

# CSE4227 Digital Image Processing

## Lecture 02 – Chapter 2: Digital Image Fundamentals

Dr. Kazi A Kalpoma

Professor, Department of CSE

Ahsanullah University of Science & Technology (AUST)

Contact: [kalpoma@aust.edu](mailto:kalpoma@aust.edu)



CSE | AUST

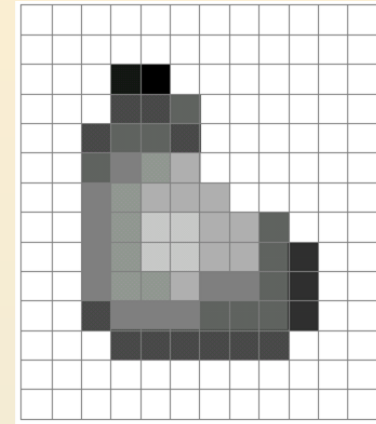
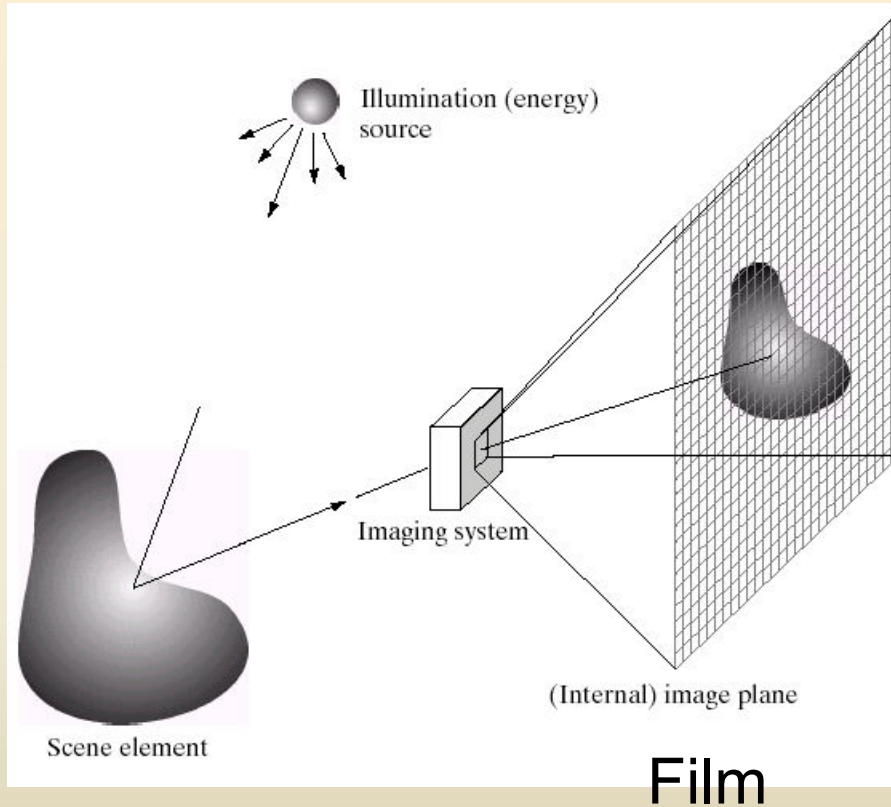
Fall 2023

# Today's Contents

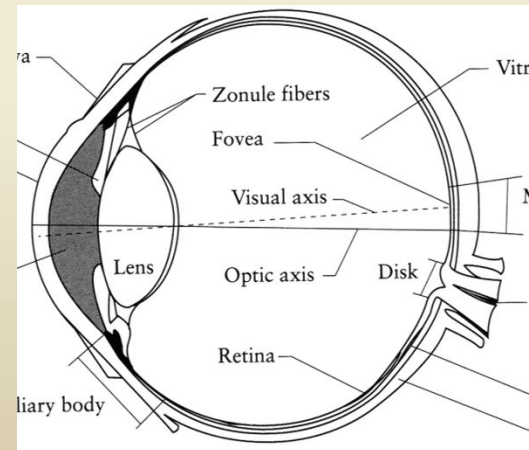
- ❑ Image Sensing and Acquisition
- ❑ Sampling and quantization
- ❑ Image Representation
- ❑ Spatial and Intensity Resolution

•Chapter 2 from R.C. Gonzalez and R.E. Woods, Digital Image Processing (3rd Edition), Prentice Hall, 2008 [ **Section 2.3, 2.4** ]

# Image Formation



Digital Camera



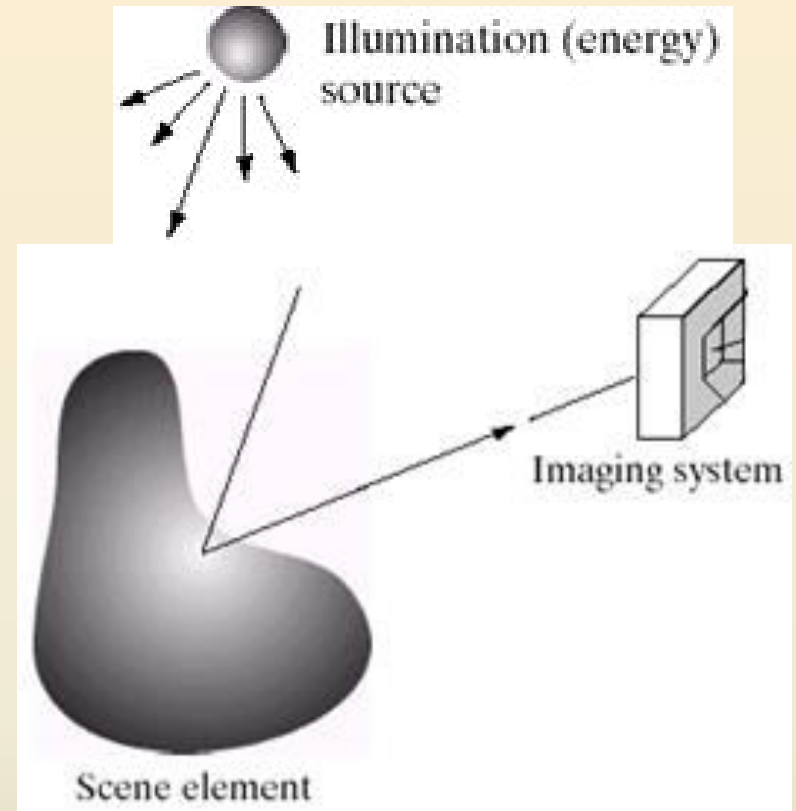
The Eye

# **IMAGE SENSING AND ACQUISITION**

# Image Acquisition

## □ Three main elements

1. **Illumination source**
2. **Scene**
3. **Sensor (imaging system)**



# Image Acquisition

## 1. Illumination source:

- Can be light energy or
- EM spectrum
- Even less tradition sources like
  - Sound, heat

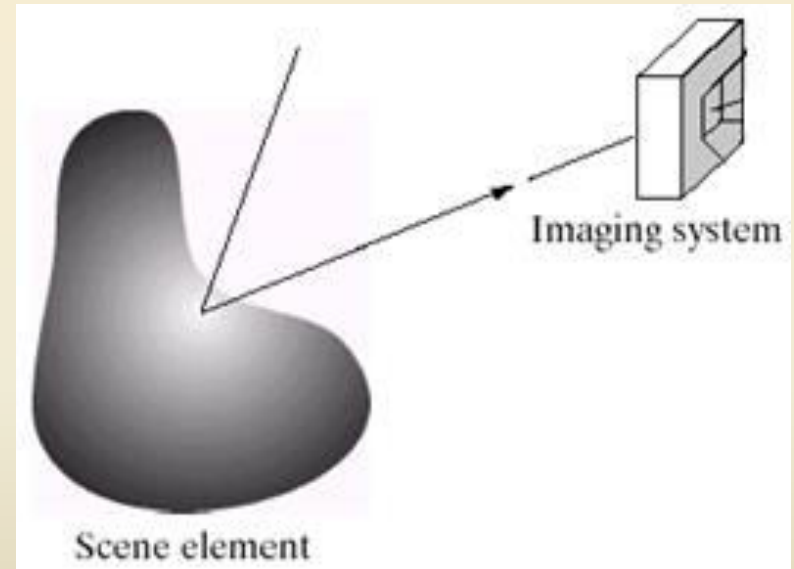


## 2. Scene:

- Any object: visible or hidden
- Source itself

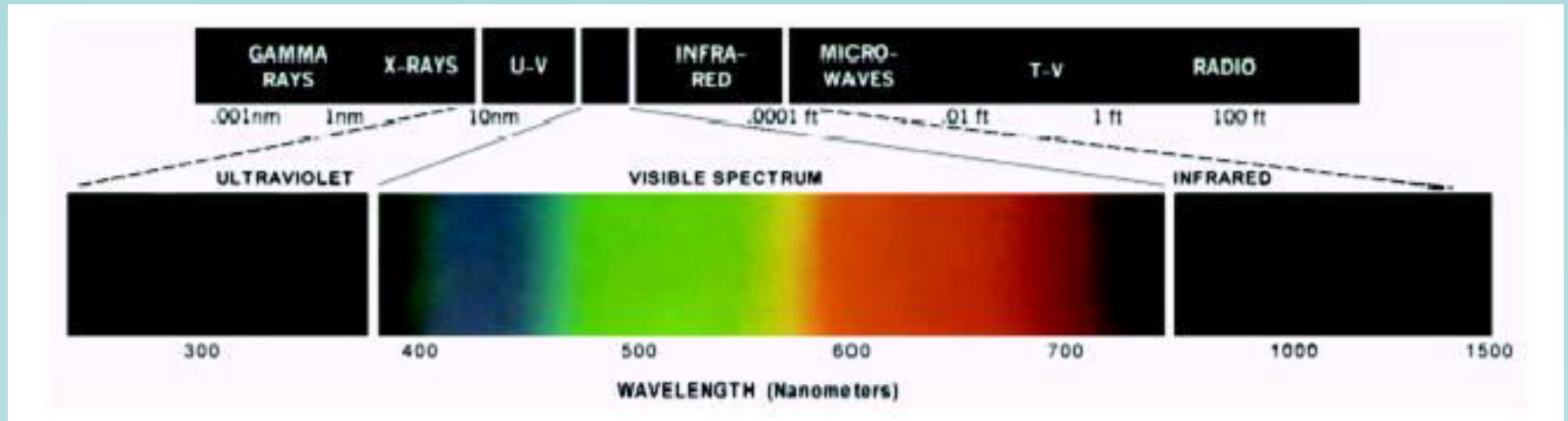
## 3. Sensor:

- Should be capable of sensing the energy



# Light And The Electromagnetic Spectrum

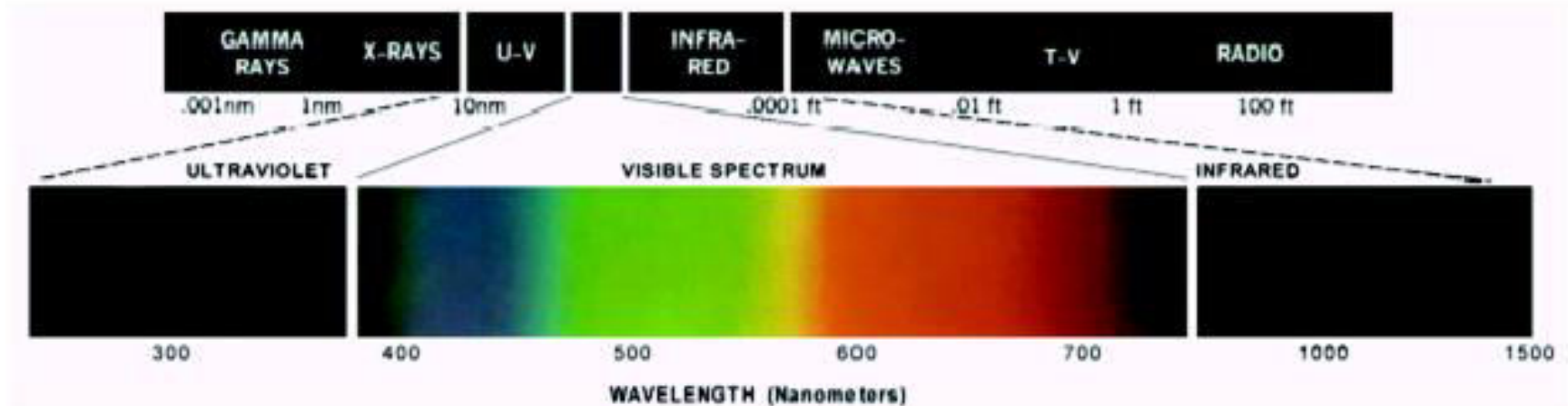
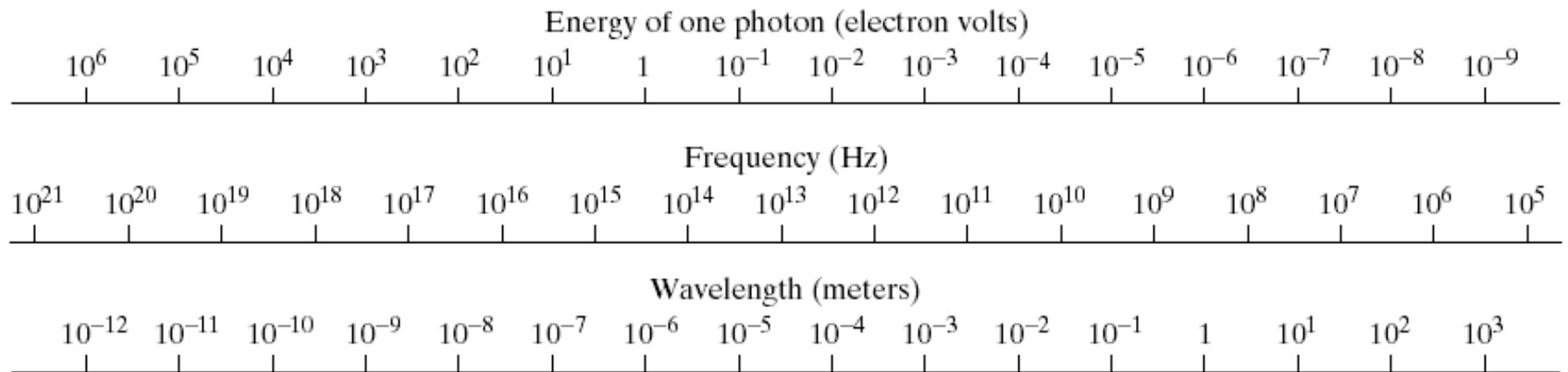
❑ In 1666 Sir Isaac Newton discovered that sunlight passed through a prism splits into a continuous spectrum of colors



❑ Light is just a particular part of the electromagnetic spectrum that can be sensed by the human eye

❑ The electromagnetic spectrum is split up according to the wavelengths of different forms of energy

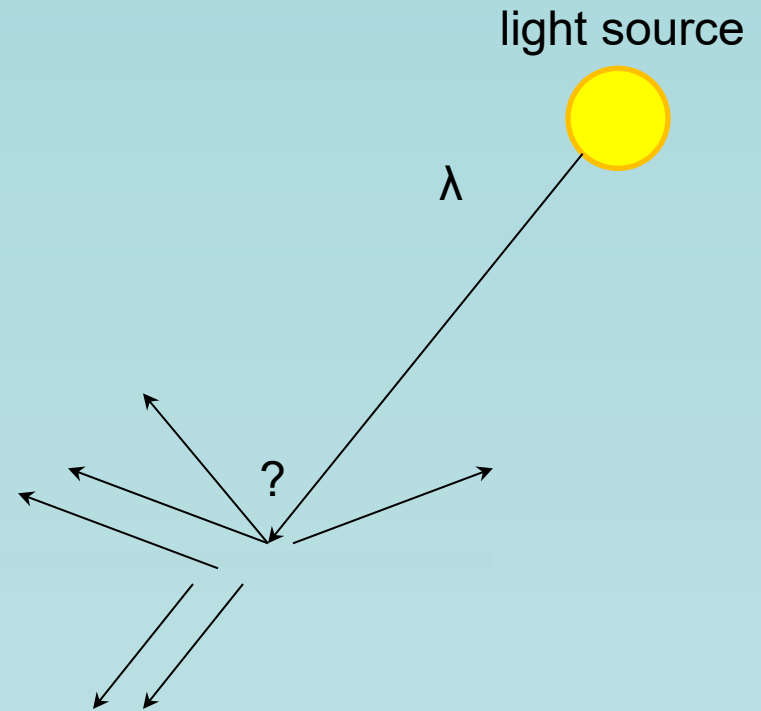
- ❑ A discrete bundle (or *quantum*) of electromagnetic (or light) energy, PHOTON is proportional to frequency.





