CSE4227 Digital Image Processing

Lecture 03 – **Chapter 2: Pixel and their Relationships**

Dr. Kazi A Kalpoma
Professor, Department of CSE

Ahsanullah University of Science & Technology (AUST)

Contact: kalpoma@aust.edu

Today’s Contents

❑ Some Basics Relationships between Pixels

- Neighborhood
- Adjacency
- Connectivity
- Paths
- Regions and boundaries

➡Chapter 2 from R.C. Gonzalez and R.E. Woods, Digital Image Processing (3rd Edition), Prentice Hall, 2008 [**Section 2.5**] [**Problems: 2.9 ~ 2.17 excluding 2.14**]

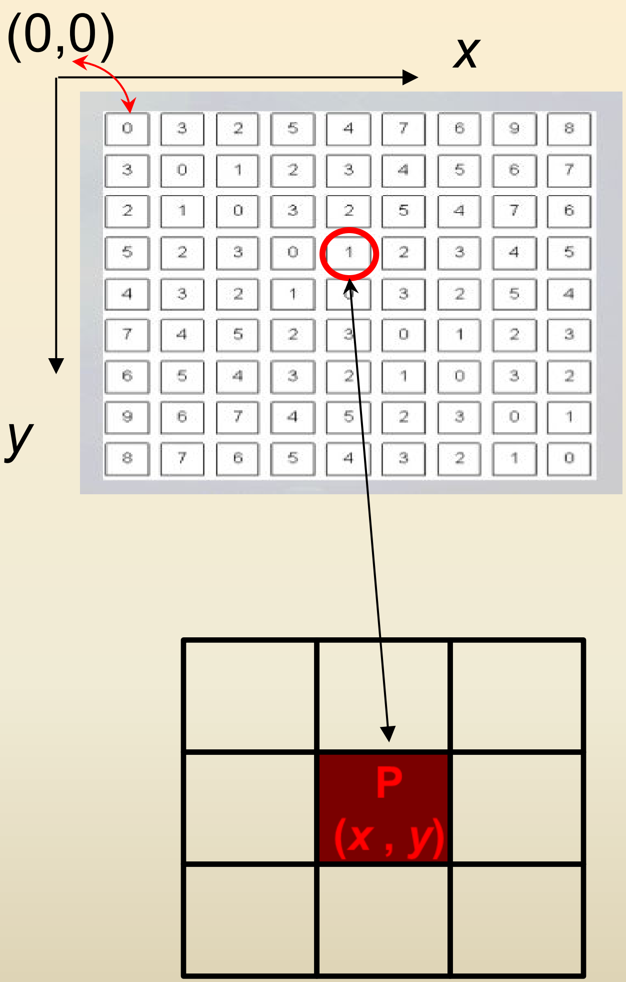
➡https://www.quora.com/How-can-we-find-the-shortest-m-connected-path-if-the-pixel-values-are-given-in-matrix-form

➡http://www.cse.iitm.ac.in/~vplab/courses/CV_DIP/PDF/NEIGH_CONN.pdf

Some Basic Relationships between Pixels

0	3	2	5	4	7	6	9	8
3	0	1	2	3	4	5	6	7
2	1	0	3	2	5	4	7	6
5	2	3	0	1	2	3	4	5
4	3	2	1	0	3	2	5	4
7	4	5	2	3	0	1	2	3
6	5	4	3	2	1	0	3	2
9	6	7	4	5	2	3	0	1
8	7	6	5	4	3	2	1	0

Conventional indexing method of Pixels



(0,0) x

y

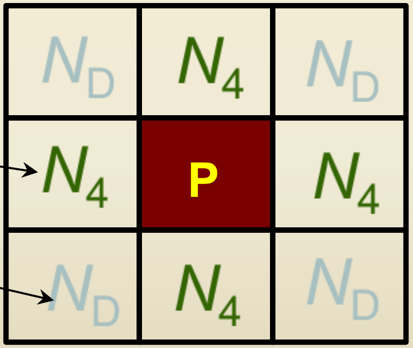
(x-1),(y-1)	**(x),(y-1)**	**(x+1),(y-1)**
(x-1),(y)	**P** **(x,y)**	**(x+1),(y)**
(x-1),(y+1)	**(x),(y+1)**	**(x+1),(y+1)**

Conventional indexing method

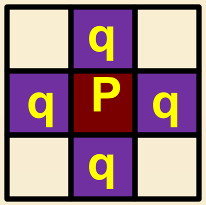
Pixel and their Relationships

- Neighbors of a pixel, p(x,y)
  - Neighborhood relation is used to tell adjacent pixels.
  - It is useful for analyzing regions.

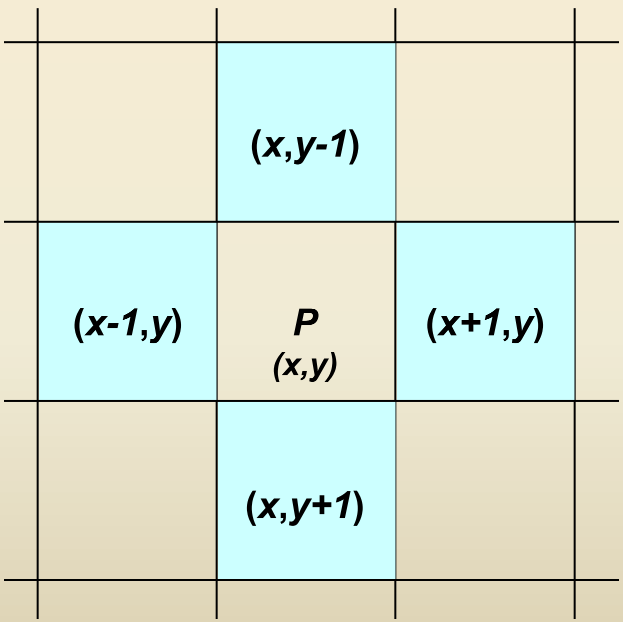
- 3 types of neighbors
  - $N_4(p)$: 4-neighbor of p
  - $N_D(p)$: diagonal neighbor of p
  - $N_8(p)$: 8-neighbor of p



$N_4(p)$: 4-neighbor of p



- ❑considers only vertical and horizontal neighbors pixels.
- ❑Note: $q \in N_4(p)$ implies $p \in N_4(q)$



4-neighbors of *p(x,y)*:

$$N_4(p) = \left\{ \begin{array}{c} (x-1,y) \\ (x+1,y) \\ (x,y-1) \\ (x,y+1) \end{array} \right\}$$

❖ Each of them is at a **<u>unit distance</u>** from P.

