

# CSC 4227 Digital Image Processing

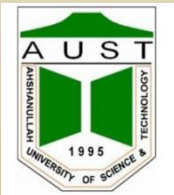
## Lecture 18 – Image Restoration : Noise Removal

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# Contents

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In this lecture we will look at image restoration techniques used for noise removal

- What is image restoration?
- Noise and images
- Noise models
- Noise removal using spatial domain filtering
- Periodic noise
- Noise removal using frequency domain filtering

# What is Image Restoration?

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- Image restoration attempts to restore images that have been degraded
  - ✓ Identify the degradation process and attempt to reverse it.
  - ✓ Almost Similar to image enhancement, but more objective.

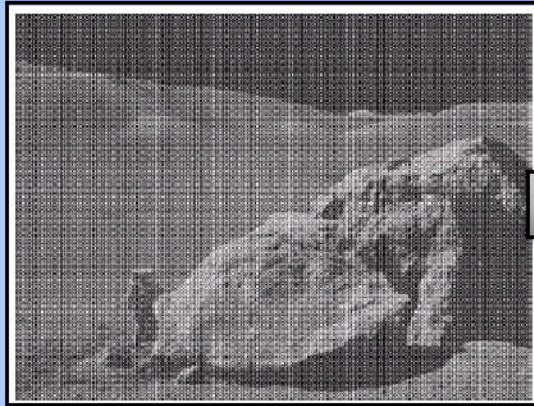


Fig: Degraded image



Fig: Restored image

# Image enhancement vs. Image Restoration

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- Image restoration assumes a degradation model that is known or can be estimated.
- Original content and quality does not mean **Good looking or appearance**.
- Image Enhancement is **subjective**, where as image restoration is **objective process**.
- Image restoration try to recover original image from degraded with **prior knowledge of degradation process**.
- Restoration involves **modeling of degradation** and applying the **inverse process in order to recover the original image**.
- Although the restore image is not the original image, its approximation of actual image.

# Noise and Images

The sources of noise in digital images arise during image acquisition (digitization) and transmission

- Imaging **sensors** can be affected by **ambient conditions**
- **Interference** can be added to an image **during transmission**



# Noise Model

We can consider a noisy image to be modelled as follows:

$$g(x, y) = f(x, y) + \eta(x, y)$$

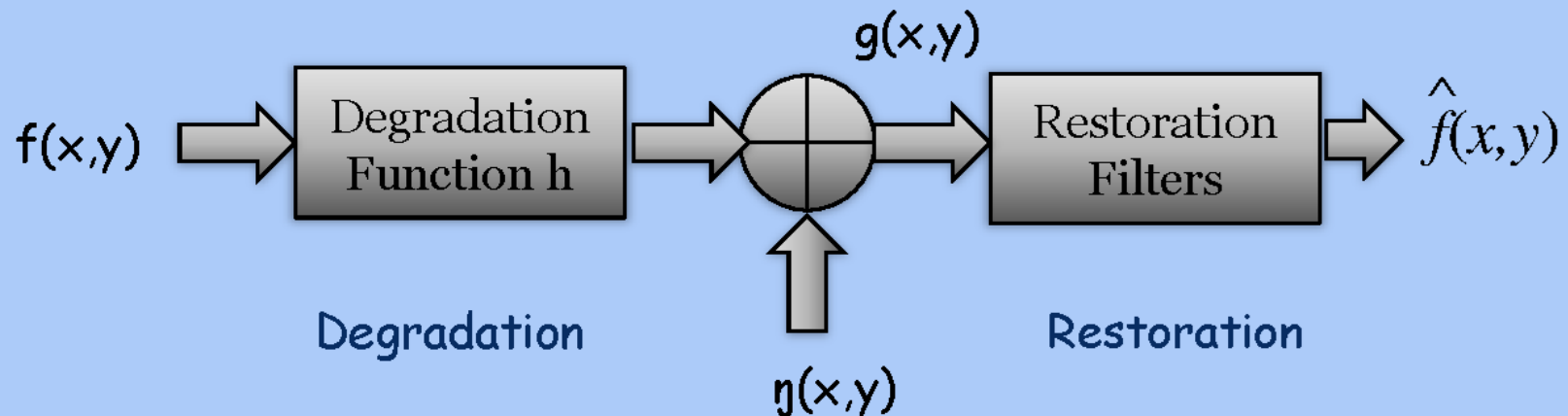
where  $f(x, y)$  is the original image pixel,  $\eta(x, y)$  is the noise term and  $g(x, y)$  is the resulting noisy pixel