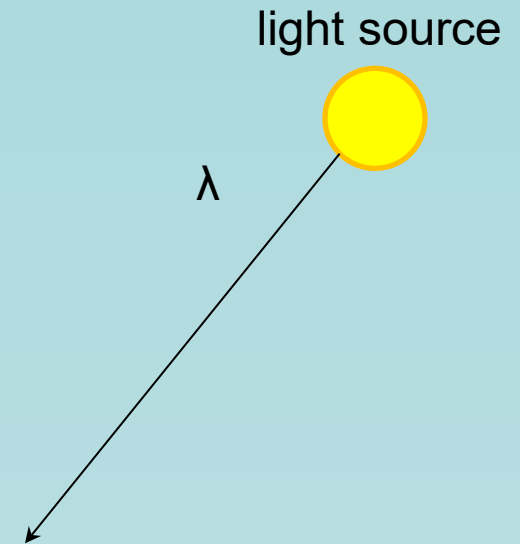
# A photon's life choices

- ☐ Absorption
- ☐ Diffusion
- ☐ Reflection
- ☐ Transparency
- ☐ Refraction
- ☐ Fluorescence
- ☐ Subsurface scattering
- ☐ Phosphorescence
- ☐ Interreflection



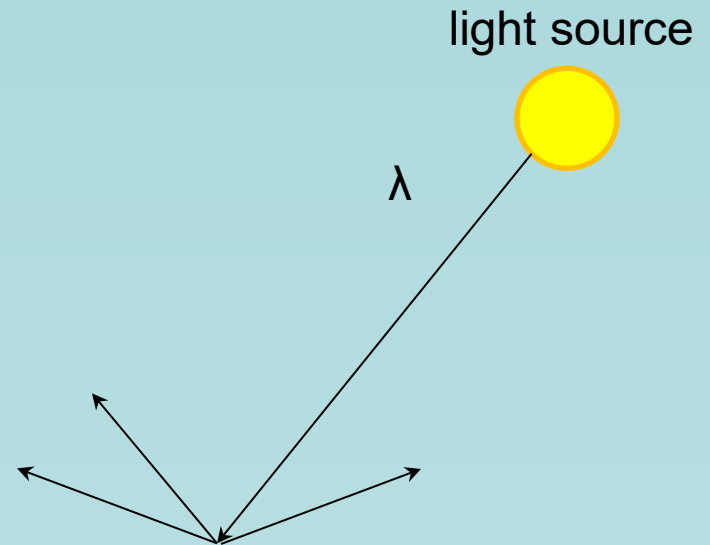
# A photon's life choices

- **Absorption**
- Diffusion
- Reflection
- Transparency
- Refraction
- Fluorescence
- Subsurface scattering
- Phosphorescence
- Interreflection



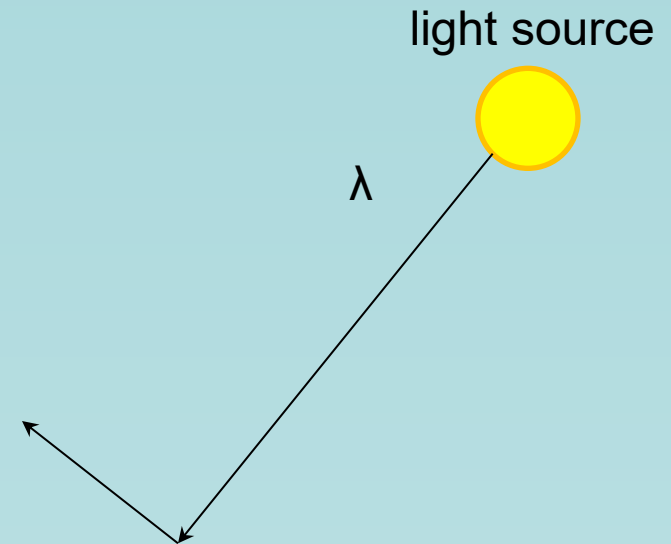
# A photon's life choices

- Absorption
- **Diffuse Reflection**
- Reflection
- Transparency
- Refraction
- Fluorescence
- Subsurface scattering
- Phosphorescence
- Interreflection



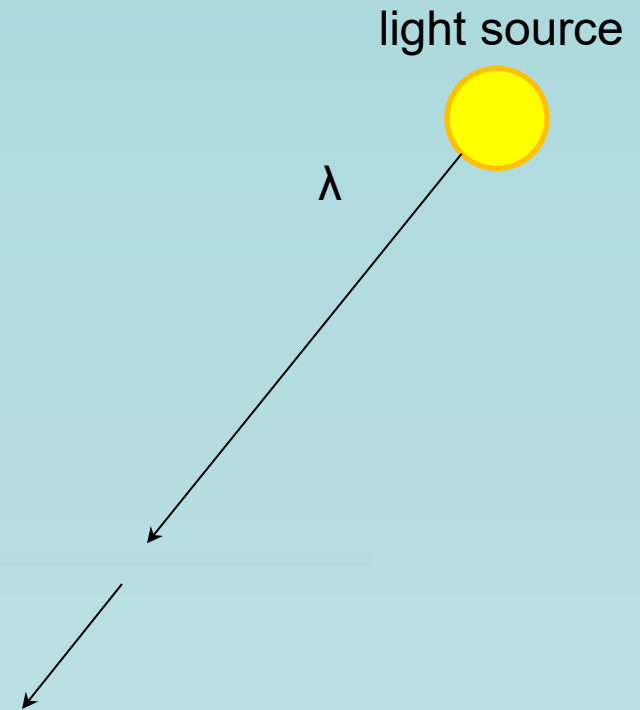
# A photon's life choices

- Absorption
- Diffusion
- **Specular Reflection**
- Transparency
- Refraction
- Fluorescence
- Subsurface scattering
- Phosphorescence
- Interreflection



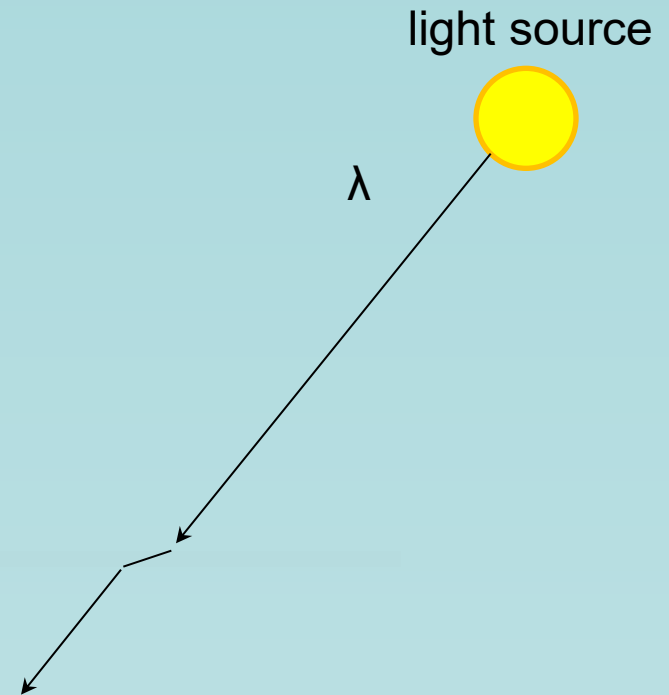
# A photon's life choices

- Absorption
- Diffusion
- Reflection
- **Transparency**
- Refraction
- Fluorescence
- Subsurface scattering
- Phosphorescence
- Interreflection



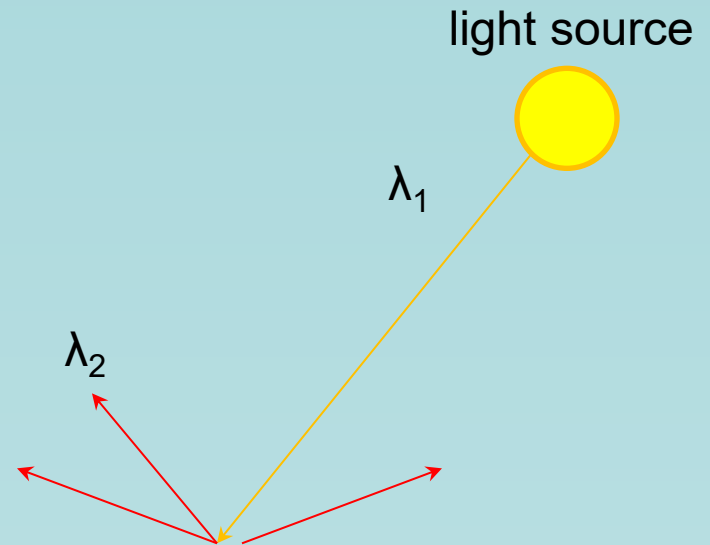
# A photon's life choices

- Absorption
- Diffusion
- Reflection
- Transparency
- **Refraction**
- Fluorescence
- Subsurface scattering
- Phosphorescence
- Interreflection



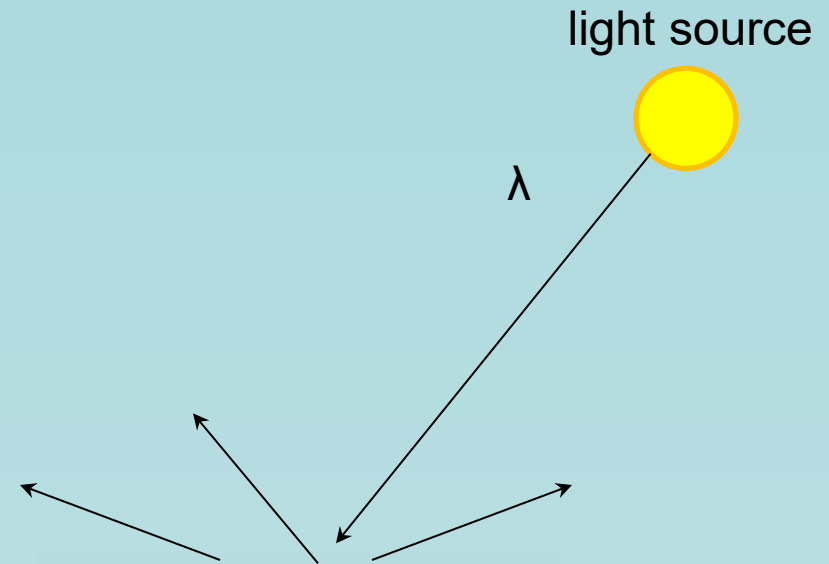
# A photon's life choices

- Absorption
- Diffusion
- Reflection
- Transparency
- Refraction
- **Fluorescence**
- Subsurface scattering
- Phosphorescence
- Interreflection



# A photon's life choices

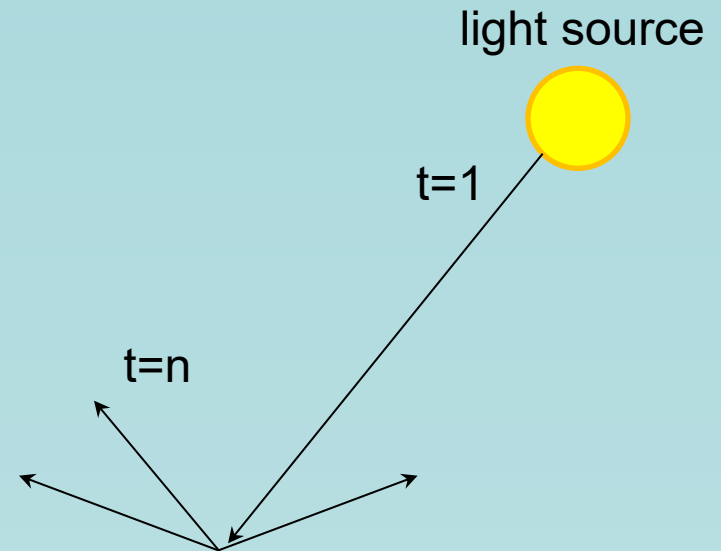
- Absorption
- Diffusion
- Reflection
- Transparency
- Refraction
- Fluorescence
- **Subsurface scattering**
- Phosphorescence
- Interreflection





