Exercise - 1

5. A sample of size

$n = 25$

$N = 150$

$\sigma = 10$

$$SE(\bar{x}) = \frac{\sigma}{\sqrt{n}}\sqrt{\frac{N - n}{N - 1}}$$

$$= \frac{10}{\sqrt{25}}\sqrt{\frac{150 - 25}{150 - 1}} = \frac{10}{5} \times 0.91 = 1.825$$

6. A random sample

$n = 65$

$\bar{x} = 3500$

$S = 200$

$$SE(\bar{x}) = \frac{s}{\sqrt{n - 1}} = \frac{200}{\sqrt{65 - 1}} = \frac{200}{8} = 25$$

7. A simple random sample

$n = 20$

$N = 75$

$X = 12$

$$P = \frac{n}{N} = \frac{12}{75} = 0.16$$

$Q = 1 - p = 0.84$

$$SE(P) = \sqrt{\frac{PQ}{n} \; \frac{N - n}{N - 1}}$$

$$= \sqrt{\frac{0.16 \times 0.84}{20} \; \frac{75 - 20}{75 - 1}}$$

$$= \sqrt{\frac{0.16 \times 0.84 \times 55}{20 \times 74}}$$

$$= \sqrt{0.00499}$$

$= 0.07$

8. A manager

$N = 300$

$n = 25$

$x = 15$

$$n = \frac{x}{r} = \frac{15}{25} = 0.6$$

$q = 1 - p = 0.4$

$$SE(P) = \sqrt{\frac{pq}{n - 1} \; \frac{N - n}{N}}$$

$$= \sqrt{\frac{0.6 \times 0.4}{25 - 1} \times \frac{300 - 25}{300}}$$

$$= \sqrt{\frac{0.6 \times 0.4 \times 275}{24 \times 300}}$$

$$= \sqrt{(0.00916)}$$

$= 0.095$

9. A candidate

$x_1 = 42$

$n_1 = 70$

$x_2 = 59$

$n_2 = 100$

$$p_1 = \frac{x_1}{r_1} \; \frac{42}{70} = 0.6$$

$$P_2 = \frac{x_2}{r_2} = \frac{59}{100} = 0.59$$

$$SE(p_1 - P_2) = \sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

$$p = \frac{n_1\,p_1 + n_2\,p_2}{n_1 + n_2}$$

$$= \frac{x_1 + x_2}{n_1 + n_2}$$

$$= \frac{42 + 59}{70 + 100} = \frac{101}{170} = 0.554$$

$Q = 1 - p = 0.406$

$\therefore SE(P_1 - n_2) = \sqrt{0.594 \times 0.406\left(\frac{1}{70} + \frac{1}{100}\right)}$

$= \sqrt{0.00578}$

$= 0.076$

10. Internet connection

$n_1 = 500$

$n_2 = 300$

$\bar{x}_1 = 0.8$

$s_1 = 0.1$

$\bar{x}_2 = 0.5$

$s_2 = 0.08$

$SE(\bar{x}_1 - \bar{x}_2) = \sqrt{s^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$

$S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}$

$= \frac{500 \times 0.1^2 + 300 \times 0.08^2}{500 + 300 - 2}$

$= \frac{5 + 1.92}{798}$

$= 0.0086$

$= SE(\bar{x}_1 - \bar{x}_2) = \sqrt{s^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$

$= \sqrt{0.008(0.0020 + 0.0033)}$

$= \sqrt{0.0086 \times 0.0053}$

$= 0.0067$

11. A random sample

$n = 64$

$\sigma = 20$

$\bar{x}_1 = 80$

$\alpha = 5\%$

C.I for $\mu = \bar{x} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

$= 80 \pm 1.96 \times \frac{20}{\sqrt{64}}$

$= 80 \pm 1.96 \times \frac{20}{8}$

$= 80 \pm 4.9$

$= (75.1, 84.9)$

Where $n = 256$

CI for $\mu = \bar{x} \pm 2_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

