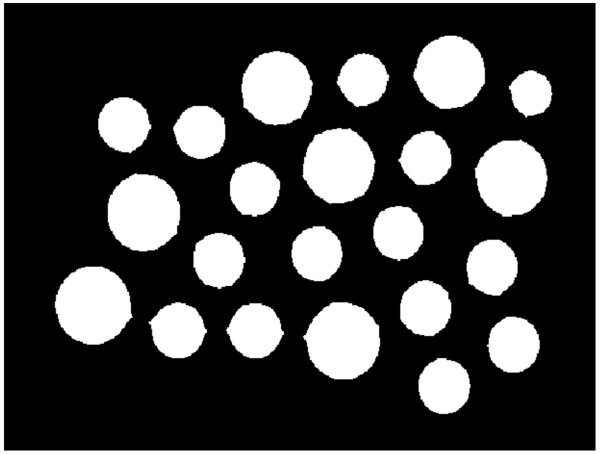


Exercise: Solution

Binarize the image

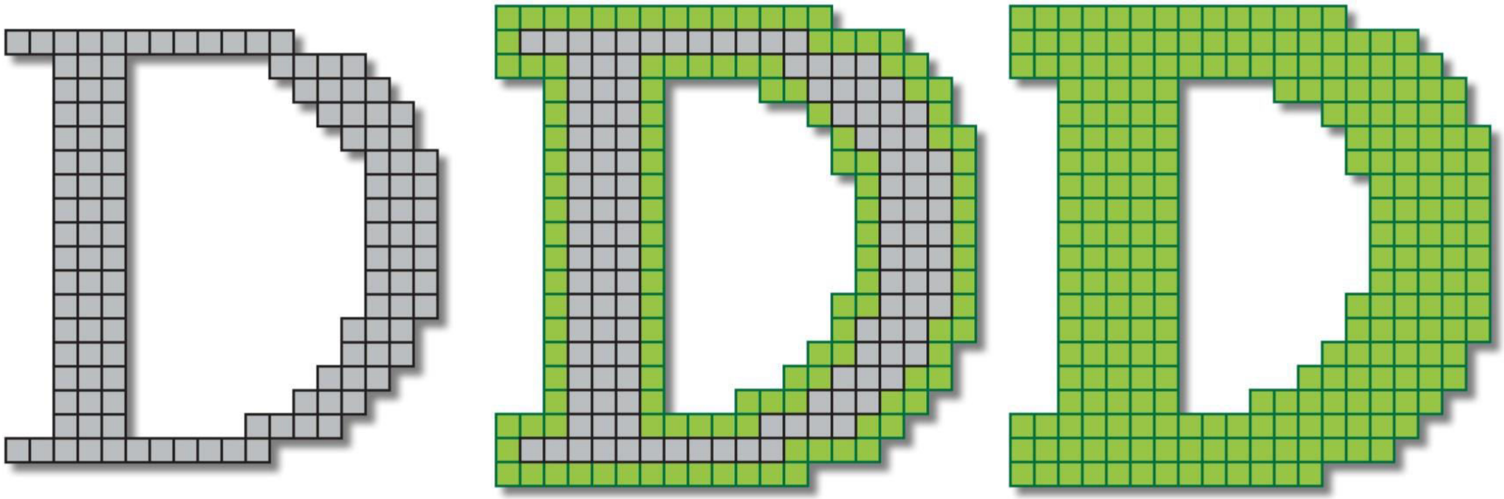


Perform Erosion



Use connected component labeling to count the number of coins

Dilation

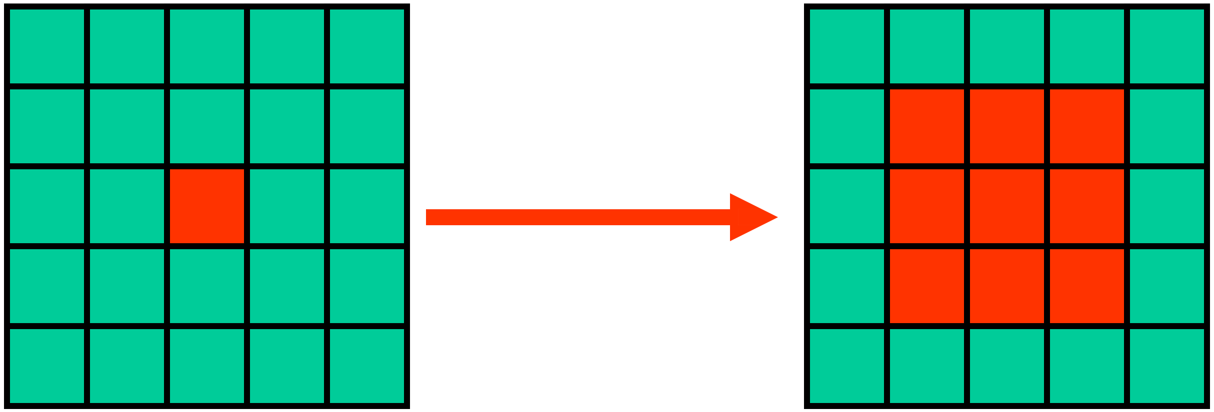


Grows the object

Enlarges foreground, shrinks background

Dilation

- Dilation is used for expanding an element A by using structuring element B.
- The dilation operator takes two pieces of data as input
    1. A binary image, which is to be dilated
    2. A structuring element (or kernel), which determines the behavior of the morphological operation



Original Image Dilated Image

Dilation

Hit: Any "***ON"*** *pixel* in the structuring element covers an "***ON"*** *pixel* in the image

Definition 1:

- Does the structuring element **hit the set?**

$$A \oplus B = \left\{ z \mid (\hat{B})_z \cap A \neq \Phi \right\}$$

- Dilation of a set A by structuring element B:
  - all z in A such that B hits A when origin of B=z
  - such that overlap A by at least one element

Dilation

Definition 2:

Dilation of image *f* by structuring element *s* is given by

$f \oplus s$

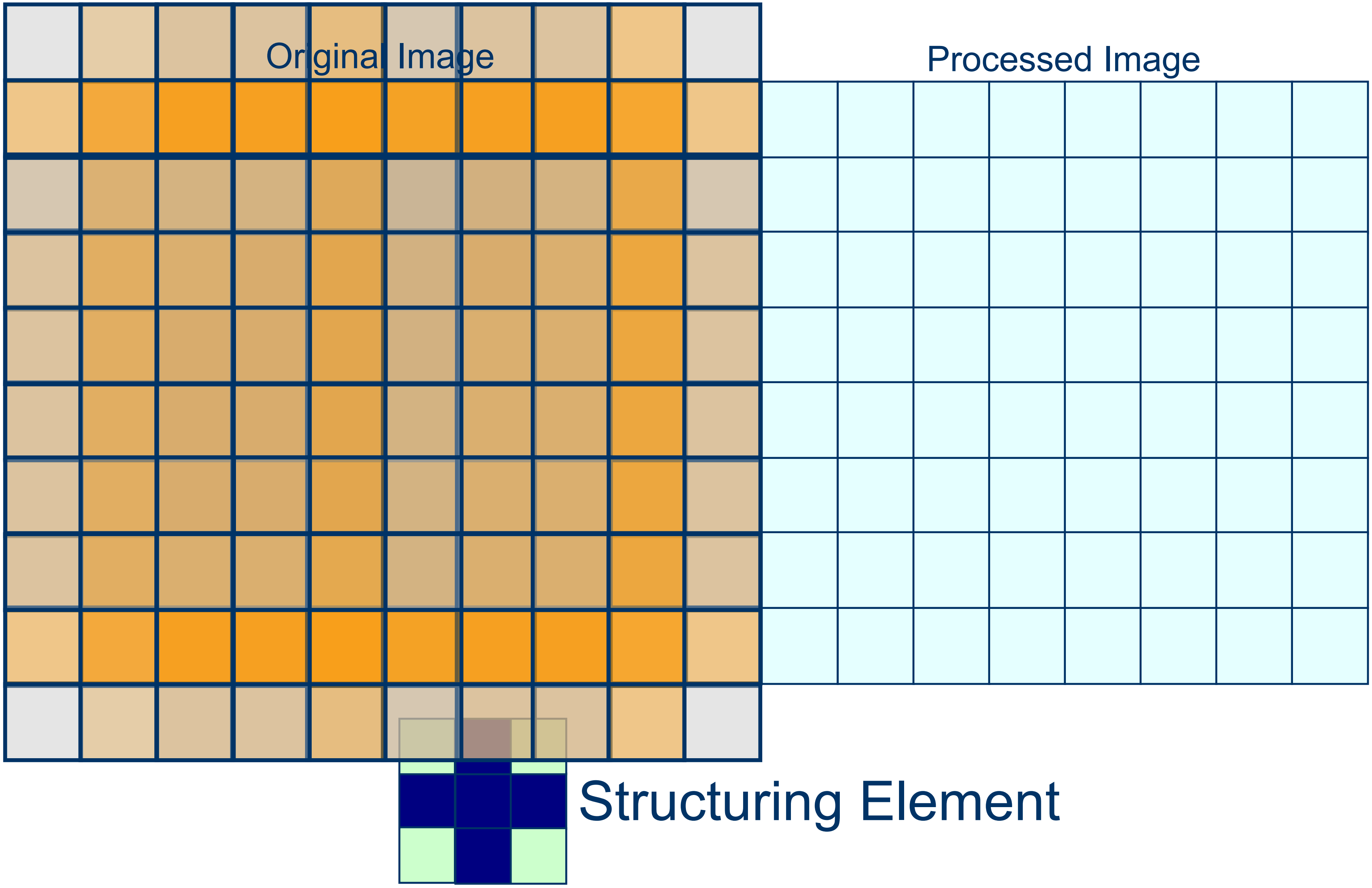
The structuring element s is positioned with its origin at *(x, y)* and the new pixel value is determined using the rule:

$$g(x,y)=\begin{cases}1 \text{ if } s \text{ hits } f\\ 0 \text{ otherwise}\end{cases}$$

Dilation – How to compute

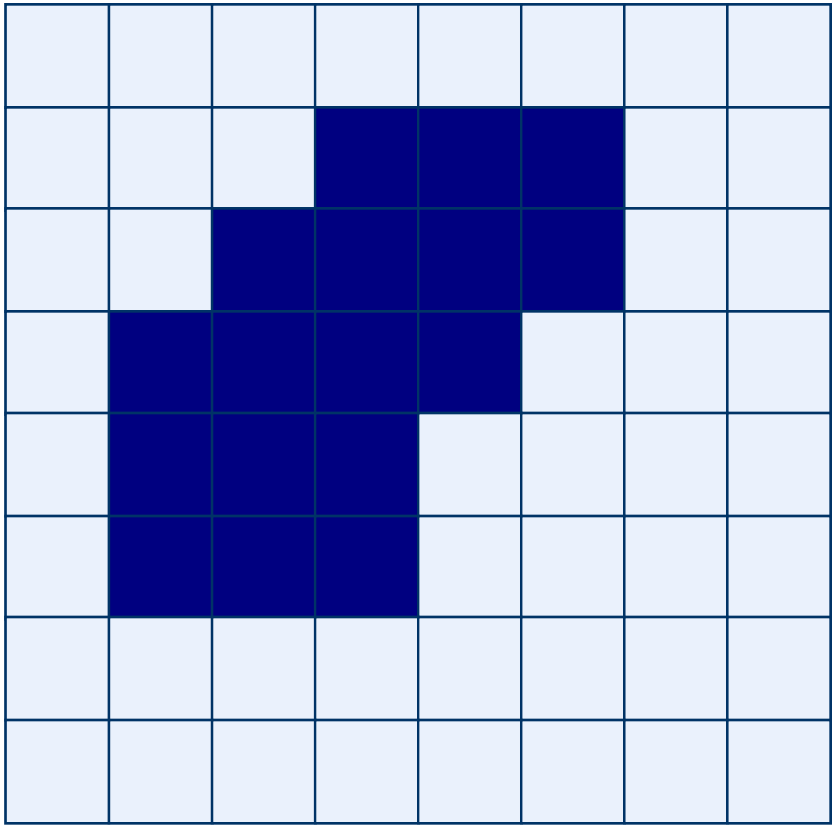
- For each background pixel (which we will call the *input pixel*)
  - Superimpose the structuring element on top of the input image so that the origin of the structuring element coincides with the input pixel position
  - If *at least one* pixel in the structuring element coincides with a foreground pixel in the image underneath, then the input pixel is set to the foreground value
  - If all the corresponding pixels in the image are background, however, the input pixel is left at the background value

Dilation: Example

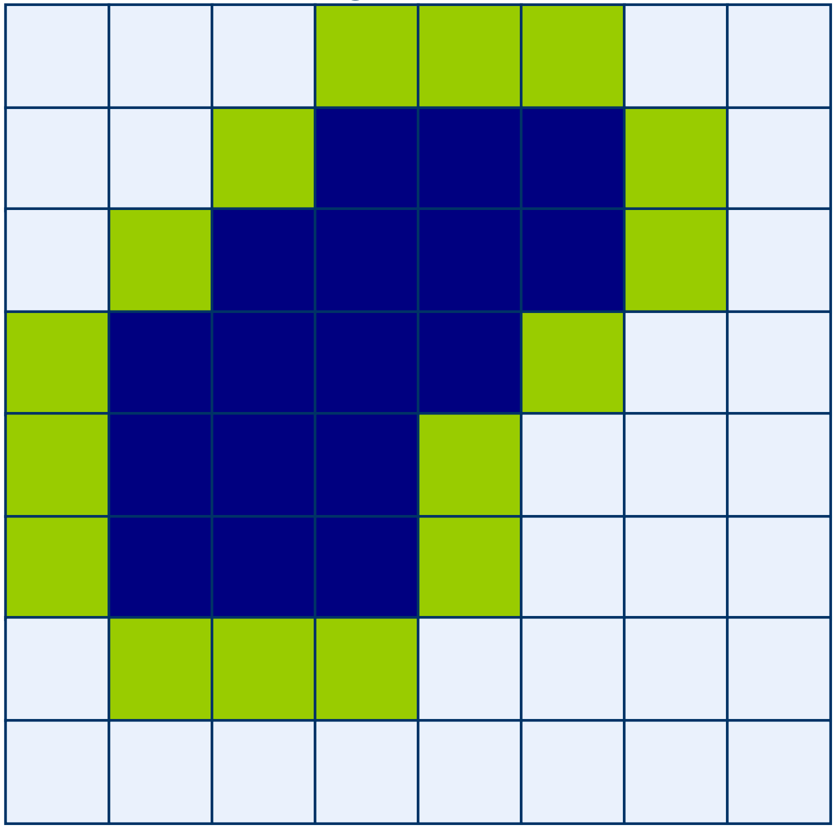


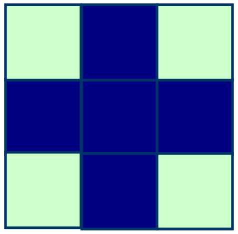
Dilation: Example 1

Original Image



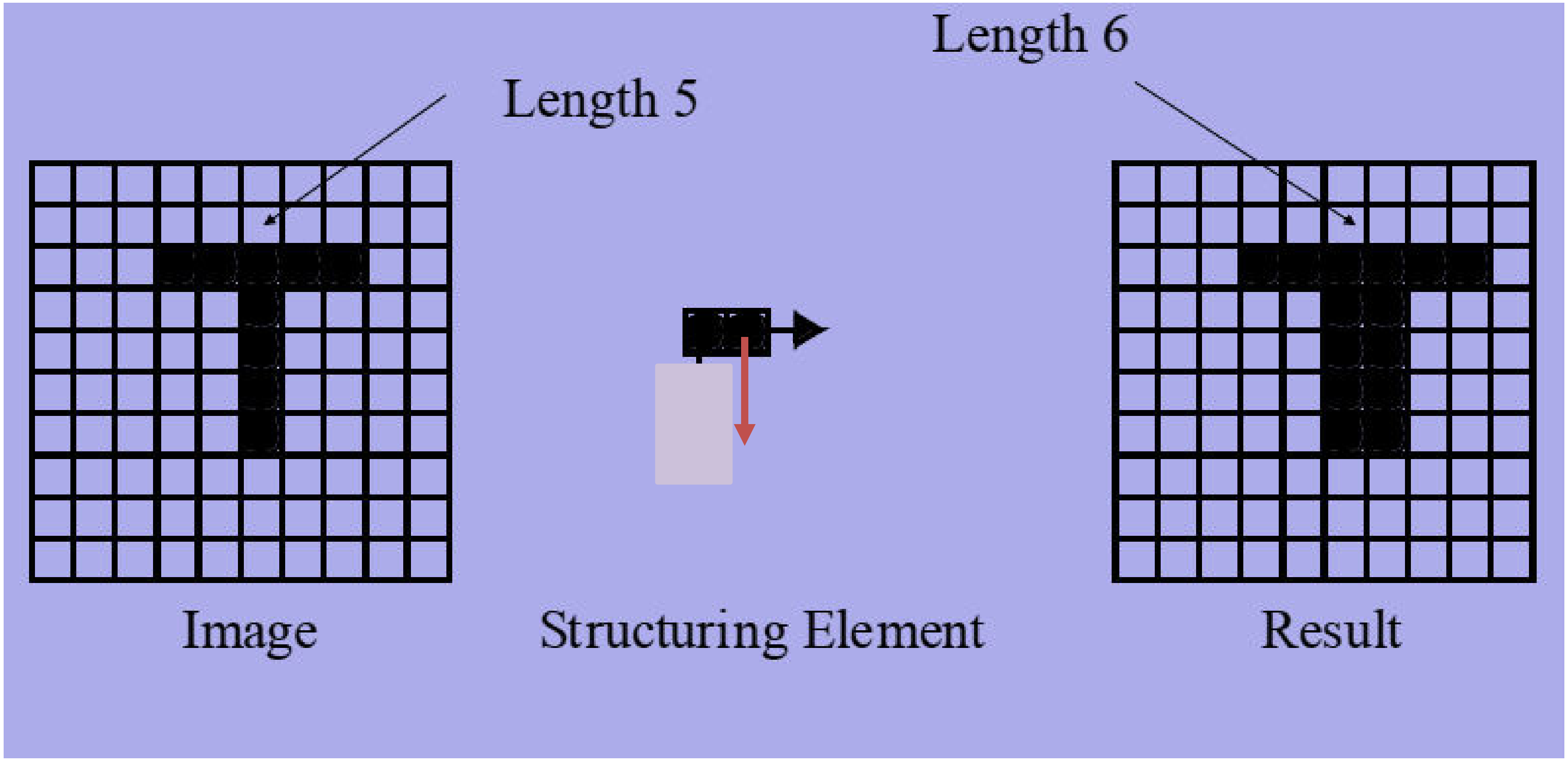
Processed Image With Dilated Pixels



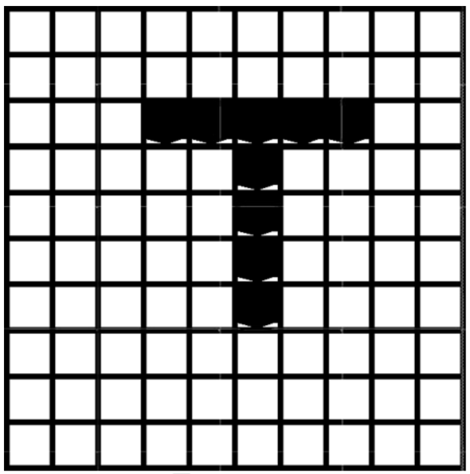


Structuring Element

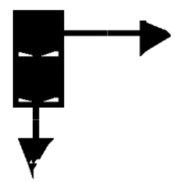
Structuring Element in Dilation Example



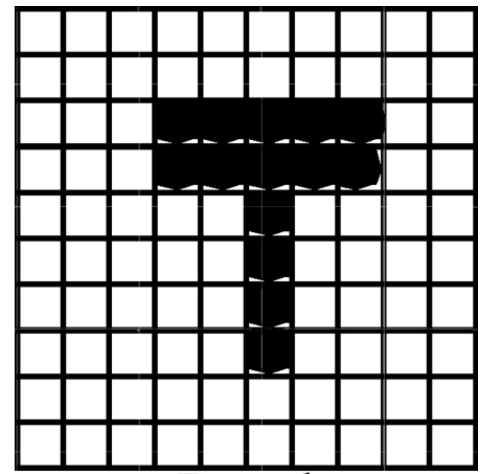
Structuring Element in Dilation Example



Image

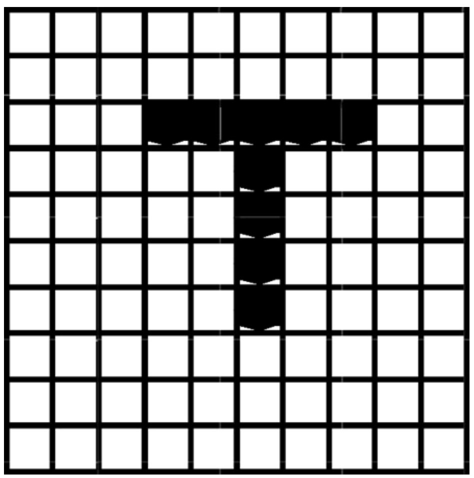


Structuring Element

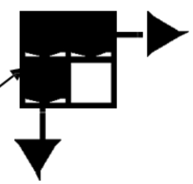


Result

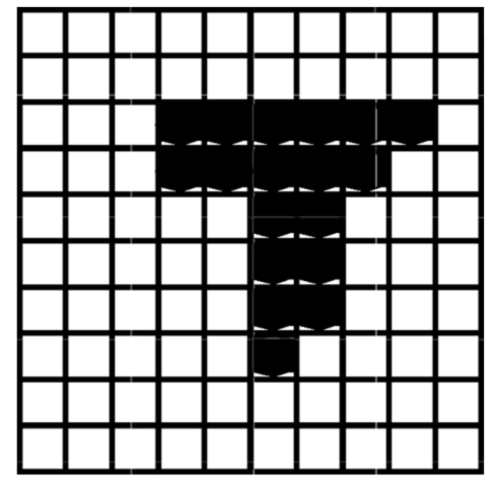
Structuring Element in Dilation Example



Image



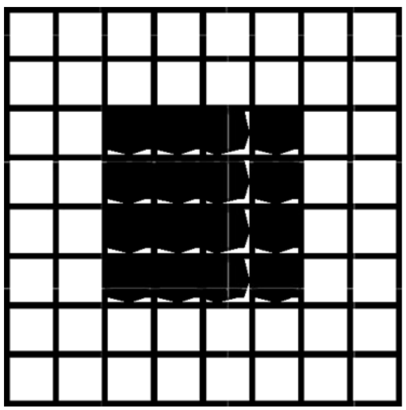
Structuring Element



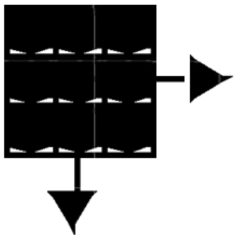
Result

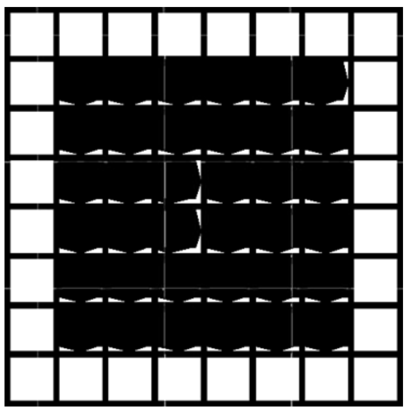
Single point in Image replaced with this in the Result

Structuring Element in Dilation Example



Image


Structuring Element


Result

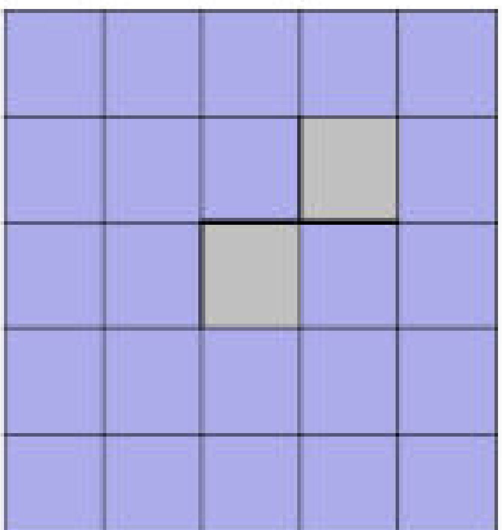
Dilation

Question: Suppose that the structuring element is a 3x3 square with the origin at its center evaluate the new image

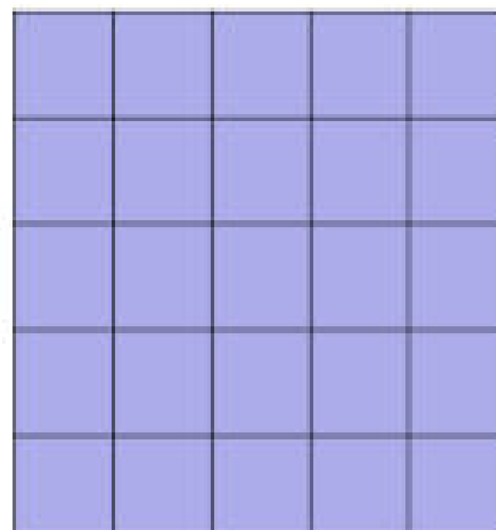
$$B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