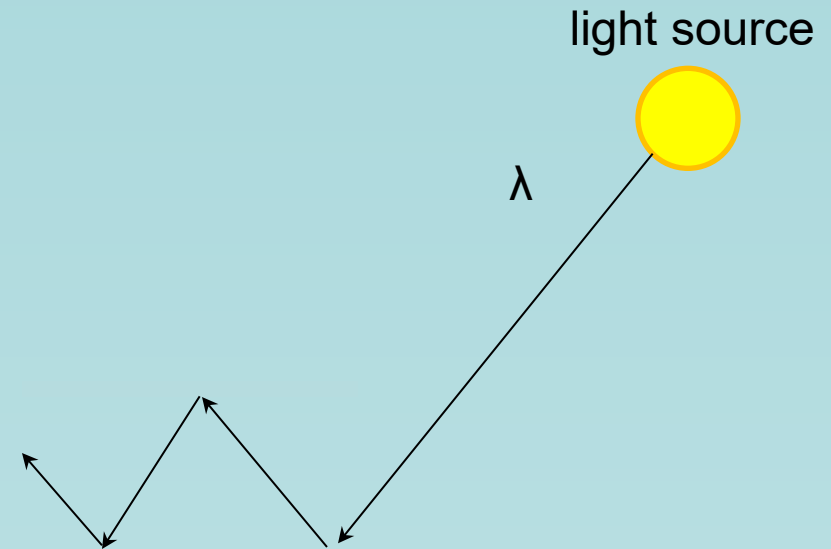
# A photon's life choices

- Absorption
- Diffusion
- Reflection
- Transparency
- Refraction
- Fluorescence
- Subsurface scattering
- **Phosphorescence**
- Interreflection



# A photon's life choices

- Absorption
- Diffusion
- Reflection
- Transparency
- Refraction
- Fluorescence
- Subsurface scattering
- Phosphorescence
- **Interreflection**

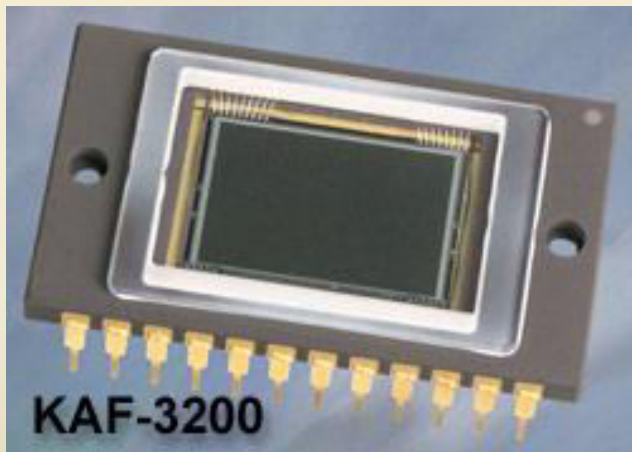


(Specular Interreflection)

# Image Sensors

- ❑ Incoming energy lands on a sensor material responsive to that type of energy and this generates a voltage
- ❑ Collections of sensors are arranged to capture images

## *Charge-Coupled Device (CCD) or Complementary Metal Oxide Semiconductor (CMOS)*



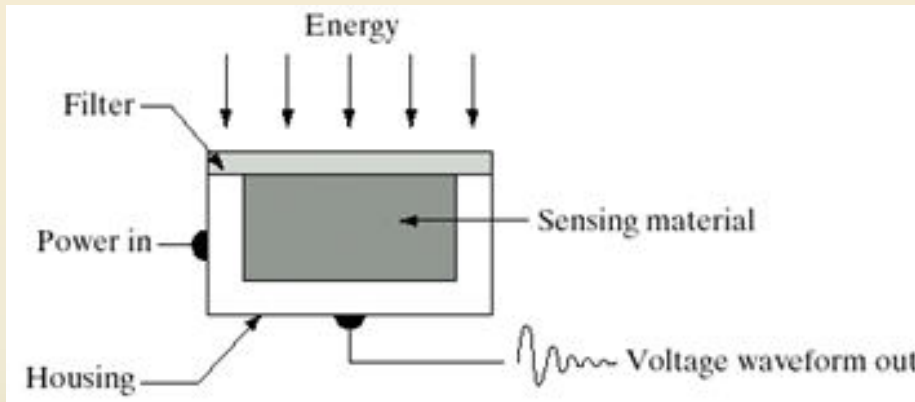
- ✓ Used for convert a continuous image into a digital image
- ✓ Contains an array of light sensors
- ✓ Converts photon into electric charges accumulated in each sensor unit

CCD KAF-3200E from  
Kodak.  
(2184 x 1472 pixels,  
Pixel size 6.8 microns<sup>2</sup>)

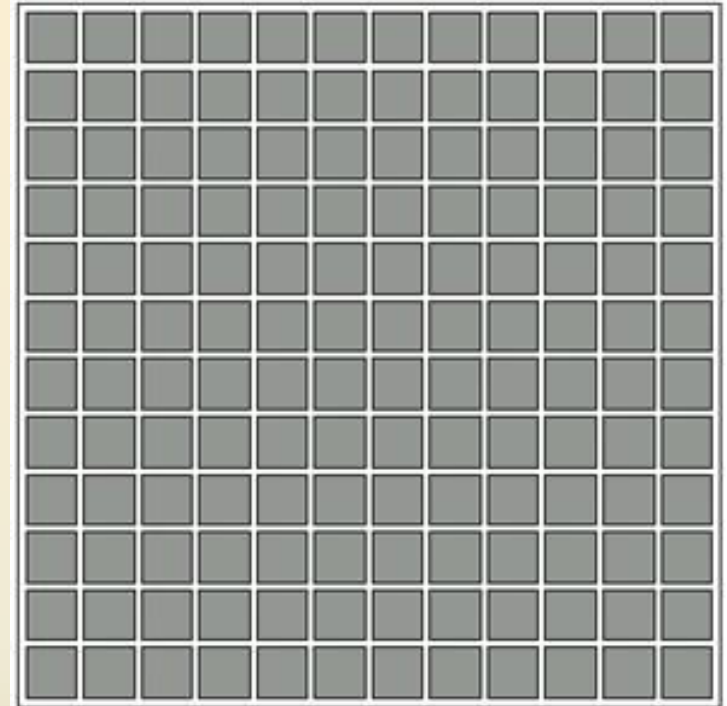
# Image Sensors used to transform illumination energy to digital images.

## □ 3 main sensor arrangements:

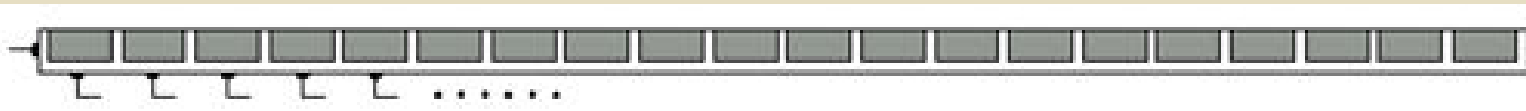
1. **Single sensor**
2. **Line sensor**
3. **Array sensor**



**Single sensor**



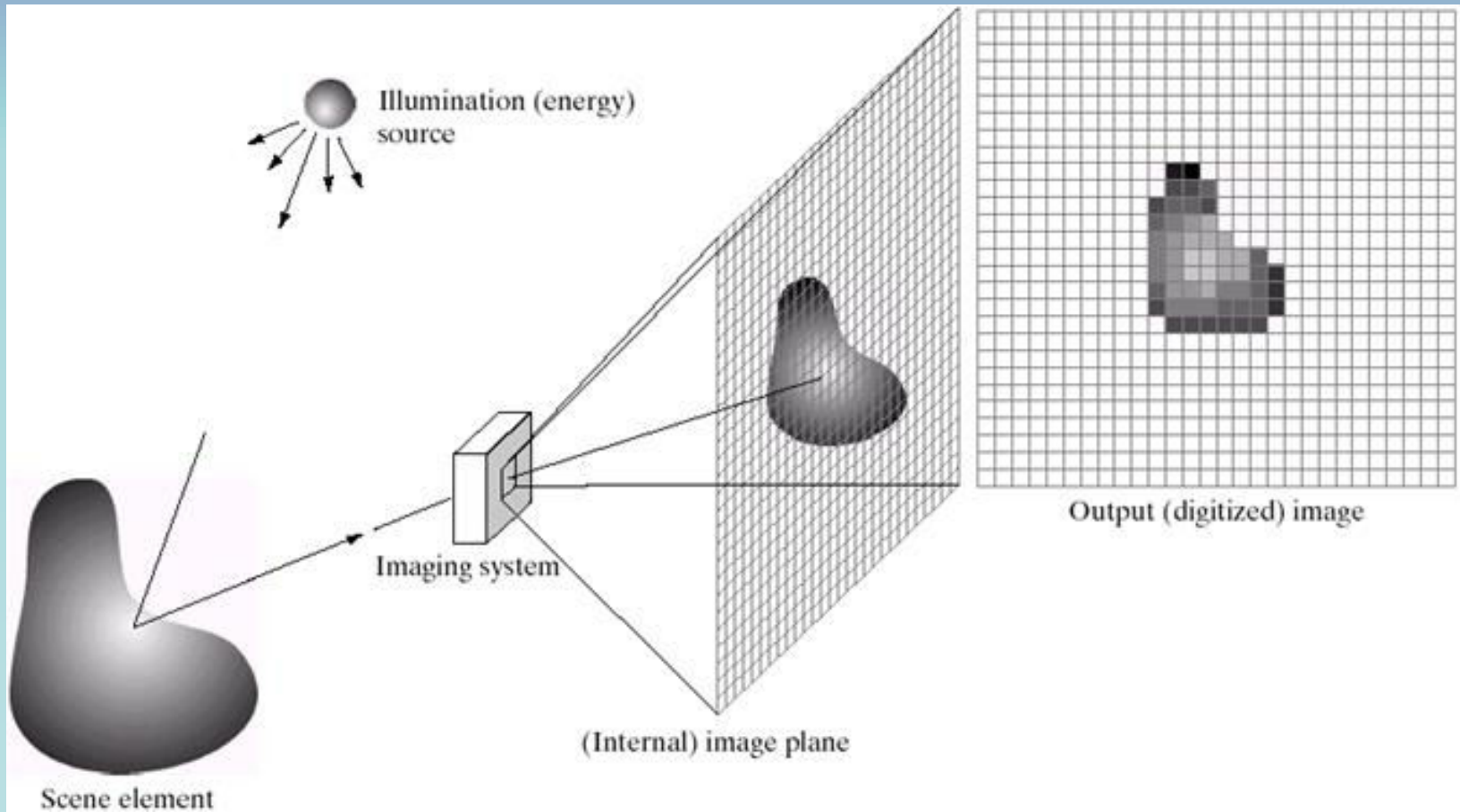
**Array sensor**



**Line sensor**

- Sensing element can be a photodiode
- Filter absorbs extra energy or acts as pass-band

# Image Acquisition using Sensor Array



- ❑ Images are typically generated by *illuminating* a scene and absorbing the energy reflected by the objects in that scene.

# Human Vision VS Computer Vision



What we see

|   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|
| 0 | 3 | 2 | 5 | 4 | 7 | 6 | 9 | 8 |
| 3 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 2 | 1 | 0 | 3 | 2 | 5 | 4 | 7 | 6 |
| 5 | 2 | 3 | 0 | 1 | 2 | 3 | 4 | 5 |
| 4 | 3 | 2 | 1 | 0 | 3 | 2 | 5 | 4 |
| 7 | 4 | 5 | 2 | 3 | 0 | 1 | 2 | 3 |
| 6 | 5 | 4 | 3 | 2 | 1 | 0 | 3 | 2 |
| 9 | 6 | 7 | 4 | 5 | 2 | 3 | 0 | 1 |
| 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |

What a computer sees

# Image Digitization

- ❑ Computers use discrete form of the pictures
- ❑ The process transforming continuous analog image into discrete approximated image is called **digitization**.



- ❑ Two steps:
  - Sampling and
  - Quantization

# Image Sampling And Quantisation

## Sampling:

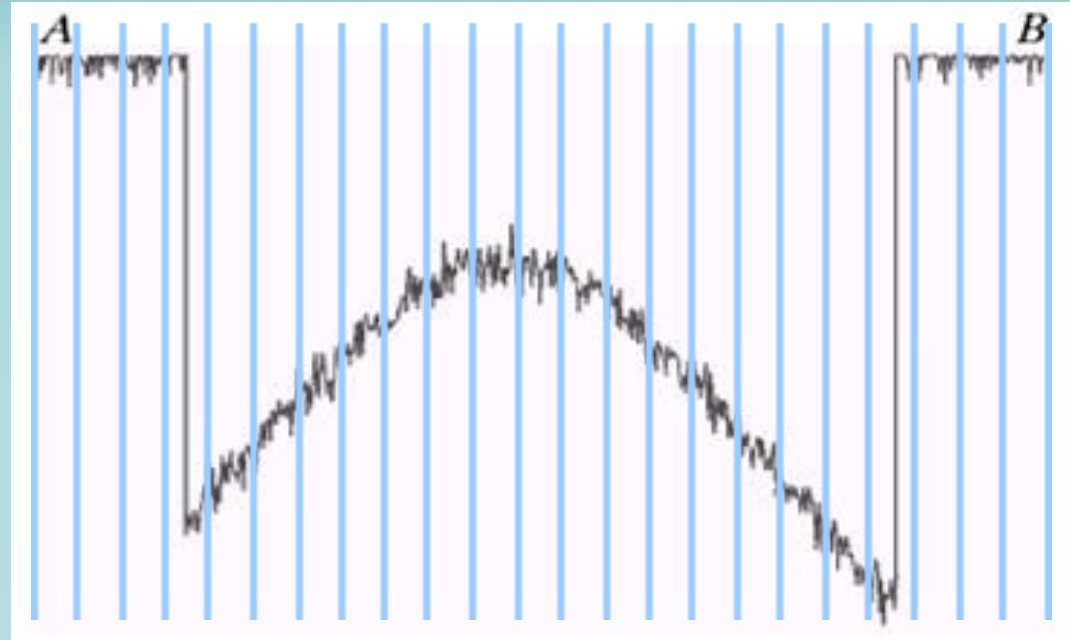
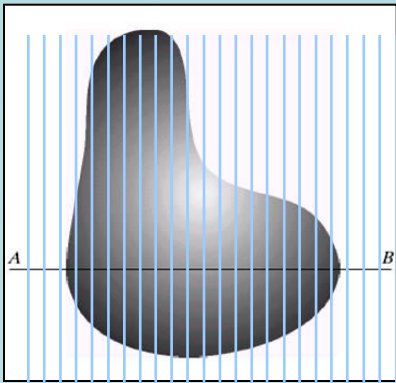
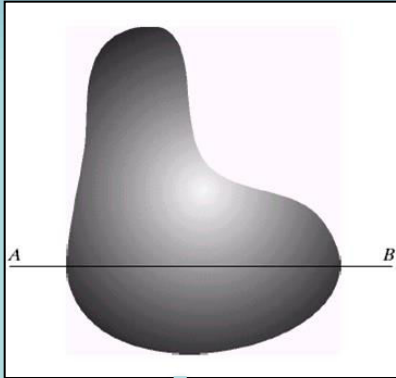
- ☐ Digitizing coordinates
- ☐ A process which converts the continuous analog space into a discrete space