$= 80 \pm 1.96 \times \frac{20}{\sqrt{256}}$

$= 80 \pm 1.96 \times \frac{20}{16}$

$= 80 \pm 2.45$

$= (77.55, 82.45)$

12. A random sample

$n = 65$

$N = 1000$

$\bar{x}_1 = 6300$

$s = 9.5$

$\alpha = 5\%$

CI for $\mu = \bar{x} \pm Z_{\alpha/2} \frac{s}{\sqrt{n-1}} \sqrt{\frac{N - n}{N}}$

$= 6300 \times 1.96 \times \frac{9.5}{\sqrt{65-1}} \sqrt{\frac{1000 - 65}{1000}}$

$= 6300 \pm 1.96 \times \frac{9.5}{8} \times 0.966$

$= 6300 \pm 2.25$

$= (6297.75, 6302.25)$

13. It is of observed

$n = 70$

$x = 28$

$p = \frac{x}{n} = \frac{28}{70} = 0.4$

$q = 1 - p = 0.6$

CI for P = $P \pm Z_{\alpha/2}$ SE (P)

$= 0.4 \pm 1.645 \sqrt{\frac{pq}{n}}$

$= 0.4 \pm 1.645 \sqrt{\frac{0.4 \times 0.6}{70}}$

$= 0.4 \pm 1.645 \times 0.058$

$= 0.4 \pm 0.096$

$= (0.304, 0.496)$

14. In a random sample

$n = 400$

$x = 20$

$p = \frac{x}{n} = \frac{20}{400} = 0.05$

$q = 1 - p = 0.95$

CI for P

$= P \pm Z_{\alpha/2} \sqrt{\frac{pq}{n}}$

$= 0.05 + 2.576 \sqrt{\frac{0.05 \times 0.95}{400}}$

$= 0.05 \pm 2.576 \times \sqrt{0.000118}$

$= 0.05 \pm 0.027$

$= (0.023, 0.077)$

15. In Laboratory experiment

$n = 400$

$x = 80$

$p = \frac{x}{n} = \frac{80}{400} = 0.2$

$q = 1 - p = 1 - 0.2 = 0.8$

$\alpha = 5\%$

CI for p = $P \pm Z_{\alpha/2} \sqrt{\frac{pq}{n}}$

$= 0.2 \pm 1.96 \sqrt{\frac{0.2 \times 0.8}{400}}$

$= 0.2 \pm 1.96 \times \sqrt{0.0004}$

$= 0.2 \pm 0.039$

$= (0.161, 0.239)$

16 . In a sample survey

$n = 100$

$p = 23\% = 0.23$

$q = 1 - p = 0.77$

$\alpha = 1\%$

CI for p = $p \pm Z_{\alpha/2} \sqrt{\frac{pq}{n}}$

$= 0.23 \pm 2.576 \times \sqrt{\frac{0.23 \times 0.77}{100}}$

$= 0.23 \pm 2.576 \times \sqrt{0.001771}$

$= 0.23 \pm 0.108$

$= (0.122, 0.339)$

17. A factory is

$N = 5000$

$n = 500$

$p = 2\% = 0.02, q = 0.98$

$\alpha = 5\%$

CI for P

$= p \pm Z_{\alpha/2} \sqrt{\frac{pq}{n-1} \frac{N-n}{N}}$

$= 0.02 \pm 1.96 \sqrt{\frac{0.02 \times 0.98}{500-1} \times \frac{50000-500}{50000}}$