$N_D(p)$: diagonal neighbor of p

q		q
	P	
q		q

- considers only diagonal neighbors pixels.
- Note: $q \in N_D(p)$ implies $p \in N_D(q)$

(*x-1*,*y-1*)		**(*x+1*,*y-1*)**
	p	
(*x-1*,*y+1*)		**(*x+1*,*y+1*)**

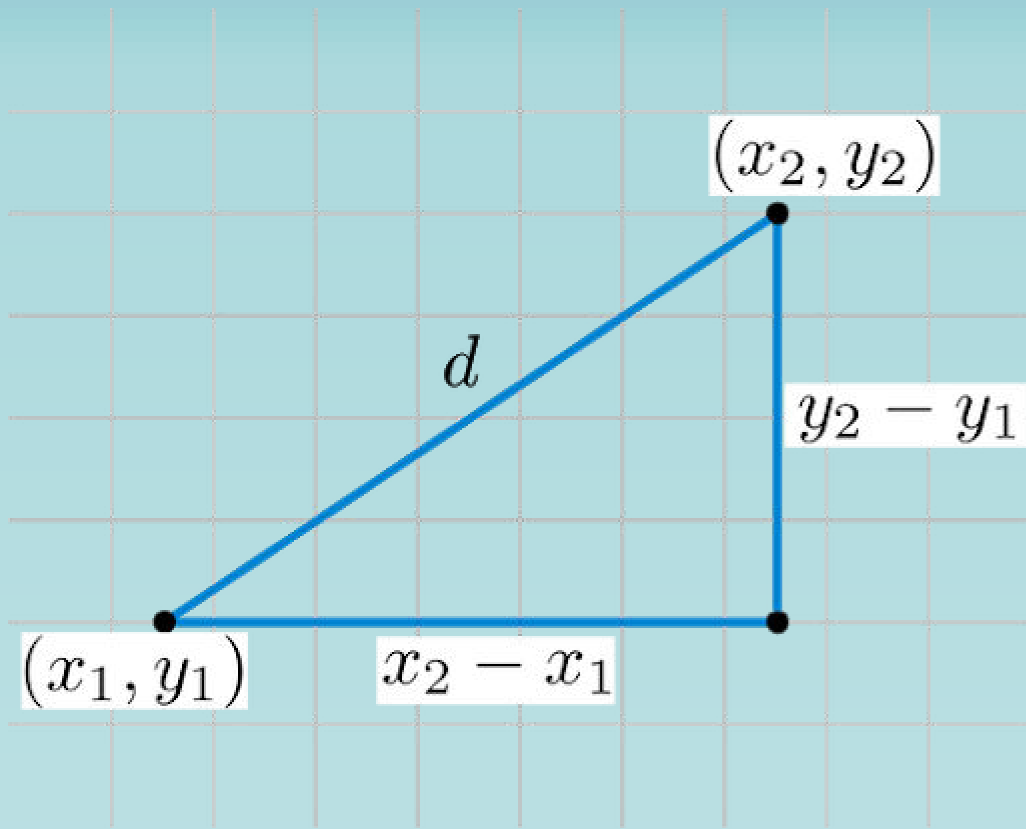
Diagonal neighbors of *p(x,y)*:

$$N_D(p) = \left\{ \begin{array}{c} (x-1,y-1) \\ (x+1,y-1) \\ (x-1,y+1) \\ (x+1,y+1) \end{array} \right\}$$

❖ Each of them are at **Euclidean distance** of 1.414 from P.

$$\text{Euclidean distance (d)} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Euclidean distance



Euclidean distance (d) $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$N_8(p)$: 8-neighbor of p

q	q	q
q	P	q
q	q	q

- considers all neighbors pixels.
- Note: $q \in N_8(p)$ implies $p \in N_8(q)$

8-neighbors of *p(x,y)*:

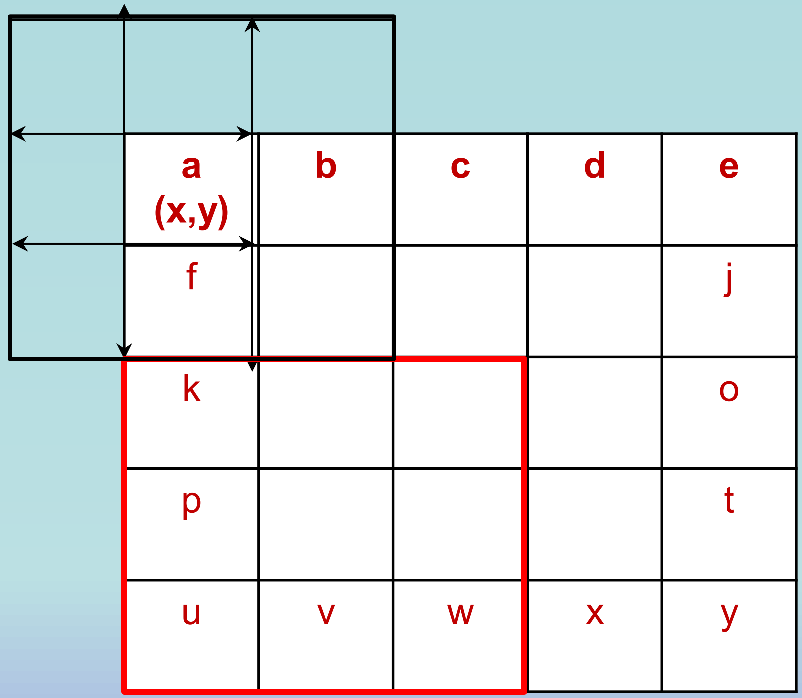
(x-1,y-1)	*(x,y-1)*	*(x+1,y-1)*
(x-1,y)	*p*	*(x+1,y)*
(x-1,y+1)	*(x,y+1)*	*(x+1,y+1)*

$$N_8(p) = \left\{ \begin{array}{c} (x-1,y-1) \\ (x,y-1) \\ (x+1,y-1) \\ (x-1,y) \\ (x+1,y) \\ (x-1,y+1) \\ (x,y+1) \\ (x+1,y+1) \end{array} \right\}$$

Missing neighbors!

- Some of the points in the N_4, N_D and N_8 may fall outside image when P lies on the **border of image**.