{ (-1,-1), (0,-1), (1,-1),
(-1,0), (0,0), (1,0),
(-1,1), (0,1), (1,1) }

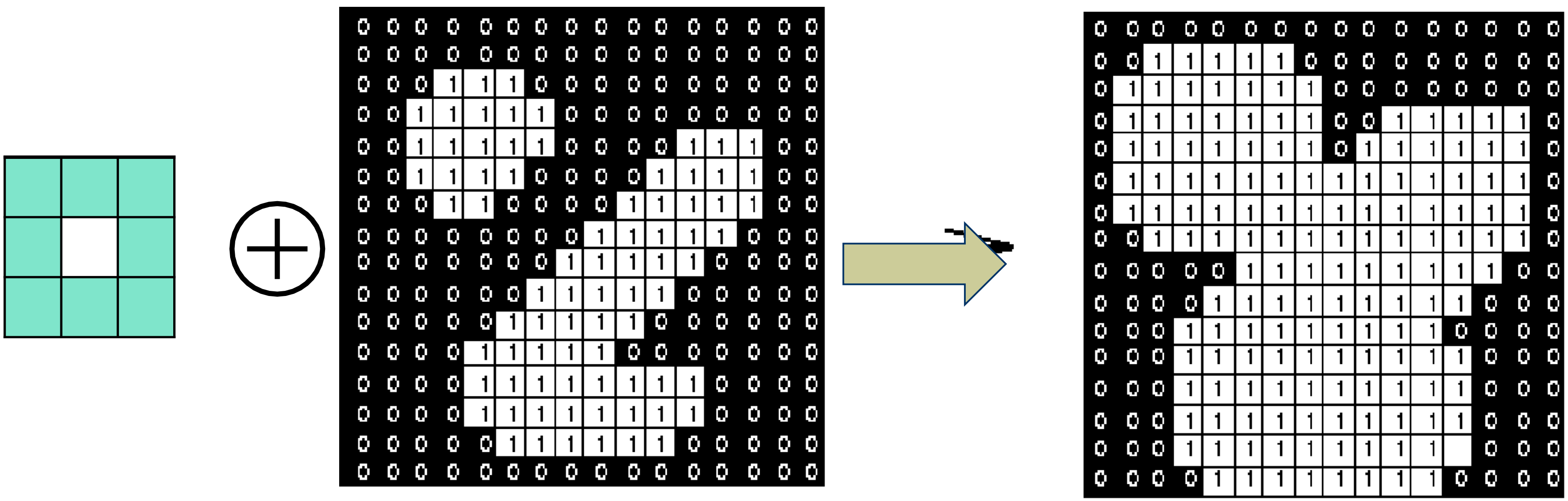
$A =$



$A \oplus B$

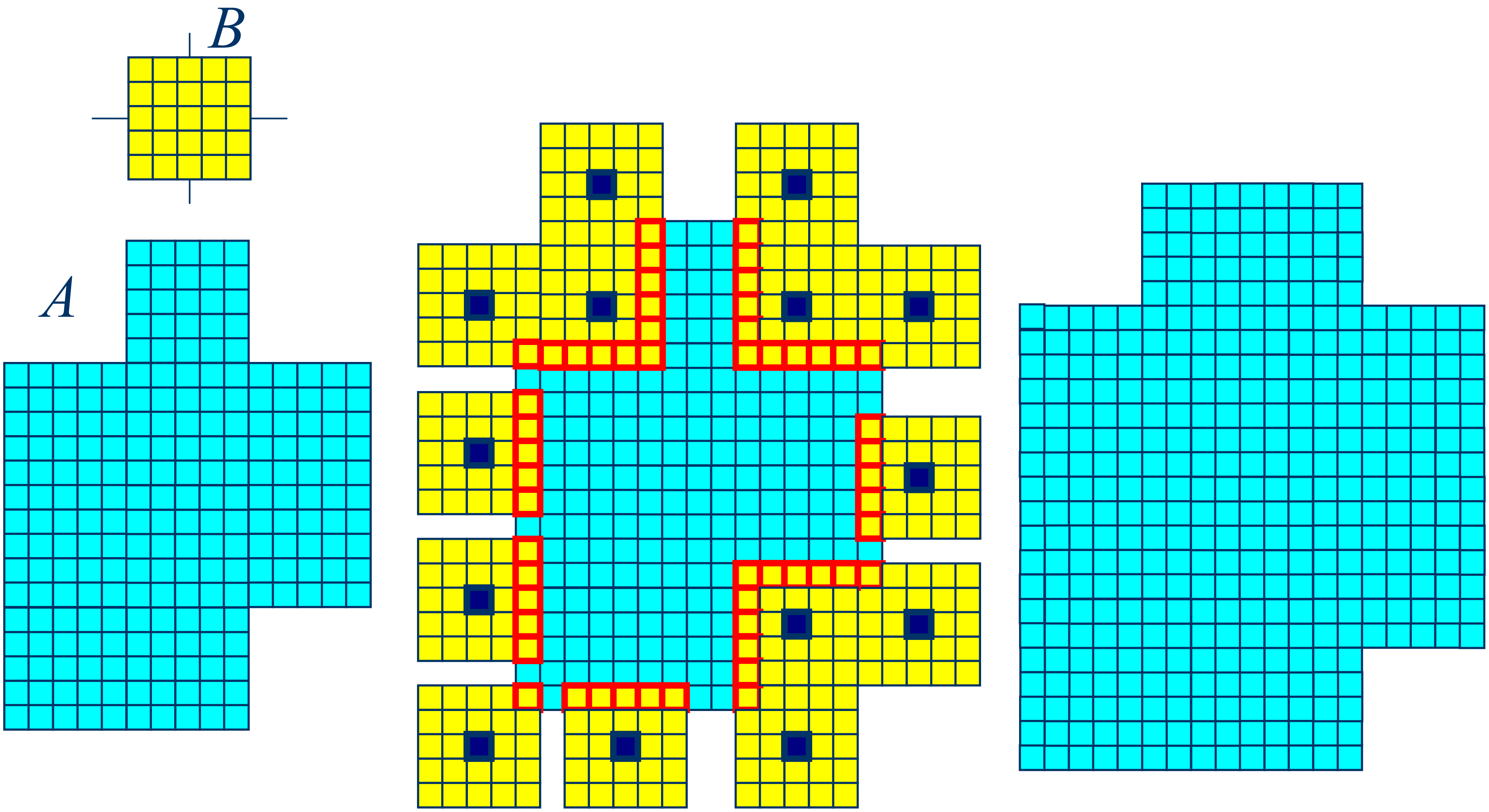


Dilation: Example 2



Effect of dilation using a 3 ×3 square structuring element

Dilation: Example 3



Dilation: Example 4

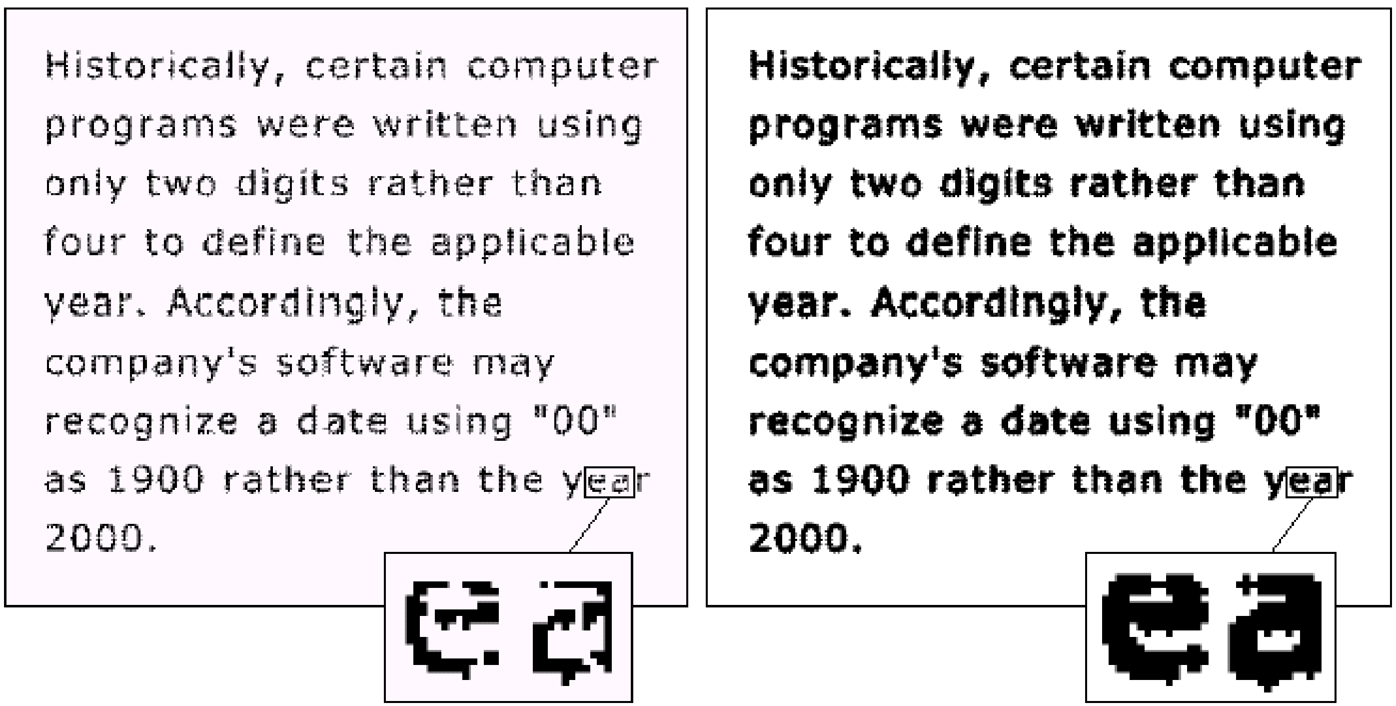
Original image

Dilation by 3*3 square structuring element

Dilation by 5*5 square structuring element

Note: In these examples a 1 refers to a black pixel!

Dilation : Bridging gaps

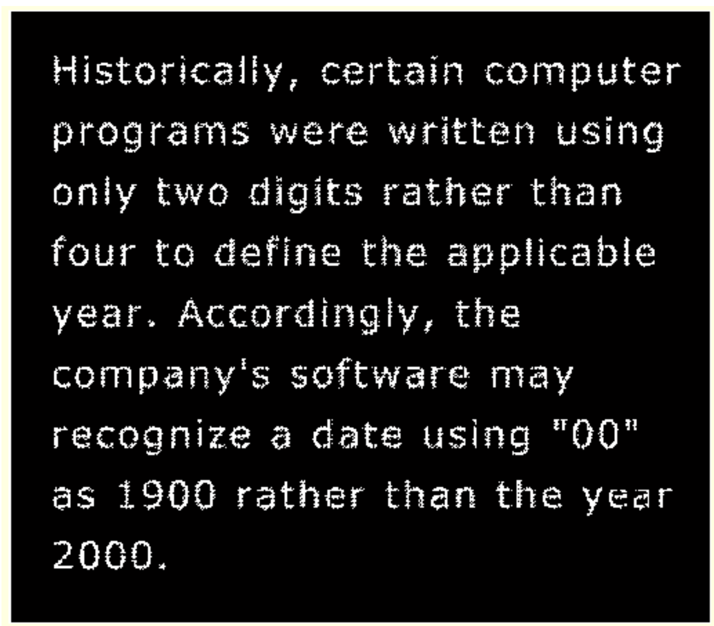


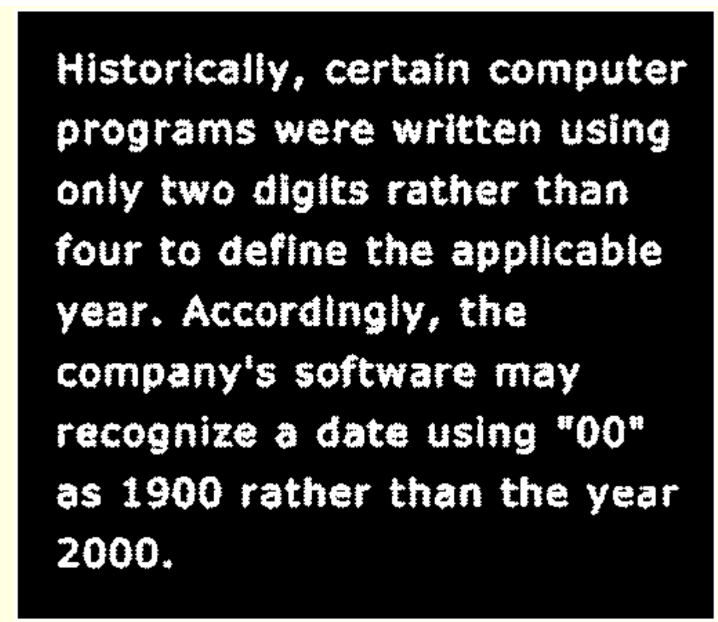
0	1	0
1	1	1
0	1	0

a c
b

FIGURE 9.5
(a) Sample text of poor resolution with broken characters (magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.

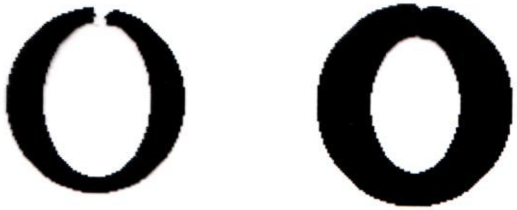
Dilation: Example 5



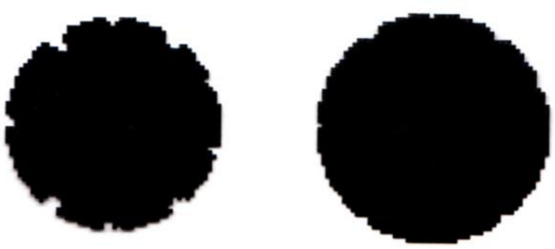


Dilation: Example 6

Dilation can repair breaks



Dilation can repair intrusions

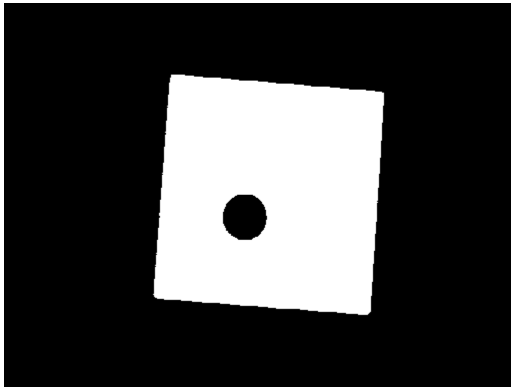
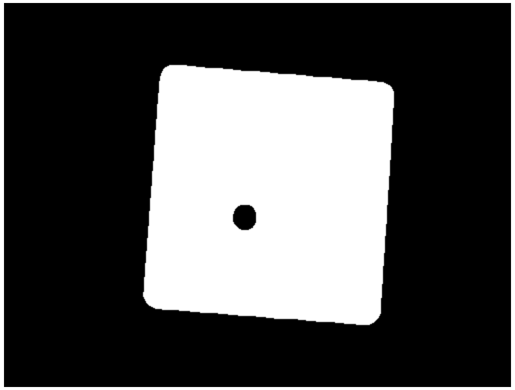
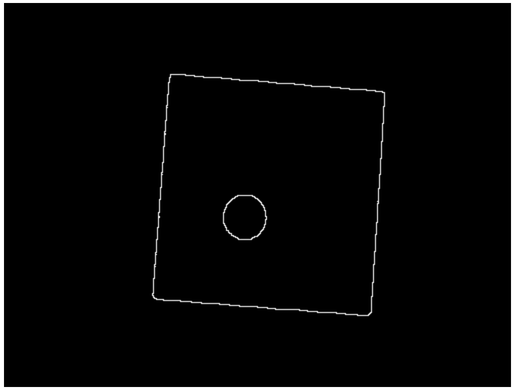


Watch out: Dilation enlarges objects

Dilation: Example 7

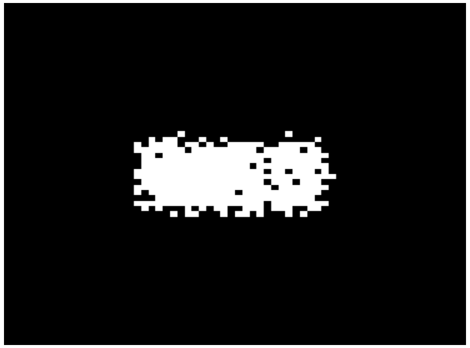
- Edge Detection

1. Dilate input image
2. Subtract input image from dilated image
3. Edges remain!

Typical use of Dilation

- Fills in holes.
- Smoothens object boundaries.
- Adds an extra outer ring of pixels onto object boundary, ie, object becomes slightly larger.



Dilation

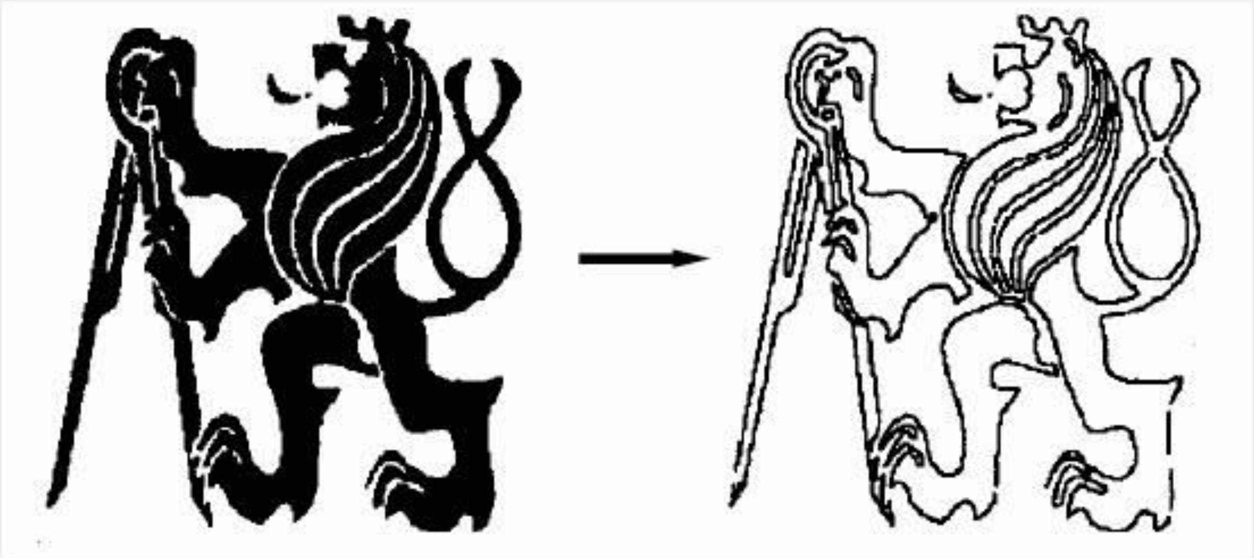
- Effects
  - Expands the size of foreground(1-valued) objects
  - Smoothes object boundaries
  - Closes holes and gaps

- Rule for Dilation
  In a binary image, if any of the pixel (in the neighborhood defined by structuring element) is 1, then output is 1

More Applications of Erosion and Dilation

Boundary Extraction

Contours can be extracted by subtraction of the eroded image from the original.

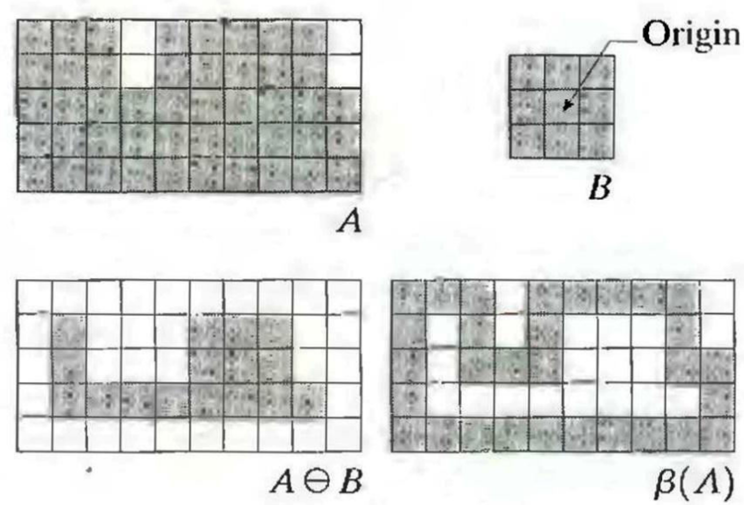


$$\beta(A) = A - (A \ominus B)$$

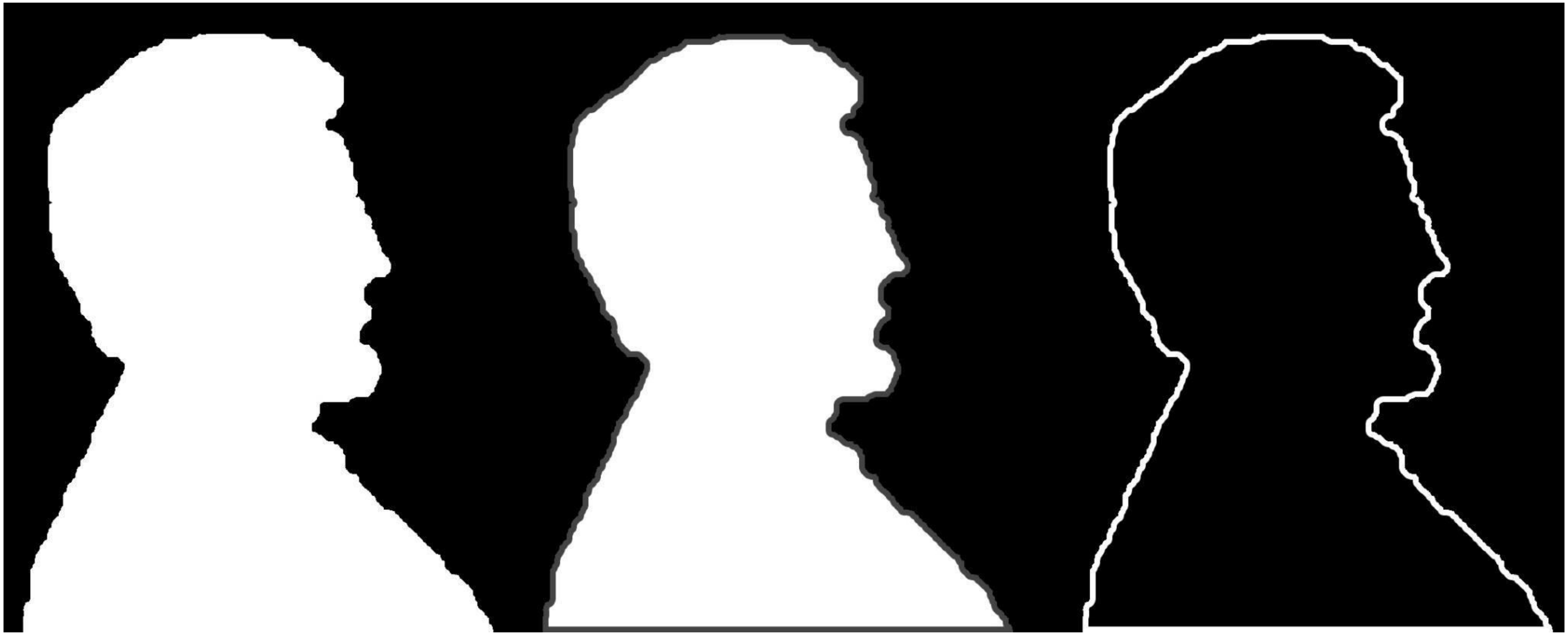
Boundary Extraction

- First, erode A by B, then make set difference between A and the erosion
- The thickness of the contour depends on the size of constructing object – B

$$\beta(A) = A - (A \ominus B)$$



Boundary Extraction



$$\beta(A) = A - (A \ominus B)$$

Edge detection

original | Dilate | Dilate - original

A | $A \oplus B$ | $A \oplus B \dot{-} A$

Duality relationship between Dilation and Erosion

- When one operation is the dual of the other, it means that one can be written in terms of the other. This does not, however, mean that they are opposites.

- Dilation and erosion are duals of each other:

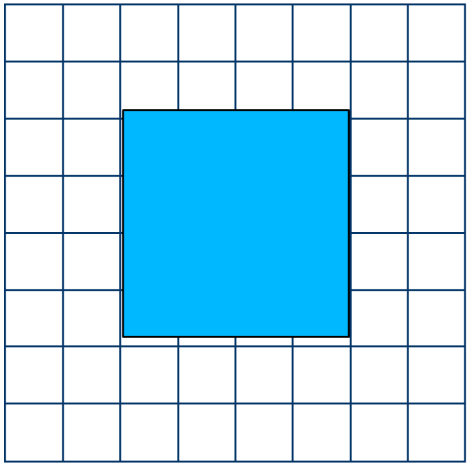
$$\left(A \ominus B\right)^c = A^c \oplus \hat{B}$$

- For a symmetric structuring element:

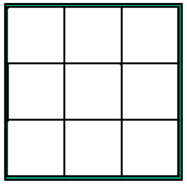
$$\left(A \oplus B\right)^c = A^c \ominus \hat{B}$$

It means that we can obtain erosion of an image A by B simply by dilating its background (i.e. A^c) with the same structuring element and complementing the result.