$= 0.02 \pm 1.96 \sqrt{\frac{0.02 \times 0.98 \times 49500}{499 \times 50000}}$

$= 0.02 \pm 1.96 \times 0.00003888$

$= 0.02 \pm 0.0122$

$= (0.0078, 0.0322)$

18. A random sample

$n = 100$

$x = 75$

$p = \frac{x}{n} = \frac{75}{100} = 0.75$

$q = 1 - p = 0.25$

$\alpha = 5\%$

$\text{CI for } p = p \pm Z_{\alpha/2}\sqrt{\frac{pq}{n}}$

$= 0.75 \pm 1.96\sqrt{\frac{0.75 \times 0.25}{100}}$

$= 0.75 \pm 1.96 \times \sqrt{0.001875}$

$= 0.75 \pm 0.0848$

$= 0.665, 0.8348$

$=(66.5\%, 83.48\%)$

19 . A sample of size

$n = 100$

$\bar{x} = 16$

$\sigma = 3$

$\alpha = 5\%$

$\text{CI for } \mu = \bar{x} + Z_{\alpha/2}\frac{\sigma}{\sqrt{x}}$

$= 16 \pm 1.96 \times \frac{3}{\sqrt{100}}$

$= 16 \pm 1.96 \times 0.3$

$= 16 \pm 0.588$

$= 15.412, 16.588$

20. Assuming population

$\sigma = 3$

$n = ?$

$d = 0.5$

$n = \frac{Z_{\alpha/2}^2\ \sigma^2}{d^2}$

$= \frac{1.96^2 \times 3^2}{0.5^2}$

$= 138.29$

≈ 138

21. The Principal of collage

$n = ?$

$d = 10\% = 0.1$

$p = 0.3$

$n = \frac{Z_{\alpha/2}^2 PQ}{d^2}$

$= \frac{3^2 \times 0.3 \times 0.7}{(0.1)^2}$

$= 189$

22. Mr. x Wants

$\sigma = 10$

$n = ?$

$\alpha = 5\%$

$d = 2$

$n = \frac{Z_{\alpha/2}^2\ \sigma^2}{d^2}$

$= \frac{1.96^2 \times 10^2}{2^2}$

$= 96.04$

≈ 96

23. The average time

$\sigma = 6.7$

$\alpha = 0.05$

$d = 1$

$n = ?$

$n = \frac{Z_{\alpha/2}^2\ \sigma^2}{d^2}$

$$= \frac{1.56^2 \times 6.7^2}{1^2}$$

$= 172.44$

≈ 173

Exercise 2 (I)

14. A sample of size

Sample size (n) = 400, sample mean ($\bar{x}$) = 99
Population mean (μ) = 100, S.D. (σ) = 8, α = 5%
Problem to test $H_0 : \mu = 100$
$H_1 : \mu \neq 100$ (two tailed)

Test statistic $Z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \dfrac{99 - 100}{8/\sqrt{400}} = \dfrac{-1 \times 20}{8} = -2.5$

Critical value
At α = 5%, Critical value $Z_{tabulated} = Z_{\alpha/2} = 1.96$
Decision
121 = 2.5 > $Z_{tabulated} = 1.96$
Reject H_0 at 5% level of significance.
Conclusion: Sample is not from population with mean 100 and S.D. 8.

15. The mean life time

n = 400, $\bar{x}$ = 1570, σ = 150, α = 1%, μ = 1600
Problem to test,
$H_0 : \mu = 1600$
$H_1 : \mu > 1600$ (one tailed)

Test statistic $Z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \dfrac{1570 - 1600}{\dfrac{150}{\sqrt{2100}}} = \dfrac{-30 \times 120}{150} = -4$

Critical value
At α = 1%, critical value is $Z_{tabulated} = Z_{\alpha/2} = 2.576$
Decision
121 = 4 > $Z_{tabulated} = 2.576$
Reject H_0 at 1% level of significance.
Conclusion: Mean life of fluorescent light is more than 1600 hrs.

16. The mean breaking strength

μ = 1800, σ = 100, n = 50, $\bar{x}$ = 1850, α = 0.01
Problem to test
$H_0 : \mu = 1800$
$H_1 : \mu > 1800$

Test statistic $Z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \dfrac{1850 - 1800}{\dfrac{100}{\sqrt{50}}} = \dfrac{50\sqrt{50}}{100} = 3.53$

Critical value
At α = 0.01 critical value is $Z_{tabulated} = Z_{\alpha} = 2.326$
Decision
Z = 3.53 > $Z_{tabulated} = 2.326$
Reject H_0 at α = 0.01
Conclusion: The claim of mean breaking strength of cables have increased is correct.