## Quantization:

- ☐ Digitizing amplitudes (gray scale values)
- ☐ A process of converting a continuous analogue signal into a digital representation of that signal



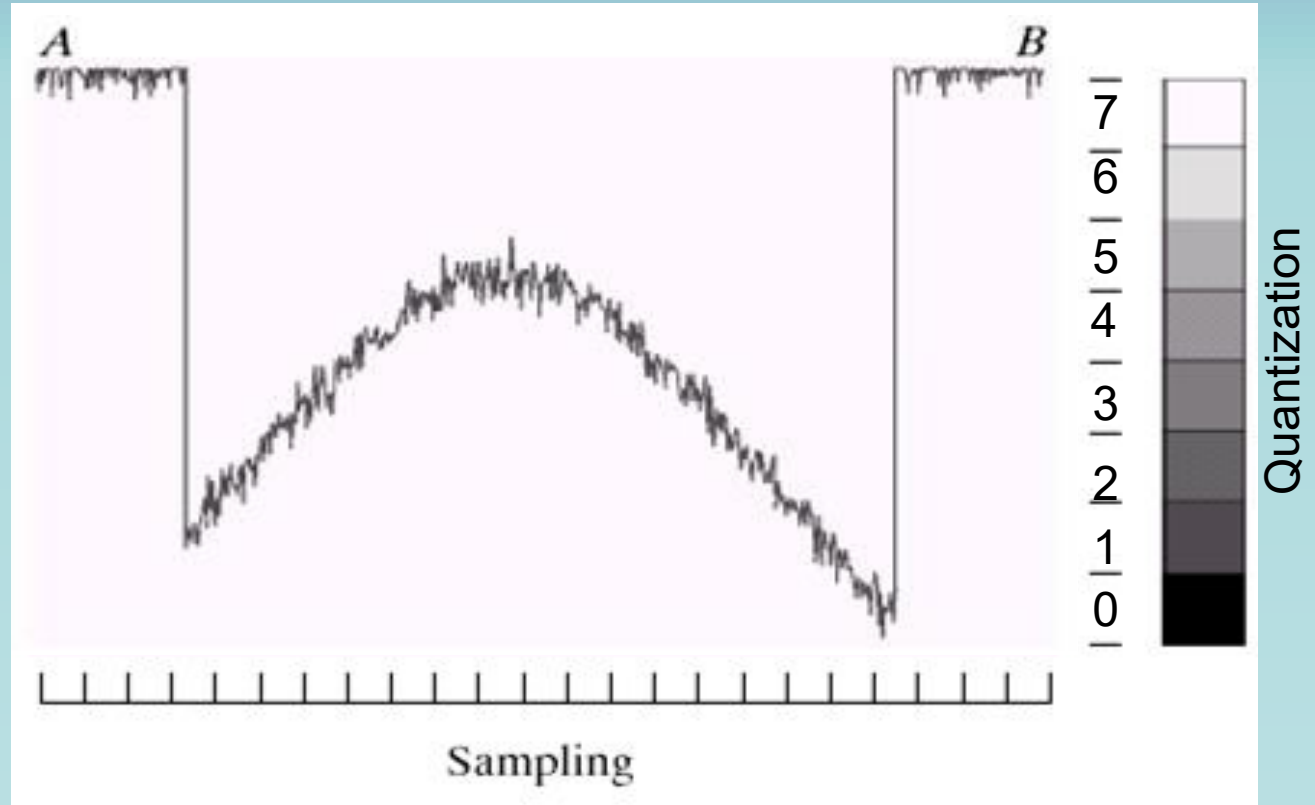
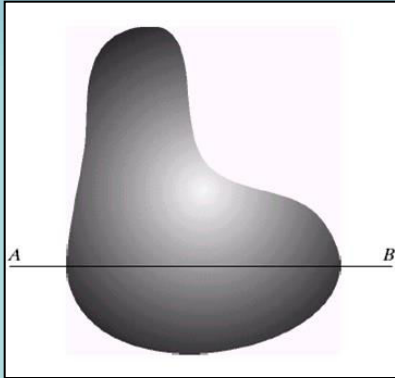
# Image Sampling



**1-Dimensional  
Sampling**

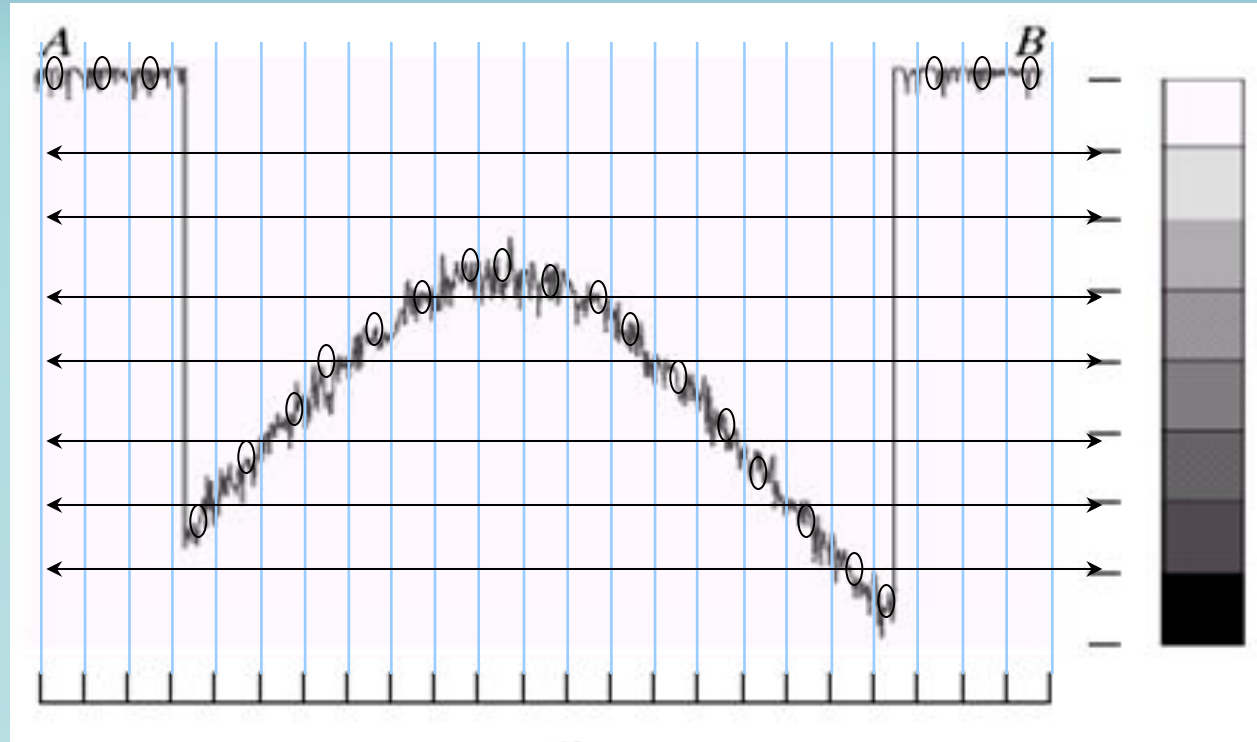
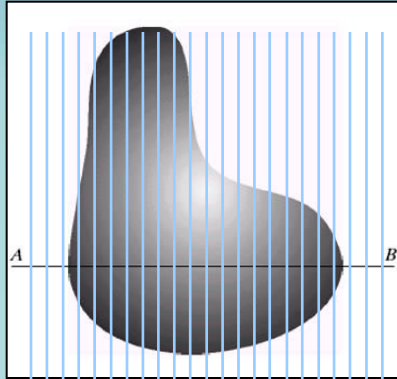
Digitizing the coordinate values

# Image Quantization



Digitizing the amplitude values

# Image Sampling And Quantisation

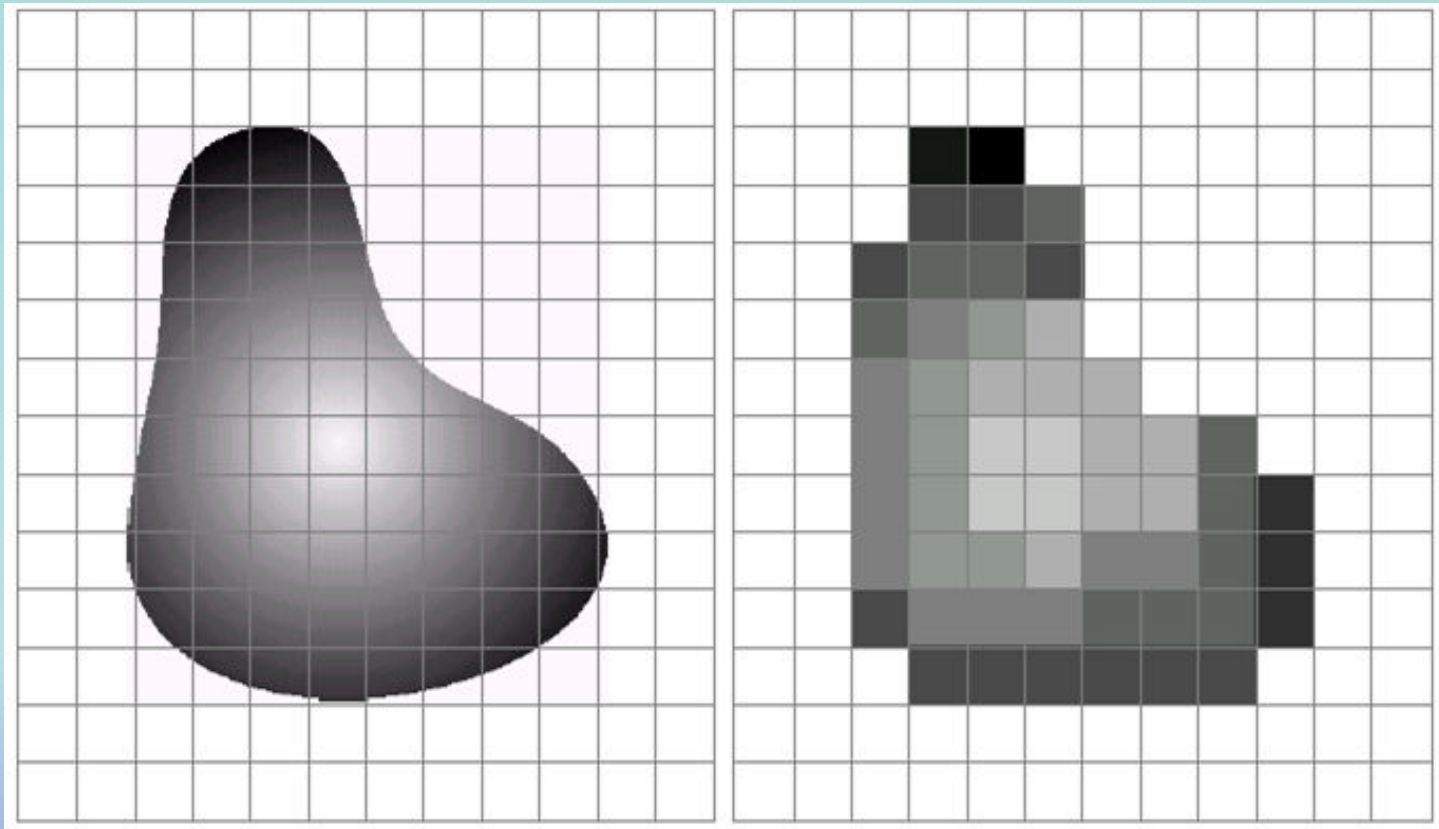


**A** 7 7 7 1 2 3 4 4 5 5 5 5 5 4 3 3 2 1 0 0 7 7 7 **B**

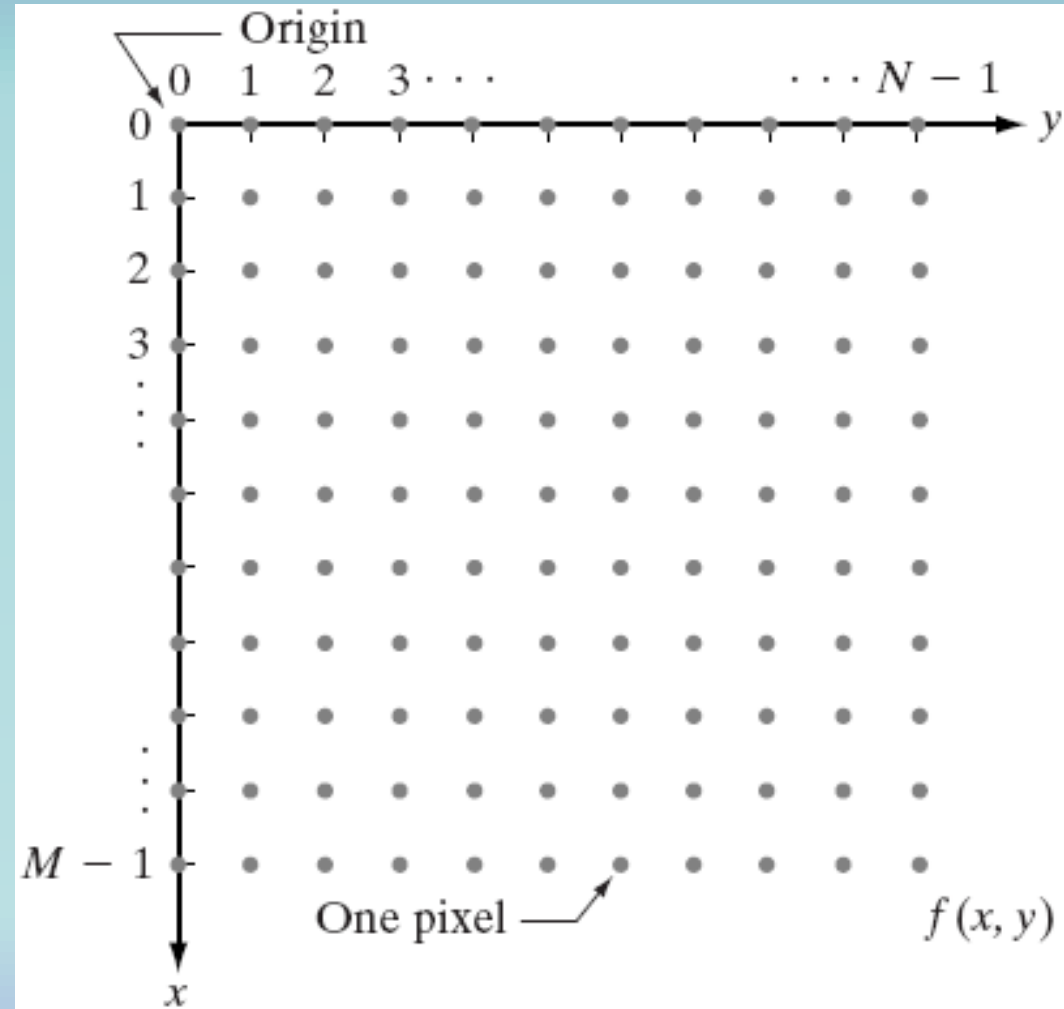
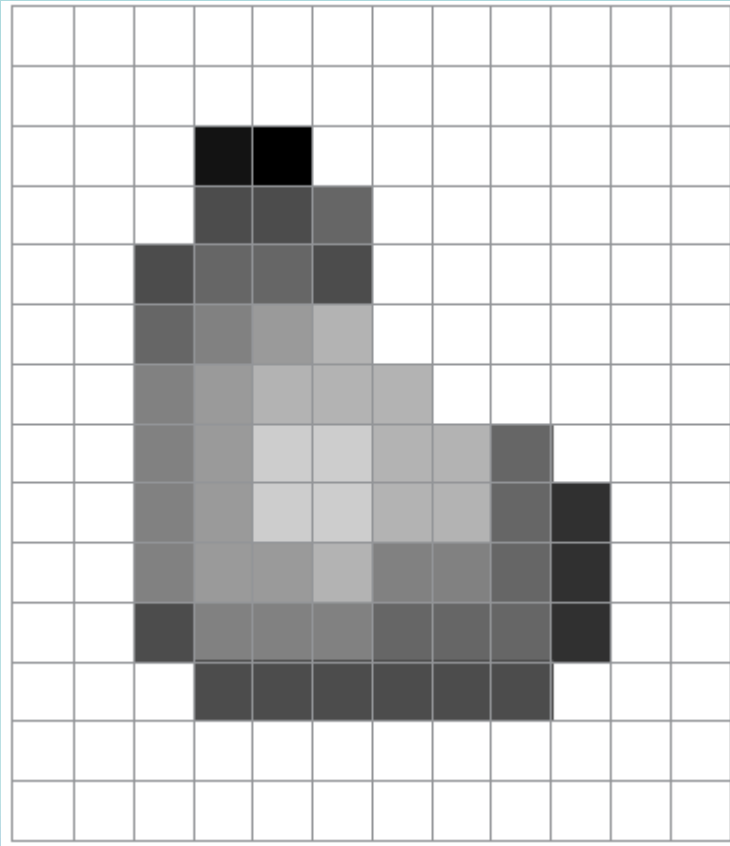
Digitization