If we can estimate the model the noise in an image is based on this will help us to figure out how to restore the image

# Degradation Model?

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- **Objective:** To restore a degraded/distorted image to its original content and quality.



- **Spatial Domain:**  $g(x,y)=h(x,y)*f(x,y)+ \eta(x,y)$
- **Frequency Domain:**  $G(u,v)=H(u,v)F(u,v)+ \eta(u,v)$
- **Matrix:**  $G=HF+\eta$

# Noise Models and Their PDF

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- Different models for the image noise term  $\eta(x, y)$

- ✓ Gaussian

- ✧ **Most common model**

- ✓ Rayleigh

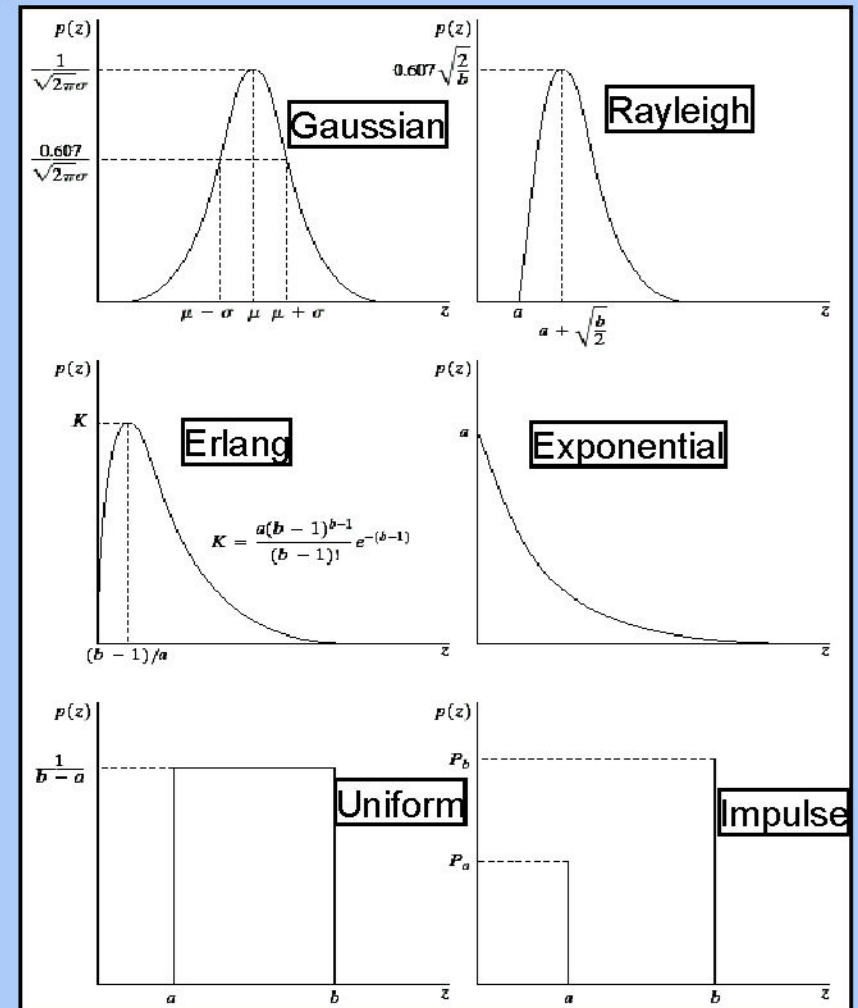
- ✓ Erlang or Gamma

- ✓ Exponential

- ✓ Uniform

- ✓ Impulse

- ✧ **Salt and pepper noise**





# Noise Models

Noise cannot be predicted but can be approximately described in statistical way using the probability density function (PDF)