17. A sample of 400 male

n = 400, $\bar{x}$ = 171.38, μ = 171.17, σ = 3.3
Let, α = 5%,
Problem to test
$H_0 : \mu = 171.17$
$H_1 : \mu \neq 171.17$

Confidence limit for $\bar{x} = \mu \pm Z_{\alpha/2} \dfrac{\sigma}{\sqrt{n}}$

$$= 171.17 \pm 1.96 \times \frac{3.3}{\sqrt{400}}$$
$$= 171.17 \pm 0.3232 = (170.84, 171.49)$$

Decision: $\bar{x}$ lies between 170.84 to 171.49
Accept H_0 at 5% level of significance.
Conclusion: Sample is from population with mean 171.17 & S.D. 3.3.

18. For a sample of 100 women

n = 100, $\bar{x}$ = 101, S = 42
Average diastolic BP = μ, μ = 75
Problem to test
$H_0 : \mu = 75$
$H_1 : \mu > 75$

Test statistics $Z = \dfrac{\bar{x} - \mu}{s/\sqrt{n}} = \dfrac{101 - 75}{\dfrac{42}{\sqrt{100}}} = \dfrac{26 \times 10}{42} = 6.19$

Critical value:
Let α = 5% then critical value is $Z_{tabulated} = Z_{\alpha} = 1.645$

Decision:
$Z = 6.19 > Z_{tabulated} = 1.645$
Reject H_0 at 5% level of significance
Conclusion:
Woman enrolled in program have diastolic BP exceeding 75.

19. Suppose that

$\mu = 13, \sigma = 0.7, n = 49, \bar{x} = 12.6$
Problem to test
$H_0 : \mu = 13$
$H_1 : \mu \neq 13$

Test statistics $Z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \dfrac{12.6 - 13}{\dfrac{0.7}{\sqrt{49}}} = -\dfrac{0.4 \times 7}{0.7} = -4$

Critical value
Let $\alpha = 5\%$ then critical value $Z_{tabulated} = Z_{\alpha/2} = 1.96$
Decision
$121 = 4 > Z_{tabulated} = 1.96$
Reject H_0 at 5% level of significance.
Conclusion: Sample from population with mean different from 13.

20. A new variety

$n = 250, \bar{x} = 82, s = 14.6, \mu = 80.2$
Problem to test: $H_0 : \mu = 80.2$
$H_1 : \mu > 80.2$

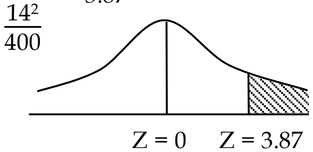
Test statistics $Z = \dfrac{\bar{x} - \mu}{s/\sqrt{n}} = \dfrac{82 - 80.2}{\dfrac{14.6}{\sqrt{250}}} = \dfrac{1.8 \times \sqrt{250}}{14.6} = 1.94$

Critical value
Let $\alpha = 5\%$ then critical value $Z_{tabulated} = Z_{\alpha} = 1.645$
Decision: $Z = 1.94 > Z_{tabulated} = 1.645$
Reject H_0 at 5% level of significance.
Conclusion: New variety is superior in yield.

21. In a certain factory

$\bar{x}_1 = 120, n_1 = 250, s_1 = 12, \bar{x}_2 = 124, n_2 = 400, s_2 = 14$
Let, μ_1 and μ_2 be mean weight from 1st process and 2nd process respectively.
Problem to test $H_0 : \mu_1 = \mu_2$
$H_1 : \mu_1 \neq \mu_2$

Test statistics $Z = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} = \dfrac{120 - 124}{\sqrt{\dfrac{12^2}{250} + \dfrac{14^2}{400}}} = -3.87$



$P = 2\,P(Z \geq Z_{cal})$
$= 2\,P(Z \geq 3.87)$
$= 2 \times (0.5 - P(0 \leq Z \leq 3.87)$
$= 2\,(0.5 - 0.49995) = 0.0001$
Decision
Here $P = 0.0001 < \alpha = 0.1$
Reject H_0 at 10% level of significance.
Conclusion: There is significant difference between mean weights.