Duality of Dilation and Erosion Example



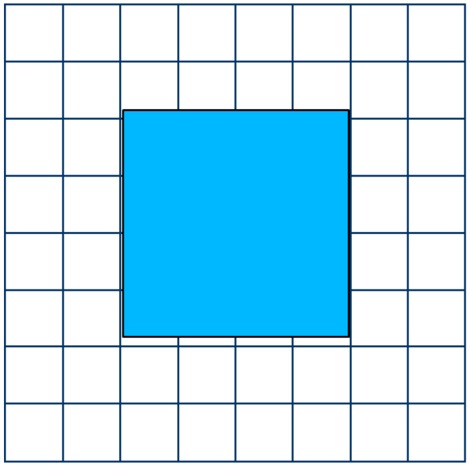
A



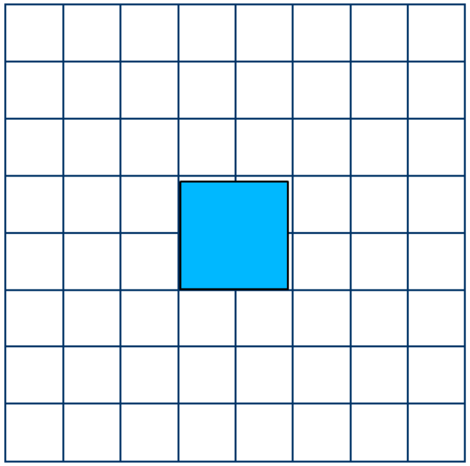
B

$$\left(A \ominus B\right)^c = A^c \oplus \hat{B}$$

Duality of Dilation and Erosion Example



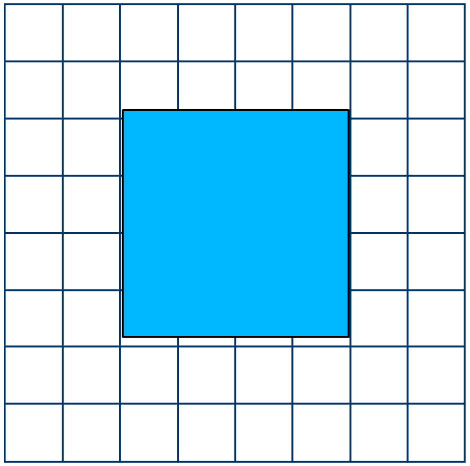
A



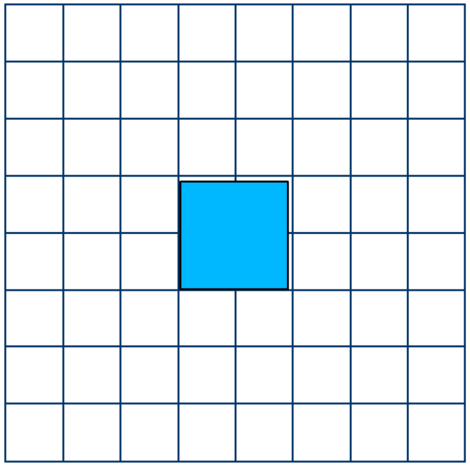
$\left(A \ominus B\right)$

$$\left(A \ominus B\right)^c = A^c \oplus \hat{B}$$

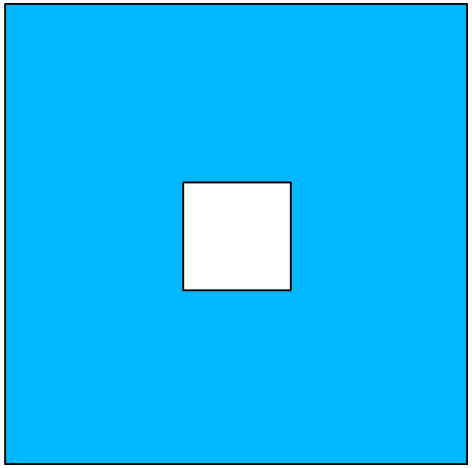
Duality of Dilation and Erosion Example



A



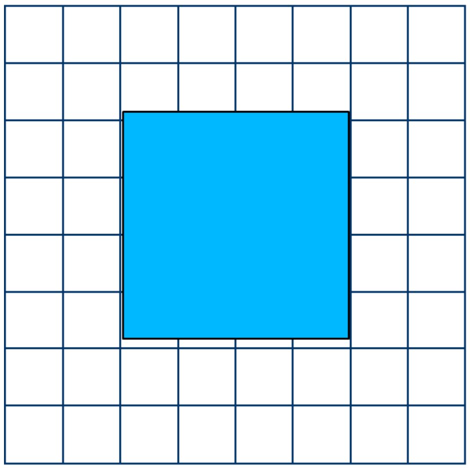
$\left(A \ominus B\right)$



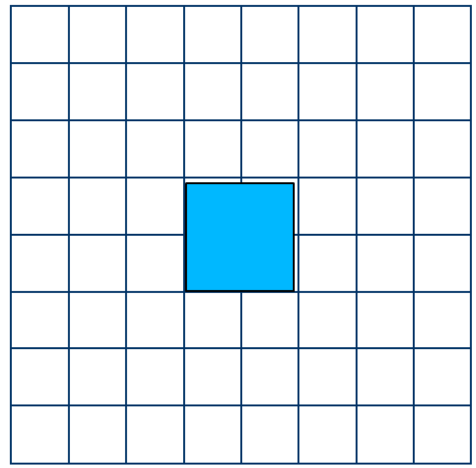
$\left(A \ominus B\right)^c$

$$\left(A \ominus B\right)^c = A^c \oplus \hat{B}$$

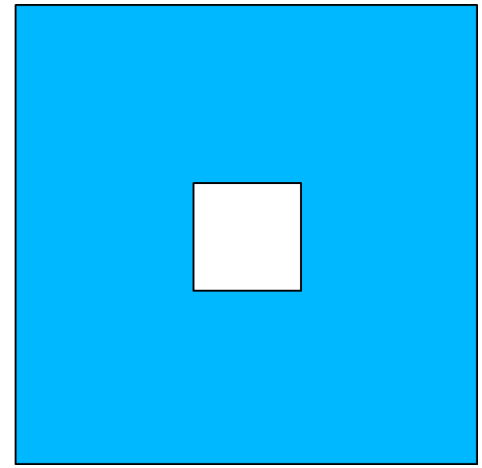
Duality of Dilation and Erosion Example



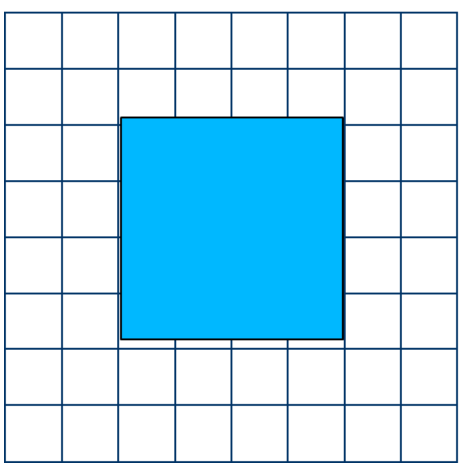
A



$(A \ominus B)$



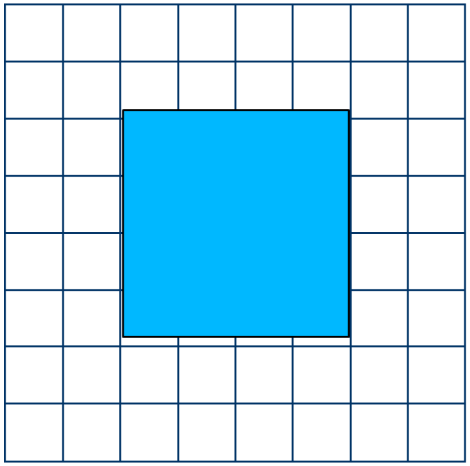
$(A \ominus B)^c$



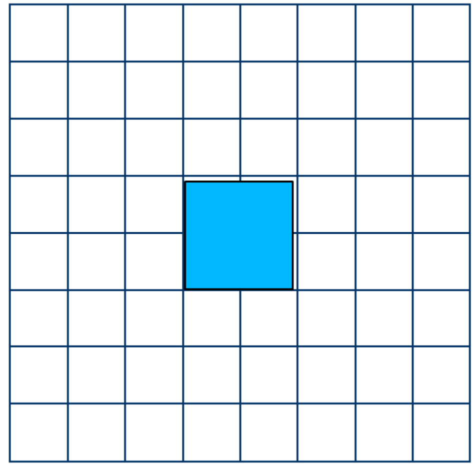
A

$$(A \ominus B)^c = A^c \oplus \hat{B}$$

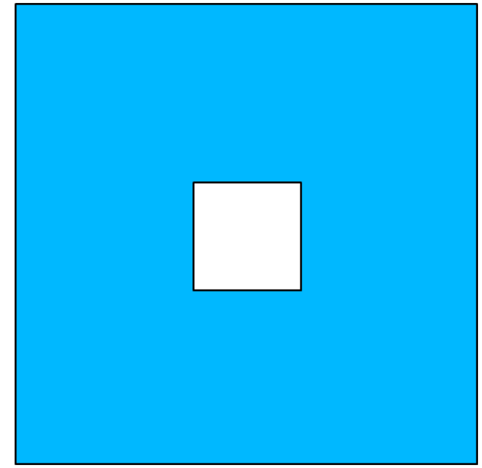
Duality of Dilation and Erosion Example



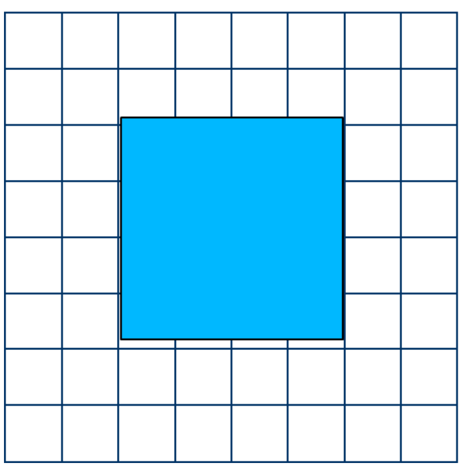
A



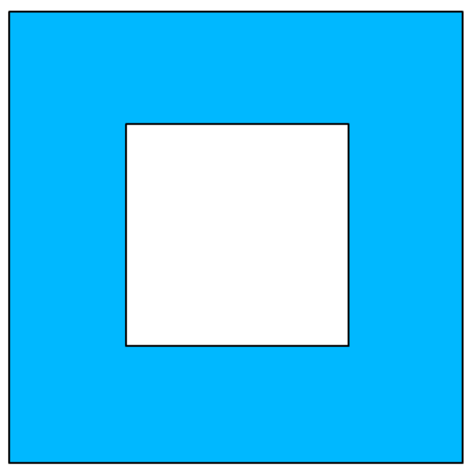
$(A \ominus B)$



$(A \ominus B)^c$

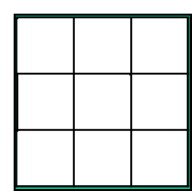


A



A^c

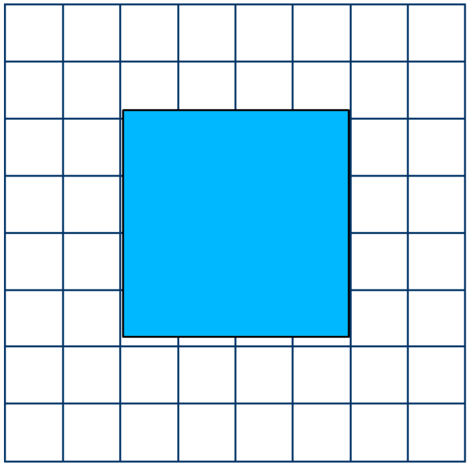
$$(A \ominus B)^c = A^c \oplus \hat{B}$$



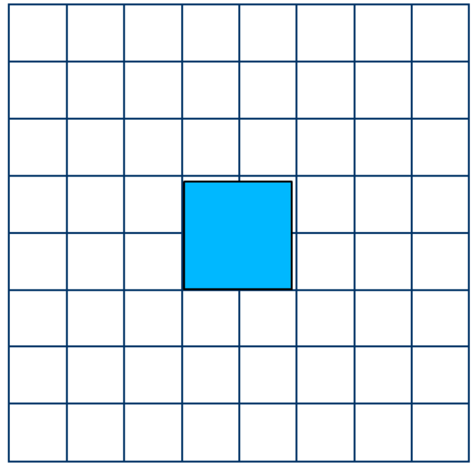
B

Duality of Dilation and Erosion Example

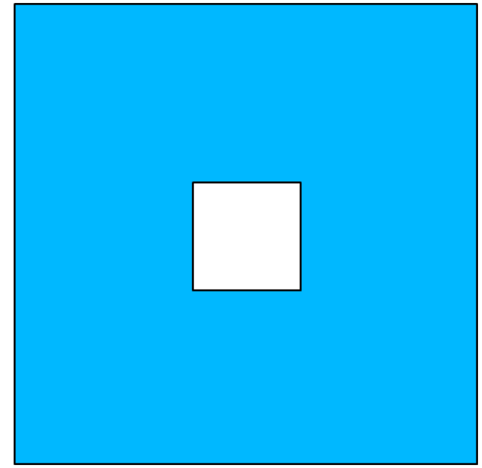
$$\boxed{(A \ominus B)^c = A^c \oplus \hat{B}}$$



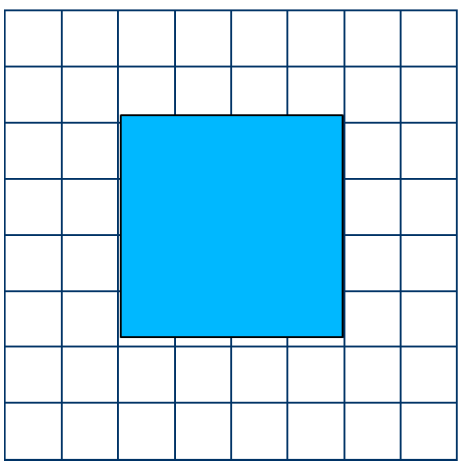
A



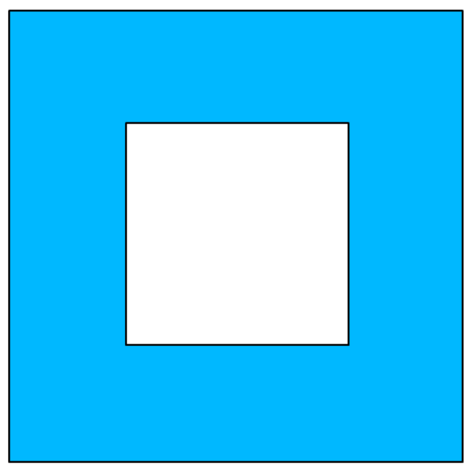
$(A \ominus B)$



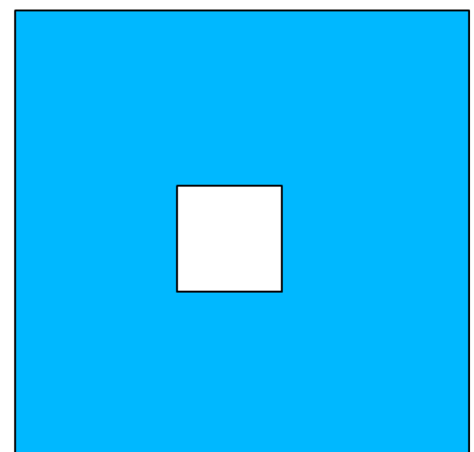
$(A \ominus B)^c$



A



A^c

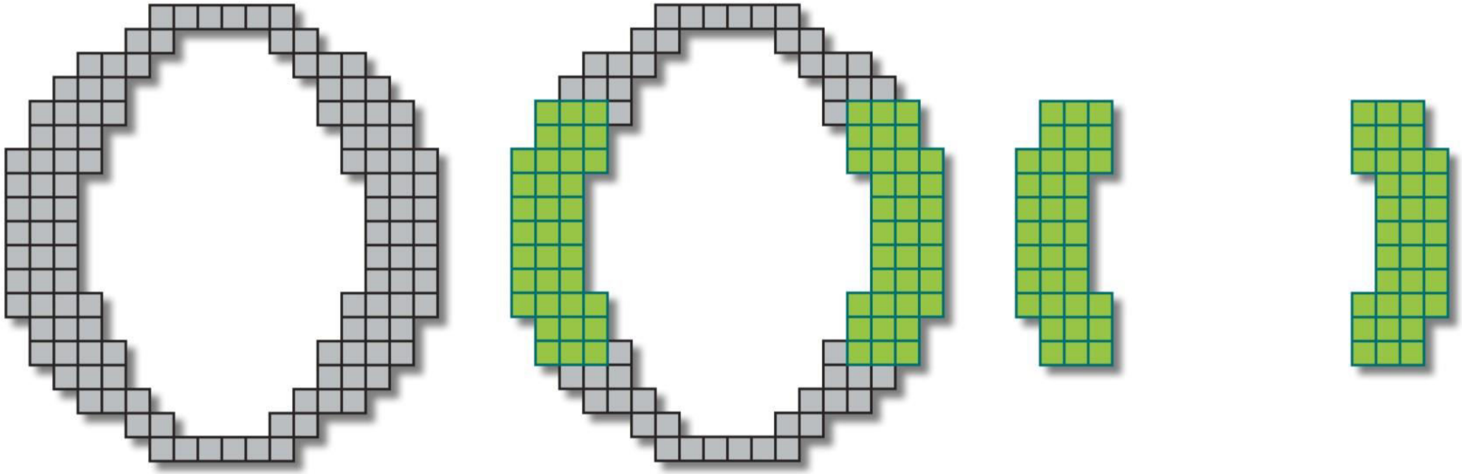


$A^c \oplus \hat{B}$

Compound Operations

- More interesting morphological operations can be performed by performing combinations of erosions and dilations
- The most widely used of these *compound operations* are:
  - Opening
  - Closing

Opening

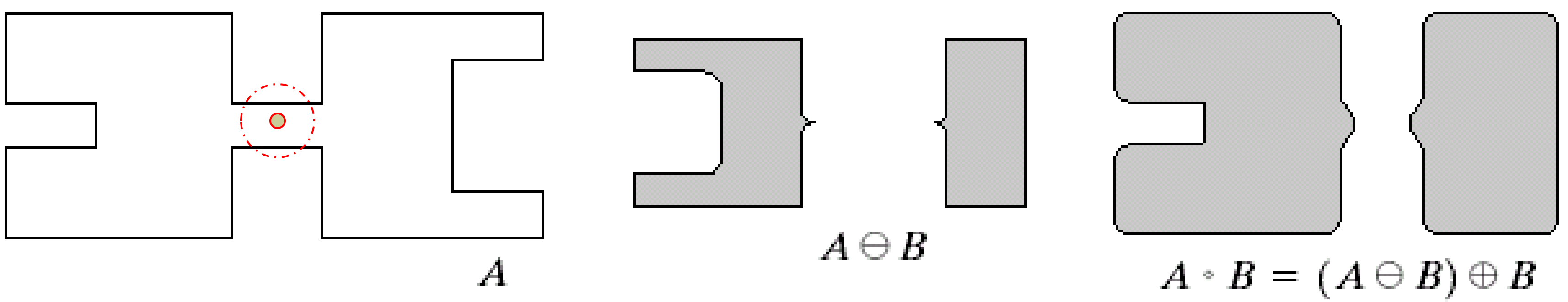


Erosion followed by a dilation

Opening

The opening of image *A* by structuring element *B,* denoted by *A*○B is simply an erosion followed by a dilation

$$A \circ B = (A \ominus B) \oplus B$$



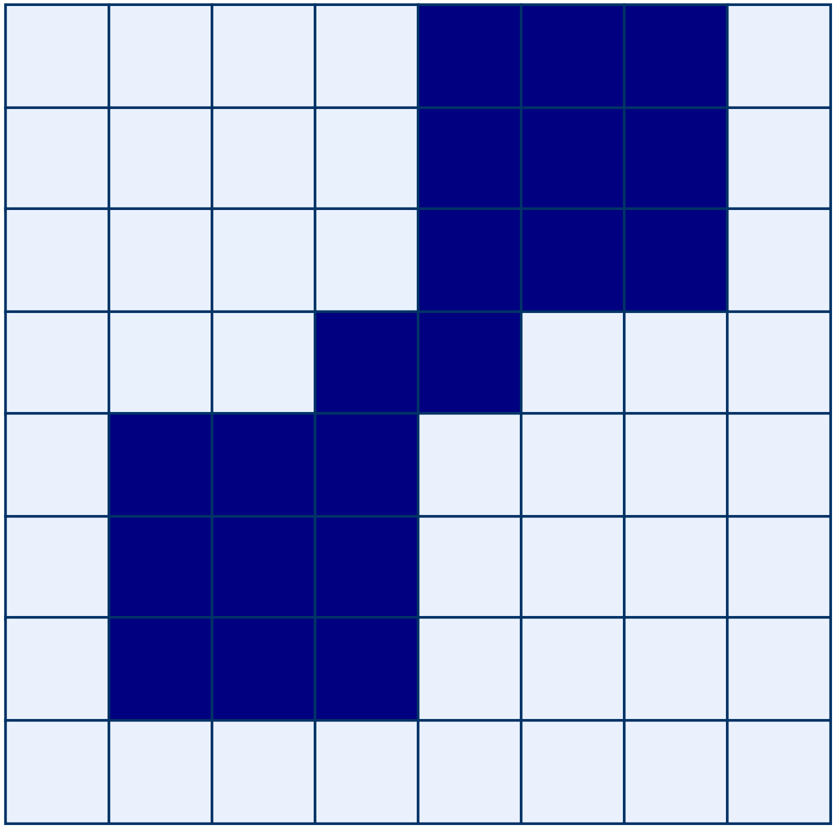
Original shape

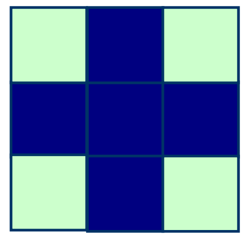
After erosion

After dilation (opening)