  - Discard
  - Zero padding
  - Pixel replication



Pixel Adjacencies

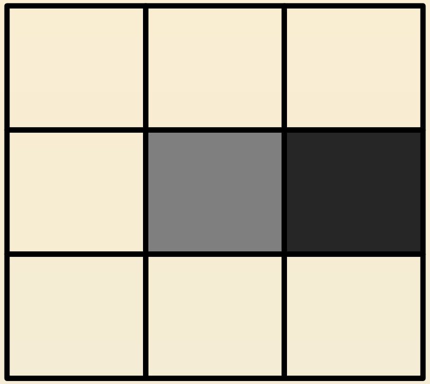
- Adjacencies depends on both
  - *neighborhood*
  - *Pixel gray values*

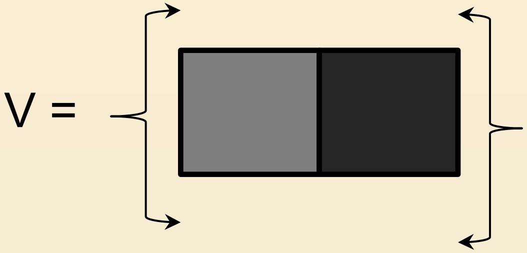
- Two pixels are connected if they are neighbors and their gray levels satisfy some specified criterion of similarity.

- For example, in a binary image two pixels are connected if they are 4-neighbors and have same gray value (0/1).

Pixel Adjacencies

Let V be the set of intensity values





- Adjacent pixels must be neighbors and have gray values from the same set, *V*
- In binary image, V = {1} if we are referring to adjacency of pixels with value 1
- In gray-scale image, V can be any subset of 256 values

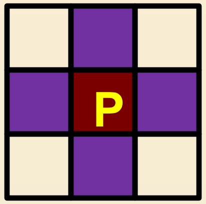
Pixel Adjacencies

- 3 types of adjacencies
  - 4-adjacency
  - 8-adjacency
  - *m*-adjacency

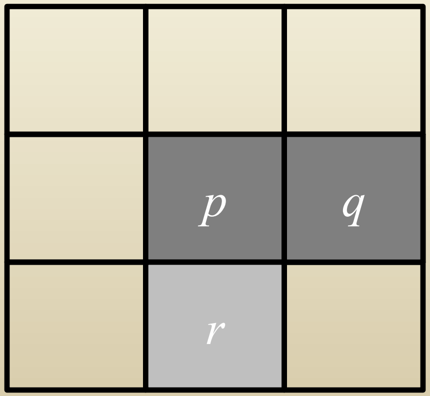
Pixel Adjacencies

- 4-adjacency
  - Two pixels *p*, *q* are 4-adjacent if
    - *q* is in the set $N_4(p)$
    - *p*, *q* have values from set *V*,

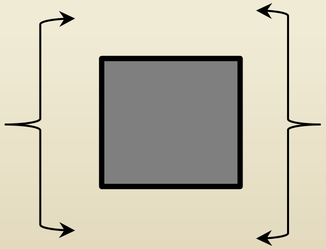
(V be the set of intensity values)



$N_4(p)$



V =

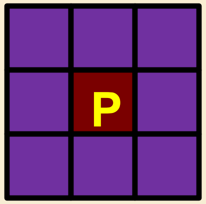


Here, p and q are 4-adjacent and connected
But p and r not 4-adjacent pixels and not connected,
r is not from set V.

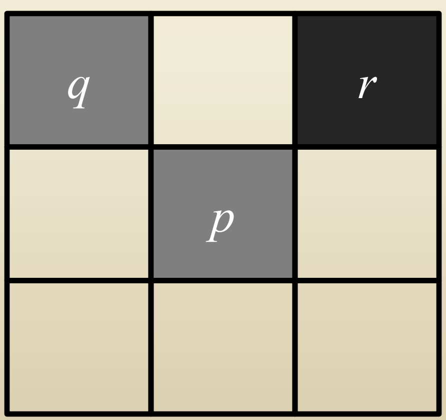
Pixel Adjacencies

- 8-adjacency
  - Two pixels *p*, *q* are 8-adjacent if
    - *q* is in the set $N_8(p)$
    - *p*, *q* have values from set *V*,

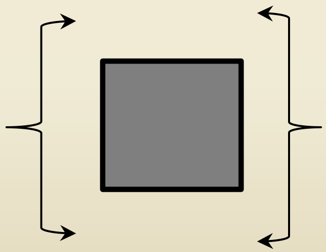
(V be the set of intensity values)



$N_8(p)$



V =



Here, p and q are 8-adjacent and connected
But p and r not 8-adjacent pixels and not connected,
r is not from set V.

m-adjacent?

4**(A)**	2**(B)**	3	2
3	3**(E)**	1	3
2	3	5**(C)**	2
2	1	2	5**(D)**

V = {0,1,2,3,5}

- Is A and B 4-adjacent? 8-adjacent?
- Is A and C 4-adjacent? 8-adjacent?
- Is B and C 4-adjacent? 8-adjacent?
- Is C and D 4-adjacent? 8-adjacent?
- Is C and E 4-adjacent? 8-adjacent?

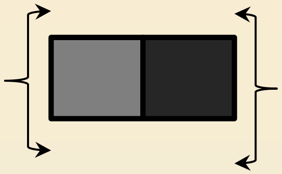
Pixel Adjacencies

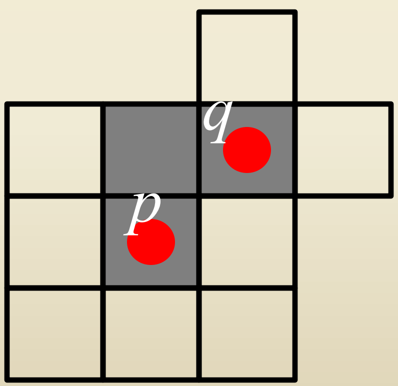
- *m*-adjacency

  Two pixels *p*, *q* are *m*-adjacent iff

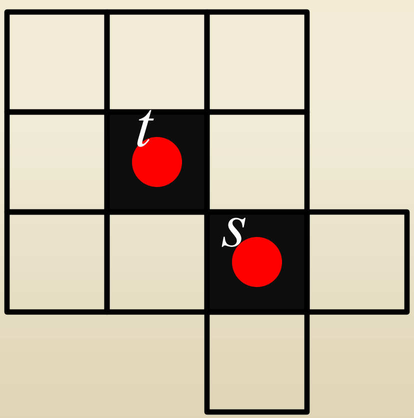
  – *p*, *q* have values from set *V*, and

  - *q* in $N_4(p)$, or
  - *q* in $N_D(p)$ and $N_4(p) \cap N_4(q)$ has no pixel with value from *V*

V = 



p, q are not m-adjacency pixels



s, t are m-adjacency pixels

m-adjacent?