22. The nicotine content
Sample size of brand A $(n_1) = 50$
Sample mean brand A $(\bar{x}_1) = 20.5$
Sample s.d. brand A $(S_1) = 2.5$
Sample size of brand B $(n_2) = 50$
Sample mean of brand B $(\bar{x}_2) = 17.5$
Sample s.d. brand B $(S_2) = 2.1$
$\alpha = 5\%$
Let μ_1 and μ_2 be population mean of brand A and brand B respectively.
Problem to test $H_0 : \mu_1 = \mu_2$
$H_1 : \mu_1 \neq \mu_2$

Test statistic $Z = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} = \dfrac{20.5 - 17.5}{\sqrt{\dfrac{2.5^2}{50} + \dfrac{2.1^2}{50}}} = \dfrac{3}{\sqrt{0.125 + 0.0882}} = 6.49$

Critical value
At $\alpha = 5\%$ critical value is $Z_{tabulated} = Z_{\alpha/2} = 1.96$
Decision: $Z = 6.49 > Z_{tabulated} = 1.96$
Reject H_0 at 5% level of significance.
Conclusion: Two brands A and B are different as nicotine context is considered.

23.
$n_1 = 300$
$\bar{x}_1 = 30$
$n_2 = 400$

$\bar{x}_2 = 28$

$\sigma_1 = \sigma_2 = 4$

Let μ_1 and μ_2 be population mean of first and second respectively problem to test

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$

Test statistic

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n^2}}}$$

$$= \frac{30 - 28}{\sqrt{\frac{4^2}{300} + \frac{4^2}{400}}}$$

$$= \frac{2}{\sqrt{0.0533 + 0.04}}$$

$$= \frac{2}{0.305} = 6.55$$

Critical value:

Let $\alpha = 5\%$ Then Critical value $Z_{\alpha/2} = 1.96$

$Z = 6.55 > Z_{\alpha/2} = 1.96$

Reject H_0 at $\alpha = 5\%$

Conclusion

Samples are not drawer from same population

24. A random sample

$n_1 = 35, \bar{x}_1 = 81, n_2 = 36, \bar{x}_2 = 76, \sigma_1 = 5.2 \ \ \sigma_2 = 3.4$

Let μ_1 and μ_2 be the average population

Problem to test:

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$

Test statistics $$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{81 - 76}{\sqrt{\frac{5.2^2}{35} + \frac{3.4^2}{36}}} = \frac{5}{\sqrt{0.772 + 0.321}} = 4.78$$

Critical value:

At $\alpha = 5\%$ critical values $Z_{\text{tabulated}} = Z_{\alpha/2} = 1.96$

Decision: $Z = 4.78 > Z_{\text{tabulated}} = 1.96$

Reject H_0 at 5% level of significance.

Conclusion: There is significant difference between two sample means.

25. Two research laboratory

$n_1 = 90, \bar{x}_1 = 8.5, S_1 = 1.8, n_2 = 80, \bar{x}_2 = 7.9, S_2 = 2.1, \alpha = 5\%$

Let μ_1 and μ_2 be average hrs of relief from first drug population and 2nd drugs of population respectively.

Problem to test $H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 > \mu_2$

Test statistic $$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{8.5 - 7.9}{\sqrt{\frac{1.8^2}{90} + \frac{2.1^2}{80}}} = \frac{0.6}{\sqrt{0.036 + 0.055}} = 1.99$$

Critical value: At $\alpha = 5\%$ critical value is $Z_{\text{tabulated}} = Z_\alpha = 1.645$

Decision: $Z = 1.99 > Z_{\text{tabulated}} = 1.645$

Reject H_0 at 5% level of significance.

Conclusion: The second drugs provide significantly shorter period of relief.

26. Two random samples of

$n_1 = 150, \bar{x}_1 = 800, S_1 = 50, n_2 = 100, \bar{x}_2 = 1250, S_2 = 30$

Let μ_1 and μ_2 be average monthly income of population from rural region and urban region respectively.

Problem to test $H_0 = \mu_1 = \mu_2$

$H_1 = \mu_1 < \mu_2$

Test statistic $$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} = \frac{800 - 1250}{\sqrt{\frac{50^2}{150} + \frac{30^2}{100}}} = \frac{-450}{\sqrt{16.666 + 9}} = -88.82$$

Critical value: Let $\alpha = 5\%$ then critical value is $Z_{\text{tabulated}} = Z_\alpha = 1.645$

Decision: $|Z| = 88.82 > Z_{\text{tabulated}} = 1.645$

Reject H_0 at 5% level of significance.