Opening: Example

Original Image

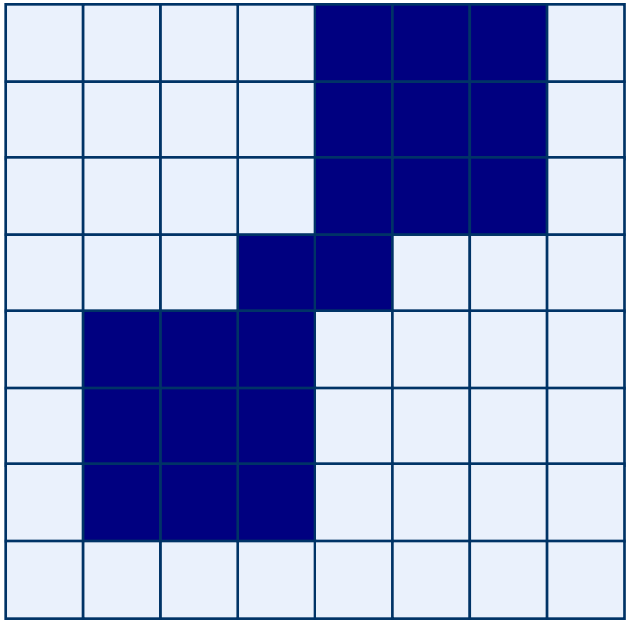




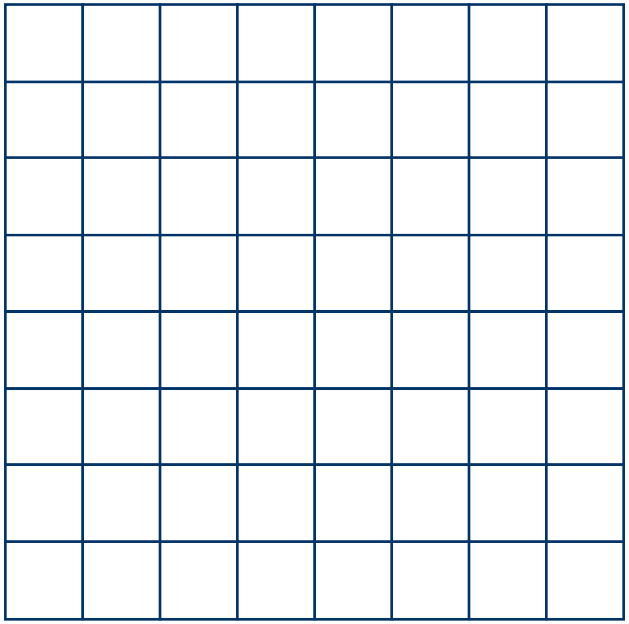
Structuring Element

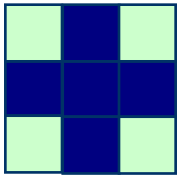
Opening: Example

Original Image



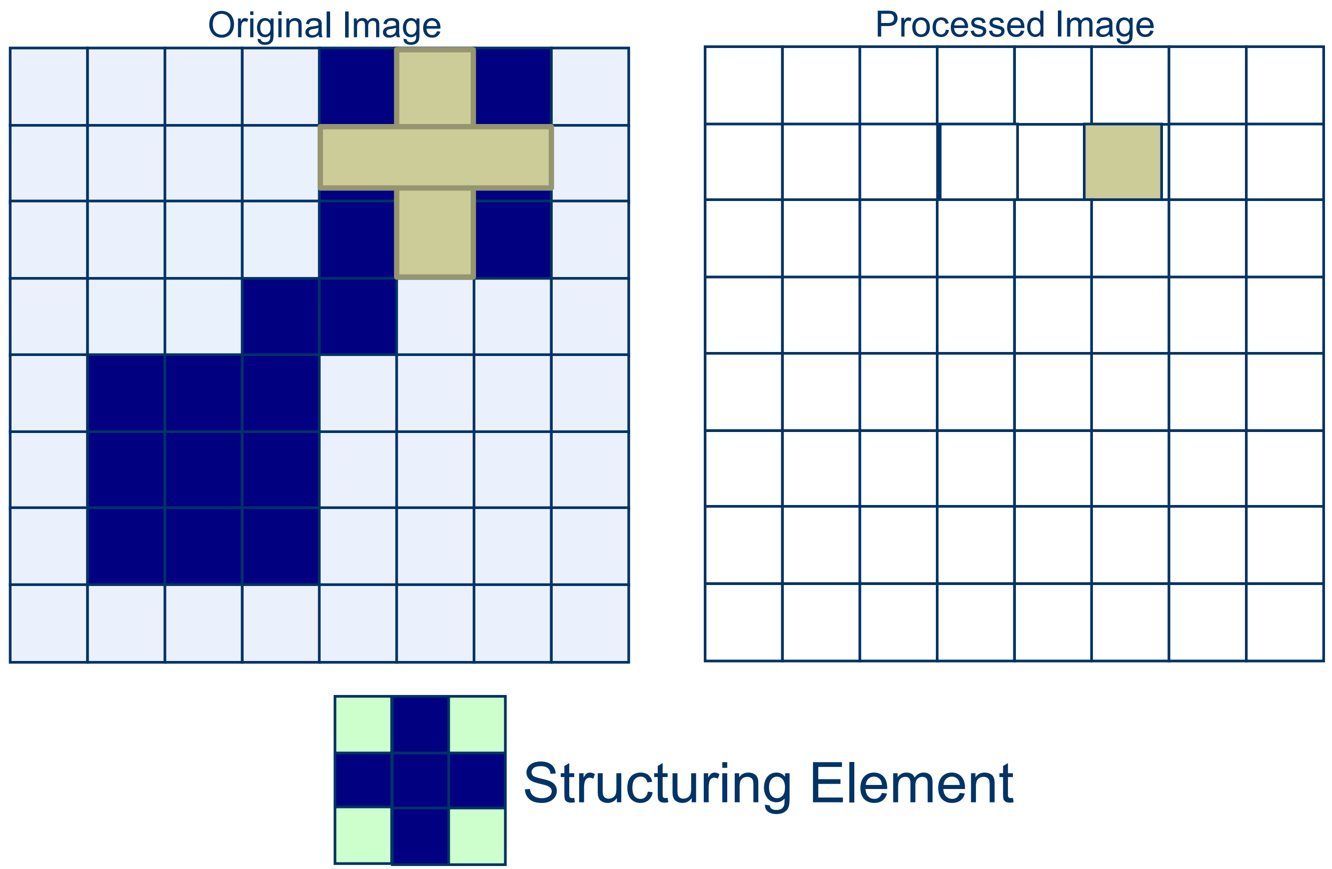
Processed Image



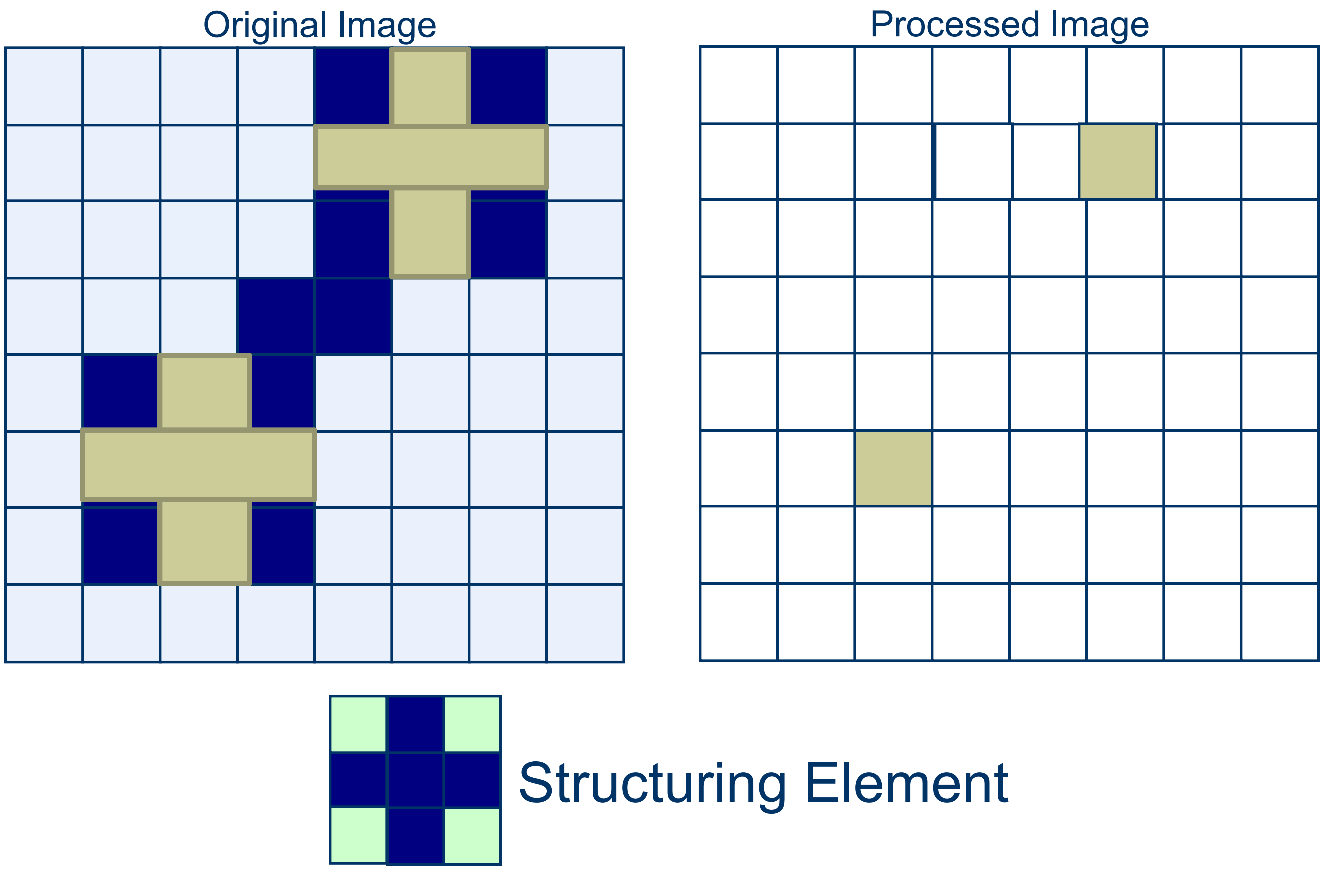


Structuring Element

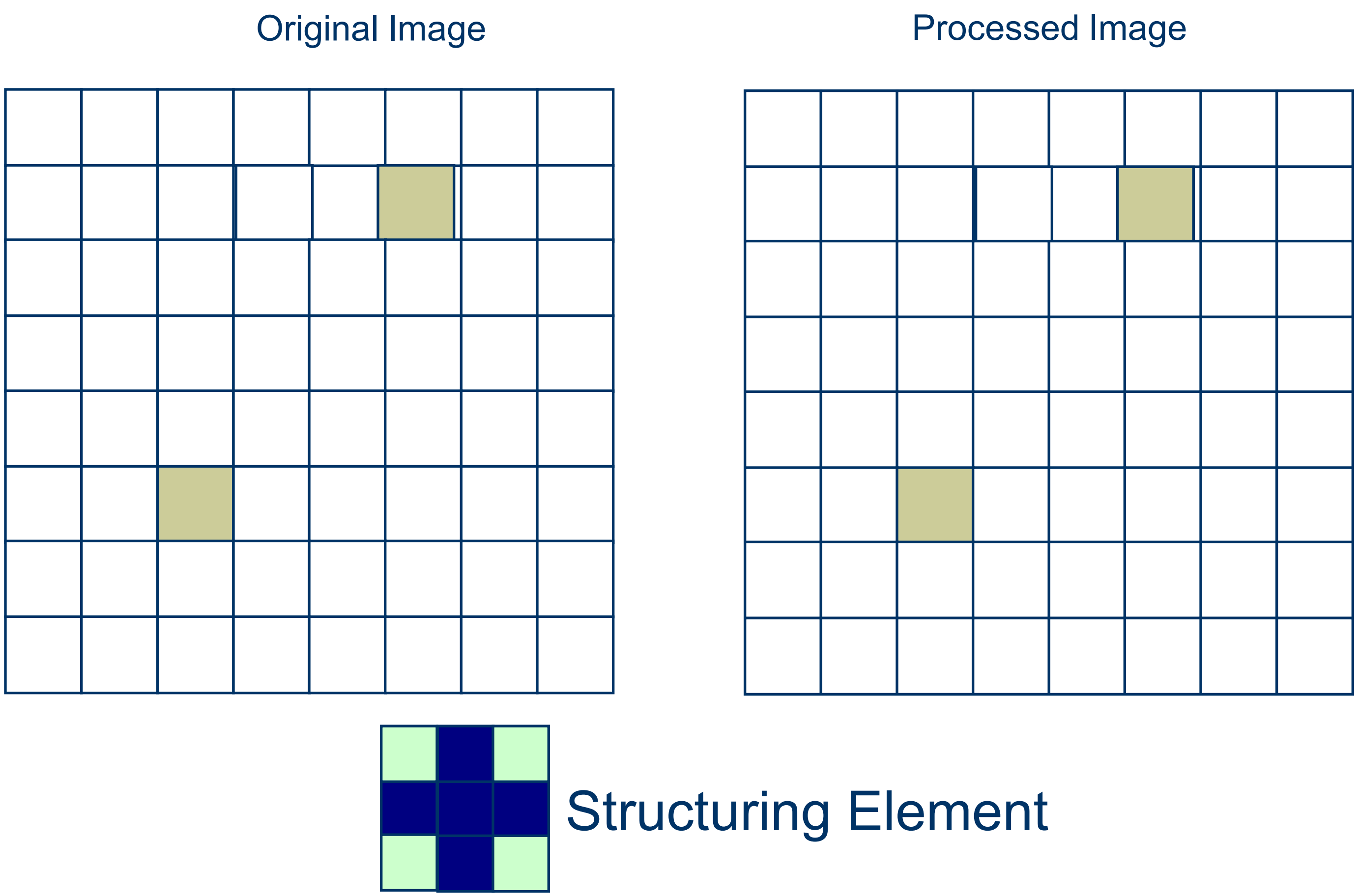
Opening: Example (performing erosion)



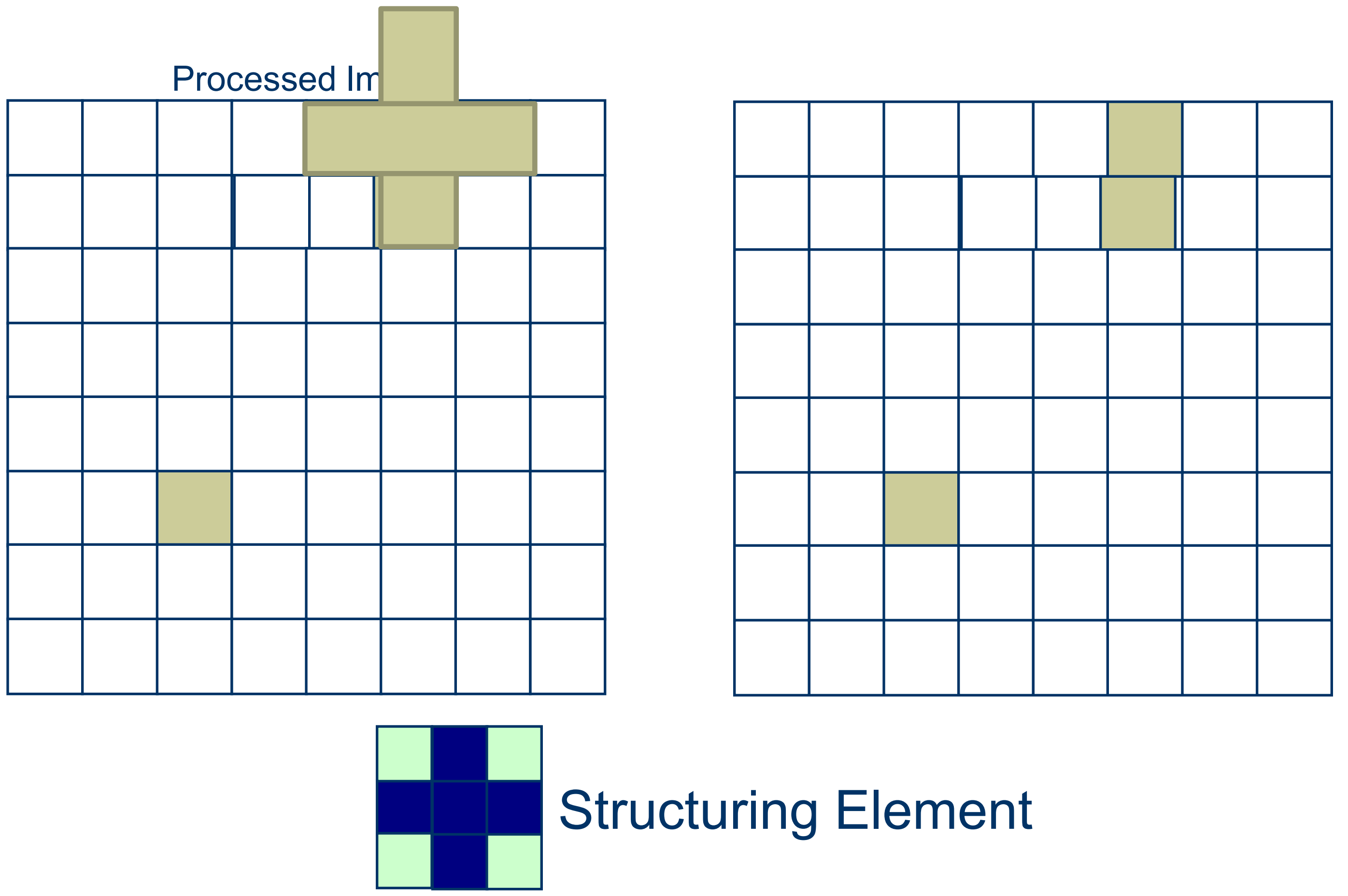
Opening: Example (performing erosion)



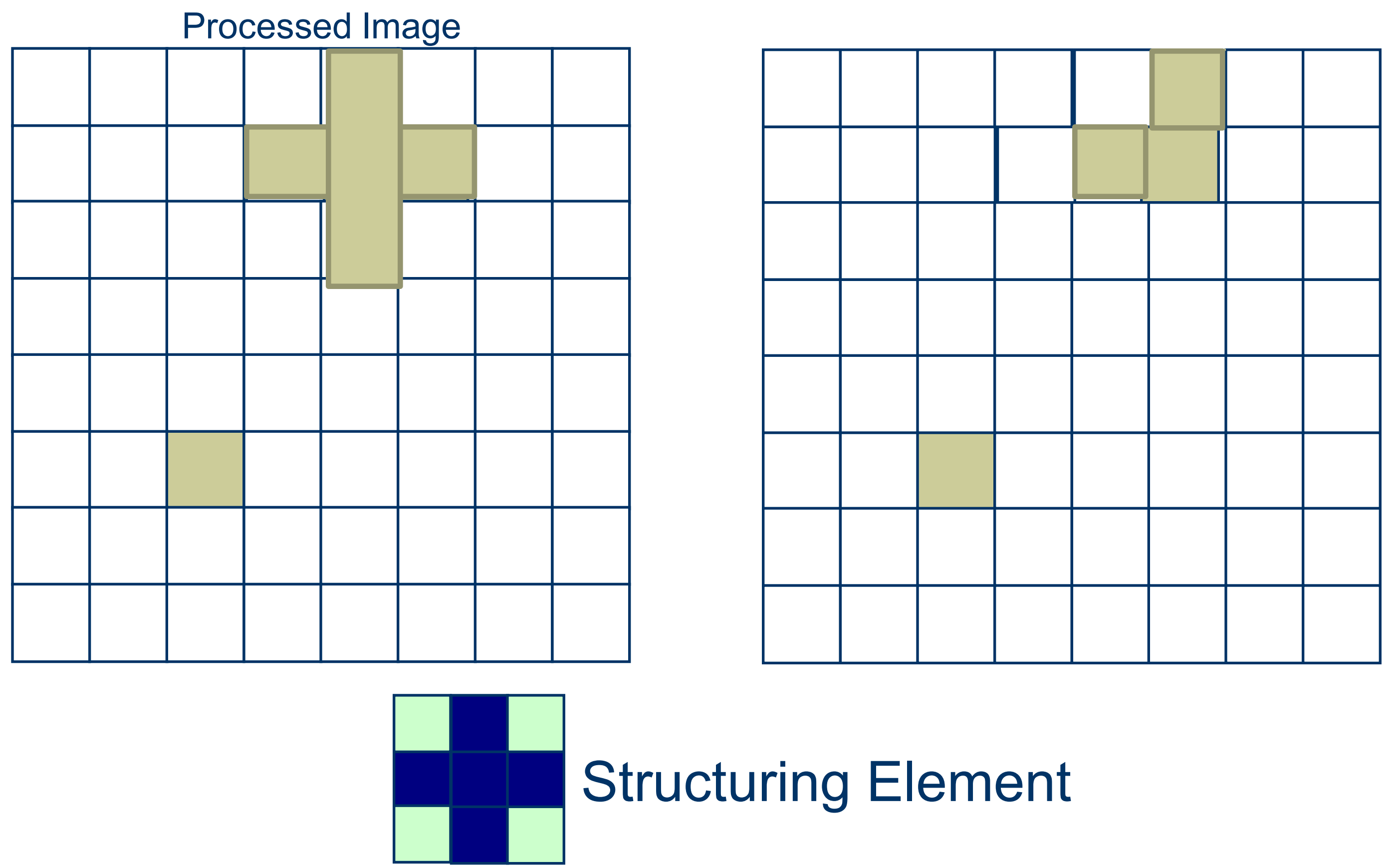
Opening: Example (performing dilation to errored image)



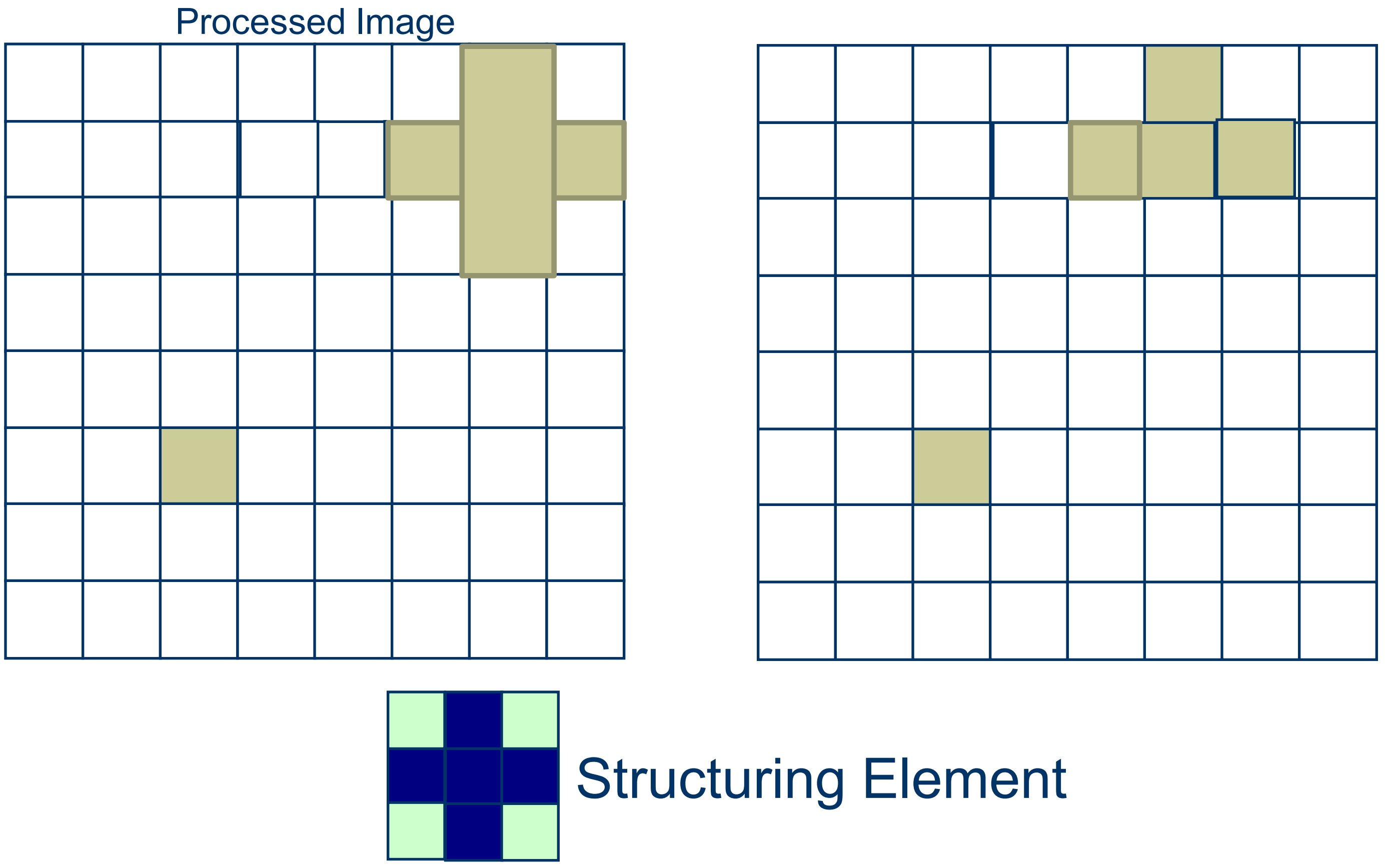
Opening: Example (performing dilation to errored image)



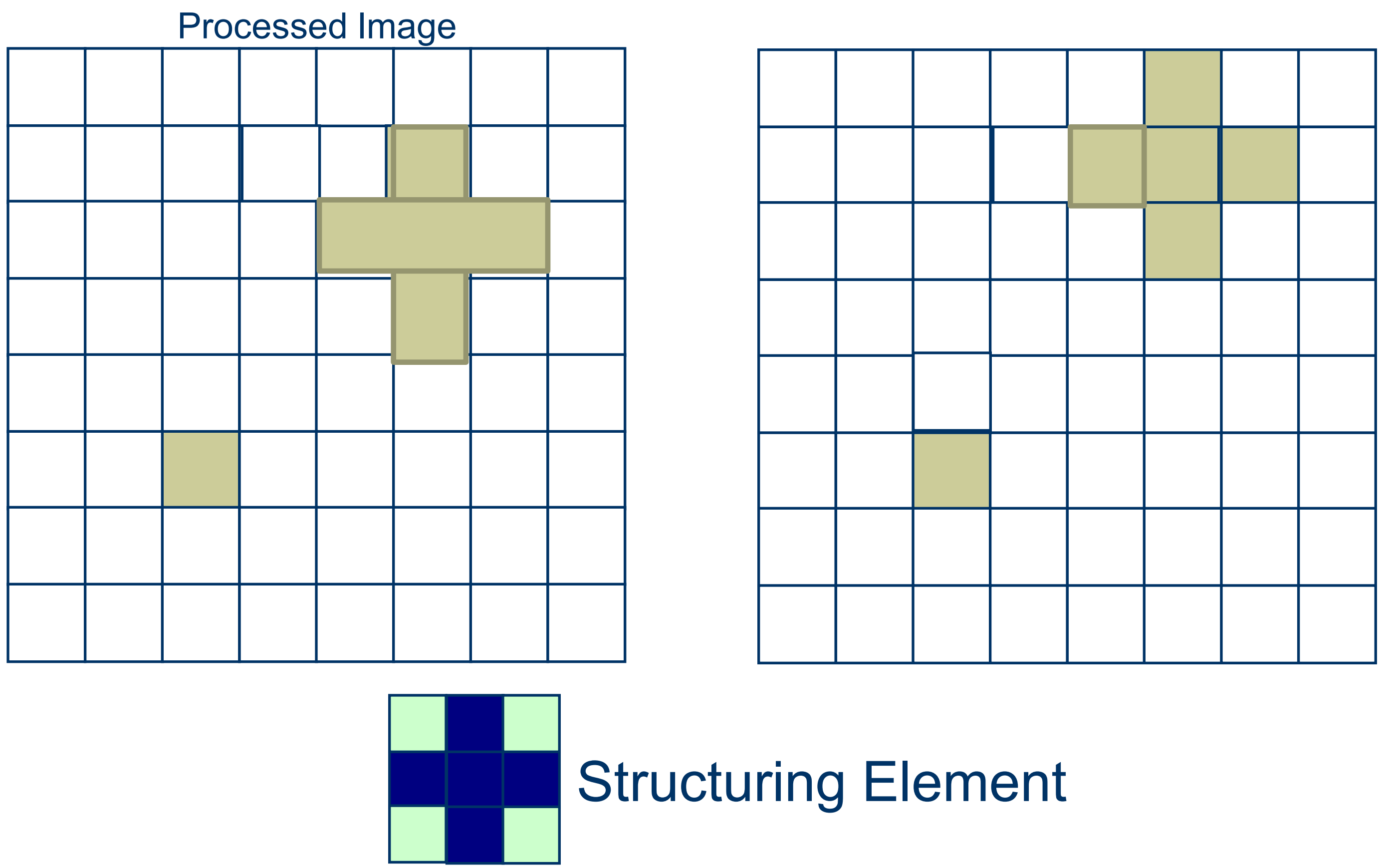
Opening: Example (performing dilation to errored image)