# Digital Image

□ Remember that a digital image is always only an **approximation** of a real world scene



# Representation of Digital Image



Matrix Representation

# Representation of Digital Image

- ▶ The representation of an  $M \times N$  numerical array as

$$f(x, y) = \begin{bmatrix} f(0,0) & f(0,1) & \dots & f(0,N-1) \\ f(1,0) & f(1,1) & \dots & f(1,N-1) \\ \dots & \dots & \dots & \dots \\ f(M-1,0) & f(M-1,1) & \dots & f(M-1,N-1) \end{bmatrix}$$

# Representation of Digital Image

- ▶ The representation of an  $M \times N$  numerical array in MATLAB

$$f(x, y) = \begin{bmatrix} f(1,1) & f(1,2) & \dots & f(1,N) \\ f(2,1) & f(2,2) & \dots & f(2,N) \\ \dots & \dots & \dots & \dots \\ f(M,1) & f(M,2) & \dots & f(M,N) \end{bmatrix}$$

# Representation of Digital Image

Formally,

- Sampling:

- Dividing (x, y) plane into grid with coordinate  $(z_i, z_j)$ , where,

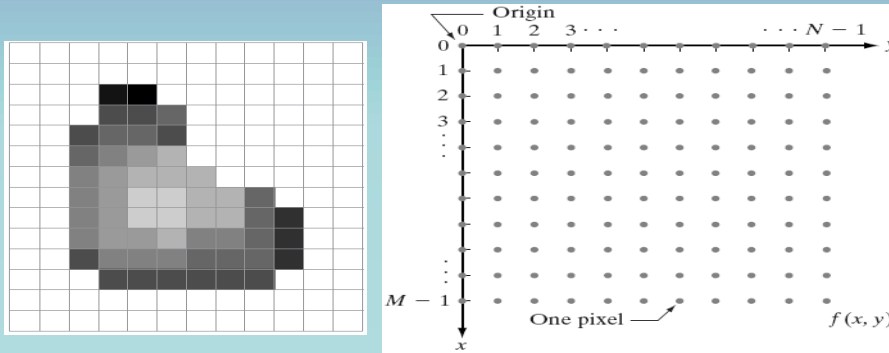
$$z_i, z_j \in Z$$

- Quantization:

- Assign gray level value from  $Z$  to  $f(z_i, z_j)$



# Spatial Resolution



❑ **Spatial resolution:**  $M \times N$

❑ Number of Rows ( $M$ ) and Columns ( $N$ ) can be any integer

❑ Bit size of an image:  $b = M \times N \times k$

❑ For Square image,  $b = N^2 k$

An image that is 2048 pixels in width and 1536 pixels in height has a total of  $2048 \times 1536 = 3,145,728$  pixels or 3.1 megapixels.

Here, spatial resolution is 3.1-megapixel.

One could refer to it as 2048 by 1536 or a 3.1-megapixel image.

# Intensity (Gray) Level Resolution

*Intensity level resolution* refers to the number of intensity levels used to represent the image

❑ **Intensity (Gray) level resolution:**  $L$

❑ Number of gray levels,  $L$ , is usually power of 2:

❑  $k$  = Bit depth  $L = 2^k$

❑ Gray level range =  $[ 0 - L-1 ]$   
=  $[ 0 - 255 ]$

Where  $k = 8$

$$L = 2^8$$

$$L = 256.$$

# Intensity (Gray) Level Resolution

- ❑ The more intensity levels used, the finer the level of detail discernable in an image
- ❑ Intensity level resolution is usually given in terms of the number of bits used to store each intensity level

| Number of Bits | Number of Intensity Levels | Examples           |
|----------------|----------------------------|--------------------|
| 1              | 2                          | 0, 1               |
| 2              | 4                          | 00, 01, 10, 11     |
| 4              | 16                         | 0000, 0101, 1111   |
| 8              | 256                        | 00110011, 01010101 |
| 16             | 65,536                     | 1010101010101010   |

# Number of storage of bits:

---

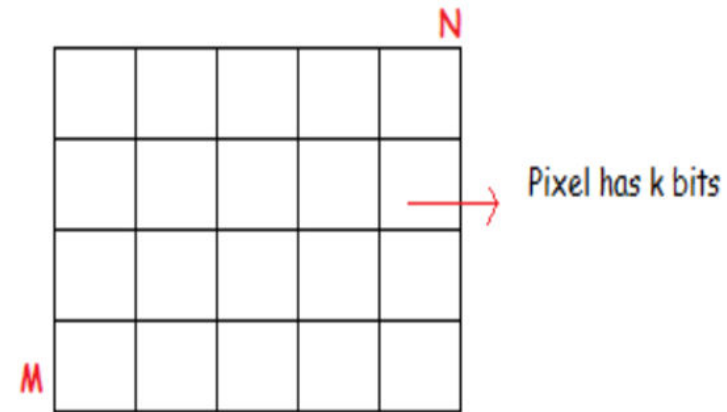
$N * M$ : the no. of pixels in all the image.

$K$ : no. of bits in each pixel

$L$ : grayscale levels the pixel can represent

$$L = 2^K$$

all bits in image =  $N * M * k$



NO of pixels =  $N * M$

NO of bits =  $N * M * k$

# Image Sizes in bits for Different Spatial and Gray Level Resolutions

| $N/k$ | 1 ( $L = 2$ ) | 2 ( $L = 4$ ) | 3 ( $L = 8$ ) | 4 ( $L = 16$ ) | 5 ( $L = 32$ ) | 6 ( $L = 64$ ) | 7 ( $L = 128$ ) | 8 ( $L = 256$ ) |
|-------|---------------|---------------|---------------|----------------|----------------|----------------|-----------------|-----------------|
| 32    | 1,024         | 2,048         | 3,072         | 4,096          | 5,120          | 6,144          | 7,168           | 8,192           |
| 64    | 4,096         | 8,192         | 12,288        | 16,384         | 20,480         | 24,576         | 28,672          | 32,768          |
| 128   | 16,384        | 32,768        | 49,152        | 65,536         | 81,920         | 98,304         | 114,688         | 131,072         |
| 256   | 65,536        | 131,072       | 196,608       | 262,144        | 327,680        | 393,216        | 458,752         | 524,288         |
| 512   | 262,144       | 524,288       | 786,432       | 1,048,576      | 1,310,720      | 1,572,864      | 1,835,008       | 2,097,152       |
| 1024  | 1,048,576     | 2,097,152     | 3,145,728     | 4,194,304      | 5,242,880      | 6,291,456      | 7,340,032       | 8,388,608       |
| 2048  | 4,194,304     | 8,388,608     | 12,582,912    | 16,777,216     | 20,971,520     | 25,165,824     | 29,369,128      | 33,554,432      |
| 4096  | 16,777,216    | 33,554,432    | 50,331,648    | 67,108,864     | 83,886,080     | 100,663,296    | 117,440,512     | 134,217,728     |
| 8192  | 67,108,864    | 134,217,728   | 201,326,592   | 268,435,456    | 335,544,320    | 402,653,184    | 469,762,048     | 536,870,912     |

❑ The calculation is for square images

❑  $N$  = number of row/column

❑  $L$  = gray level resolution

❑  $K$  = bit required to represent gray levels

## Answer the questions:

Q1: Suppose a pixel has 1 bit, how many gray levels can it represent?

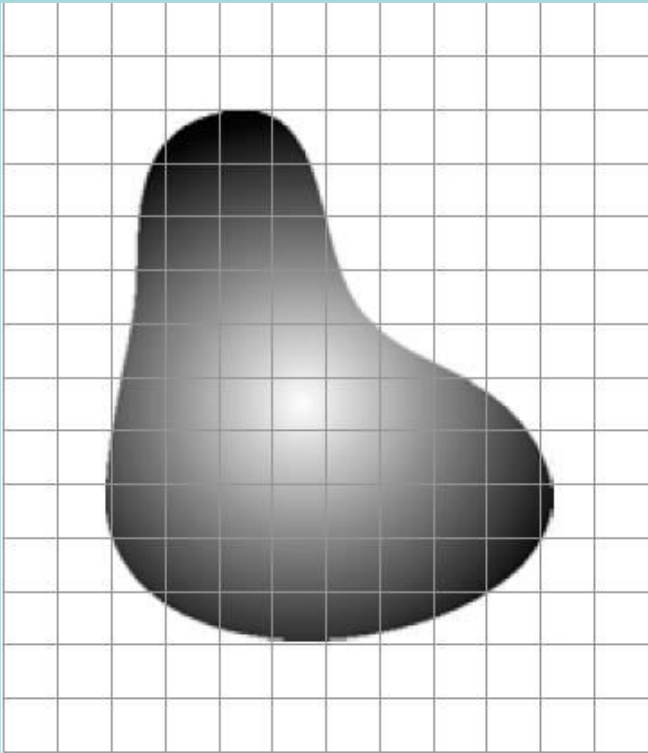
Q2: Suppose a pixel has 2 bit, how many gray levels can it represent?

Q3: Suppose a pixel has  $p$  bit, how many gray levels can it represent?

Q4: if we want to represent 256 intensities of grayscale, how many bits do we need?

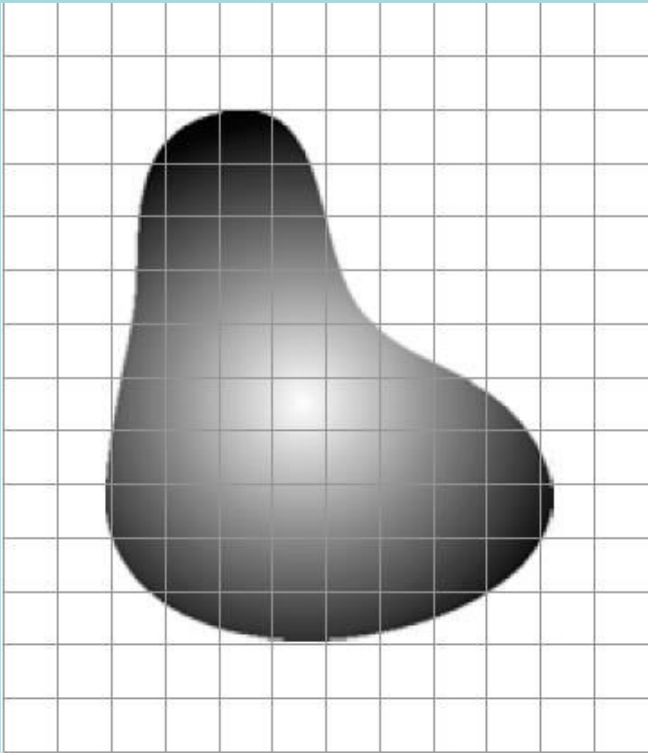
Q5: What is the **Gray Level Resolution** and **Spatial Resolution** of a 3 bit 5x5 image? What will be the image size in bits? How many gray levels can be there in the image? What is bit depth?

# Effect of Spatial and Gray Level Resolutions



- ❑ 14X12 resolution means samples from **14X12**
- ❑ What happens, if we sample from **8X8** locations?