4(A)	2(B)	3	2
3	3	1	3
2	3	5(C)	2
2	1	2	5(D)

V = {0,1,2,3,5}

- Is A and B m-adjacent?
- Is A and C m-adjacent?
- Is B and C m-adjacent?
- Is C and D m-adjacent?

- *m*-adjacency

  Two pixels *p*, *q* are *m*-adjacent iff

  – *p*, *q* have values from set *V*, and

  - *q* in $N_4(p)$, or
  - *q* in $N_D(p)$ and $N_4(p) \cap N_4(q)$ has no pixel with value from *V*

Connectivity :

To determine whether the pixels are adjacent in some sense.

Let V be the set of gray-level values used to define connectivity; then Two pixels p, q that have values from the set V are:

a. 4-connected, if q is in the set $N_4(p)$

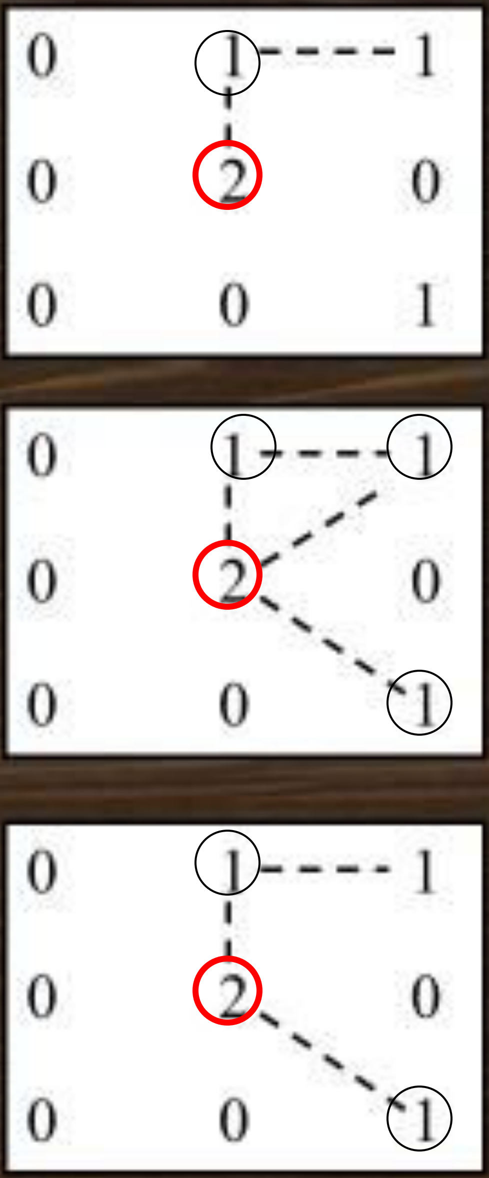
b. 8-connected, if q is in the set $N_8(p)$

c. m-connected, iff

   i. q is in $N_4(p)$ or

   ii. q is in $N_D(p)$ and the set $N_4(p) \cap N_4(q)$ is empty

$V = \{1, 2\}$



a. b. c.

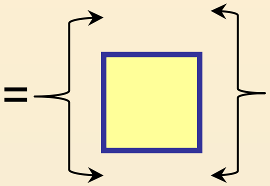
A digital path

- A digital path from pixel p with coordinates (x,y) to pixel q with coordinates (s,t) is a sequence of distinct pixels with coordinates (x_0,y_0), (x_1,y_1), …, (x_n,y_n),
  where
  $(x_0,y_0)= (x,y)$ and $(x_n,y_n)=(s,t)$,
  and pixels (x_i,y_i) and (x_{i-1},y_{i-1}) are adjacent for $1 \le i \le n$.

- n is the length of the path
- If $(x_0,y_0) = (x_n, y_n)$, the path is closed.
- We can specify 4-, 8- or m-paths depending on the type of adjacency specified.

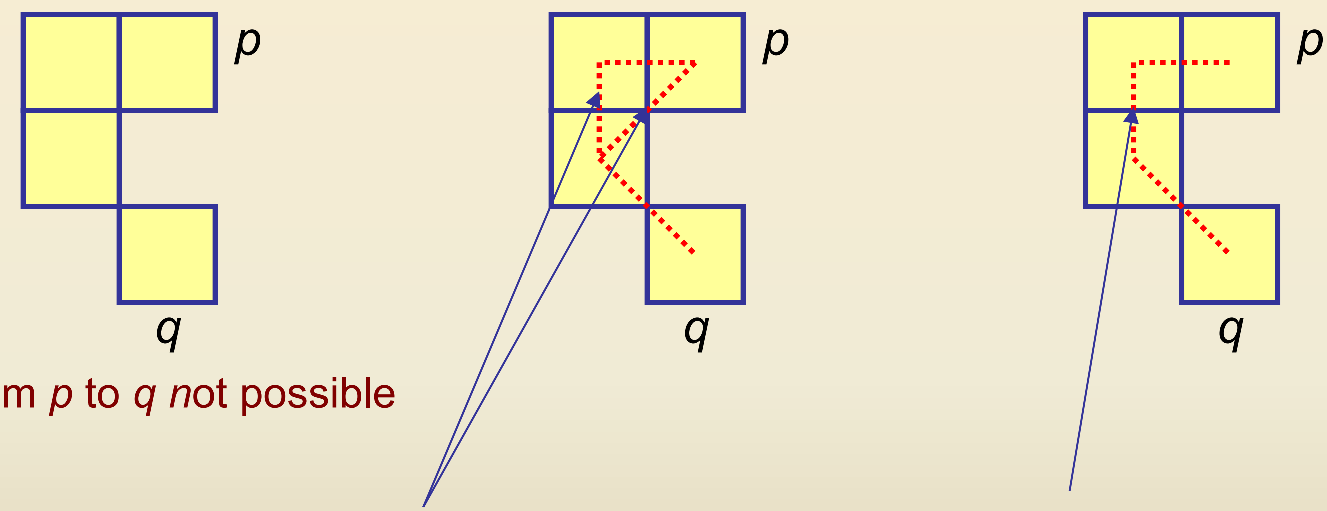
A Digital Path

Look for a path from p to q:

When, V = 

8-path

m-path



4-path from *p* to *q* *n*ot possible

8-path from *p* to *q*
results in some ambiguity

m-path from *p* to *q*
solves this ambiguity

Example of m-connected path:

- Find out the m-connected path between p and q, with **V={1, 2}** from the following image matrix.