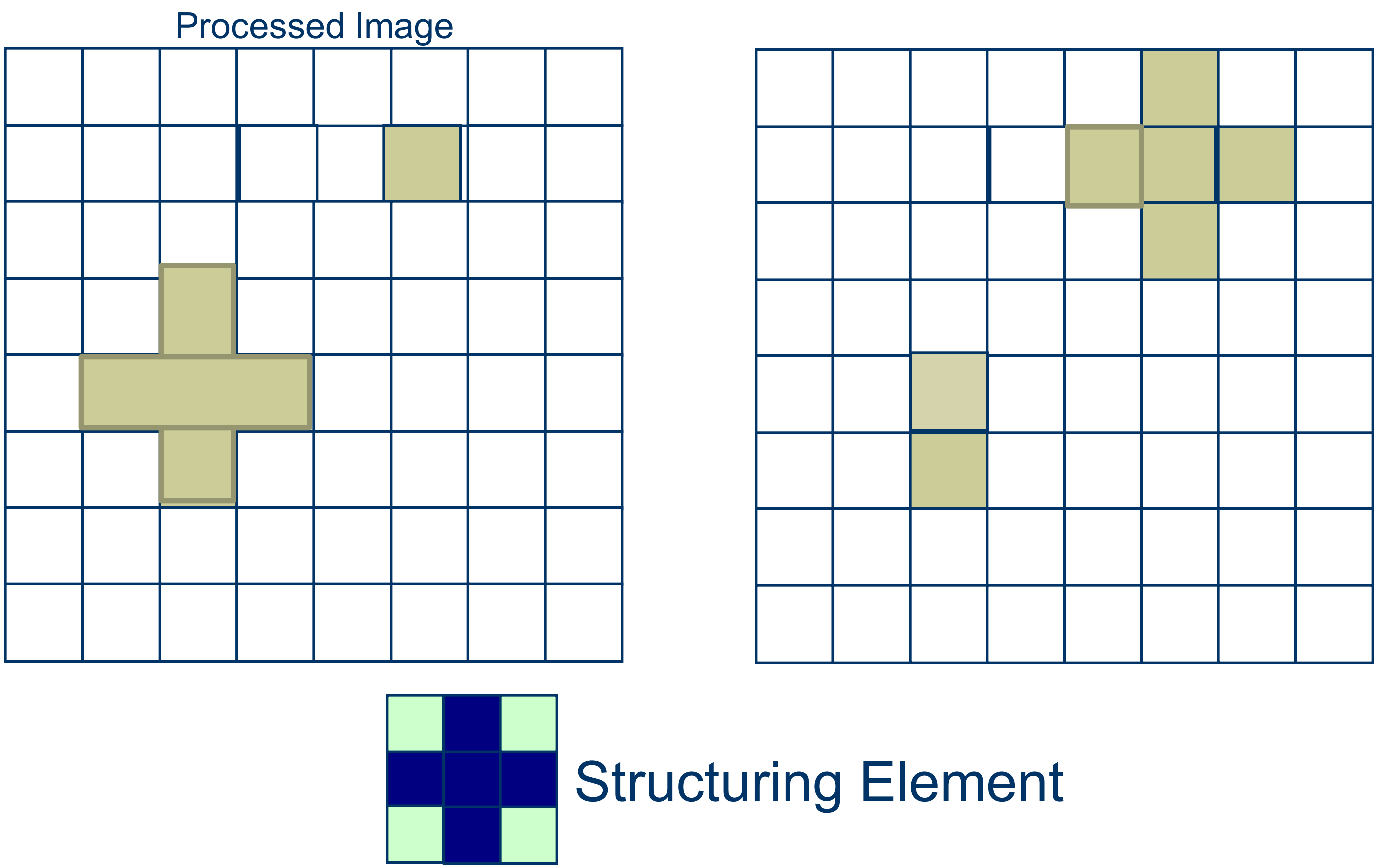
Opening: Example (performing dilation to errored image)



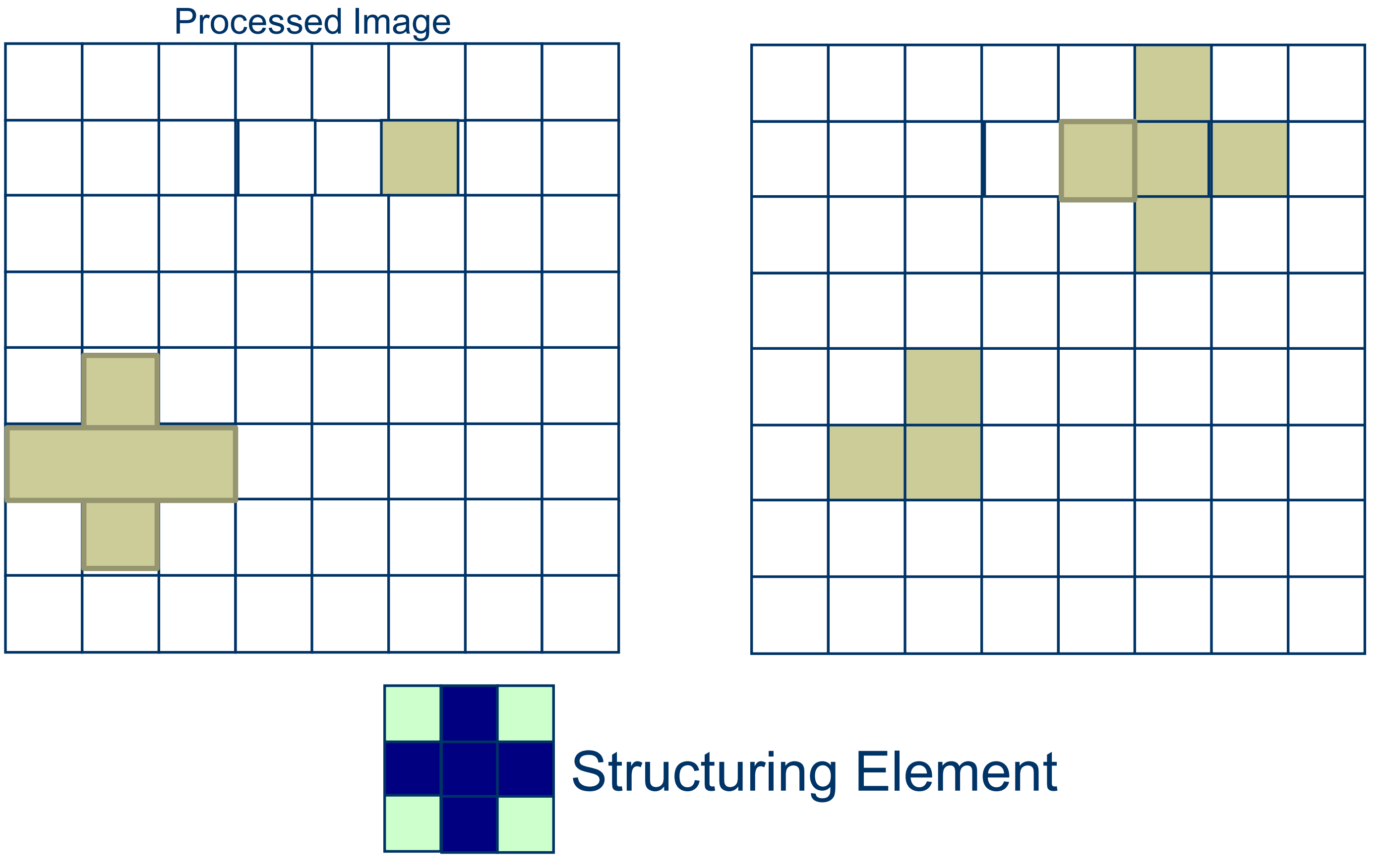
Opening: Example (performing dilation to errored image)



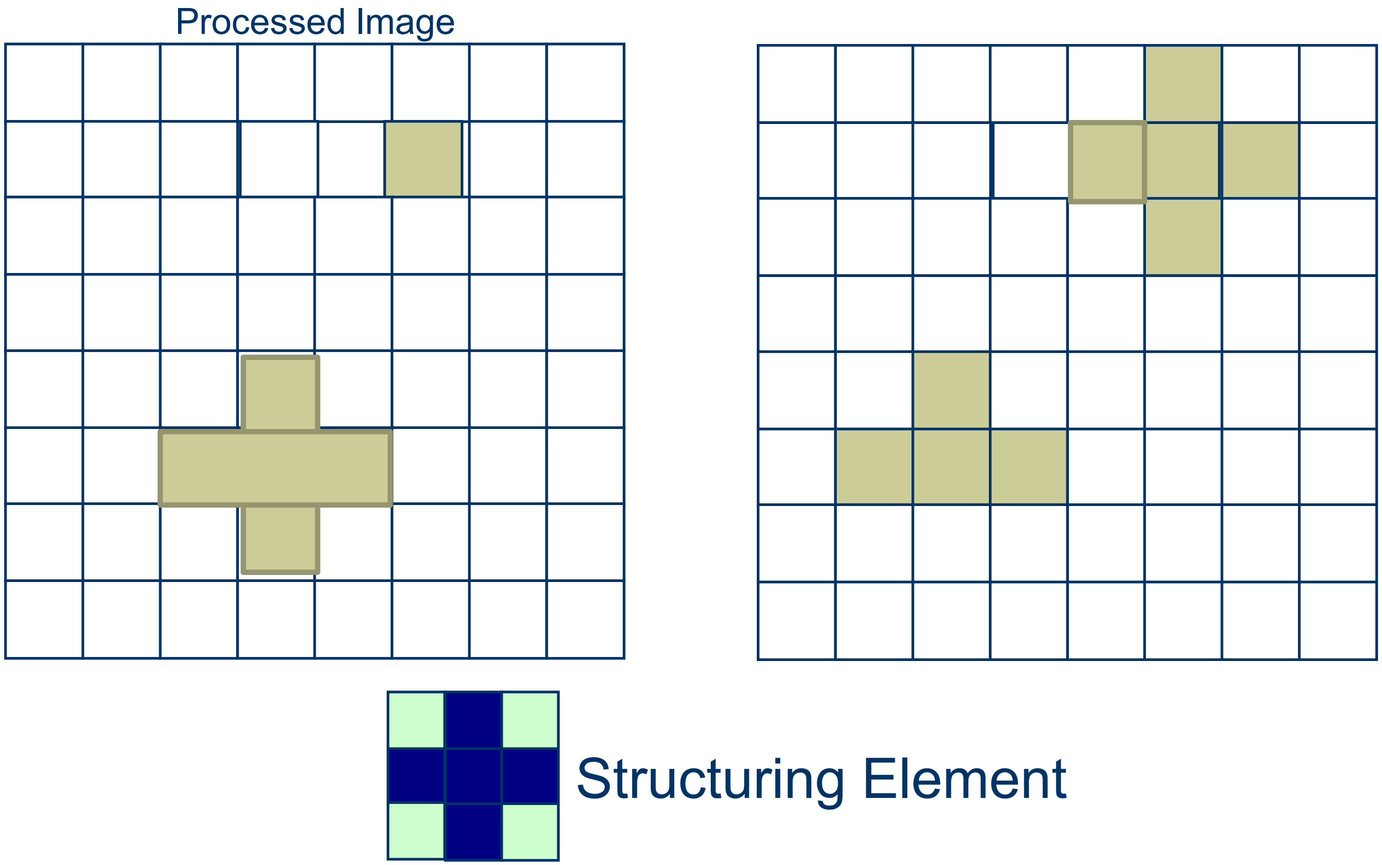
Opening: Example (performing dilation to errored image)



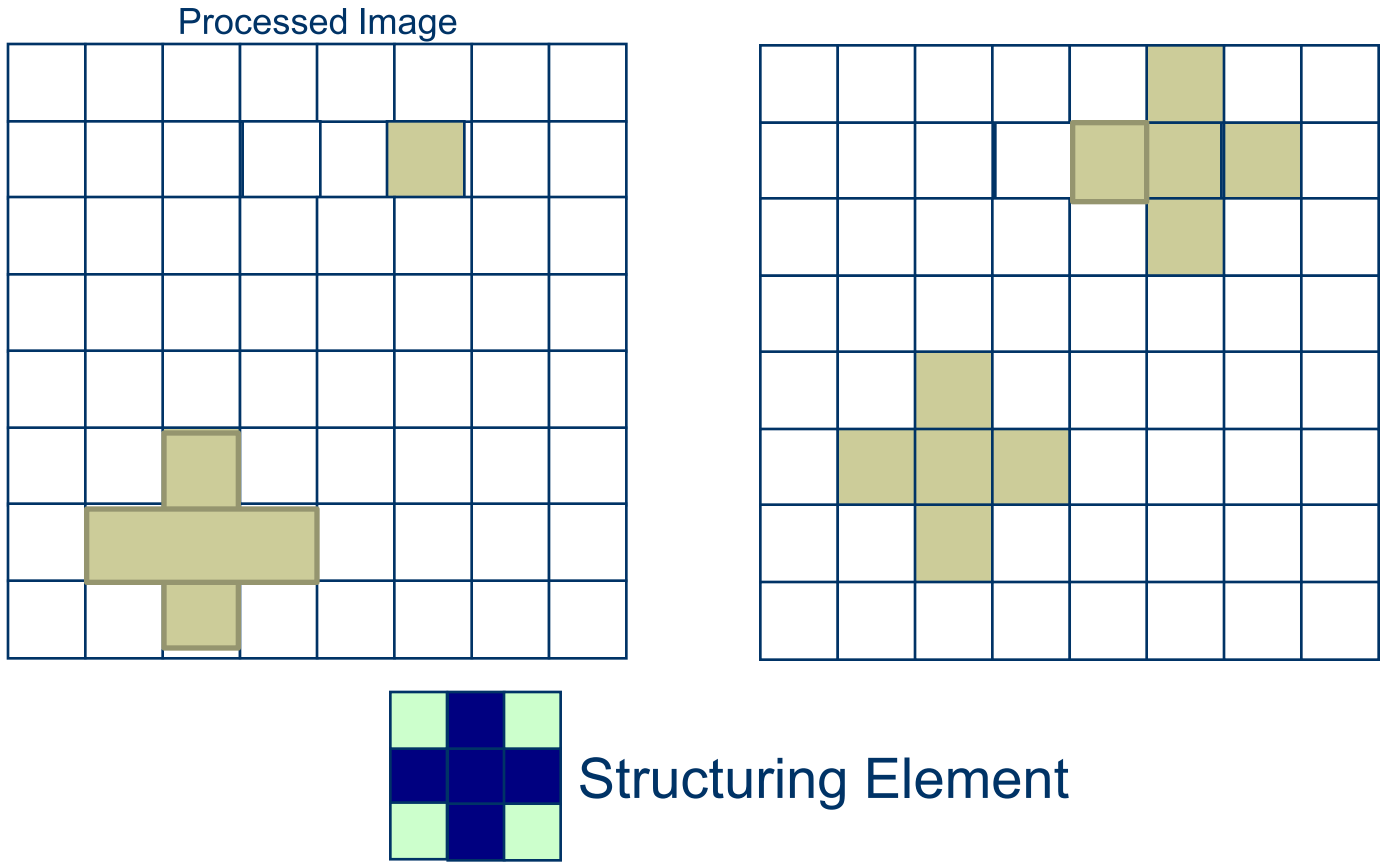
Opening: Example (performing dilation to errored image)



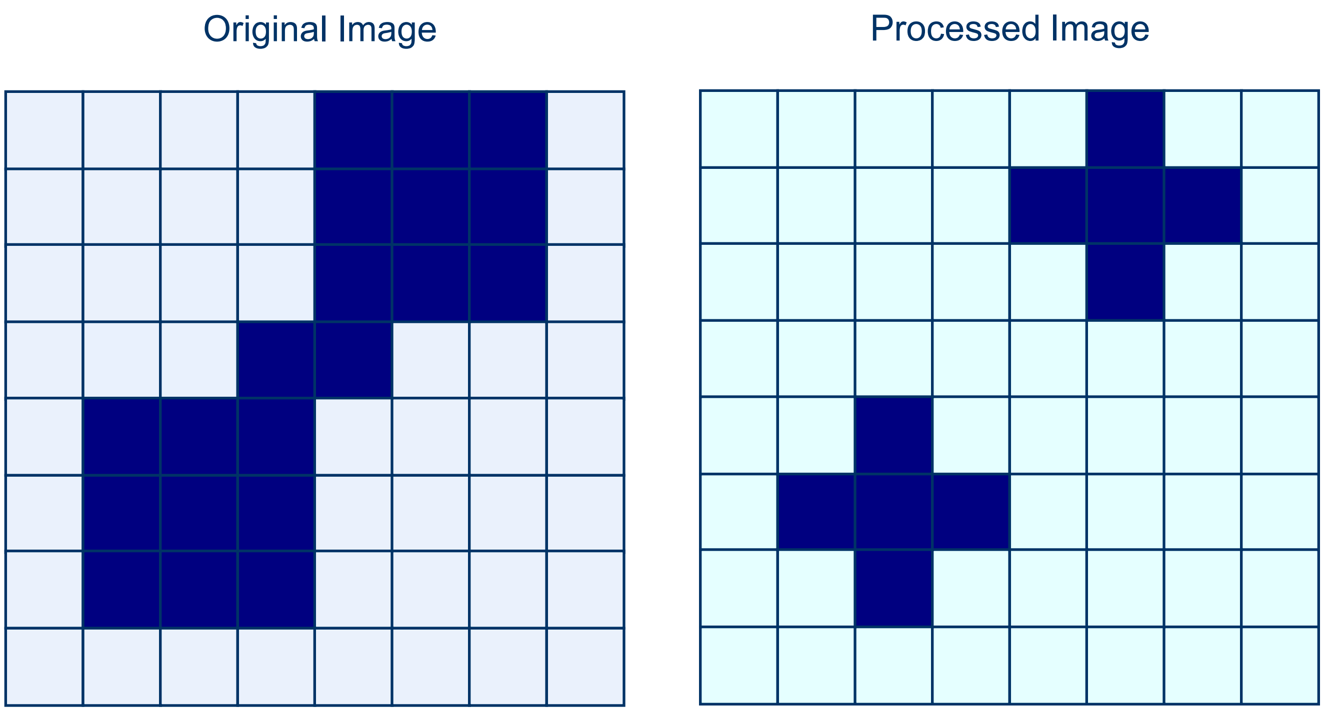
Opening: Example (performing dilation to errored image)



Opening: Example (performing dilation to errored image)



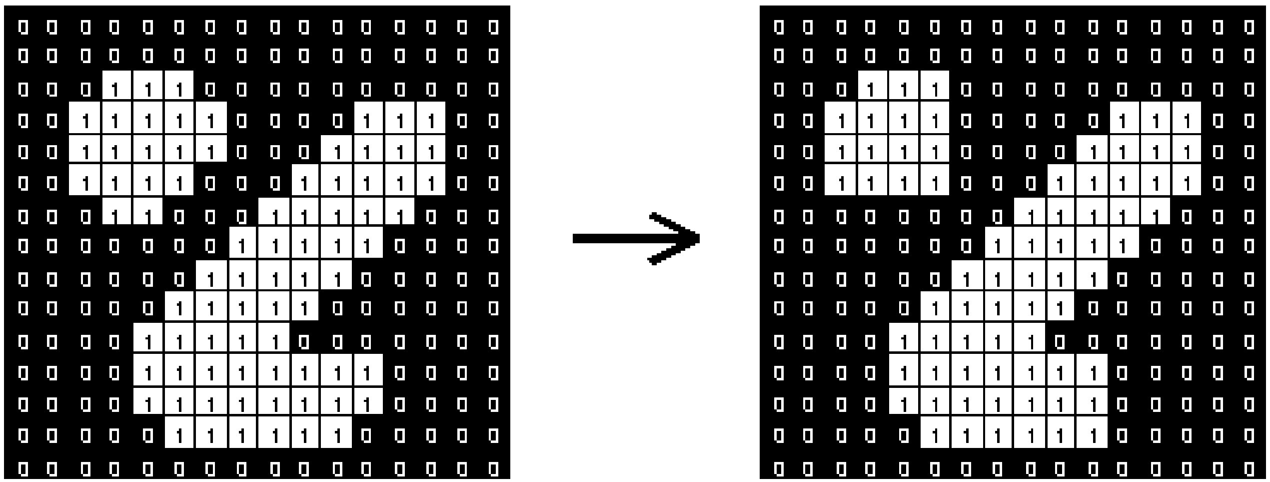
After Opening:



Effect of opening

- Structuring element: 3x3 square

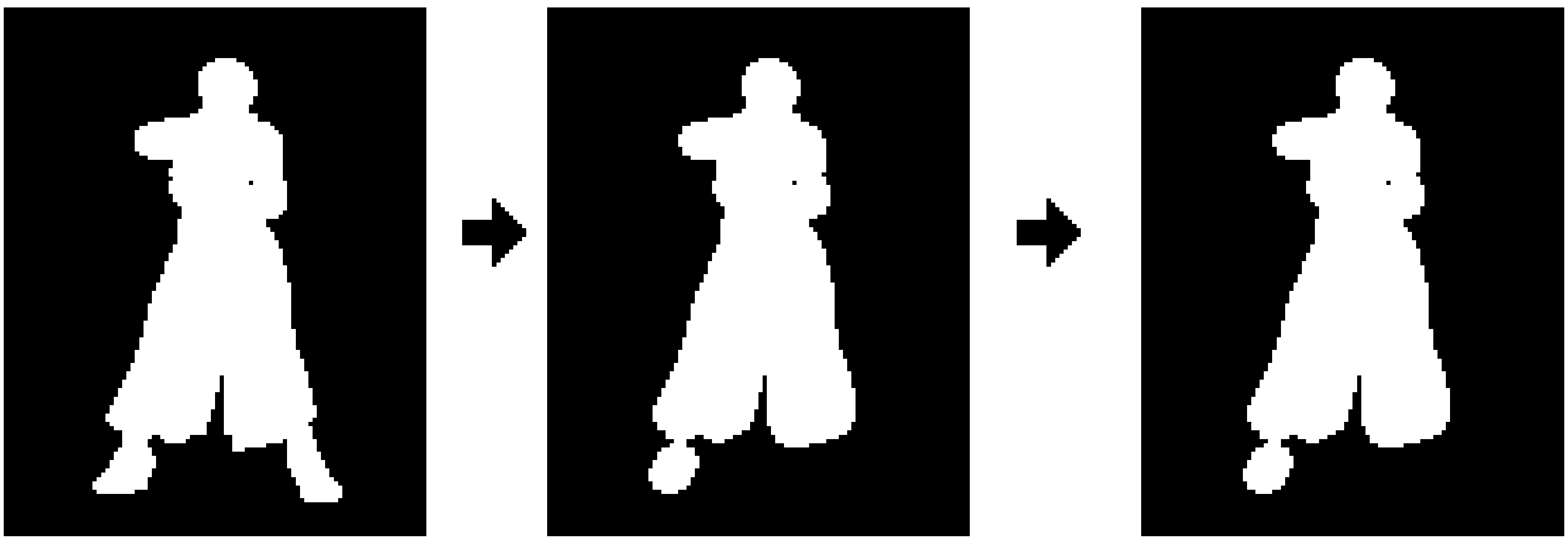
1	1	1
1	1	1
1	1	1



Effect of opening

- Smoothed the outline, by rounding off any sharp points, and
- Remove any parts that is smaller than the shape (SE) used.
- It will also disconnect or 'open' any thin bridges.
- It does not remove any 'holes', or gaps that may be present in the image.
- It does not make the basic 'core' size of the shape larger or smaller.

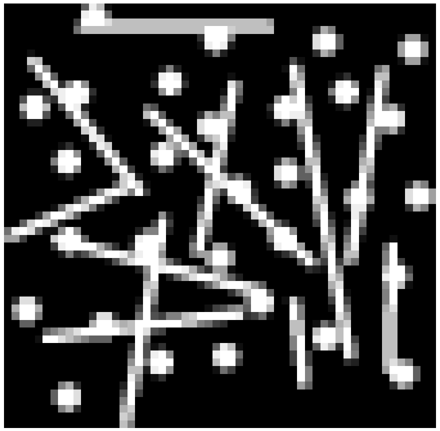
Note that performing an 'Open' on a shape that has already been opened, with the same kernel will result in no further change to the shape. For example...



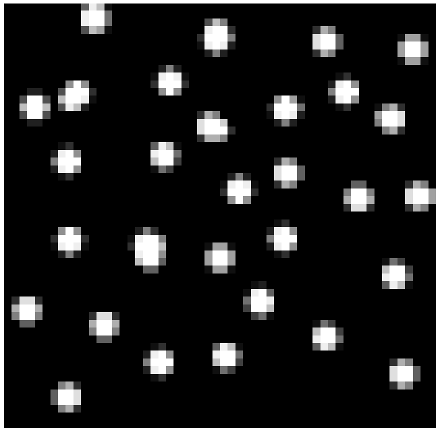
That is repeating a 'Open' operation, with the same kernel, has no effect on the result.