Conclusion: Average monthly income of rural people is significantly less than that of urban people.

27. An investigation of two

$n_1 = 71, \bar{x}_1 = 83.2, S_1 = 19.3, n_2 = 75, \bar{x}_2 = 90.8, S_2 = 21.4$

Let μ_1 and μ_2 be the population average time of repair of first kind of equipment and second kind of equipment respectively.

Problem to test $H_0 : \mu_1 = \mu_2$

$H_2 : \mu_1 \neq \mu_2$

Test statistic $Z = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{S_1^2}{n_1} + \dfrac{S_2^2}{n_2}}} = \dfrac{83.2 - 90.8}{\sqrt{\dfrac{19.3^2}{71} + \dfrac{21.4^2}{75}}} = -2.25$

Critical value

Let $\alpha = 5\%$ be level of significance then critical value is $Z_{tabulated} = Z_{\alpha} = 1.96$

Decision: $|Z| = 2.25 > Z_{tabulated} = 1.96$

Reject H_0 at 5% level of significance.

Conclusion: On average it takes different amount of time for repair.

Exercise 2 (II)

28. A dice was thrown

A dice was thrown 9000

No. of times dice thrown (n) = 9000

No. of times 3 or 4 yielded (x) = 3220

Sample proportion of 3 or 4 (p) = $\frac{x}{n} = \frac{3220}{9000} = 0.357$

Population proportion of 3 or 4 (P) = $\frac{2}{6} = \frac{1}{3} = 0.333$, Q = 0.666

Problem to test $H_0 : P = 0.333$

$H_1 : P \neq 0.333$

Test statistic: $Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.357 - 0.333}{\sqrt{\frac{0.333 \times 0.666}{9000}}} = \frac{0.0247}{0.00496} = 4.979$

Critical value

Let $\alpha = 5\%$ then critical value is $Z_{tabulated} = Z_{\alpha} = 1.96$

Decision: $Z = 4.979 > Z_{tabulated} = 1.96$

Reject H_0 at 5% level of significance.

Conclusion: The dice is biased.

29. A sample of size

Sample size (n) = 600,

Sample proportion of male (p) = 53% = 0.53

Population proportion of male (P) = $\frac{1}{2} = 0.5$, Q = 0.5

Problem to test $H_0 : P = \frac{1}{2}$

$H_1 : P \neq \frac{1}{2}$

Test statistic $Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.53 - 0.50}{\sqrt{\frac{0.5 \times 0.5}{600}}} = \frac{0.3}{0.0204} = 1.47$

Critical value: Let $\alpha = 5\%$ then critical value is $Z_{tabulated} = Z_{\alpha/2} = 1.96$

Decision: $Z = 1.47 < Z_{tabulated} = 1.96$

Accept H_0 at 5% level of significance.

Conclusion: The belief of population proportion of male is $\frac{1}{2}$ in confirmed by observation.

30. The coordinator in a college claimed

Population proportion of student

Submit assignment (P) = 98% = 0.98, Q = 0.02

Sample size (n) = 250, $\alpha = 10\%$

No. of students not submitting assignment = 15

No. of students submitting assignment (x) = 200 – 15 = 235

Sample proportion of students submitting assignment

(p) = $\frac{x}{n} = \frac{235}{250} = 0.94$

Problem to test $H_0 : P \geq 0.98$

$H_1 : P < 0.98$

Test statistic $Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.94 - 0.98}{\sqrt{\frac{0.98 \times 0.02}{250}}} = \frac{-0.04}{0.00885} = -4.519$

Critical value: Let $\alpha = 10\%$, Critical value is $Z_{tabulated} = 1.282$

Decision: $|Z| = 4.519 > Z_{tabulated} = 1.282$

Reject H_0 at 10% level of significance.

Conclusion: The claim of coordinator is not true.