Gaussian noise:

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(z-\mu)^2 / 2\sigma^2}$$

Rayleigh noise

$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-(z-a)^2/b} & \text{for } z \geq a \\ 0 & \text{for } z < a \end{cases}$$

Erlang (Gamma) noise

$$p(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!} e^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases}$$

## Noise Models (cont.)

Exponential noise

$$p(z) = ae^{-az}$$

Uniform noise

$$p(z) = \begin{cases} \frac{1}{b-a} & \text{for } a \leq z \leq b \\ 0 & \text{otherwise} \end{cases}$$

Impulse (salt & pepper) noise

$$p(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases}$$

# Noise Types (optional to watch)

**Digital Image Processing, 3rd ed.**  
Gonzalez & Woods  
www.ImageProcessingPlace.com  
Chapter 5  
Image Restoration and Reconstruction

Handwritten notes on the left:

$$p(z) = \begin{cases} \frac{1}{b} (z-a) e^{-\frac{(z-a)^2}{b}} & z \geq a \\ 0 & z \leq a \end{cases}$$
$$\bar{z} = a + \sqrt{\frac{\pi b}{4}}$$
$$\sigma^2 = \frac{b(4 - \pi)}{4}$$

Figure 5.2 shows six probability density functions:

- 1. Gaussian: A bell-shaped curve centered at  $\mu$  with standard deviation  $\sigma$ .
- 2. Rayleigh: A curve starting at  $a$  and peaking at  $\mu = \sqrt{\pi/2} \sigma$ .
- 3. Gamma: A curve starting at  $a$  and peaking at  $\mu = a + \frac{1}{\lambda}$ .
- 4. Exponential: A curve starting at  $a$  and decaying towards zero.
- 5. Uniform: A constant value between  $a$  and  $b$ .
- 6. Impulse: A single vertical line at  $a$  with height  $P_0$ .

FIGURE 5.2 Seven important probability density functions.

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<https://www.youtube.com/watch?v=vZNyZgdEhgQ>

# Noise Models Effects

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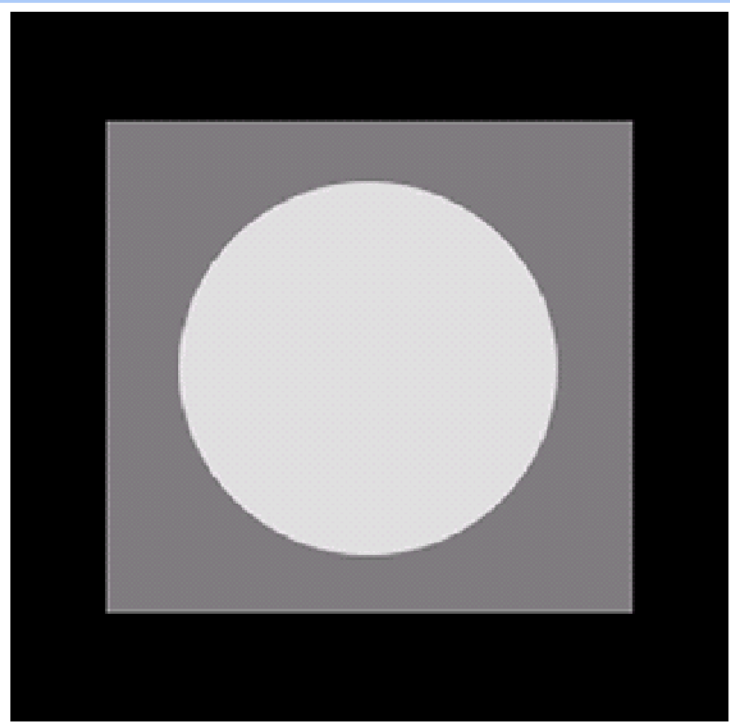