Effect of opening

Original Image

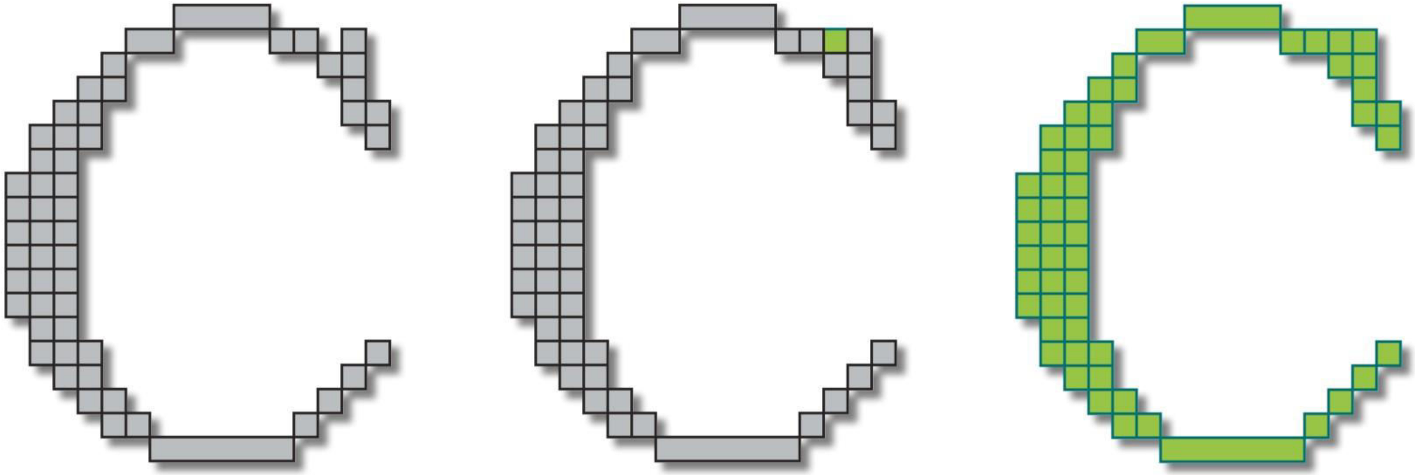


Processed Image



- Opening with a disk shaped structuring element 11 pixels in diameter gives the separation of circles from the lines.
- Some of the circles are slightly distorted, but
- In general, the lines have been almost completely removed while the circles remain almost completely unaffected.

Closing

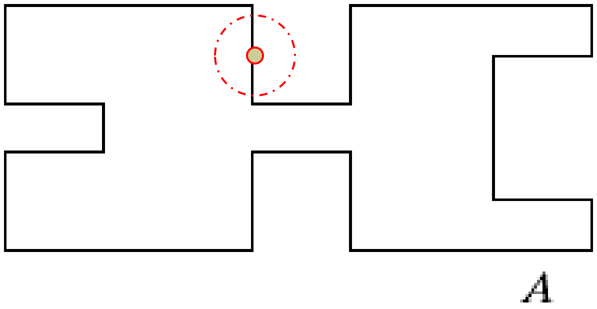


Dilation followed by an erosion

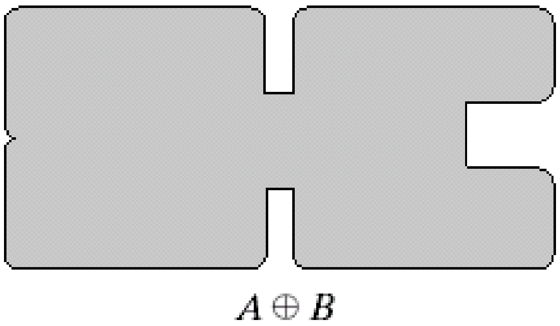
Closing

The closing of image *f* by structuring element *s*, denoted by *A* • *B* is simply a dilation followed by an erosion

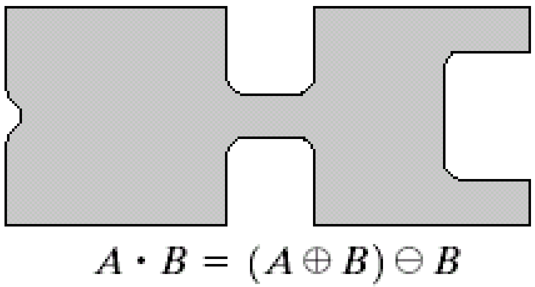
$$A \cdot B = (A \oplus B) \ominus B$$



Original shape



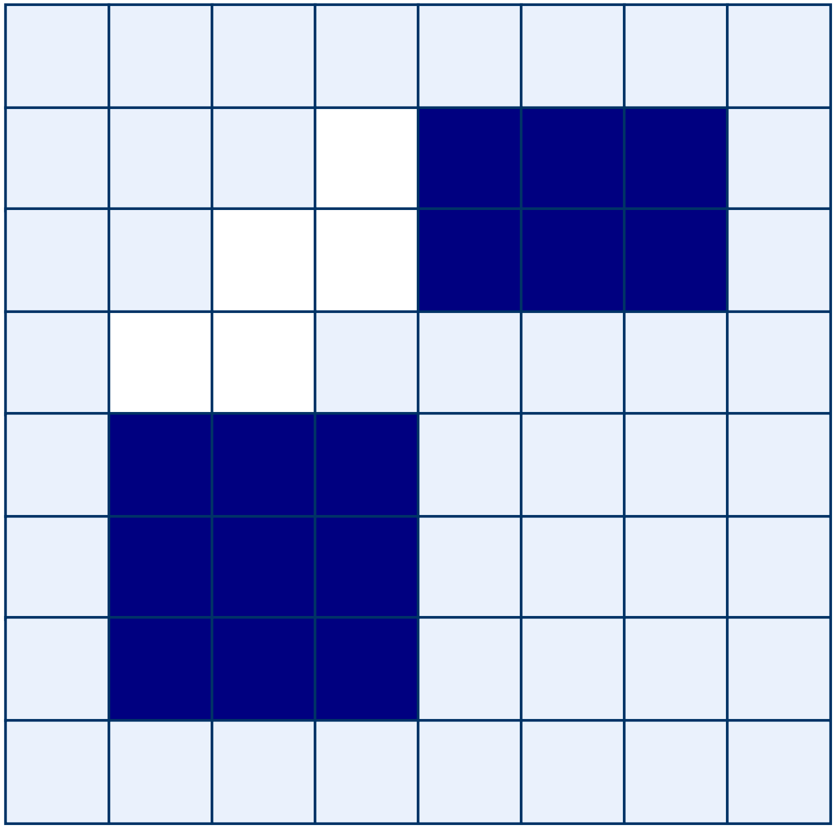
After dilation

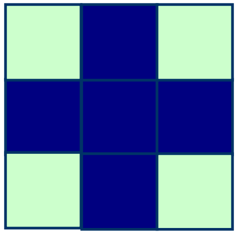


After erosion (closing)

Closing: Example

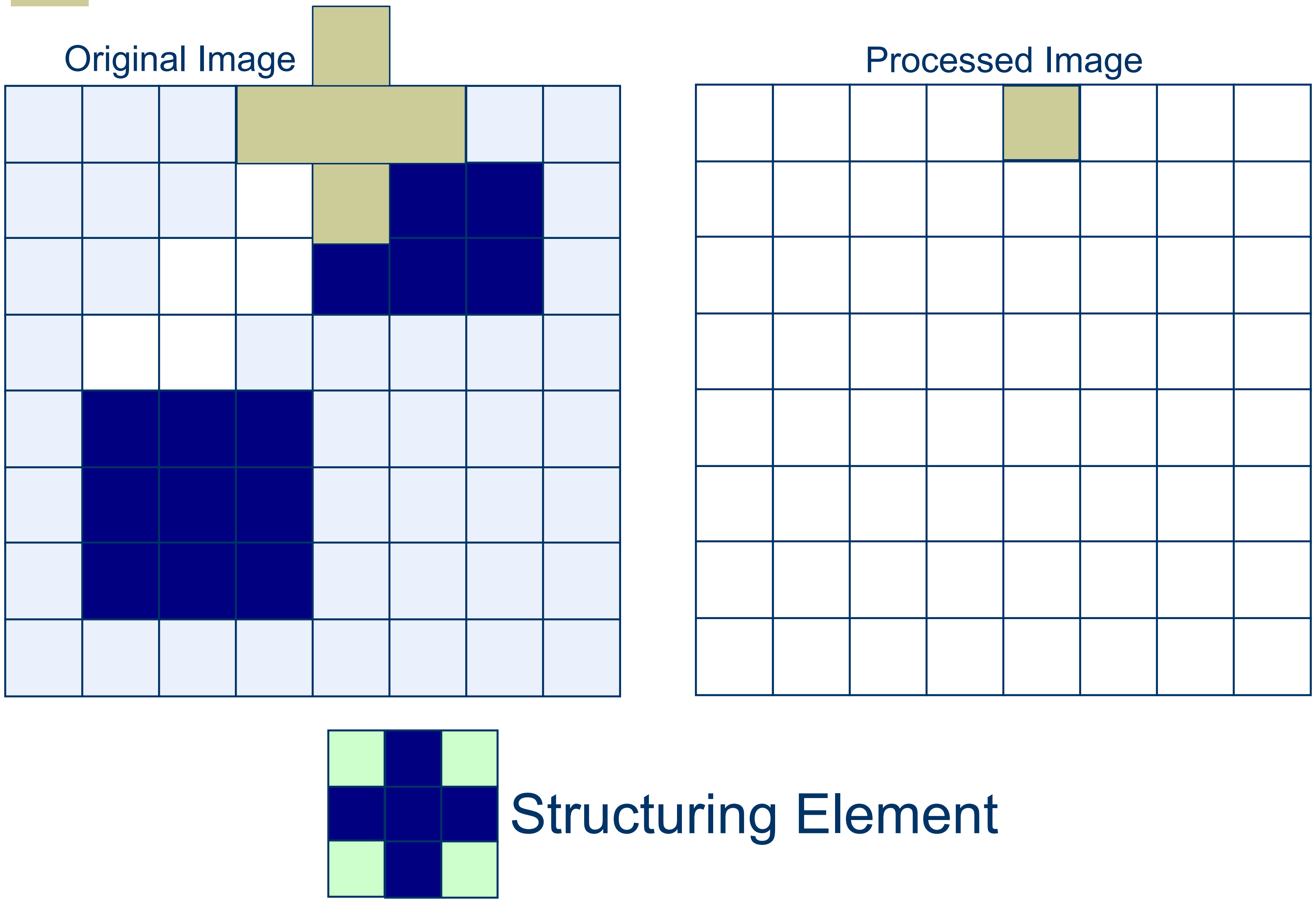
Original Image



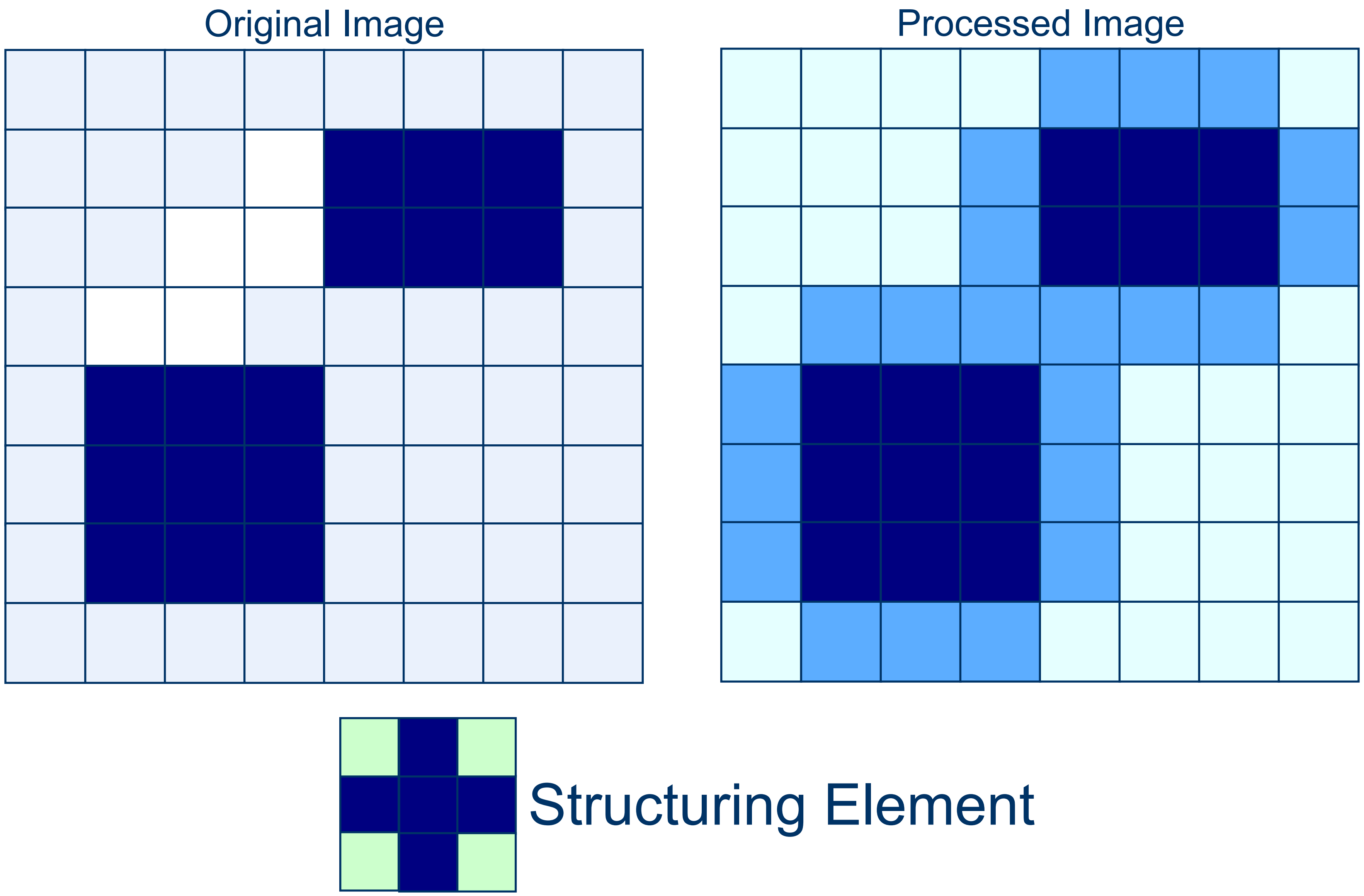


Structuring Element

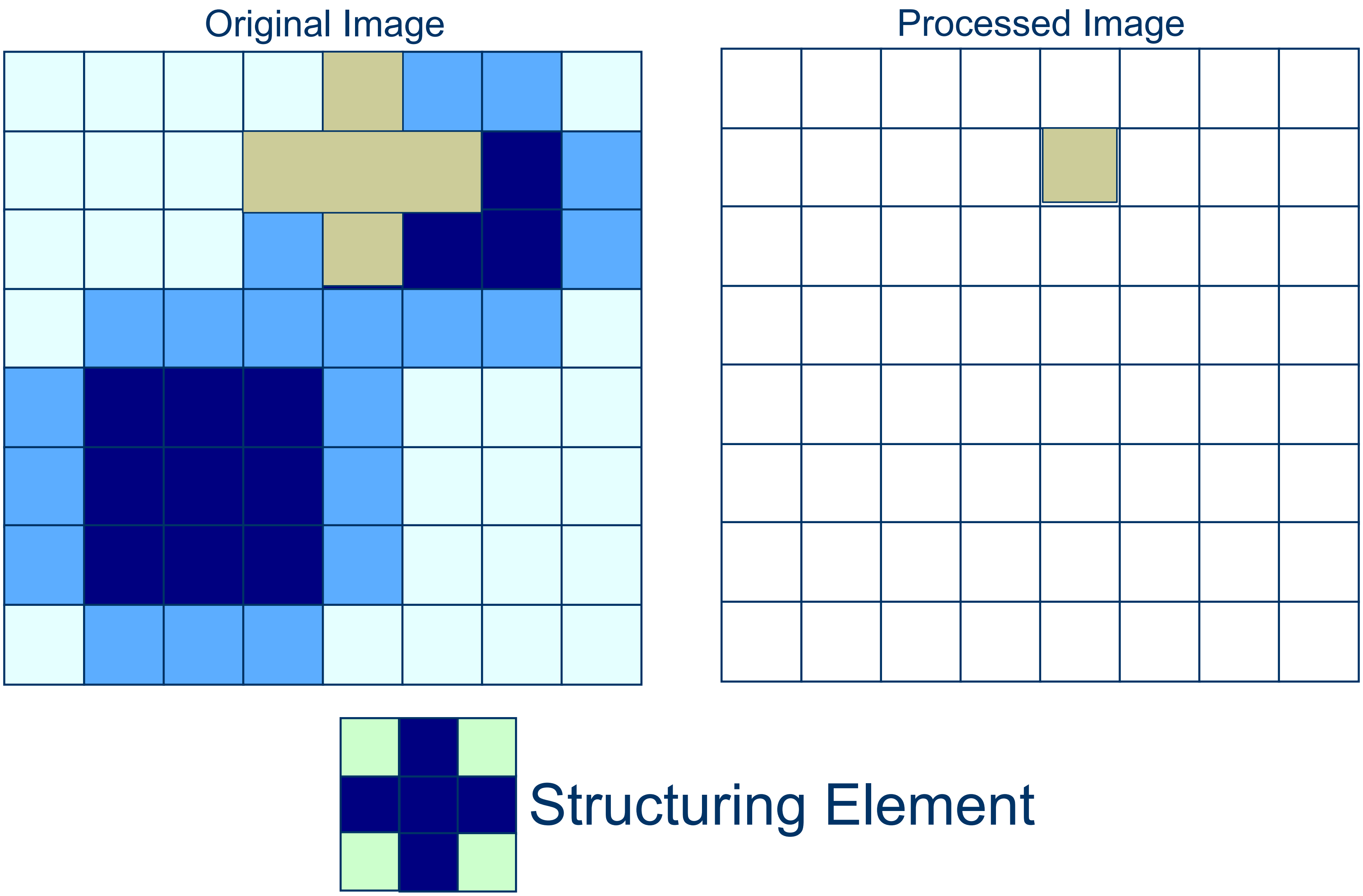
Closing: Example (performing Dilation)



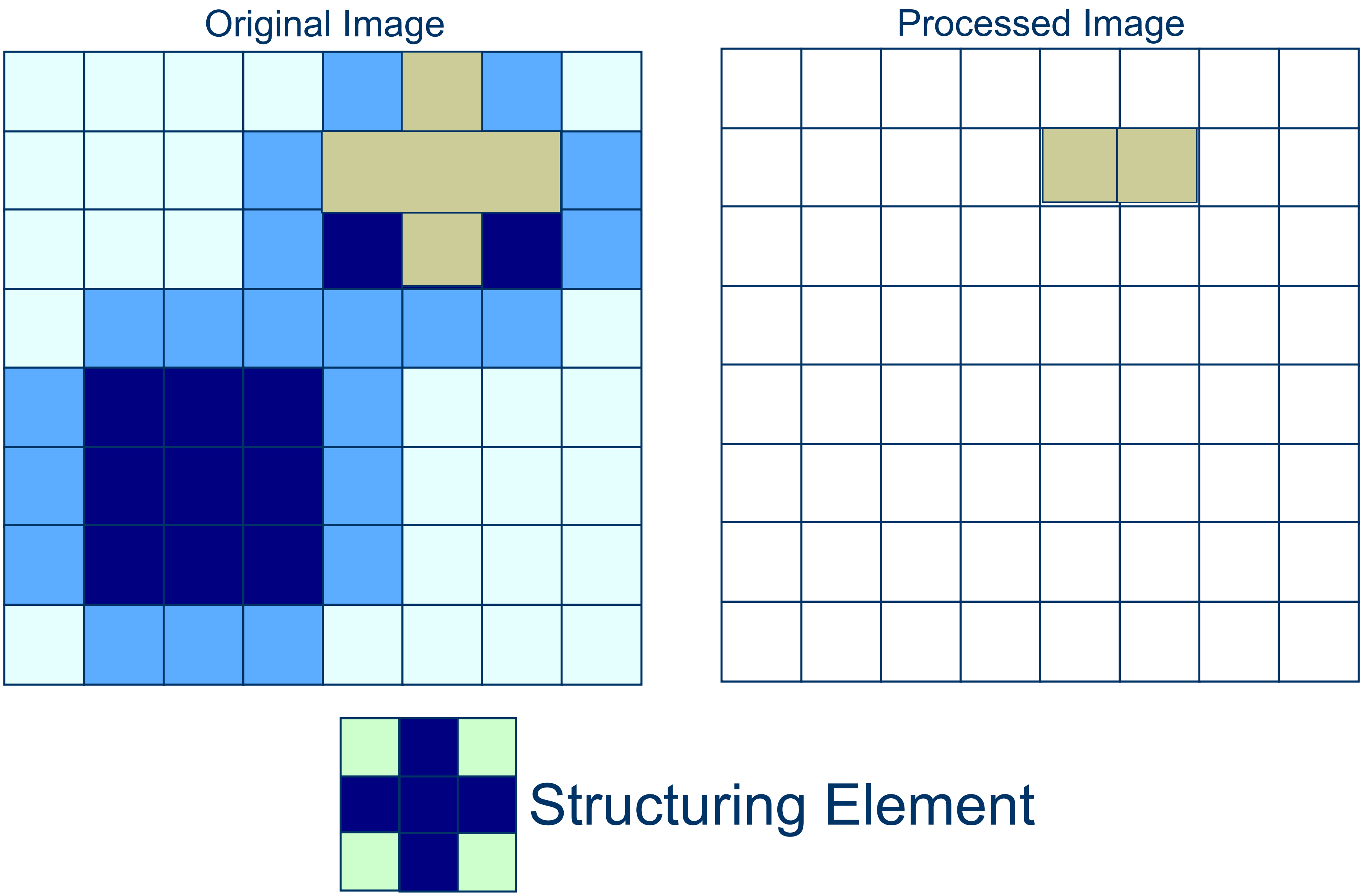
Closing: Example (after Dilation)



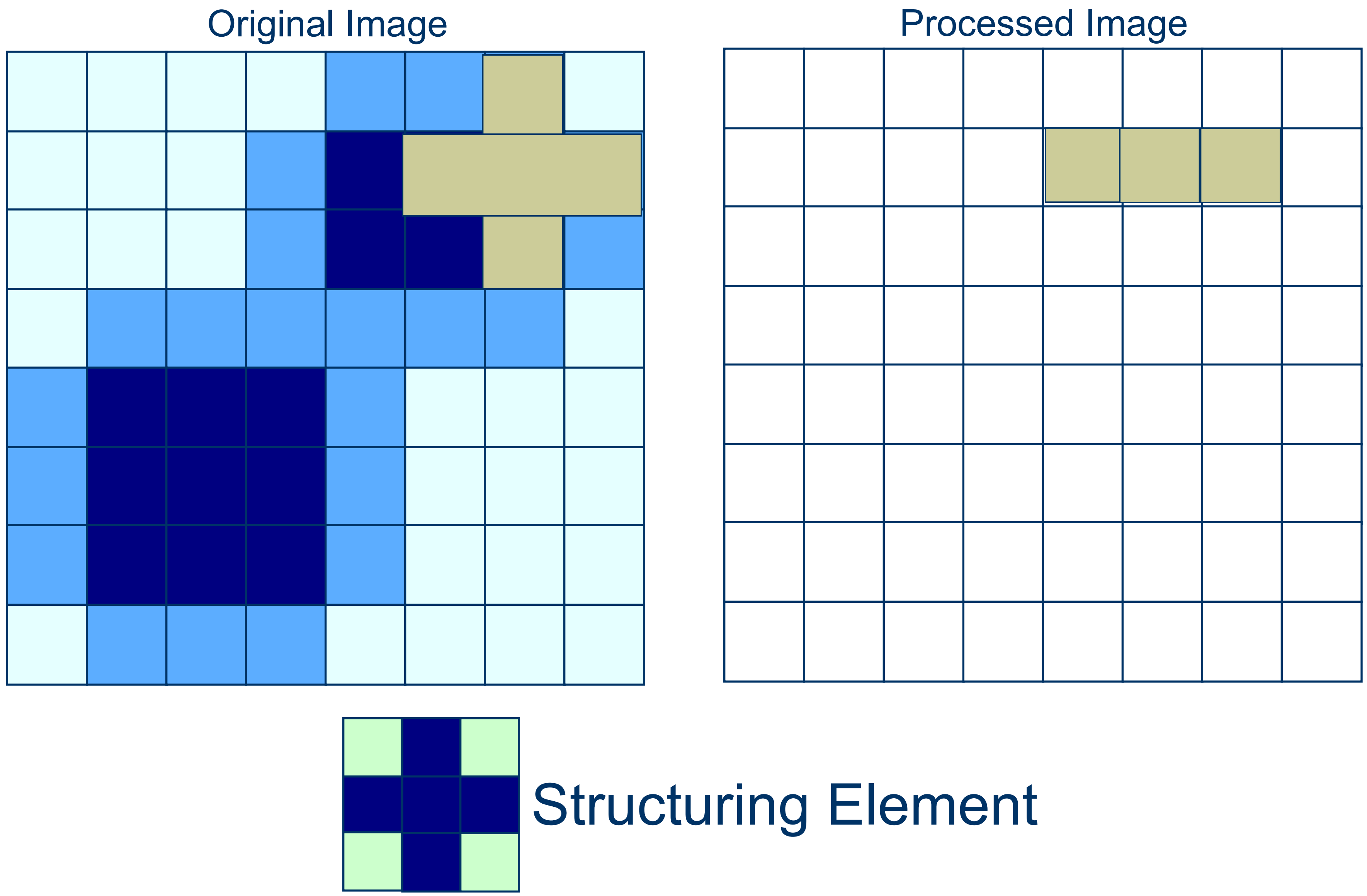
Closing: Example(performing erosion on Dilated image)