31. To check on an

Emergency calls or ambulance service (P) = 40% = 0.4, Q = 0.6

Sample size (n) = 150,

No. of life threatening calls (x) = 49

Sample proportion of life threatening calls (p) = $\frac{x}{n} = \frac{49}{150} = 0.326$

$\alpha = 1\% = 0.01$

Problem to test $H_0 : P \geq 0.4$

$H_1 : P < 0.4$

Test statistic $Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.326 - 0.4}{\sqrt{\frac{0.4 \times 0.6}{150}}} = \frac{-0.0733}{0.04} = -1.832$

$P = \text{Prob.} (Z > |Z_{Cal}|)$

$= \text{Prob.} (Z > 1.83)$

$= 0.5 - P(0 < Z < 1.83)$

$= 0.5 - 0.4664 = 0.0336$

Decision: $P = 0.0336 > \alpha = 0.01$

Accept H_0 at α = 1% level of significance.

Conclusion: The claim of ambulance service is not correct.

32. It is claimed that

Population proportion of tea consumer $(P) = \frac{1}{2}$, $Q = \frac{1}{2}$

Sample size (n) = 600

No. of tea consumer (x) = 310

Sample proportion of tea consumer $(p) = \frac{x}{n} = \frac{310}{600} = 0.516$

$\alpha = 1\%$

Problem to test $H_0 : P = \frac{1}{2}$

$H_1 : P \neq \frac{1}{2}$

Test statistic $Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.516 - 0.5}{\sqrt{\frac{0.5 \times 0.5}{600}}} = \frac{0.016}{0.0204} = 0.78$

Critical value

At α = 1% critical value is $Z_{tabulated} = Z_{\alpha/2} = 2.576$

Decision

$Z = 0.78 < Z_{tabulated} = 2.576$

Accept H_0 at 1% level of significance.

Conclusion: Tea and coefficient are equally popular in Kathmandu.

33. In a certain process

Population proportion of defective (P) = 10% = 0.1, Q = 0.9

Sample (n) = 400, No. of defective units (x) = 34

Sample proportion of defective (p) $= \frac{x}{n} = \frac{34}{400} = 0.085$

$\alpha = 1\%$

Problem to test $H_0 = P = 0.1$

$H_1 = P < 0.1$

Test statistic $Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.085 - 0.1}{\sqrt{\frac{0.1 \times 0.9}{400}}} = -\frac{0.015 \times 20}{0.3} = -1$

Critical value: At $\alpha = 1\%$, critical value is $Z_{tabulated} = Z_\alpha = 2.326$

Decision:

$|Z| = 1 < Z_{tabulated} = 2.326$

Accept H_0 at 1% level of significance.

Conclusion: Supplier claim as accepted.

34. A machine produces

Sample size (n_1) = 400, Defective articles (x_1) = 20

Sample proportion of defective (p_1) $= \frac{x_1}{n_1} = \frac{20}{400} = 0.05$

After overhauled

Sample size (n_2) = 300, Defective articles (x_2) = 10

Sample proportion of defective (p_2) $= \frac{x_2}{n_2} = \frac{10}{300} = 0.033$

Defective before and after overhauled of machine

Problem to test $H_0 : P_1 = P_2$

$H_1 : P_1 > P_2$

Test statistics $Z = \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$

$P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{20 + 10}{400 + 300} = \frac{30}{700} = 0.042$

$Q = 1 - P = 0.958$

$\therefore \quad Z = \frac{0.05 - 0.033}{\sqrt{0.042 \times 0.958\left(\frac{1}{400} + \frac{1}{300}\right)}} = \frac{0.017}{0.0153} = 1.11$

Critical value

$Z = \alpha = 5\%$ then critical value is $Z_{tabulated} = Z_\alpha = 1.645$

Decision: $Z = 1.11 < Z_{tabulated} = 1.645$

Accept H_0 at 5% level of significance.

Conclusion: The machine is improved.

35. At a certain date

Sample size of men (n_1) = 500, Smoker (x_1) = 400, $\alpha = 5\%$

Sample proportion of smokers (p_1) $= \frac{x_1}{n_1} = \frac{400}{500} = 0.8$

After tax on tobacco increased.

Sample size of men (n_2) = 600

Smokers (x_2) = 400

Sample proportion of smokers (p_2) $= \frac{x_2}{n_2} = \frac{400}{600} = 0.666$

Let, P_1 and P_2 be the population proportion of smoker before and after tax increase.