Fig: Original Image

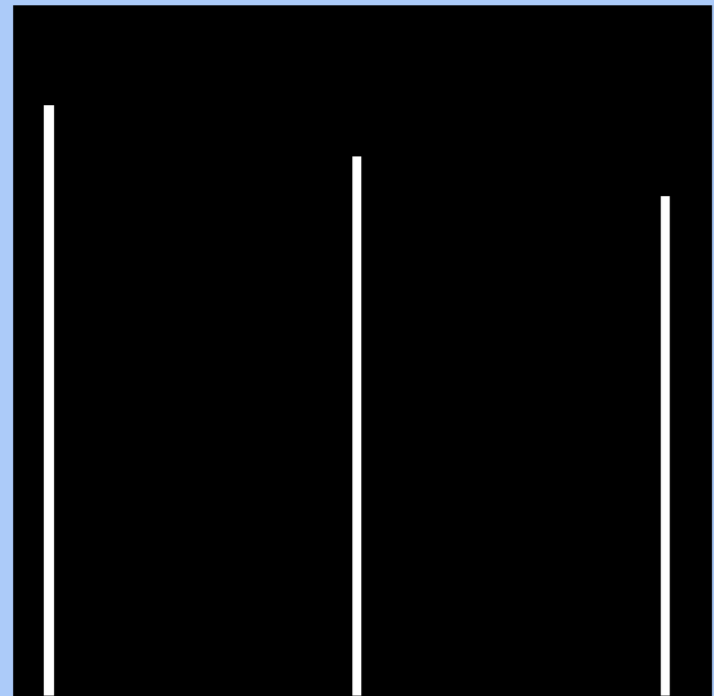
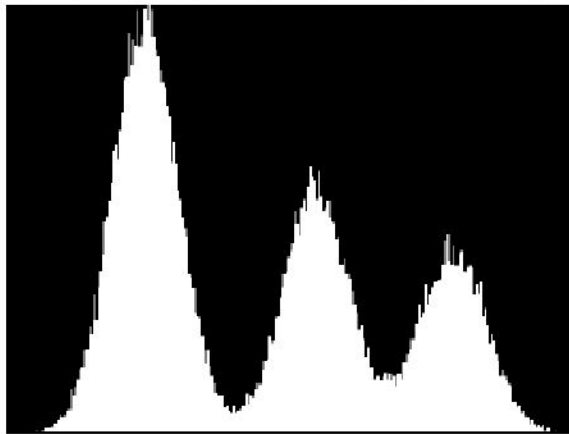
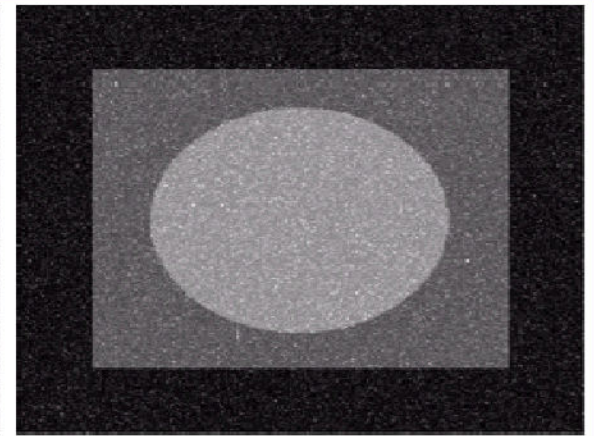
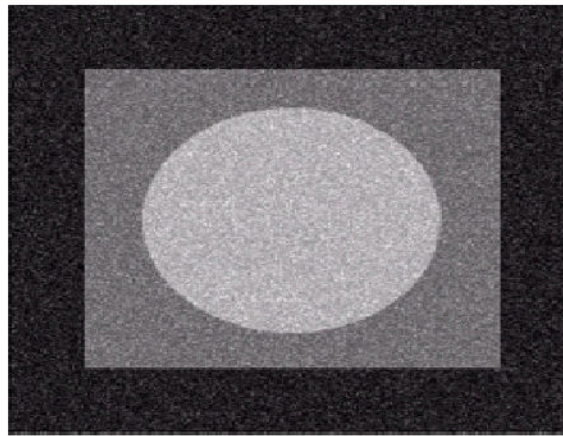
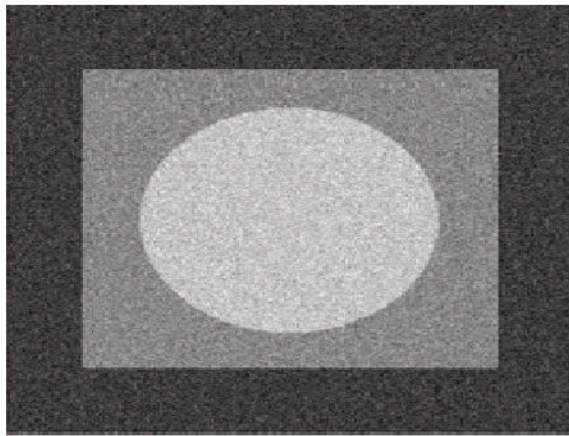