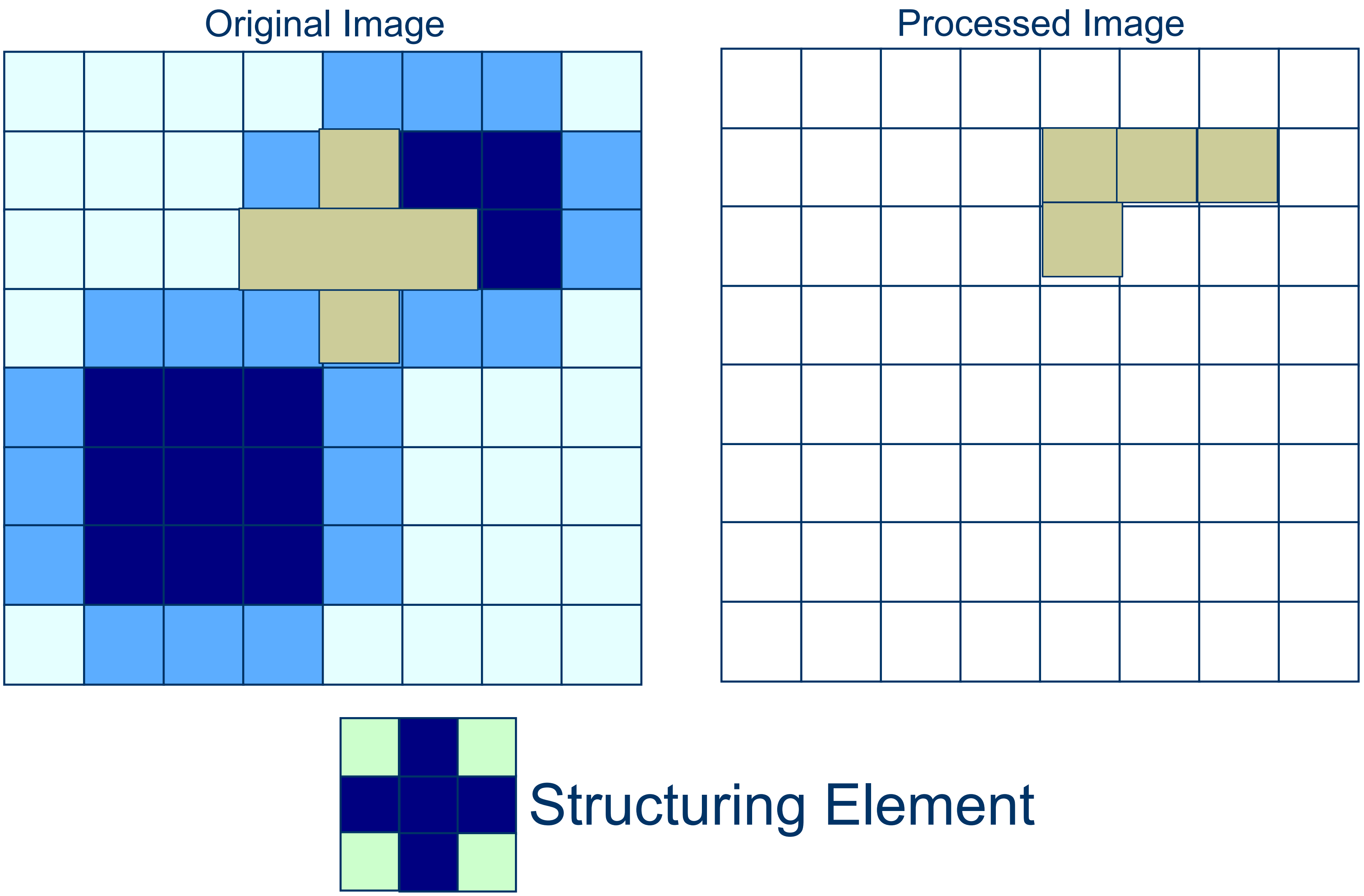
Closing: Example(performing erosion on Dilated image)



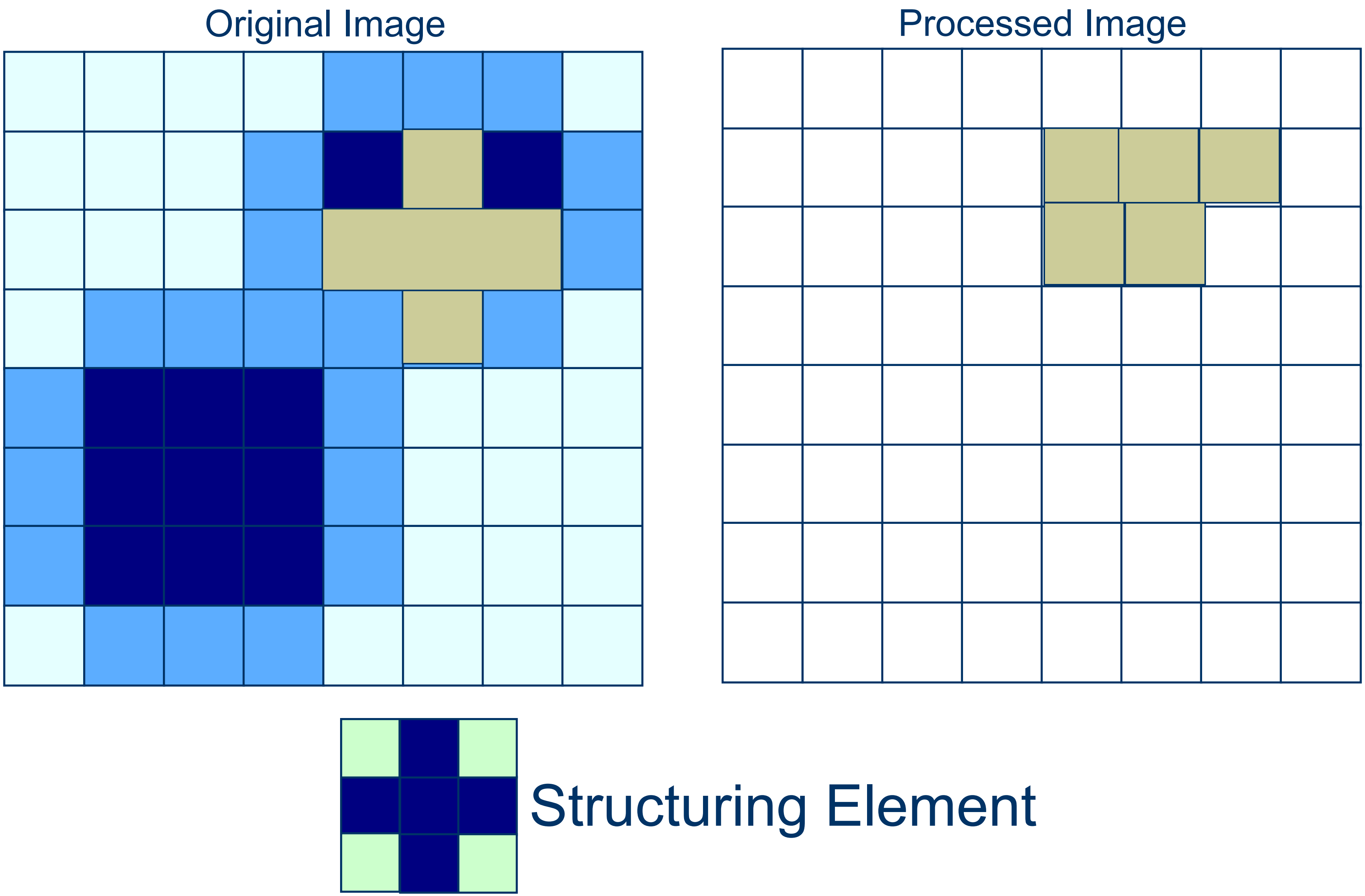
Closing: Example(performing erosion on Dilated image)



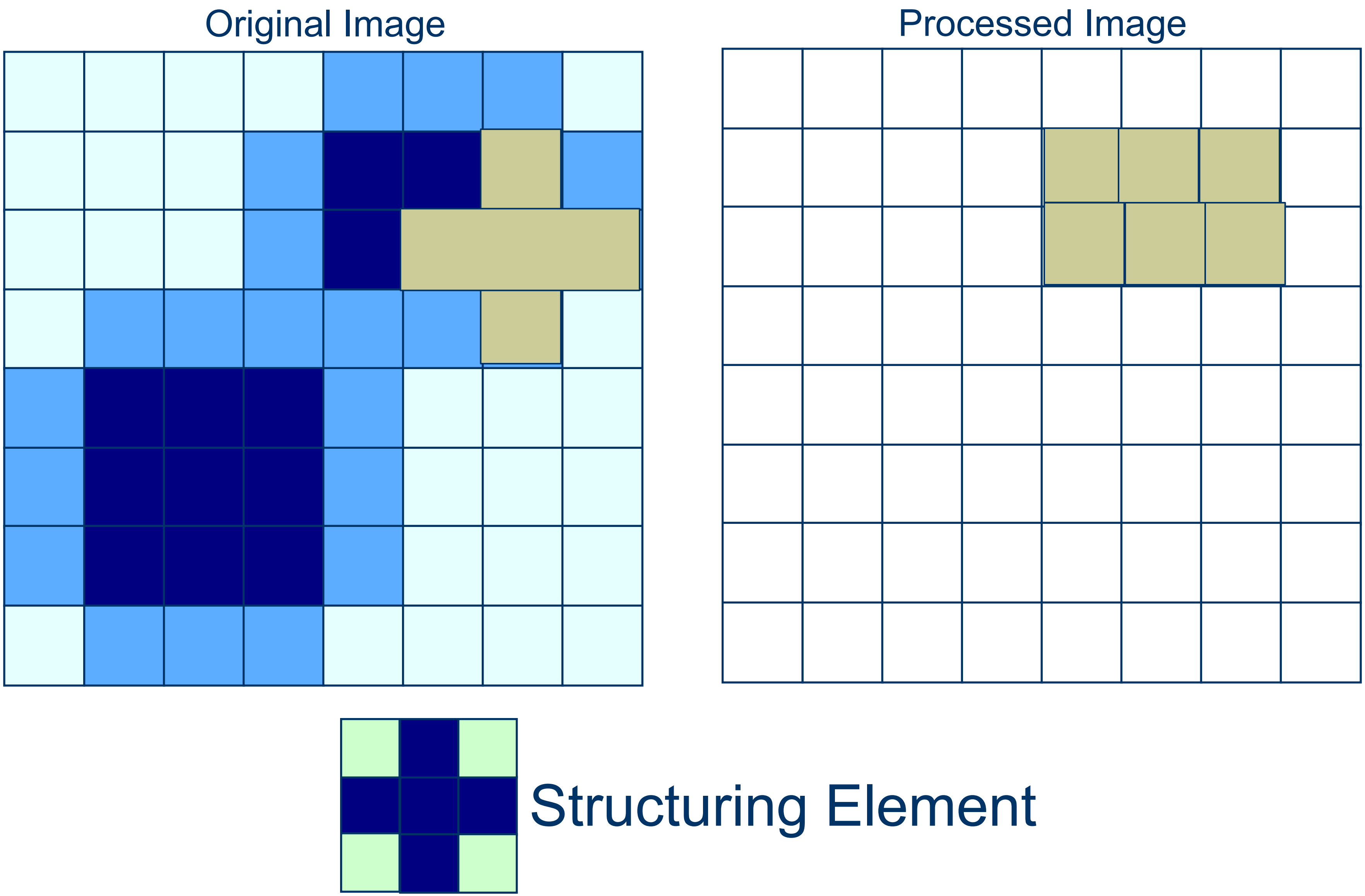
Closing: Example(performing erosion on Dilated image)



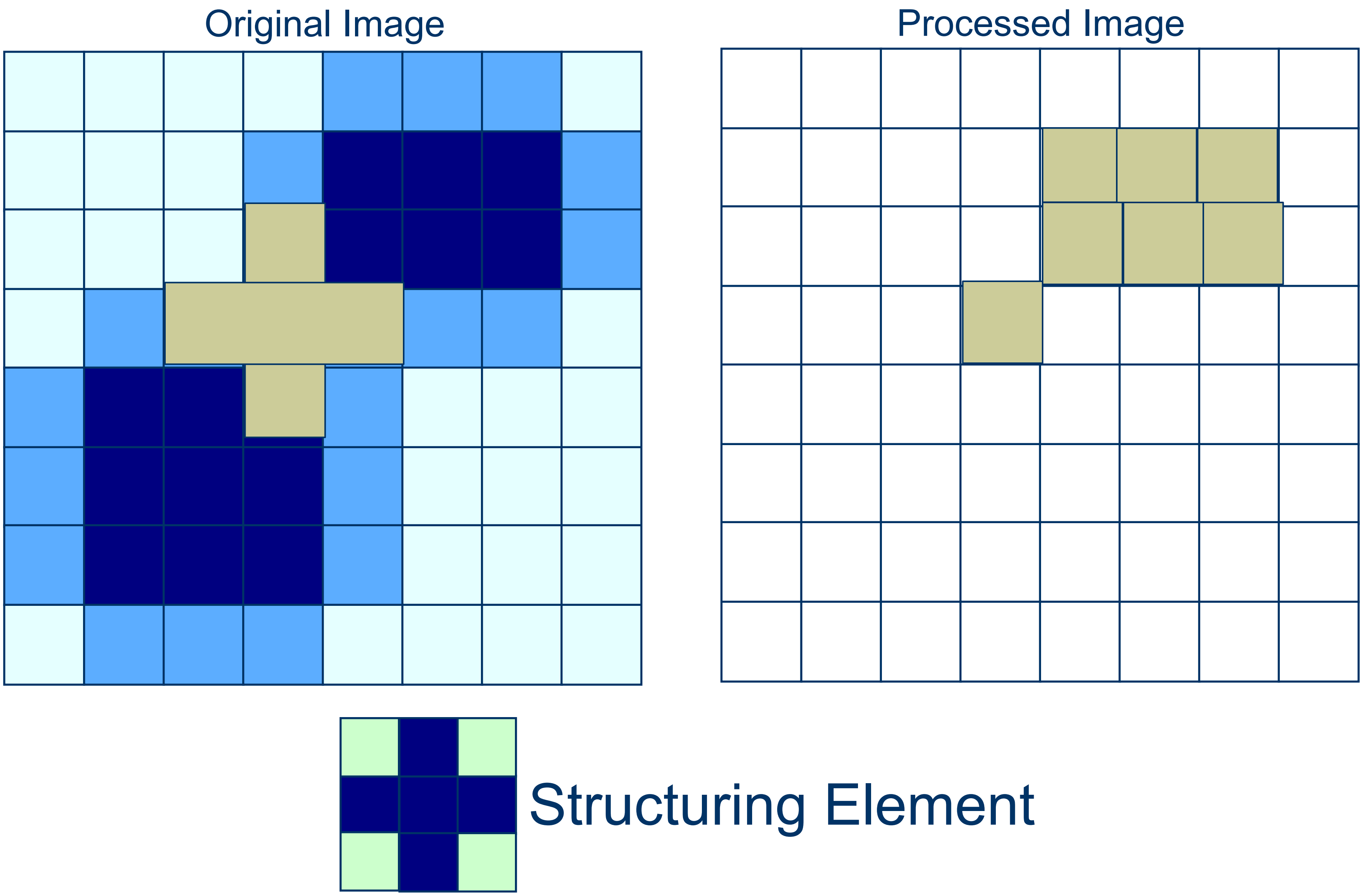
Closing: Example(performing erosion on Dilated image)



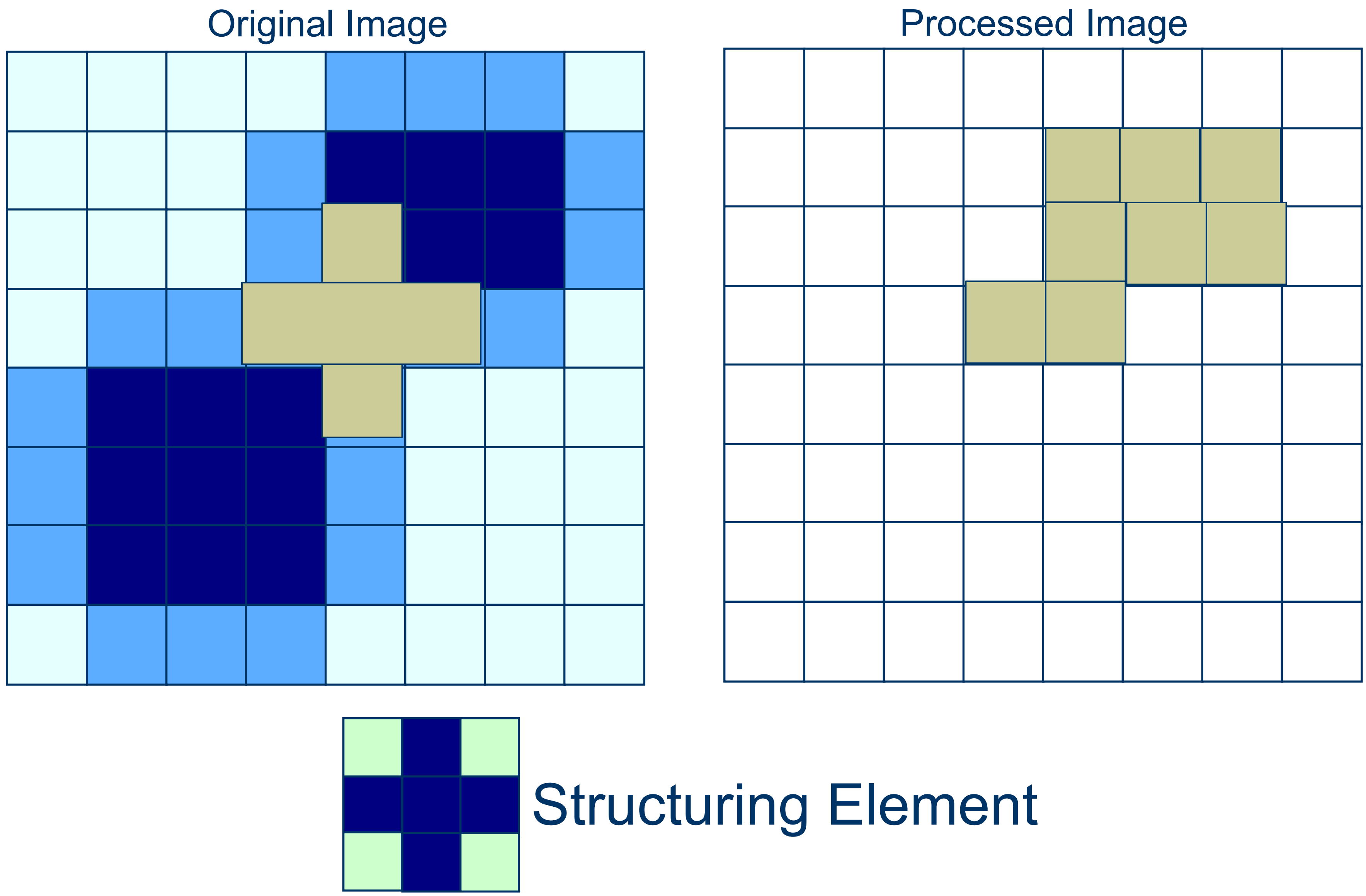
Closing: Example(performing erosion on Dilated image)



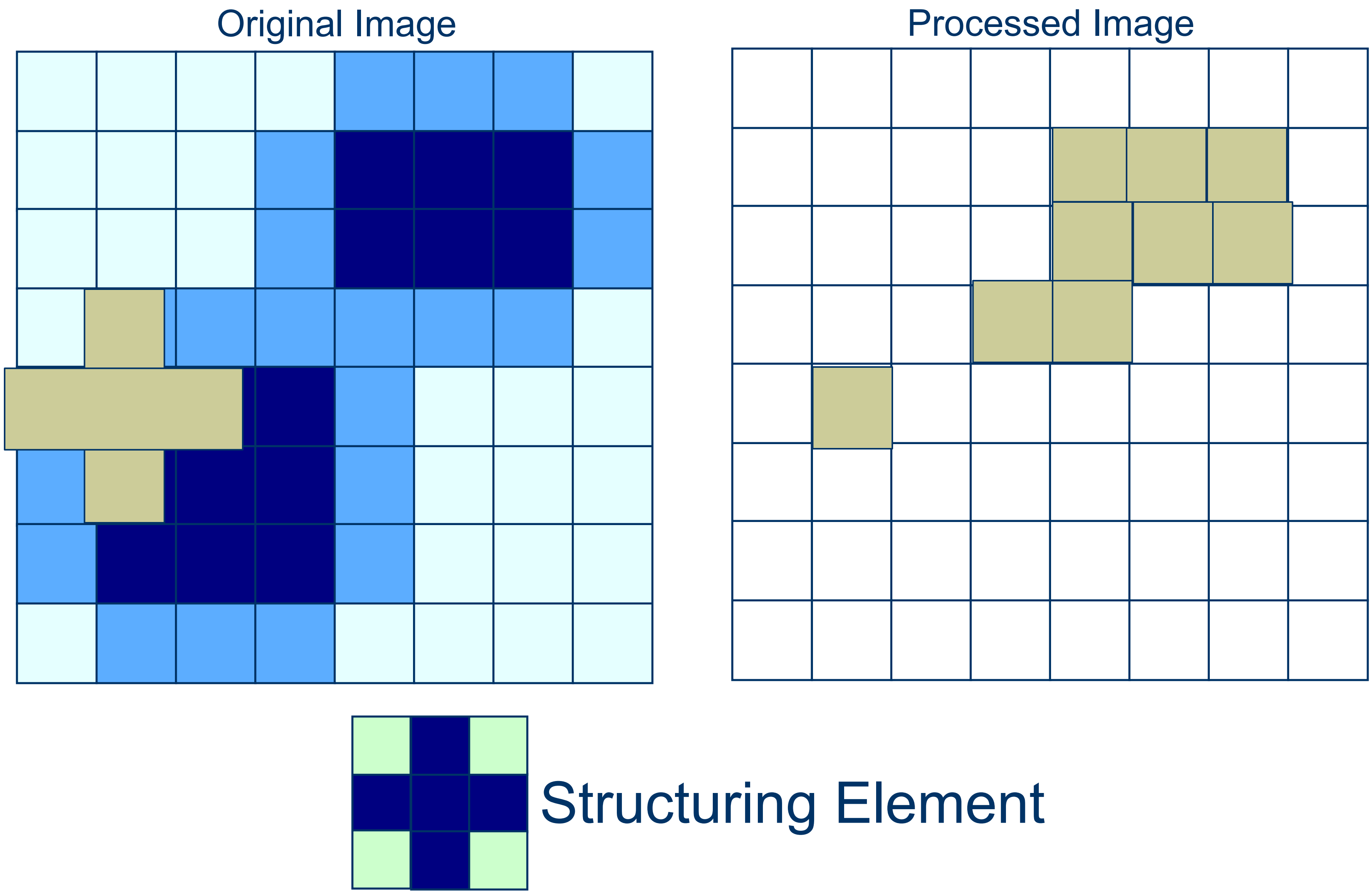
Closing: Example(performing erosion on Dilated image)



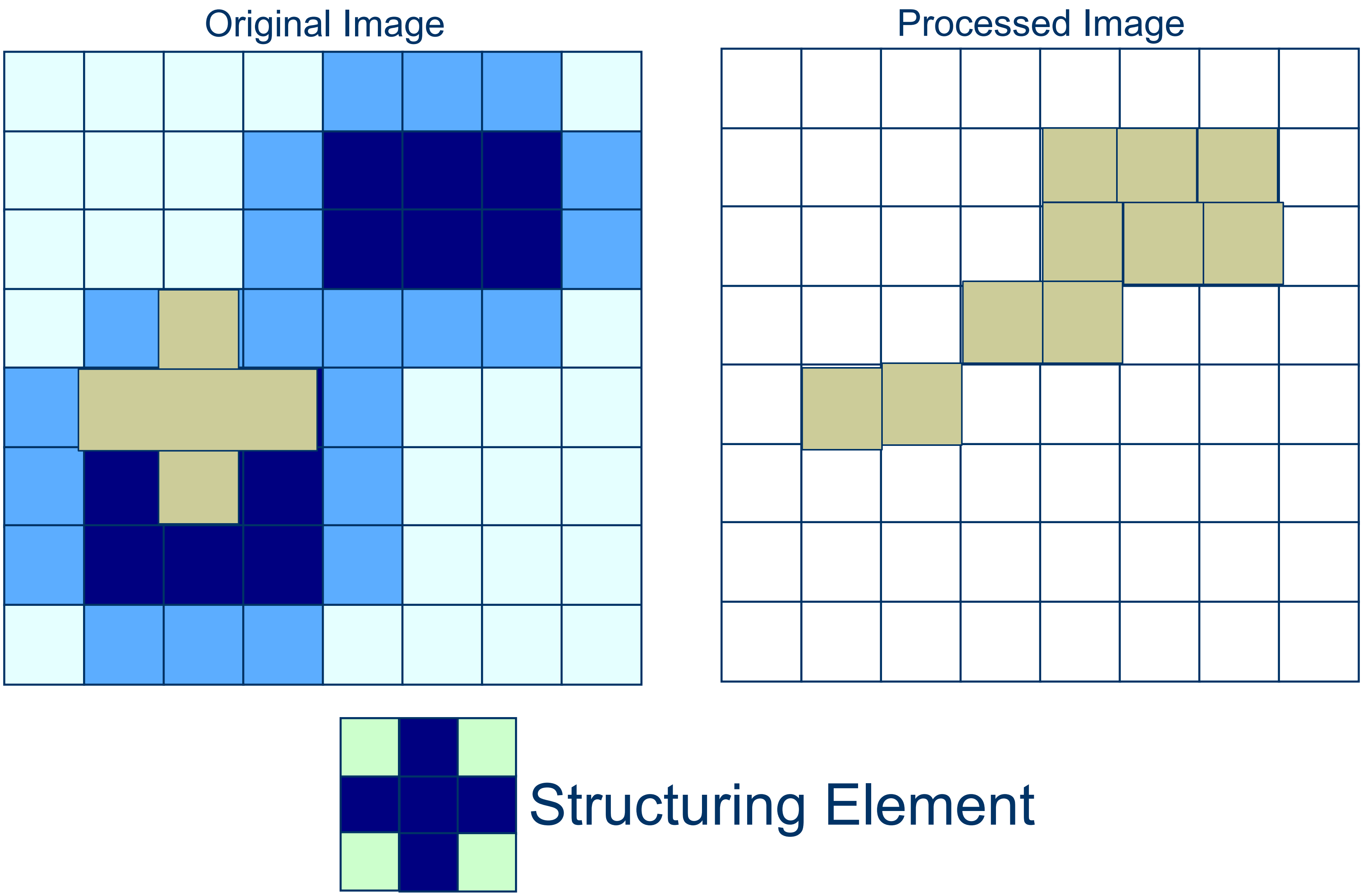
Closing: Example(performing erosion on Dilated image)