4	**2**	**3**	**2(q)**
3	**3**	**1**	**3**
2	**3**	**2**	**2**
2(p)	**1**	**2**	**3**

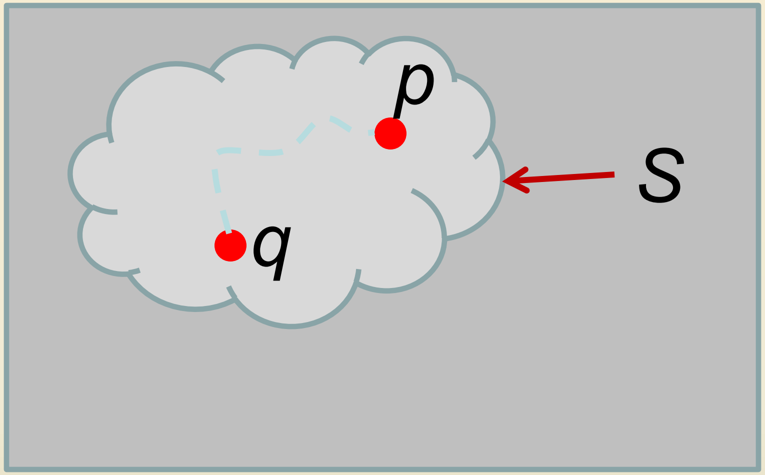
- Is 4-connected path between p and q possible?

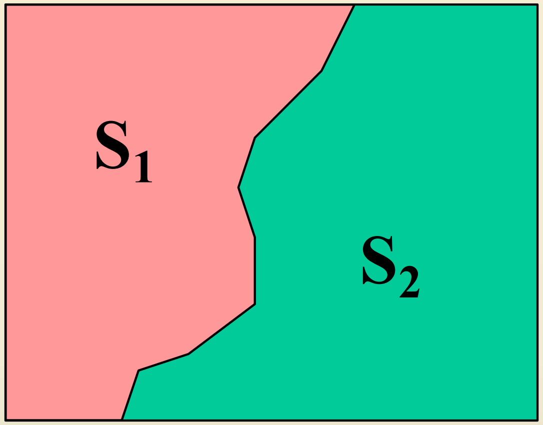
- The m-connected path between p and q is:

2(p) -> 1 -> 2 -> 2 -> 1 -> 2(q)

Adjacency / Connectivity

- Let **S** represent a subset of pixels in an image,
  - Two pixels *p* and *q* are connected in *S* if there is a path between *p* and *q* entirely in *S*

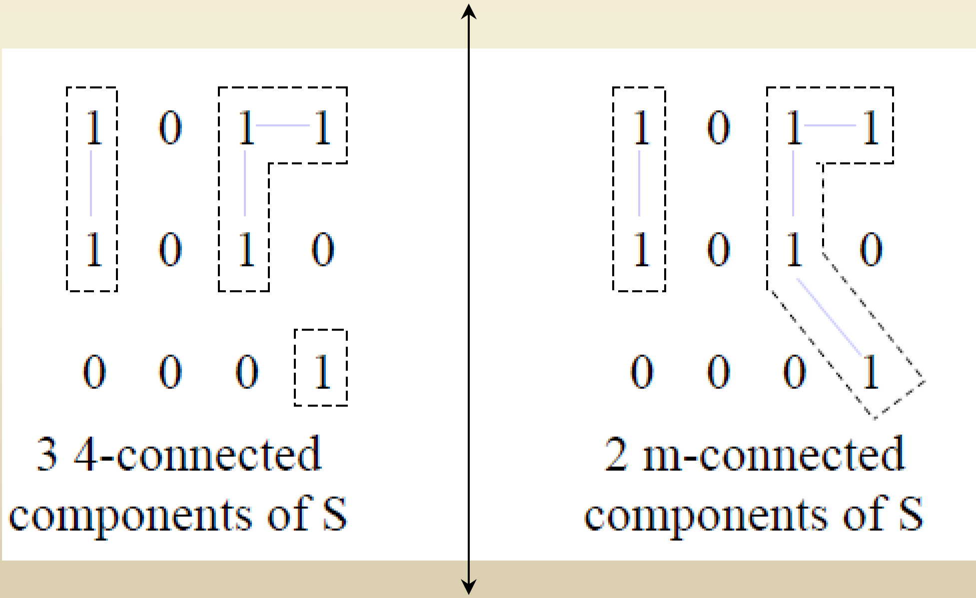




- Two image subsets **S1** and **S2** are adjacent **if** some pixel in **S1 is adjacent** to some pixel in **S2**

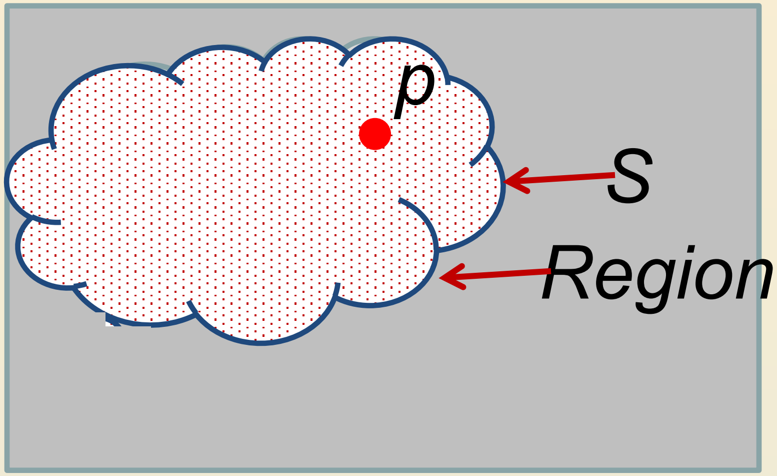
Connected Component

- Connected Component in a set of pixels, *S*
  - For any pixel *p* in *S*, the set of pixels that are connected to it in *S* is called a *connected component* of *S*