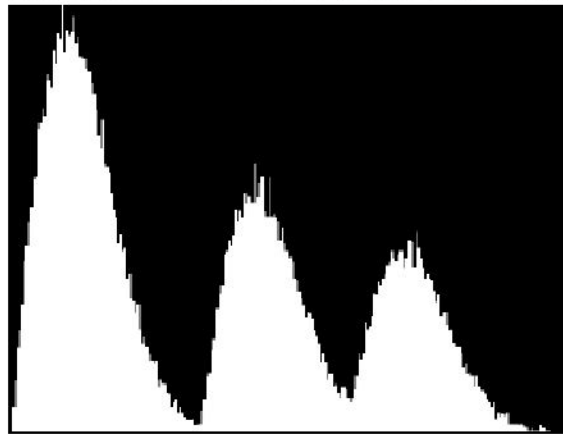
Fig: Original Image histogram

# Noise Models Effects contd1...

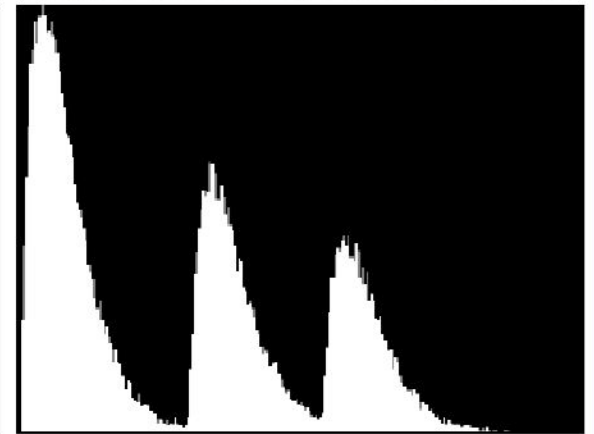
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Gaussian