# Effect of Spatial and Gray Level Resolutions

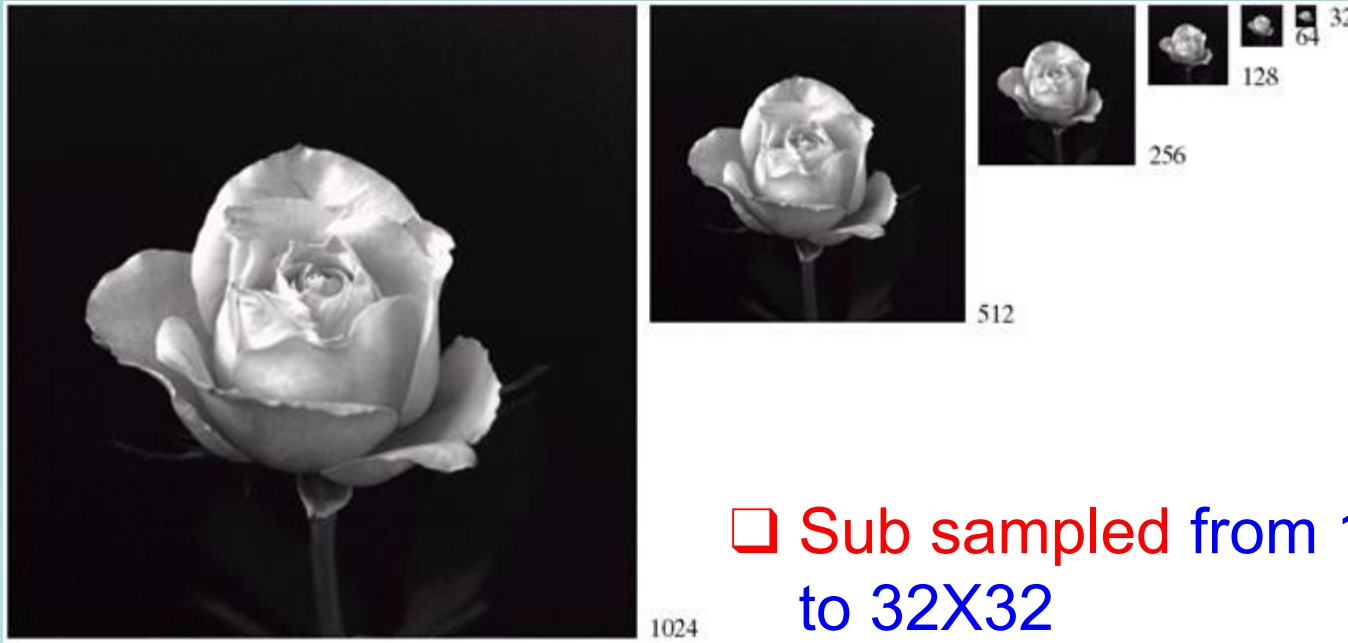


- ❑ 14X12 resolution means samples from **14X12**
- ❑ What happens, if we sample from **8X8** locations?

- ❑ Larger cell size
- ❑ Lower spatial resolution
- ❑ Lower features spatial accuracy
- ❑ Lower file size, faster display and faster processing time



# Effect of Spatial Resolutions



❑ Sub sampled from 1024X1024 up to 32X32

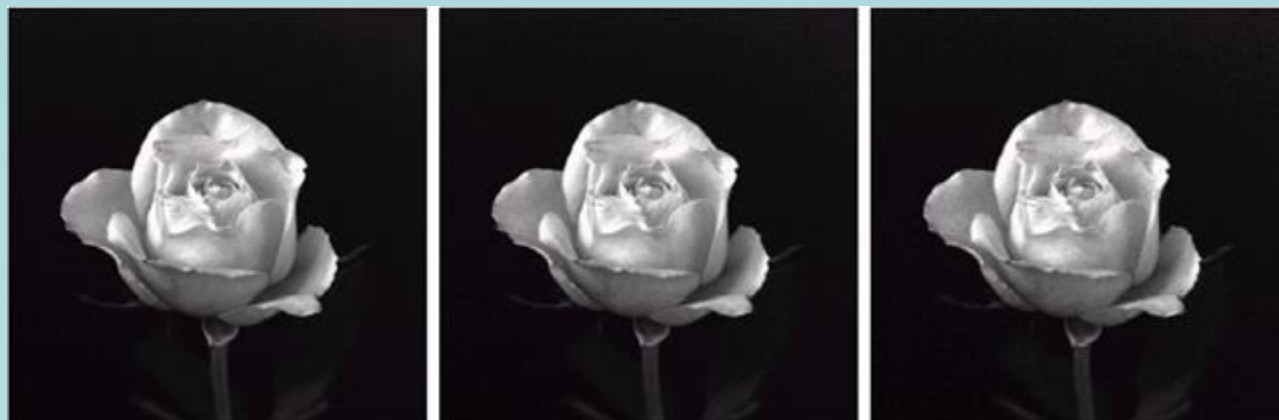
❑ unchanged gray level (intensity)

❑ **Delete** alternate row and column while  
**sub sampling**

# Effect of Spatial Resolutions

Resized to 1024 x 1024

From → 1024X1024      512X512      256X256



From → 128X128      64X64      32X32

Using column  
duplication process

# Can we increase spatial resolution ?

From  $\longrightarrow$

128X128

64X64

32X32

Using nearest  
neighbor



Using bilinear  
interpolation



From  $\longrightarrow$

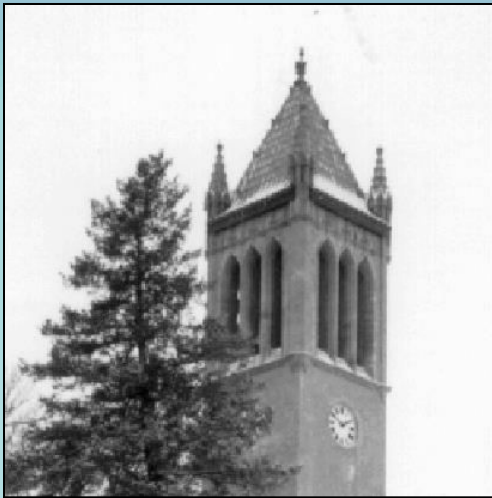
128X128

64X64

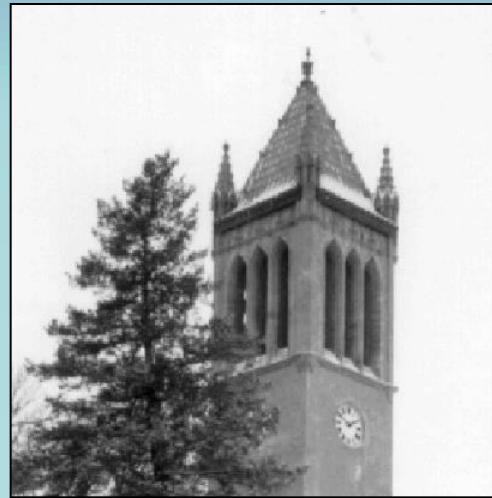
32X32

Down sampling is an irreversible process.

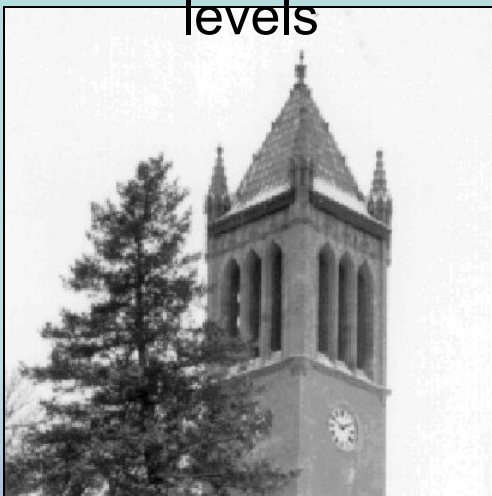
# Effect of Gray level Resolutions



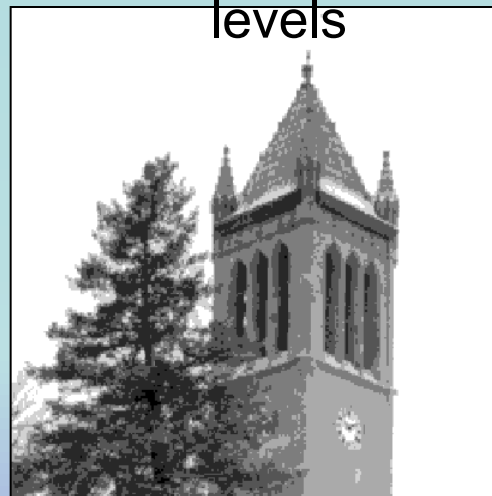
256  
levels



128  
levels



64  
levels

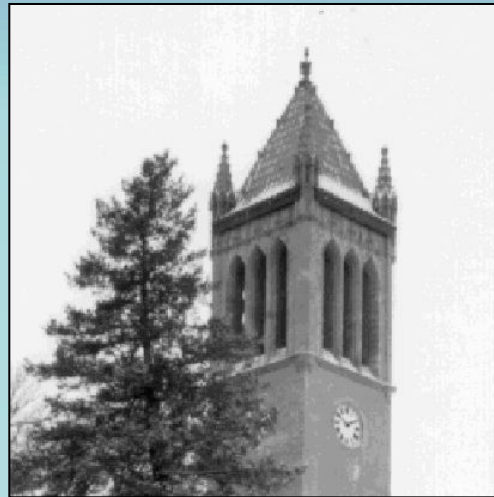


32  
levels

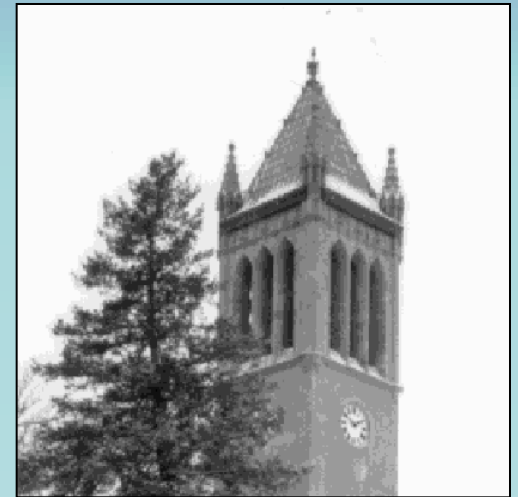
- ☐ unchanged spatial resolution
- ☐ Gray level changed from 256 to 32

# Effect of Gray level Resolutions

□ Gray level  
changed  
from 16 to 2

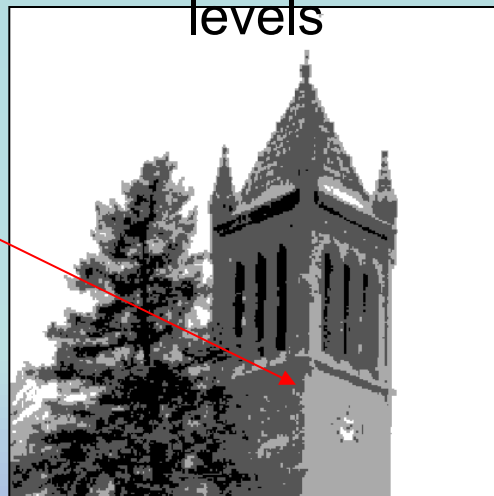


16  
levels

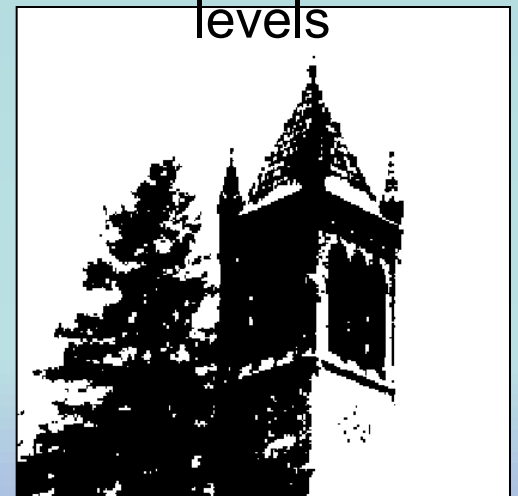


8  
levels

In this  
image,  
it is easy to  
see  
false contour.