Definition of Region

- Connected Set, *S*
  - If all pixels of *S* are connected to *p*

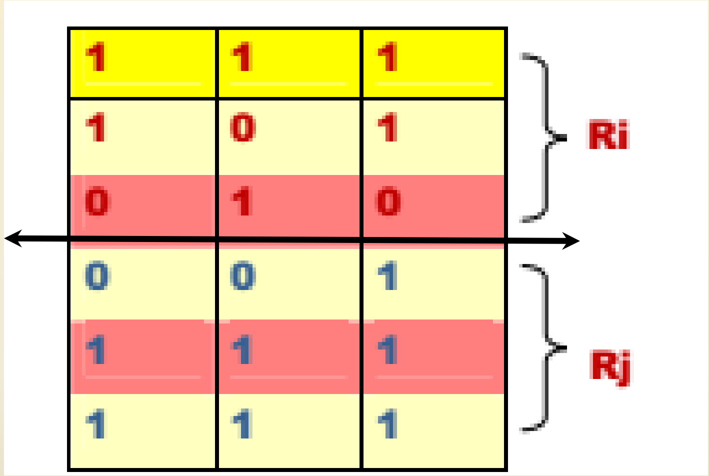


- If it only has one connected component,
  - then set ***S*** is called a *connected set*.
  - And we call ***S*** a region of the image R

Region is a Connected Set

❑Region that are not adjacent are said to be **disjoint**.

❑***Example*: the two regions (of *1s*) in figure, are adjacent only if 8-adjacany is used.**

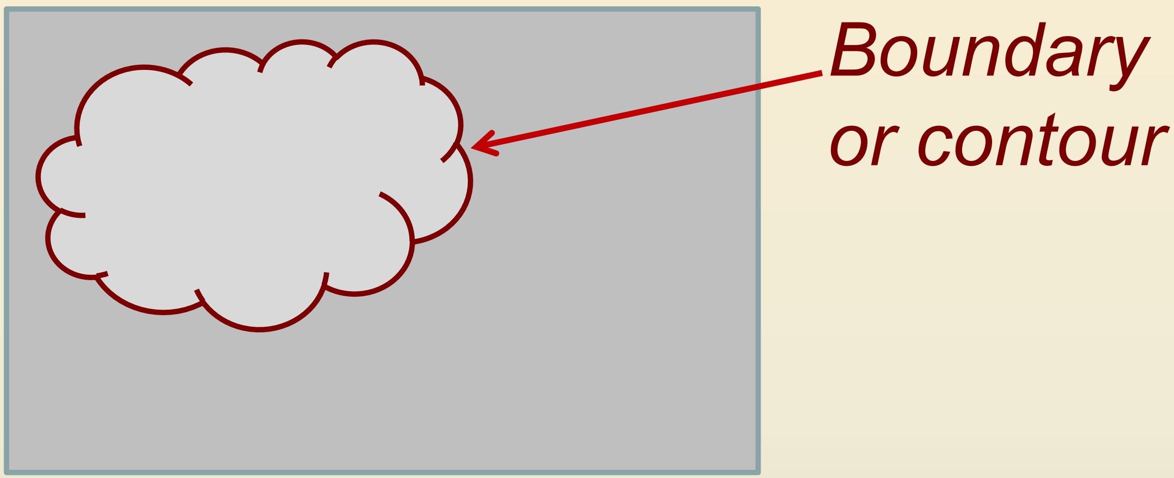


V = {1}

❑ ***4-path* between the two regions does not exist, (so their union is not a 4-connected set).**

Boundary of a region

- Boundary of a region, *R*
  - Set of pixels whose at least one neighbor is not in *R*
  - *also called border or contour) of a region R*

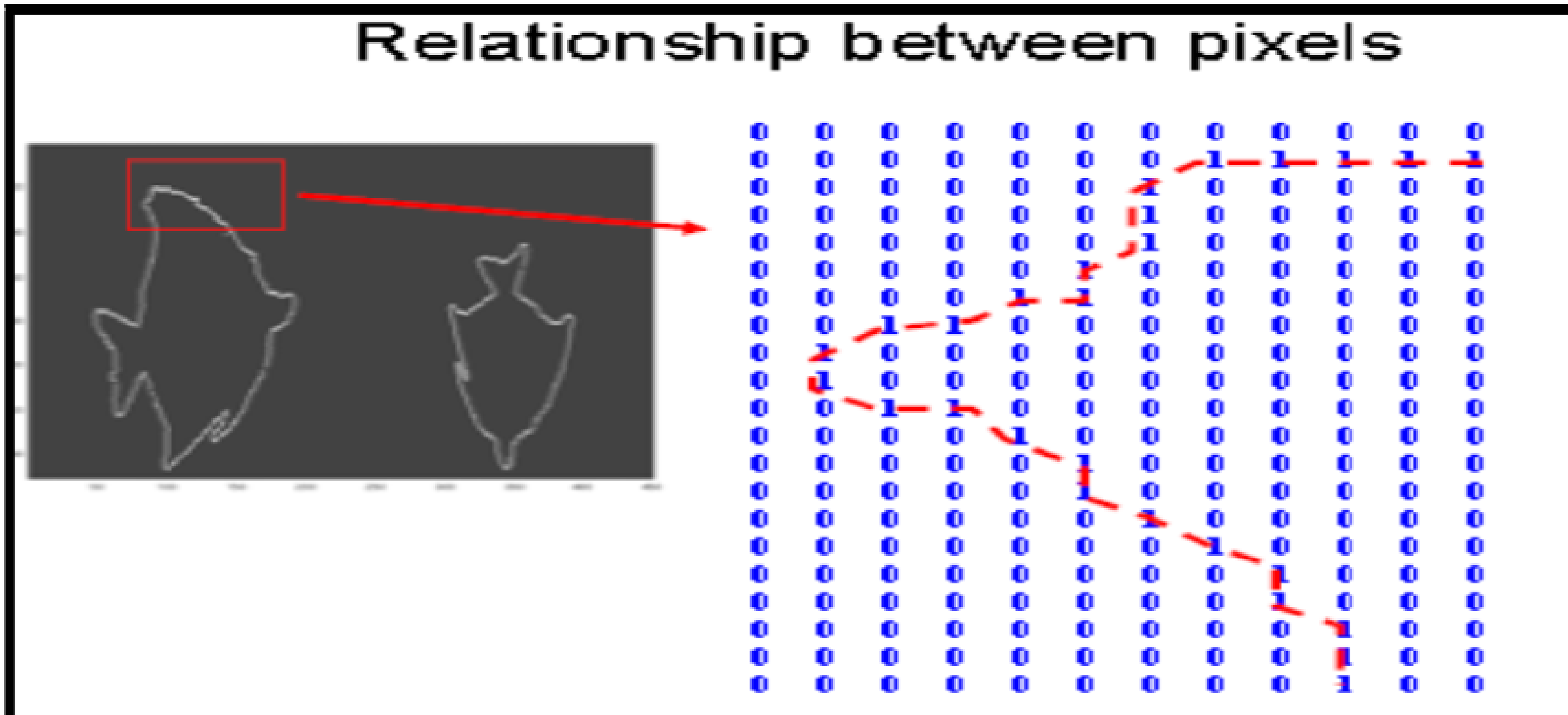


a	b	c	d	e
f				j
k				o
p				t
u	v	w	x	y

❑ If *R* happens to be an entire image, then its boundary is defined as the set of pixels in the first and last rows and columns in the image.