Rayleigh

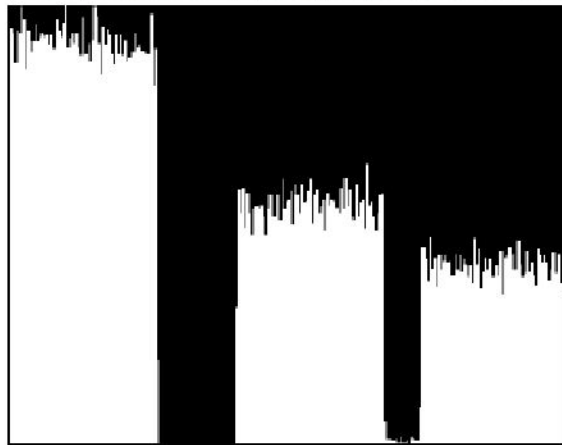
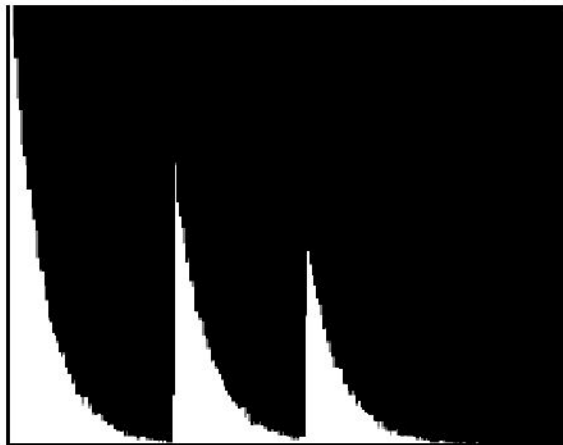
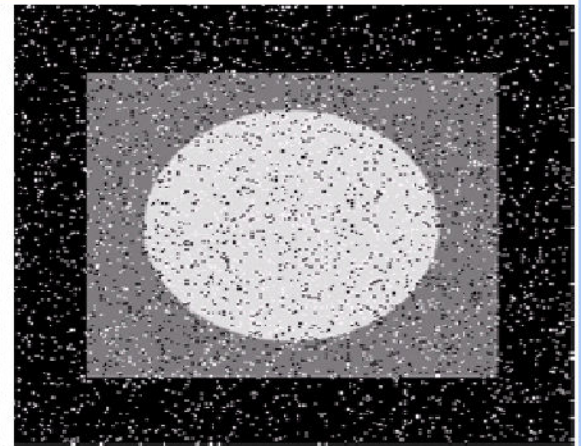
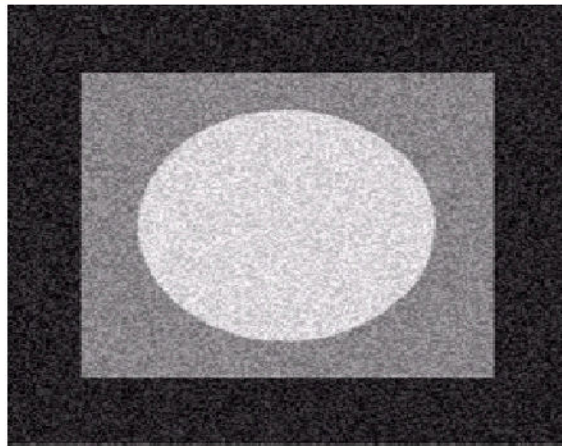
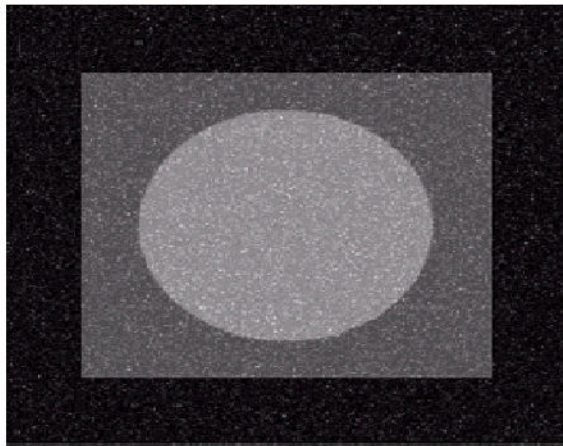