Closing: Example(performing erosion on Dilated image)

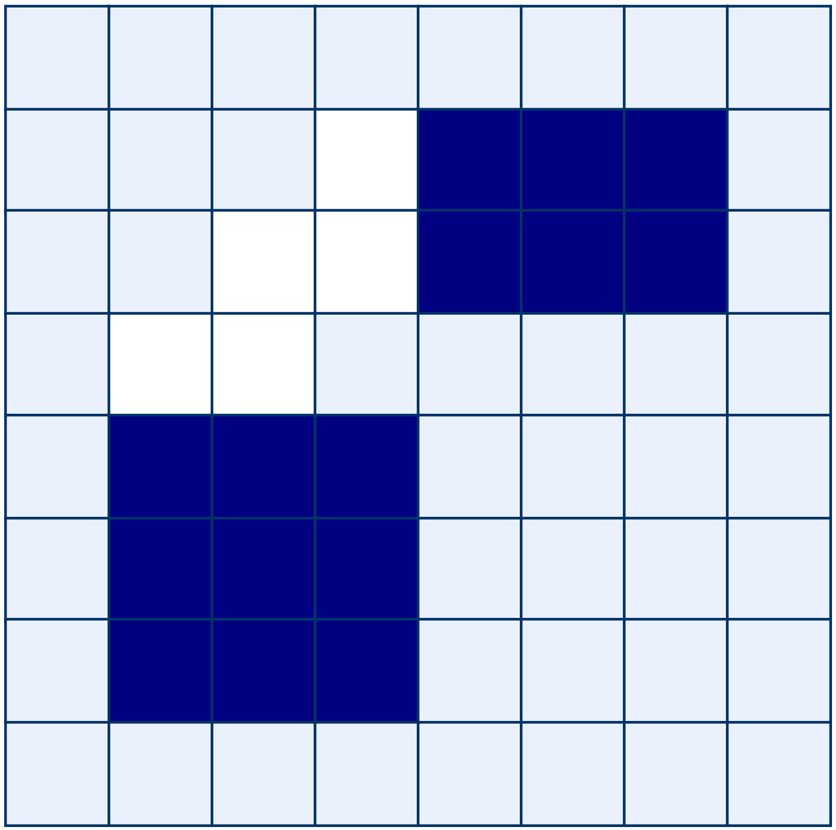


Closing: Example(performing erosion on Dilated image)

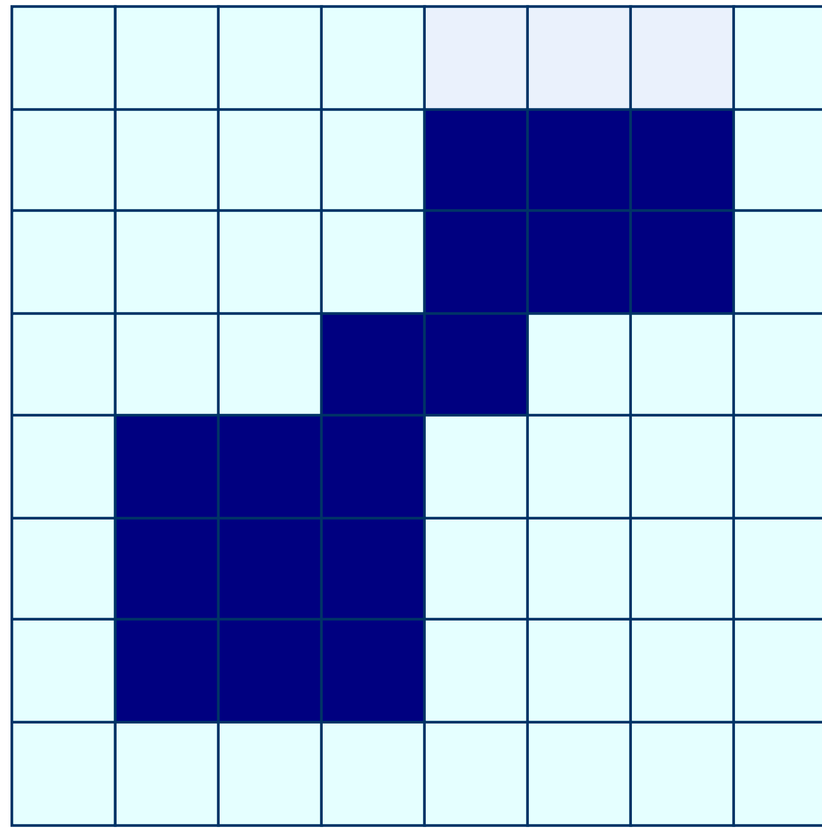


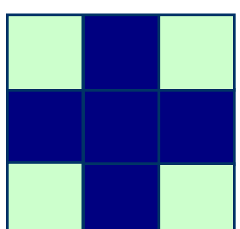
Closing: Output

Original Image



Processed Image

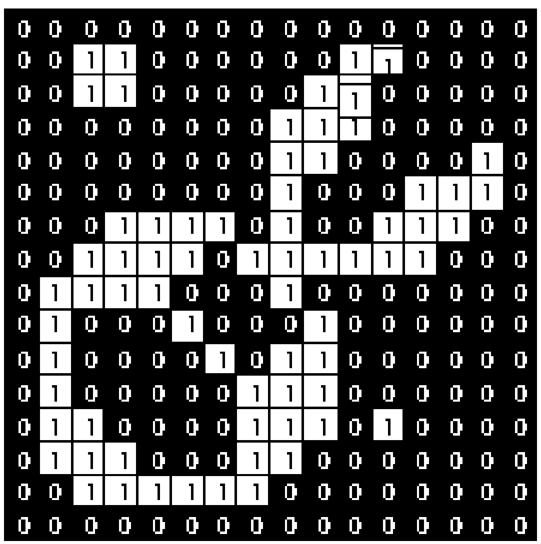


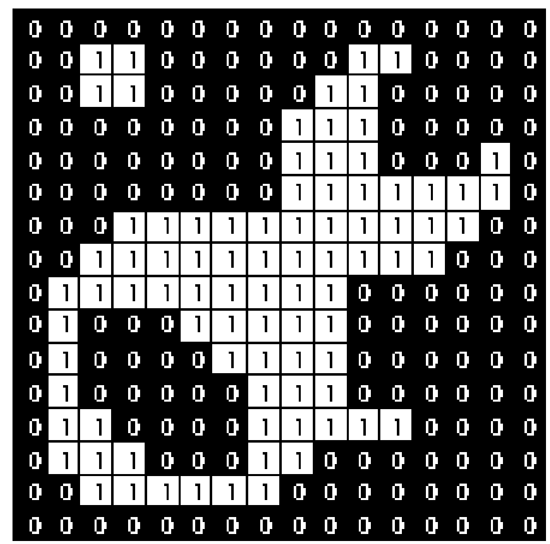


Structuring Element

Closing

- Structuring element: 3x3 square





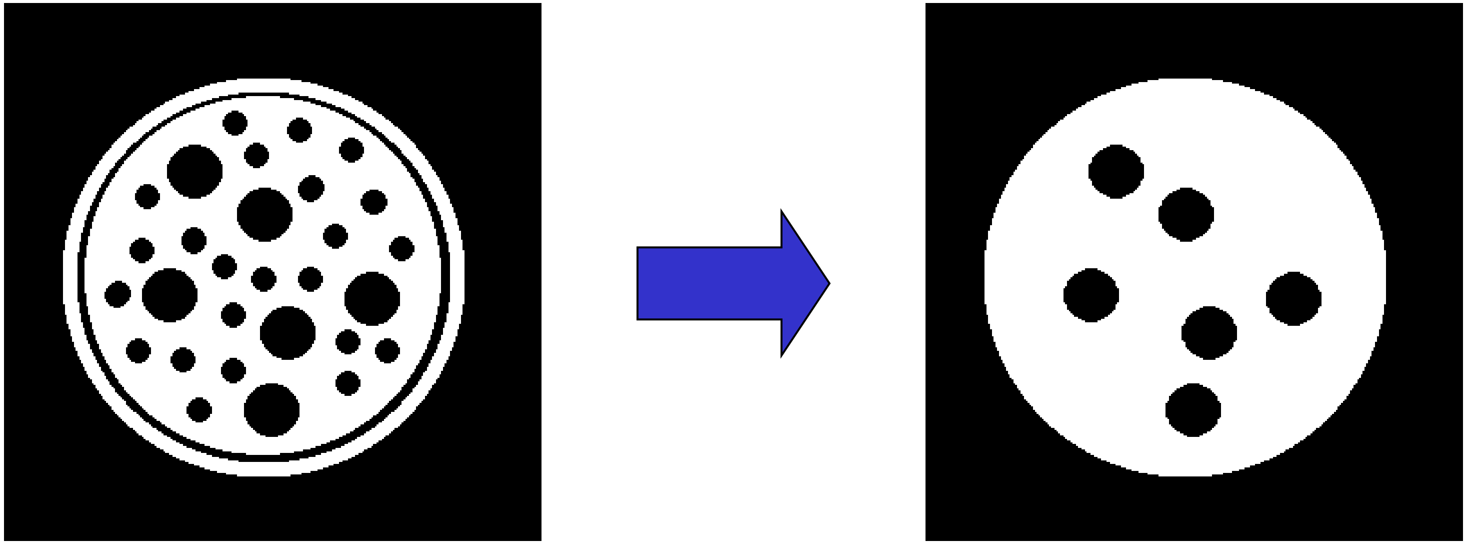
Effect of closing

- Smoothed the outline, by filling in (closing) any holes and indentations.
- It also will form connecting 'bridges' to other shapes that are close enough for the kernel (SE) to touch both simultaneously.
- But it does not make the basic 'core' size of the shape larger or smaller.

As with **Open**', repeating the '**Close**' method with the same kernel does not make any further changes to the image.

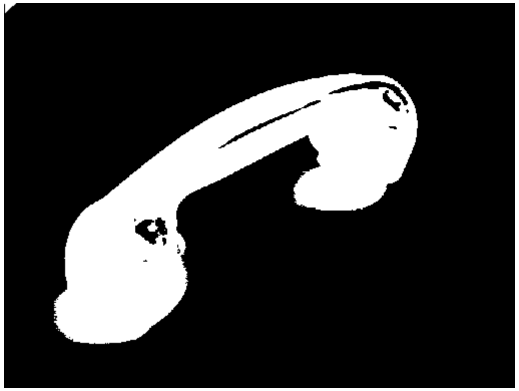
Closing Example

- Closing operation with a 22 pixel disc
- Closes small holes in the foreground

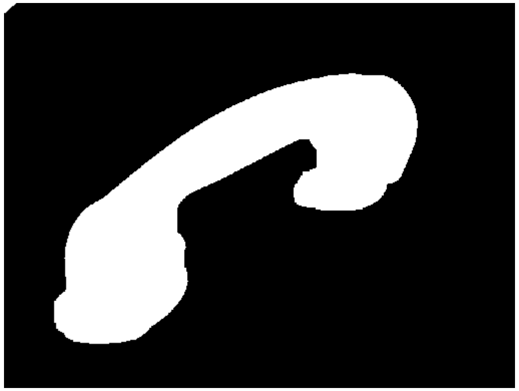


Closing Example 1

1. Threshold
2. Closing with disc of size 20



Thresholded



closed

Opening & Closing

- Opening is the *dual* of closing
- *i.e.* opening the foreground pixels with a particular structuring element
- is equivalent to closing the background pixels with the same element.

***Home Work: Make an example with simulations.

Next class

Outlook:

Hit-and-miss Transformation,

Thinning, and

Thickening

Basic Set Theory

- The set space of binary image is Z^2
  - Each element of the set is a 2D vector whose coordinates are the *(x,y)* of a black (or white, depending on the convention) pixel in the image
- The set space of gray level image is Z^3
  - Each element of the set is a 3D vector: *(x,y)* and intensity level.

NOTE:

Set Theory   Logical operations

Basic Set Theory

- Let A be a set in Z^2. if $a = (a_1,a_2)$ is an element of A, then we write

$$a \in A$$

- If a is not an element of A, we write

$$a \notin A$$

- Set representation

$$A = \{(a_1, a_2), (a_3, a_4)\}$$

- Empty or Null set

$$A = \varnothing$$

Basic Set Theory

- Subset: if every element of A is also an element of another set B, the A is said to be a subset of B

$$A \subseteq B$$

- Union: The set of all elements belonging either to A, B or both

$$C = A \bigcup B$$

- Intersection: The set of all elements belonging to both A and B

$$D = A \bigcap B$$

Basic Set Theory

- Two sets A and B are said to be disjoint or mutually exclusive if they have no common element

$$A \bigcap B = \varnothing$$

- Complement: The set of elements not contained in A

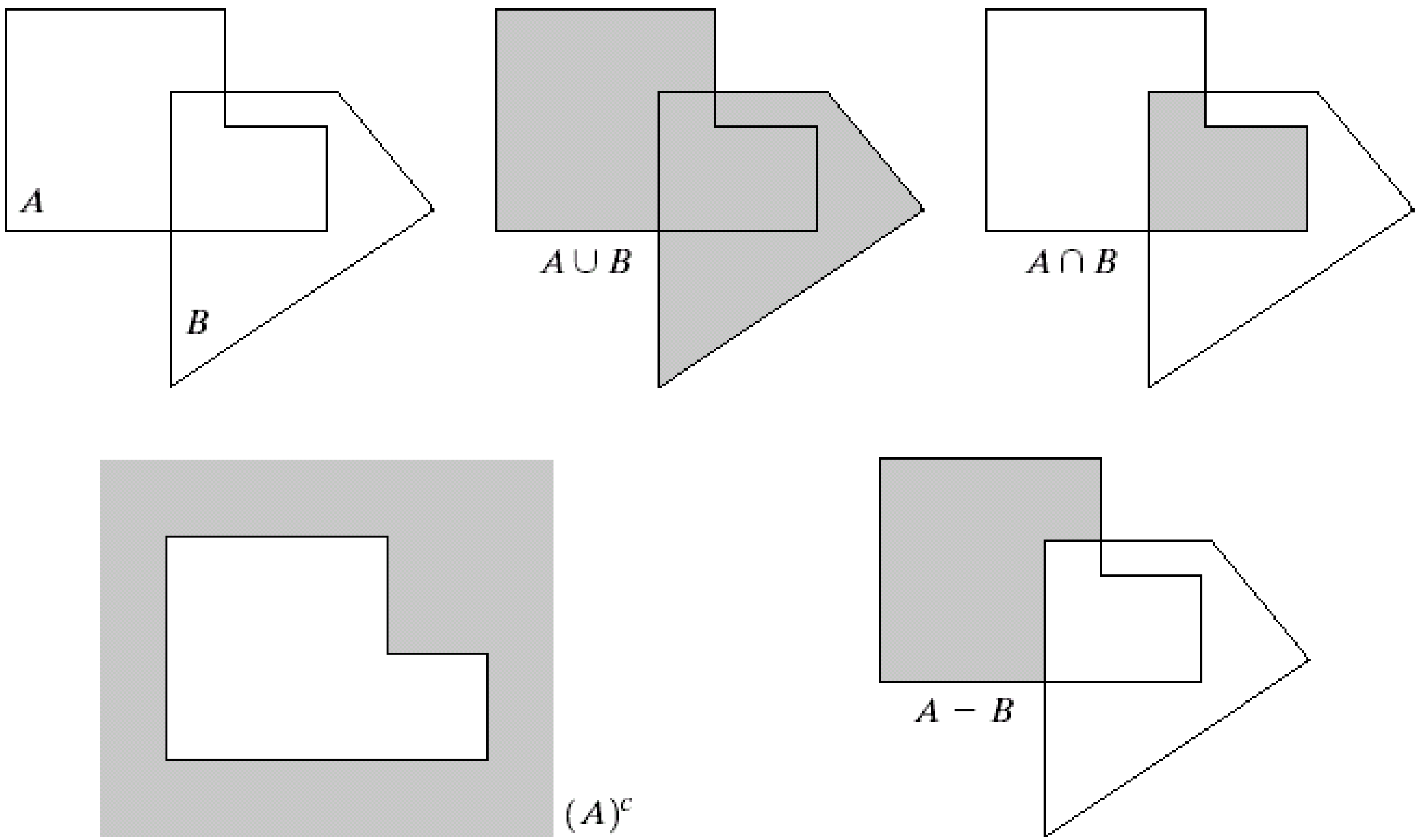
$$A^c = \{w \mid w \notin A\}$$

- Difference of two sets A and B, denoted by A – B, is defined as

$$A - B = \{w \mid w \in A, w \notin B\} = A \bigcap B^c$$

i.e. the set of elements that belong to A, but not to B

Basic Set Theory



a b c
d e

FIGURE 9.1
(a) Two sets A and B. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B.

Example of some logic operations:

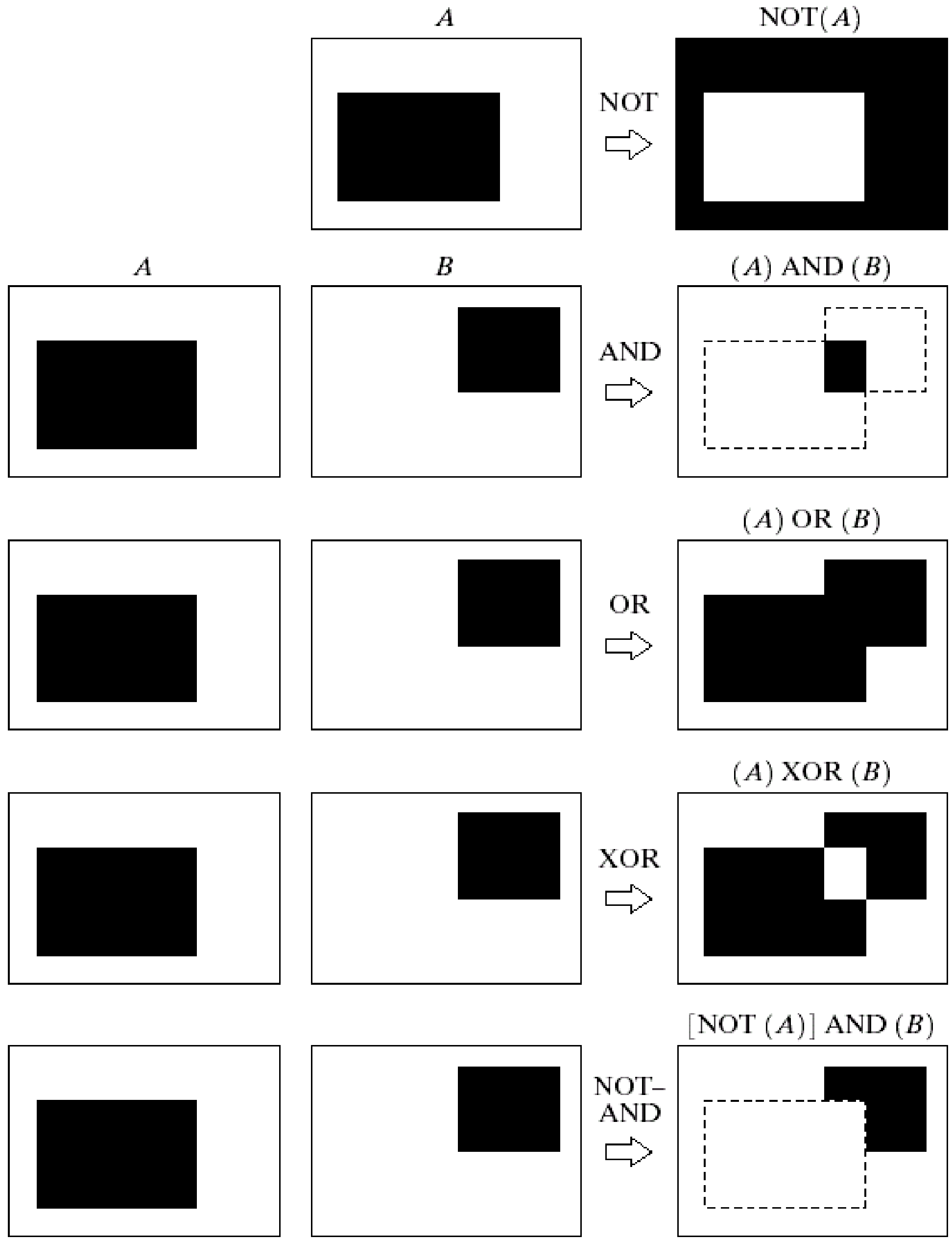


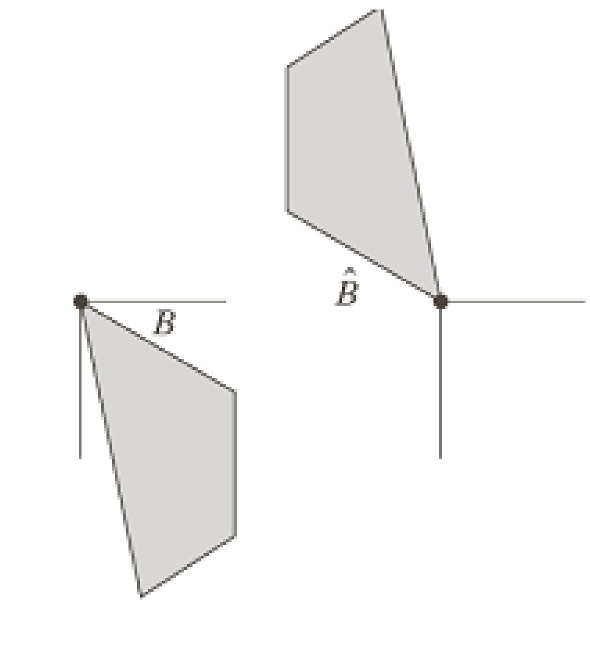
FIGURE 9.3 Some logic operations between binary images. Black represents binary 1s and white binary 0s in this example.

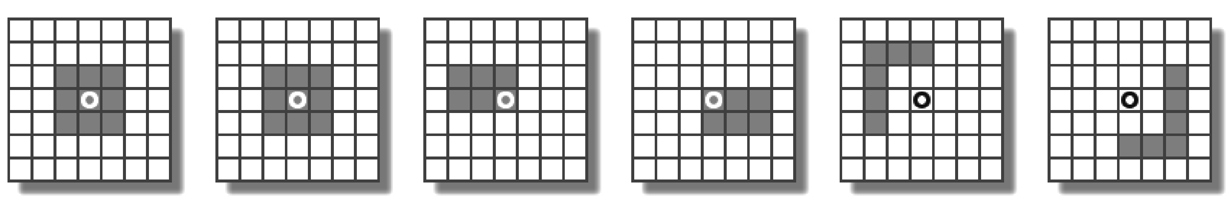
Reflection of set B

- Reflection of set B

$$B = \{w \mid w = -b, \quad \text{for } b \in B\}$$

i.e. the set of element *w*, such that *w* is formed by multiplying each of two coordinates of all the elements of set *B* by -1



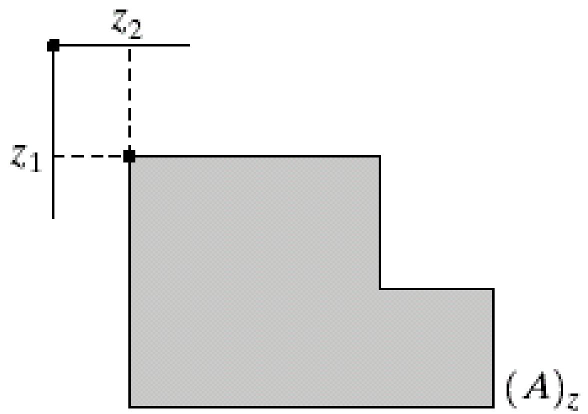


B_1 $\widehat{B}_1$ B_2 $\widehat{B}_2$ B_3 $\widehat{B}_3$

Translation of set A

- Translation of set A by point $z = (z_1, z_2)$, denoted $(A)_z$, is defined as

$$(A)_z = \{w \mid w = a + z, \ \text{for } a \in A\}$$



Structuring Element: Translation

Let I be an image and *B* a SE.

(B)$_z$ means that *B* is moved so that its origin coincides with location z in image *I*.

(B)$_z$ is the *translate* of *B* to location *z* in *I*.