- **<u>Boundary (border or contour)</u>** of a region **R** is the set of points that are adjacent to points in the **complement** of **R** (another way: the border of a region is the set of pixels in the region that have at least are background neighbor).

We must specify the connectivity being used to define adjacency



Distance Metrics

If we have 3 pixels: p,q,z respectively

p with (x,y)
q with (s,t)
z with (v,w)

D, distance function has following properties:

- $D(p,q) \geq 0$ [$D(p,q) = 0$ iff $p = q$] and
- $D(p,q) = D(q,p)$ and
- $D(p,z) \leq D(p,q) + D(q,z)$

Distance Metrics

We can divide distance between pixels in following categories

- City Block Distance - D_4
- Chess Board Distance - D_8
- Euclidean Distance - D_E

Distance measures

p with (*x*,*y*)
q with (*s*,*t*)

- Euclidean distance

$$D_e(p,q) = [(x-s)^2 + (y-t)^2]^{\frac{1}{2}}$$

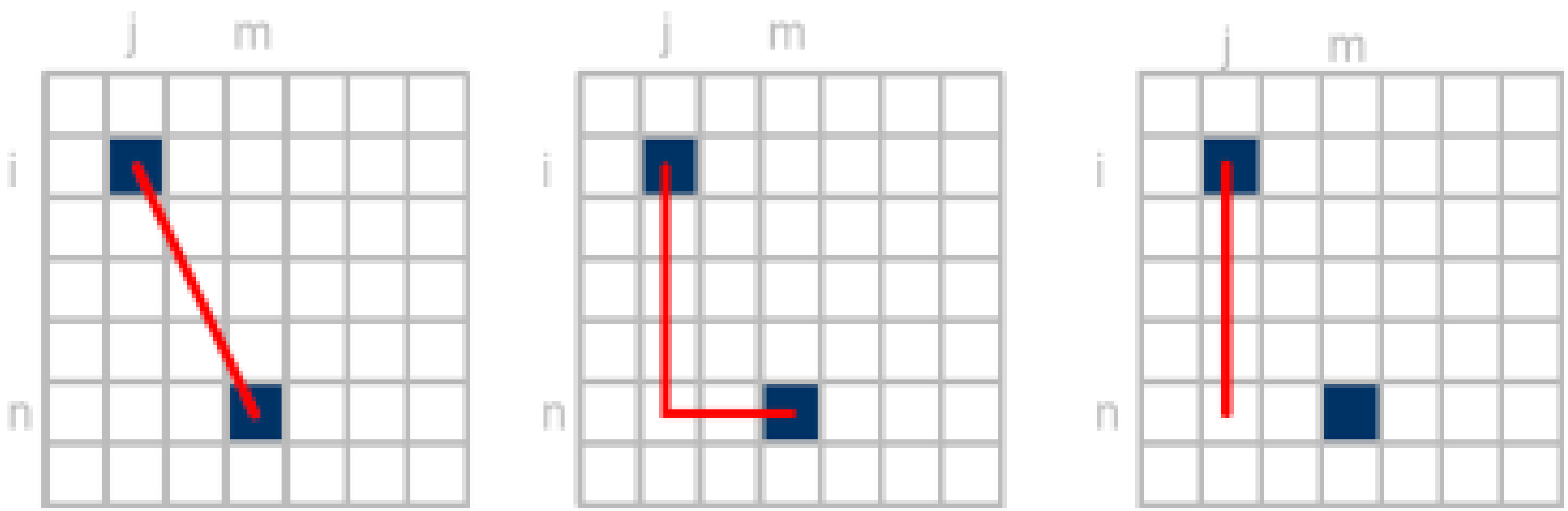
- City-block distance

$$D_4(p,q) = |(x-s)| + |(y-t)|$$

- Chessboard distance

$$D_8(p,q) = \max(|(x-s)|, |(y-t)|)$$

Distance measures



Euclidean Distance $= \sqrt{(i-n)^2 + (j-m)^2}$

City Block Distance $= |i-n| + |j-m|$

Chessboard Distance $= \max[\,|i-n|, |j-m|\,]$

Example:

Compute the distance between the two pixels using the three distances :

q:(1,1)

P: (2,2)

Euclidian distance : $((1-2)^2+(1-2)^2)^{1/2}$ = sqrt(2).

D4(City Block distance): |1-2| +|1-2| =2

D8(chessboard distance) : max(|1-2|,|1-2|)= 1

(because it is one of the 8-neighbors)

	1	2	3
1	q		
2		p	
3			

Distance measures

Example :

Use the city block distance to prove 4-neighbors?

Pixel a : | 2-2| + |1-2| = 1

Pixel b: | 3-2|+|2-2|= 1

Pixel c: |2-2|+|2-3| =1

Pixel d: |1-2| + |2-2| = 1

	1	2	3
1		d	
2	a	p	c
3		b	

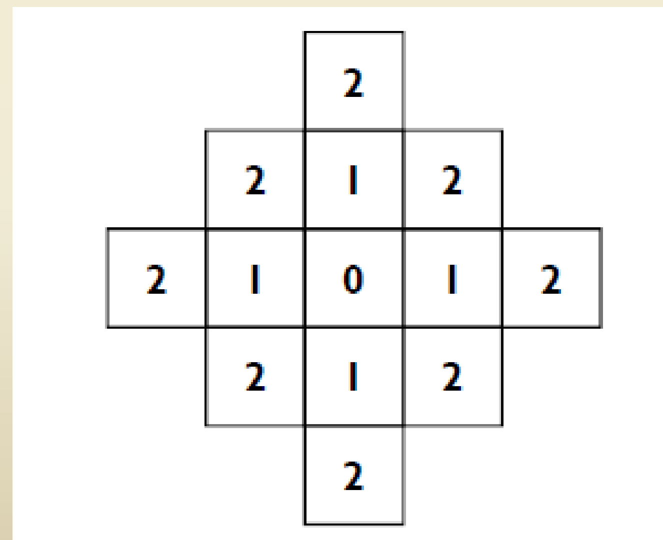
Now as a homework try the chessboard distance to proof the 8- neighbors!!!!