Gamma



# Noise Models Effects contd2...

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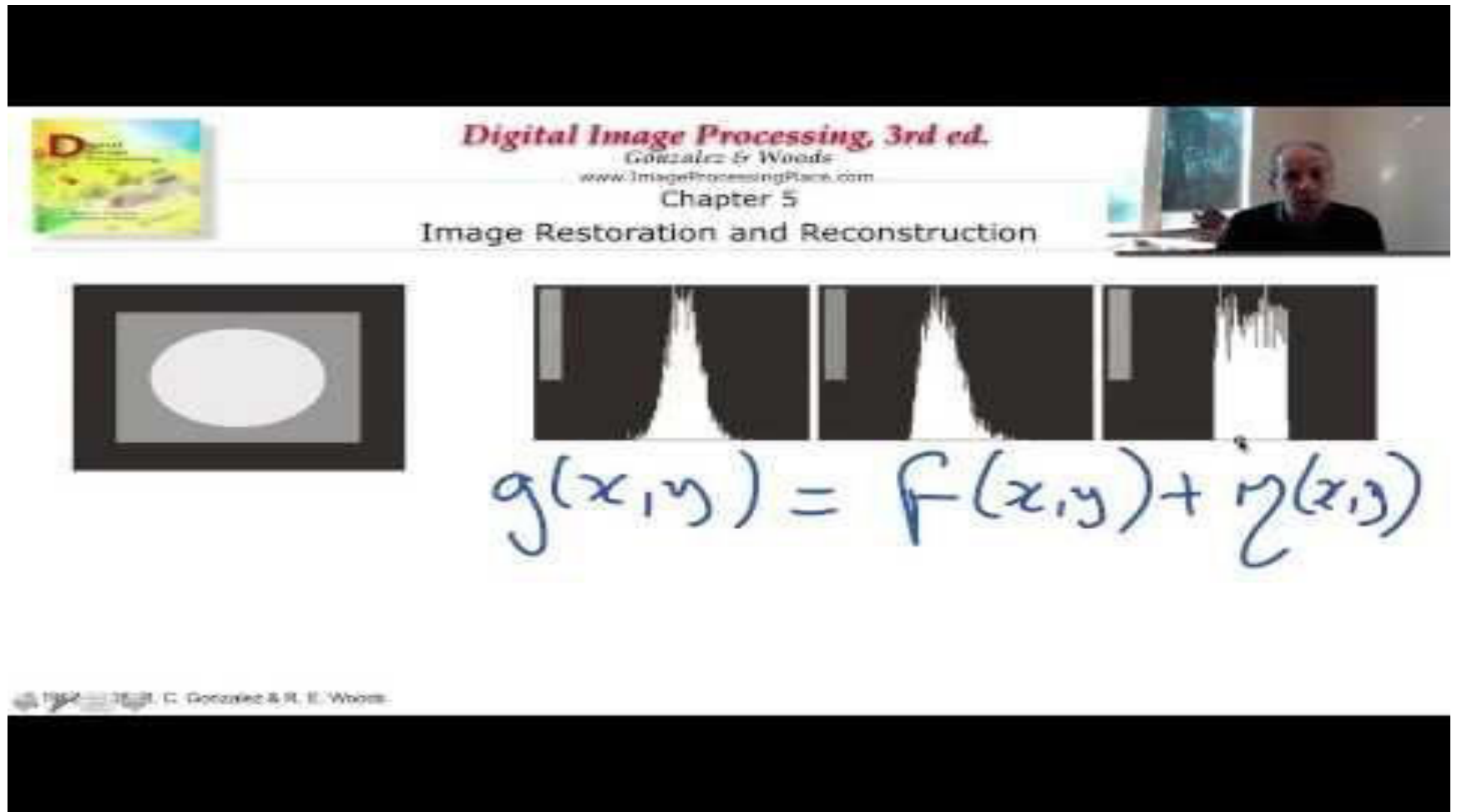


Exponential


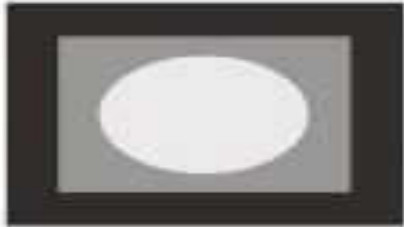
Uniform

Salt & Pepper

# Estimating noise



**Digital Image Processing, 3rd ed.**  
Gonzalez & Woods  
www.ImageProcessingPlace.com  
Chapter 5  
Image Restoration and Reconstruction


$$g(x,y) = f(x,y) + n(x,y)$$

© 1994-2005 R. C. Gonzalez & R. E. Woods.