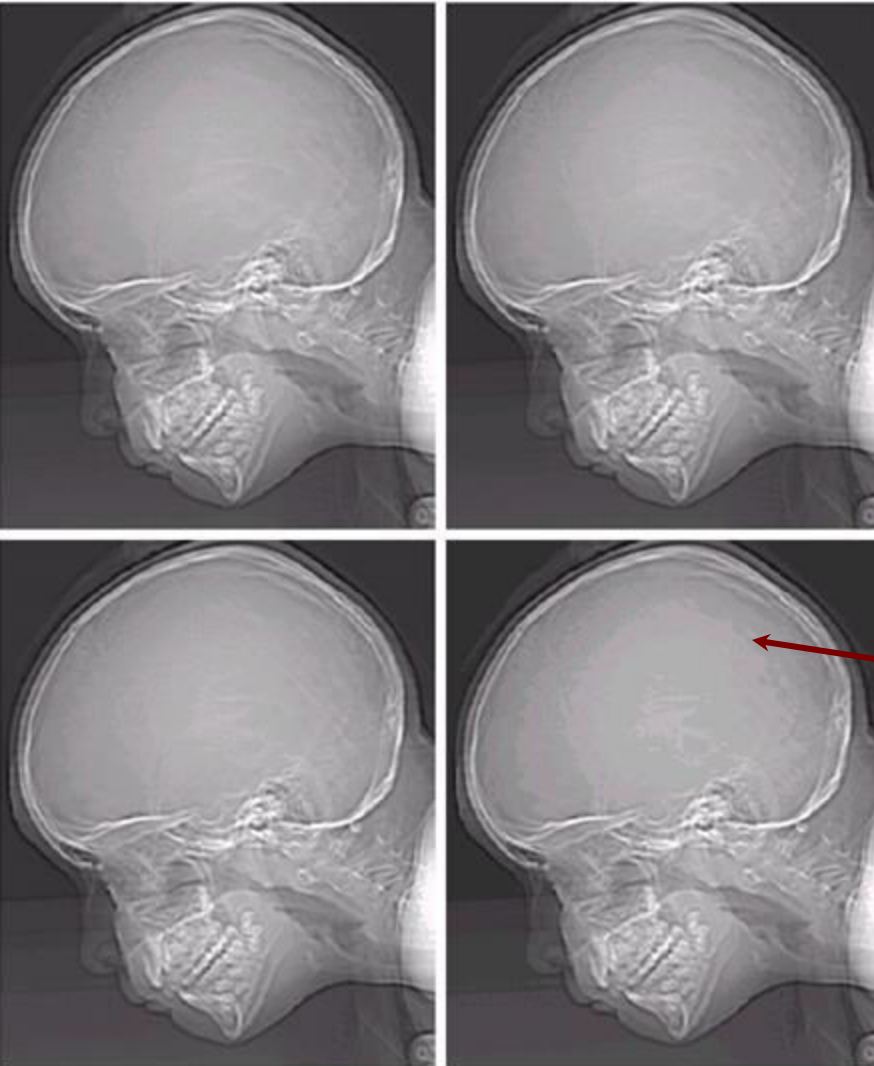


4  
levels



2  
levels

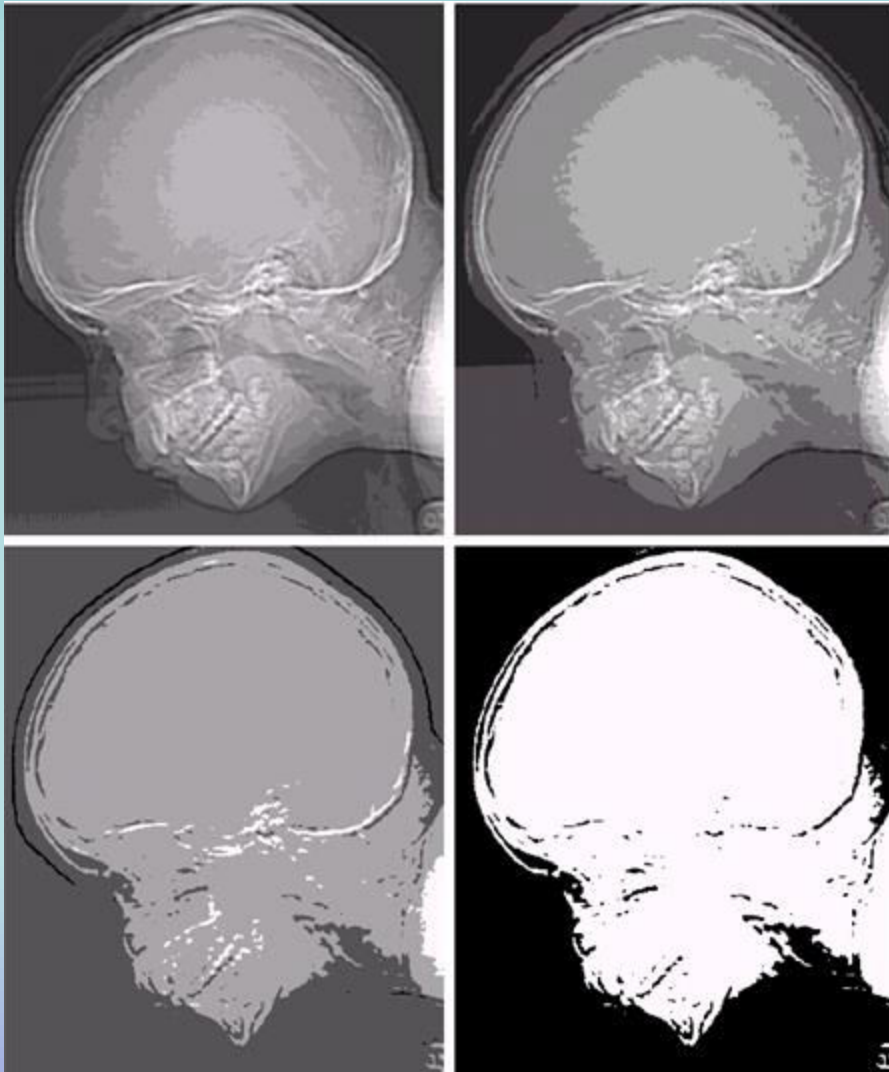
# Effect of Gray level Resolutions



- unchanged spatial resolution
- Gray level changed from 256 to 32

Ridge-like structure in the smooth area

# Effect of Spatial and Gray Level Resolutions



- Gray level changed from 16 to 2
- Ridge-like structure is more prominent
- Reason: insufficient number of gray levels used



# Resolution: How Much Is Enough?

□ The big question with resolution is always *how much is enough?*

- This all depends on what is in the image and what you would like to do with it
- Key questions include
  - Does the image look aesthetically pleasing?
  - Can you see what you need to see within the image?



# Resolution: How Much Is Enough? (cont...)



- ❑ The picture on the right is fine for counting the number of cars, but not for reading the number plate

## *How to select the suitable size and pixel depth of images*

The word “suitable” is subjective: depending on “subject”.



Low detail image

Lena image



Medium detail image

Cameraman image

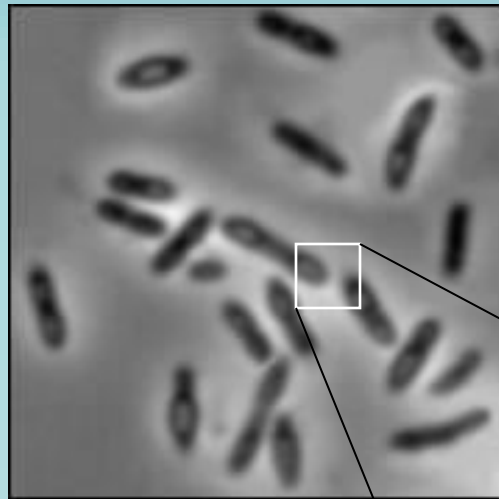


High detail image

### To satisfy human mind

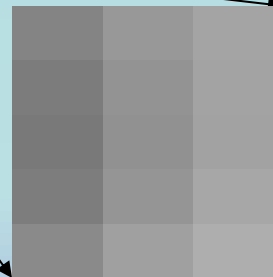
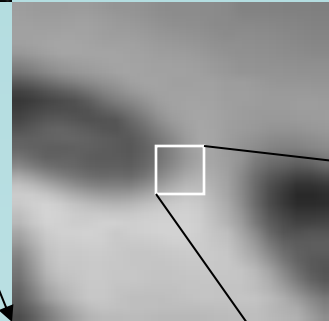
1. For images of the same size, the low detail image may need more pixel depth.
2. As an image size increase, fewer gray levels may be needed.

# Digital Image Types : Intensity Image

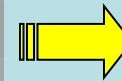


## Intensity image or monochrome image

each pixel corresponds to light intensity  
normally represented in gray scale (gray level).



Gray scale  
values




|    |    |    |    |
|----|----|----|----|
| 10 | 10 | 16 | 28 |
| 9  | 6  | 26 | 37 |
| 15 | 25 | 13 | 22 |
| 32 | 15 | 87 | 39 |



## components

components


$$\begin{bmatrix} 10 & 65 & 70 & 56 & 43 \\ 9 & 6 & 99 & 26 & 70 & 37 & 56 & 78 \\ 15 & 32 & 99 & 34 & 96 & 67 & 67 \\ 32 & 21 & 60 & 13 & 90 & 22 & 96 & 67 \\ & 15 & 54 & 47 & 42 & & & \\ & 54 & 85 & 87 & 39 & 43 & 39 & 92 \\ & & 32 & 65 & 87 & 99 & & \end{bmatrix}$$

# Image Types : Binary

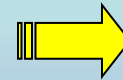
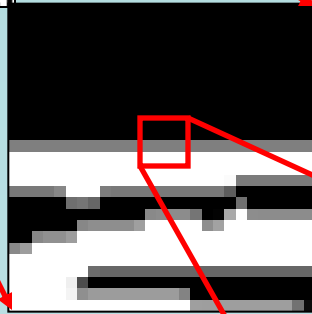
## Image

**Binary image or black and white image**

Each pixel contains one bit :

1 represent white

0 represents black



Binary

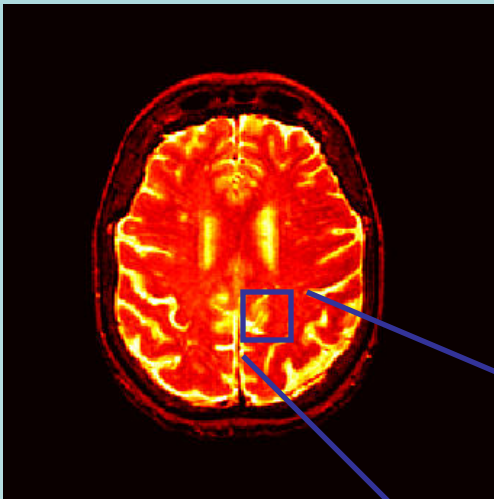
data

|   |   |   |   |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |

# Image Types : Index Image

## Index image

Each pixel contains index number pointing to a color in a color table



$\begin{bmatrix} 1 & 4 & 9 \\ 6 & 4 & 7 \\ 6 & 5 & 2 \end{bmatrix}$

Index  
value

Color Table

| Index No. | Red component | Green component | Blue component |
|-----------|---------------|-----------------|----------------|
| 1         | 0.1           | 0.5             | 0.3            |
| 2         | 1.0           | 0.0             | 0.0            |
| 3         | 0.0           | 1.0             | 0.0            |
| 4         | 0.5           | 0.5             | 0.5            |
| 5         | 0.2           | 0.8             | 0.9            |
| ...       | ...           | ...             | ...            |

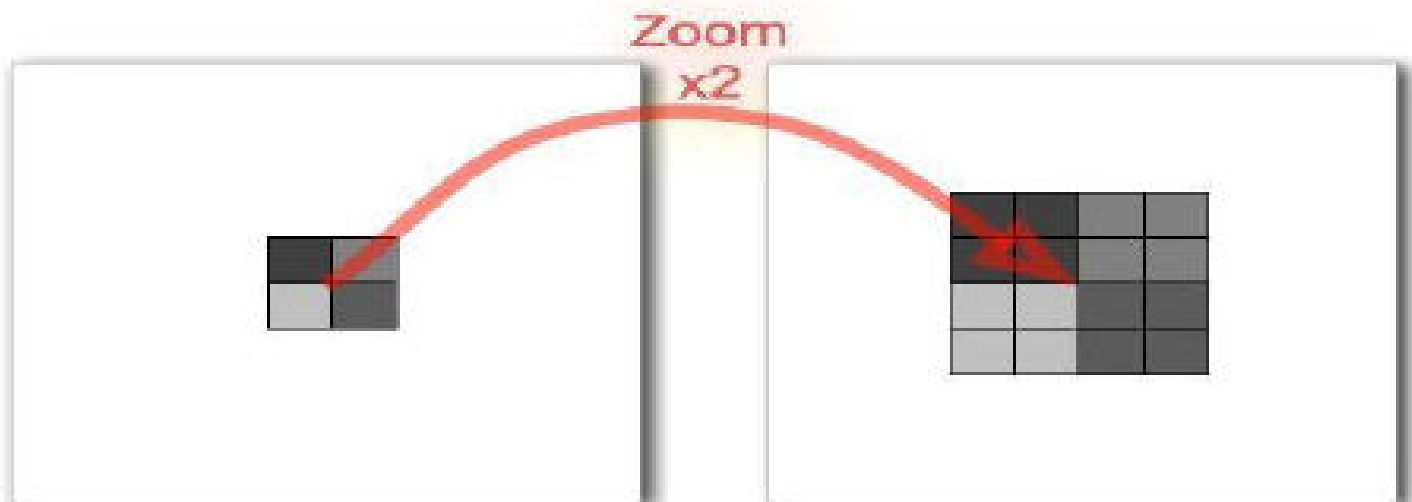
# Image Scaling (Zooming & Shrinking)

- Zooming/up scaling/resizing upward can be achieved by the following techniques:
  - Pixel Replication
  - Interpolation
    - Nearest Neighbor Interpolation
    - Bilinear Interpolation
    - Bicubic Interpolation

# Image Scaling (Zooming and Shrinking)

## ▪ Pixel Replication

- ❑ Pixel replication is applicable when we want to increase the size of an image an integer number of times.
- ❑ For example to double the size of an image we can duplicate each column, this doubles the size of image in horizontal direction. Then we duplicate each row of the enlarged image to double the size in the vertical direction
- ❑ The same procedure can be applied to enlarge the image by any integer number of times (triple, quadruple and so on)
- ❑ The gray level assignment of each pixel is predetermined by the fact that new locations are exact duplicate of old locations



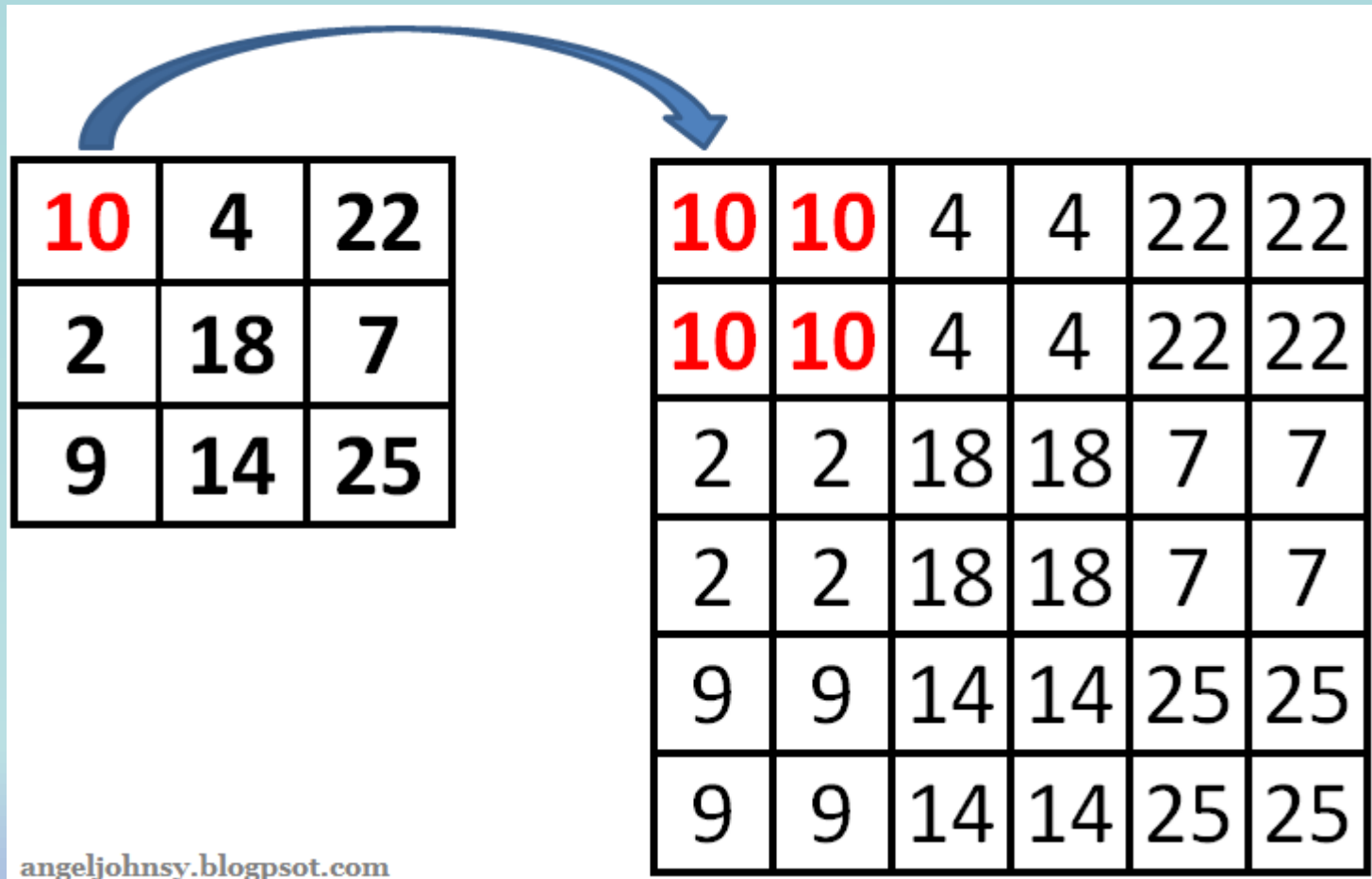


# Image Scaling (Zooming and Shrinking)

## ▪ Nearest neighbor Interpolation

- ❑ Suppose that we have an image of size 500 x 500 and we want to enlarge it to 1.5 times 750 x 750 pixels.
- ❑ For any zooming approach we have to create an imaginary grid of the size which is required over the original image. In that case we will have an imaginary grid of 750 x 750 over an original image.
- ❑ Obviously the spacing in the grid would be less than one pixel because we fitting it over a smaller image. In order to perform gray level assignment for any point in the overlay, we look for the closest pixel in the original image and assign its gray level to new pixel in the grid.
- ❑ When finished with all points in the grid, we can simply expand it to the originally specified size to obtain the zoomed image. **This method of gray level assignment is called nearest neighbor interpolation**