City block distance (D_4 distance)

$$D_4(p,q) = |x-s| + |y-t|$$

- The pixels with distance $D_4 \le 2$ from (x,y) form the following contours of constant distance, diamond with center at *(x,y)*

- $D_4 = 1$ are the 4 neighbors of pixel *p(x,y)*



Chessboard distance (D8 distance)

$$D_8(p,q) = \max(|x-s|, |y-t|)$$

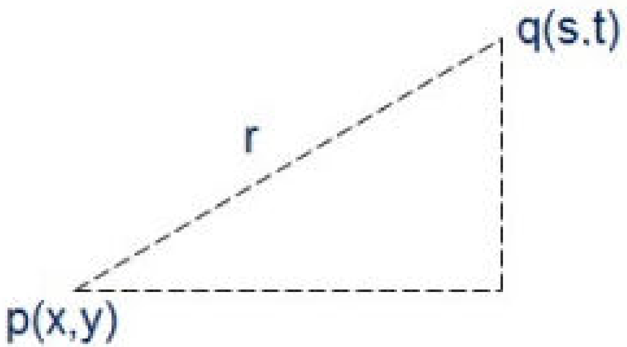
- D8 distance ≤ 2 from (x,y) form the following contours of constant distance, square centered at *p(x,y)*
- $D_8 = 1$ are the 8 neighbors of pixel *p(x,y)*

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

Euclidean Distance

$$D_e(p,q) = \sqrt{(x-s)^2 + (y-t)^2}$$

5	8	7	4	6
9	15	10	q(s,t)	2
17	14	1	5	18
27	2	9	19	22
p(x,y)	4	1	10	12



D = {(1-4)^2 + (1-4)^2}^0.5

D = 4.24

A circle with radius *r* centered at *(x,y)*

Two grid point: $P = (x,y)$ and $Q = (u,v)$

Euclidean Distance

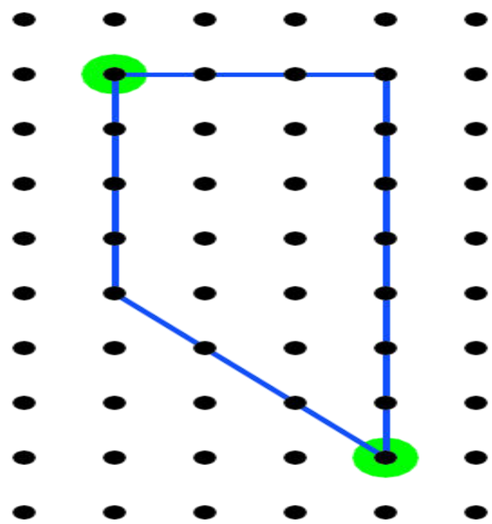
$$d_e(P,Q) = \sqrt{(x-u)^2 + (y-v)^2}$$

City Block Distance

$$d_4(P,Q) = |(x-u)| + |(y-v)|$$

Chessboard Distance

$$d_8(P,Q) = \max(|(x-u)| , |(y-v)|)$$



Neighborhood based arithmetic/Logic :

Value assigned to a pixel at position 'e' is a function of its neighbors and a set of window functions.

		⋮		
	a	b	c	
…	d	e	f	…
	g	h	i	
		⋮		

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

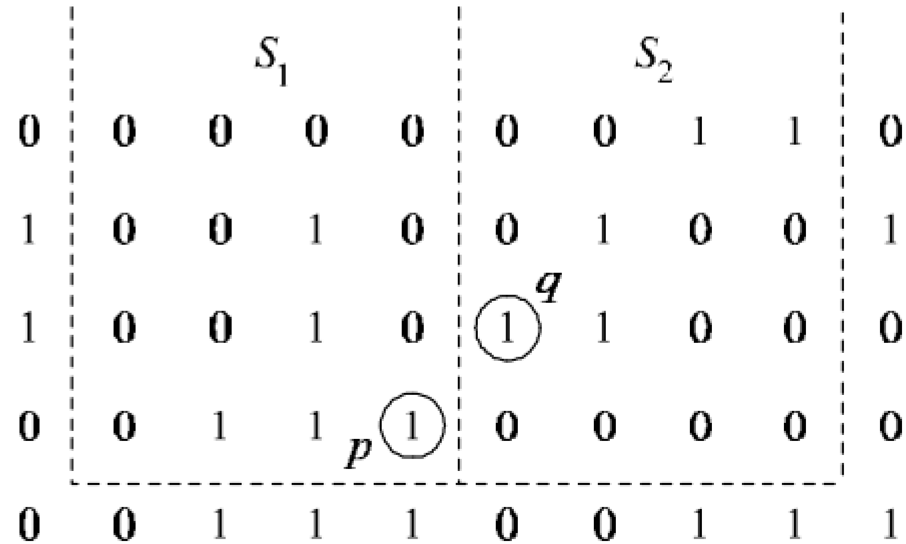
$$p = (w_1 a + w_2 b + w_3 c + w_4 d + w_5 e + w_6 f + w_7 g + w_8 h + w_9 i)$$
$$= \sum w_i f_i$$

Relationship between pixels (Contd..)

Arithmetic/Logic Operations:

– **Addition :**	$p + q$
– **Subtraction:**	$p - q$
– **Multiplication:**	$p*q$
– **Division:**	p/q
– **AND:**	p AND q
– **OR :**	p OR q
– **Complement:**	NOT(q)

Problem 2.11



Let p and q be as shown in Fig. P2.11. Then, (a) S_1 and S_2 are not 4-connected because q is not in the set $N_4(p)$u (b) S_1 and S_2 are 8-connected because q is in the set $N_8(p)$u (c) S_1 and S_2 are *m*-connected because (i) q is in $N_D(p)$, and (ii) the set $N_4(p) \setminus N_4(q)$ is empty.

Arithmetic/Logic Operations

- Tasks done using neighborhood processing:
  - Smoothing / averaging
  - Noise removal / filtering
  - Edge detection
  - Contrast enhancement