<https://www.youtube.com/watch?v=Cla60r2E2VI>

# Restoration Techniques.

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- Inverse Filtering.
- Minimum Mean Squares Errors.
  - ✓ **Weiner Filtering.**
- Constrained Least Square Filter.
- Non linear filtering
- Advanced Restoration Technique.



# Filter used for Restoration Process

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- Mean filters
  - Arithmetic mean filter
  - Geometric mean filter
  - Harmonic mean filter
  - Contra-harmonic mean filter
- Order statistics filters
  - Median filter
  - Max and min filters
  - Mid-point filter
  - alpha-trimmed filters
- Adaptive filters
  - Adaptive local noise reduction filter.
  - Adaptive median filter

# Filtering to Remove Noise-AMF

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- Use spatial filters of different kinds to remove different kinds of noise
- **Arithmetic Mean :**

$$\hat{f}(x, y) = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} g(s, t)$$

- This is implemented as the simple smoothing filter Blurs the image to remove noise.

$1/9$	$1/9$	$1/9$
$1/9$	$1/9$	$1/9$
$1/9$	$1/9$	$1/9$

# Filtering to Remove Noise-GMF

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- **Geometric Mean:**

$$\hat{f}(x, y) = \left[ \prod_{(s,t) \in S_{xy}} g(s, t) \right]^{\frac{1}{mn}}$$

- Achieves similar smoothing to the arithmetic mean, but tends to lose less image detail.

# Filtering to Remove Noise-HMF

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- Harmonic Mean:

$$\hat{f}(x, y) = \frac{mn}{\sum_{(s,t) \in S_{xy}} \frac{1}{g(s,t)}}$$

- Works well for salt noise, but fails for pepper noise
- Satisfactory result in other kinds of noise such as Gaussian noise

# Filtering to Remove Noise-CHMF

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- **Contra-harmonic Mean:**

$$\hat{f}(x, y) = \frac{\sum_{(s,t) \in S_{xy}} g(s, t)^{Q+1}}{\sum_{(s,t) \in S_{xy}} g(s, t)^Q}$$

- $Q$  is the *order* of the filter and adjusting its value changes the filter's behaviour.
- Positive values of  $Q$  eliminate pepper noise.
- Negative values of  $Q$  eliminate salt noise.

# Order Statistics Filters

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- Spatial filters that are based on ordering the pixel values that make up the neighbourhood operated on by the filter
- Useful spatial filters include
  - Median filter.
  - Maximum and Minimum filter.
  - Midpoint filter.
  - Alpha trimmed mean filter.

# Median Filter

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- **Median Filter:**

$$\hat{f}(x, y) = \underset{(s,t) \in S_{xy}}{\text{median}}\{g(s, t)\}$$

- Excellent at noise removal, without the smoothing effects that can occur with other smoothing filters
- Best result for removing salt and pepper noise.

# Maximum and Minimum Filter

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- **Max Filter:**

$$\hat{f}(x, y) = \max_{(s,t) \in S_{xy}} \{g(s, t)\}$$

- **Min Filter:**

$$\hat{f}(x, y) = \min_{(s,t) \in S_{xy}} \{g(s, t)\}$$

- Max filter is good for **pepper noise** and min is good for **salt noise**



# Midpoint Filter

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- **Midpoint Filter:**

$$\hat{f}(x, y) = \frac{1}{2} \left[ \max_{(s,t) \in S_{xy}} \{g(s, t)\} + \min_{(s,t) \in S_{xy}} \{g(s, t)\} \right]$$

- Good for random Gaussian and uniform noise

# Alpha-Trimmed Mean Filter

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- **Alpha-Trimmed Mean Filter:**

$$\hat{f}(x, y) = \frac{1}{mn - d} \sum_{(s,t) \in S_{xy}} g_r(s, t)$$

- Here deleted the  $d/2$  lowest and  $d/2$  highest grey levels, so  $g_r(s, t)$  represents the remaining  $mn - d$  pixels

# Answer these questions:

- Can we **detect** what type of **noise** is present in an **image**?
- How can we estimate **noise** from an **image**?
- How do you improve the quality of a **degraded image**?