# 3x3 matrix is interpolated to 6x6 matrix



## Steps to be performed:

1. Consider a matrix  $A = \begin{bmatrix} 10 & 4 & 22 \\ 2 & 18 & 7 \\ 9 & 14 & 25 \end{bmatrix}$  where number of rows = 3 and number of columns = 3

2. Define the new size for the matrix , number of rows = 6 and number of columns = 6

3. Find the Ratio of the new size and old size,

$$Ratio_{Row} = \frac{3}{6} \text{ and } Ratio_{Column} = \frac{3}{6}$$

4. Normalize the row-wise pixel positions and column wise pixel positions based on the new size

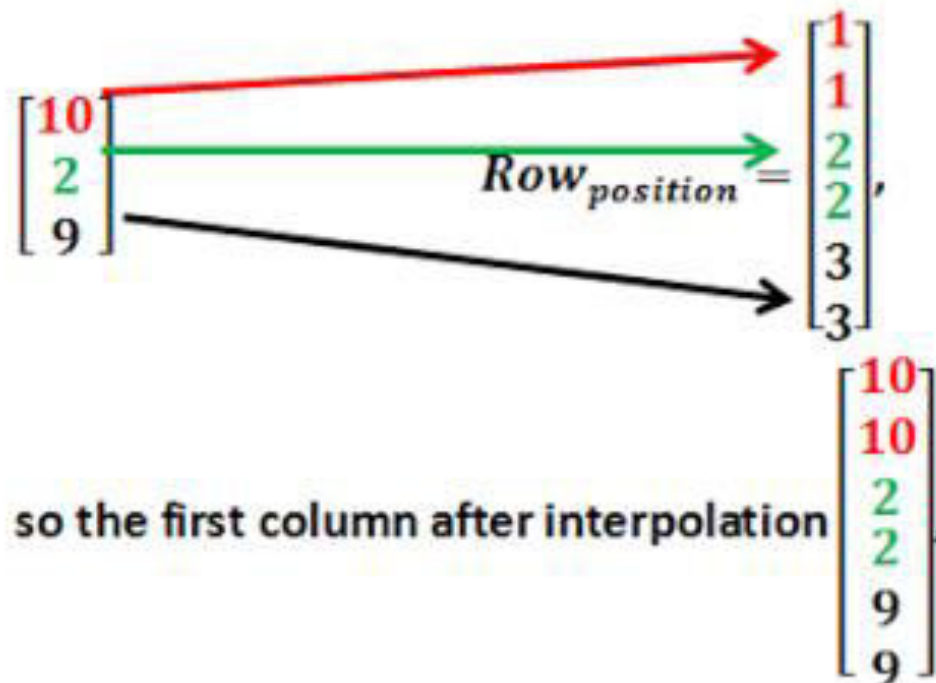
$$Row_{Positions} = \frac{[1 \ 2 \ 3 \ 4 \ 5 \ 6]}{Ratio_{Row}} = [0.5 \ 1 \ 1.5 \ 2 \ 2.5 \ 3]$$

5. By using ceil function that gives the least integer greater than or equal to

the value, the  $Row_{position} = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 2 \\ 3 \\ 3 \end{bmatrix}$  and

$Column_{position} = [1 \ 1 \ 2 \ 2 \ 3 \ 3]$

6. Perform row-wise interpolation on the columns of the matrix,  
For instance, consider the first column of the matrix,




7. Similarly, perform the row-wise interpolation on all the columns of the matrix,

$$\begin{bmatrix} 10 & 4 & 22 \\ 10 & 4 & 22 \\ 2 & 18 & 7 \\ 2 & 18 & 7 \\ 9 & 14 & 25 \\ 9 & 14 & 25 \end{bmatrix}$$

8. Perform column wise interpolation,

Take the first row from the row-wise interpolated matrix  $A, [10 \ 4 \ 22]$  and based on the `column_position`, the values are interpolated. The column position 1 indicates the 1<sup>st</sup> element i.e. 10, 2 indicates the 2<sup>nd</sup> element (i.e) 4 so the interpolated first row is  $[10 \ 10 \ 4 \ 4 \ 22 \ 22]$

*First row* =  $[10 \ 4 \ 22]$ , *Column<sub>position</sub>* =  $[1 \ 1 \ 2 \ 2 \ 3 \ 3]$



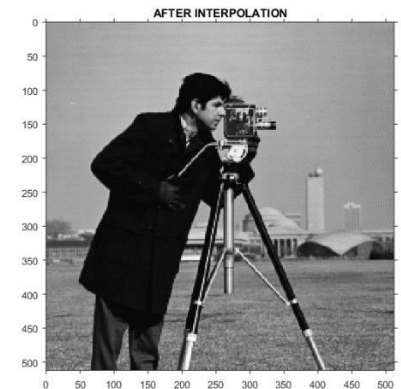
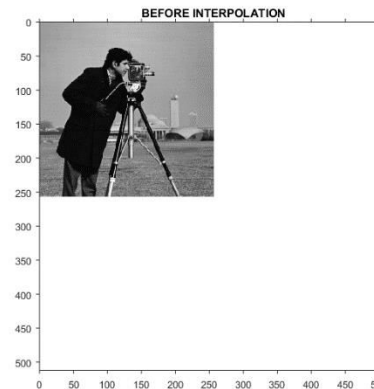
*Interpolated first row is*  $[10 \ 10 \ 4 \ 4 \ 22 \ 22]$

9. Similarly, perform this column wise interpolation on all the row wise interpolated rows,

10. The interpolated 6x6 matrix is

$$\begin{bmatrix} 10 & 10 & 4 & 4 & 22 & 22 \\ 10 & 10 & 4 & 4 & 22 & 22 \\ 2 & 2 & 18 & 18 & 7 & 7 \\ 2 & 2 & 18 & 18 & 7 & 7 \\ 9 & 9 & 14 & 14 & 25 & 25 \\ 9 & 9 & 14 & 14 & 25 & 25 \end{bmatrix}$$

angeljohnsy.blogspot.com

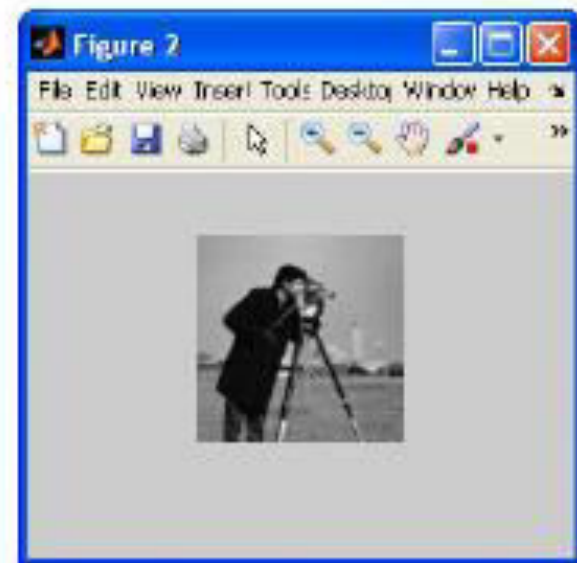




# Image Scaling (Zooming and Shrinking)

- **Shrinking (Down scaling, resizing downward)**

- ☐ Image shrinking is done in the similar manner as zooming with one difference as now the process of pixel replication is row column deletion. Now we can delete every second column and row for shrinking



# Class Work:

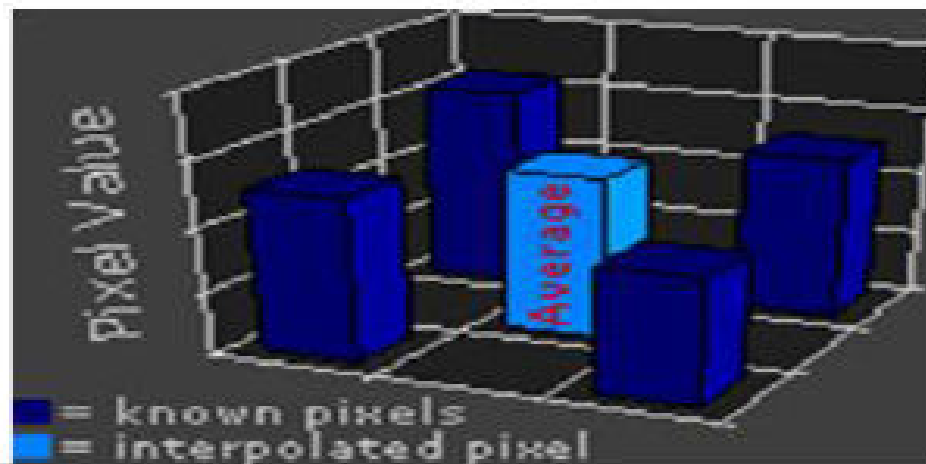
- Interpolate a 2x2 matrix [1, 4, 3, 7] to 4x4
- Interpolate a 3x3 matrix [1,4,3,7,2,3,5,6,0] to 9x9
- Interpolate a 2x2 matrix [1, 2, 3, 4] to 8x8
- Interpolate a 3x3 matrix [1,4,3, 7, 5, 6] to 6x6



# Image Scaling (Zooming and Shrinking)

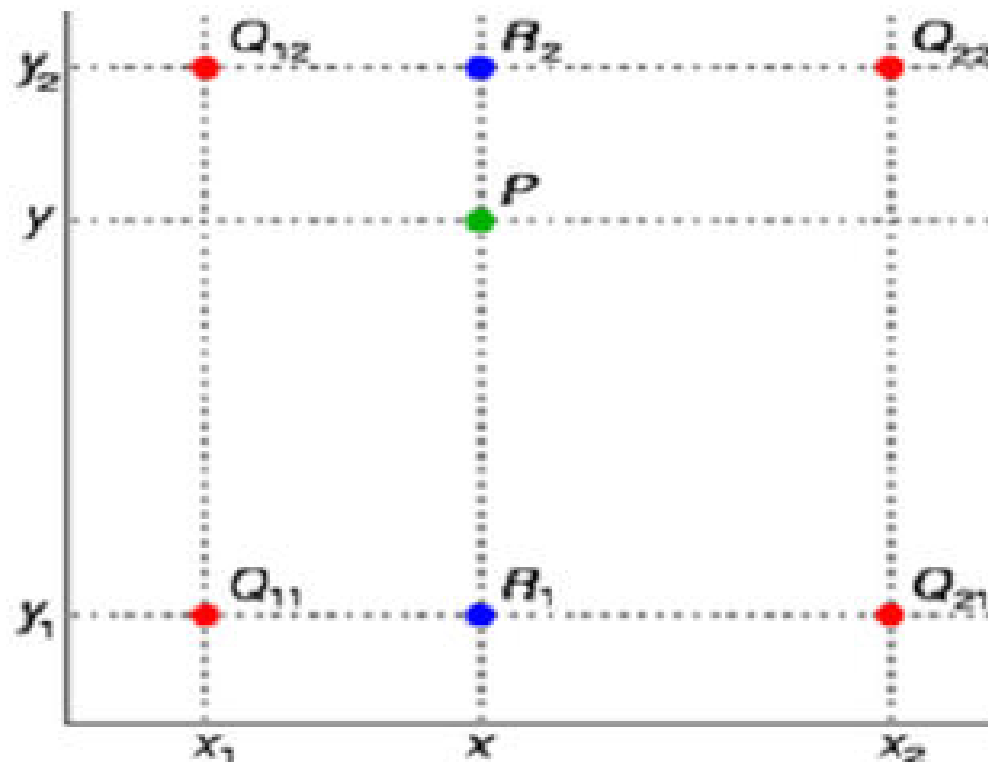
## ▪ Bilinear Interpolation

- ❑ Bilinear interpolation considers the closest 2x2 neighborhood of known pixel values surrounding the unknown pixel.
- ❑ It then takes a weighted average of these 4 pixels to arrive at its final interpolated value. This results in much smoother looking images than nearest neighbor.
- ❑ The diagram below is for a case when all known pixel distances are equal, so the interpolated value is simply their sum divided by four.
- ❑ In case the distance varies then The closer pixels are given more weightage in the calculation



# Image Scaling (Zooming and Shrinking)

- The key idea is to perform linear interpolation first in one direction, and then again in the other direction.
- Suppose that we want to find the value of the unknown function  $f$  at the point  $P = (x, y)$ .
- It is assumed that we know the value of  $f$  at the four points  $Q_{11} = (x_1, y_1)$ ,  $Q_{12} = (x_1, y_2)$ ,  $Q_{21} = (x_2, y_1)$ , and  $Q_{22} = (x_2, y_2)$ .



# Image Scaling (Zooming and Shrinking)

We first do linear interpolation in the  $x$ -direction. This yields

$$f(R_1) \approx \frac{x_2 - x}{x_2 - x_1} f(Q_{11}) + \frac{x - x_1}{x_2 - x_1} f(Q_{21})$$

where  $R_1 = (x, y_1)$ ,

$$f(R_2) \approx \frac{x_2 - x}{x_2 - x_1} f(Q_{12}) + \frac{x - x_1}{x_2 - x_1} f(Q_{22})$$

where  $R_2 = (x, y_2)$ .

$$f(P) \approx \frac{y_2 - y}{y_2 - y_1} f(R_1) + \frac{y - y_1}{y_2 - y_1} f(R_2).$$

We proceed by interpolating in the  $y$ -direction.

# Image Scaling (Zooming and Shrinking)

This gives us the desired estimate of  $f(x, y)$ .

$$\begin{aligned} f(x, y) \approx & \frac{f(Q_{11})}{(x_2 - x_1)(y_2 - y_1)} (x_2 - x)(y_2 - y) \\ & + \frac{f(Q_{21})}{(x_2 - x_1)(y_2 - y_1)} (x - x_1)(y_2 - y) \\ & + \frac{f(Q_{12})}{(x_2 - x_1)(y_2 - y_1)} (x_2 - x)(y - y_1) \\ & + \frac{f(Q_{22})}{(x_2 - x_1)(y_2 - y_1)} (x - x_1)(y - y_1). \end{aligned}$$

<https://theailearner.com/2018/12/29/image-processing-bicubic-interpolation//>