Problem to test $H_0 : P_1 = P_2$

$H_1 : P_1 > P_2$

Test statistic $Z = \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$

Since $P = \frac{x_1 + x_2}{n_1 + n_2} = \frac{400 + 400}{500 + 600} = 0.727$

$Q = 1 - P = 0.273$

$Z = \frac{0.8 - 0.666}{\sqrt{0.727 \times 0.273\left(\frac{1}{500} + \frac{1}{600}\right)}} = \frac{0.134}{0.0269} = 4.98$

Critical value

At $\alpha = 5\%$ critical value is $Z_{tabulated} = Z_{\alpha} = 1.645$

Decision: $Z = 4.98 > Z_{tabulated} = 1.645$

Reject H_0 at $\alpha = 5\%$

Conclusion: Proportion of smokers decreased after increased tax.

36. In a random samples

Sample of men from a city $(n_1) = 600$

No. of literate men from a city $(x_1) = 400$

Sample proportion of literal men in a city $(p_1) = \frac{x_1}{n_1} = \frac{400}{600} = 0.66$

Sample of men from next city $(n_2) = 1000$

No. of literate men from next city $(x_2) = 600$

Sample proportion of literate men in next city $(p_2) = \frac{x_2}{n_2} = \frac{600}{1000} = 0.6$

Let P_1 and P_2 be the population proportion of literate in a city and next city respectively.

Problem to test $H_0 : P_1 = P_2$

$H_1 : P_1 \neq P_2$

$$P = \frac{x_1 + x_2}{n_1 + n_2} = \frac{400 + 600}{600 + 1000} = 0.625$$

$Q = 1 - P = 0.375$

$$\text{Test statistics } Z = \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$= \frac{0.66 - 0.6}{\sqrt{0.625 \times 0.375\left(\frac{1}{600} + \frac{1}{1000}\right)}} = \frac{0.06}{0.0249} = 2.409$$

Critical value

Let $\alpha = 5\%$ then critical value $Z_{tabulated} = Z_{\alpha/2} = 1.96$

Decision: $Z = 2.409 > Z_{tabulated} = 1.96$

Reject H_0 at $\alpha = 5\%$.

Conclusion: There is no significant difference in percentage of literacy.

37. Random samples of

Sample bolts of machine $A(n_1) = 250$

Defective bolts of machine $A(x_1) = 24$

Sample bolts machine $B(n_2) = 200$

Defective bolts machine $B(x_2) = 10$

Sample proportion of defective bolt by machine A $(p_1) = \frac{x_1}{n_1} = \frac{24}{250} = 0.096$

Sample proportion of defective bolt by machine B $(p_2) = \frac{x_2}{n_2} = \frac{10}{200} = 0.05$

Let P_1 and P_2 be population proportion of defective both by machine A and B respectively.

$$P = \frac{x_1 + x_2}{n_1 + n_2} = \frac{24 + 10}{250 + 200} = 0.0755, \qquad Q = 1 - P = 0.9245$$

Problem to test $H_0 : P_1 = P_2$

$H_1 : P_1 = P_2$

$$\text{Test statistic } Z = \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$= \frac{0.096 - 0.05}{\sqrt{0.0755 \times 0.9245\left(\frac{1}{250} + \frac{1}{200}\right)}}$$

$$= \frac{0.046}{0.025} = 1.84$$

Critical value

At $\alpha = 5\%$ critical value is $Z_{tabulated} = Z_{\alpha/2} = 1.96$

Decision

$Z = 1.84 < Z_{tabulated} = 1.96$

Accept H_0 at $\alpha = 5\%$

Conclusion: Machine A and B show same quality of performance.

38. In a sample of

No. of sample students in a college (n_1) = 600

No. of students use dot pen (x_1) = 400

Sample proportion of student use dot pen (p_1) $= \frac{x_1}{n_1} = \frac{400}{600} = 0.666$

No. of sample student in another college (n_2) = 900

Use dot pen (x_2) = 450

Sample proportion of student use dot pen (p_2) $= \frac{x_2}{n_2} = \frac{450}{500} = 0.5$

Let P_1 and P_2 be the population proportion of student use dot pen in a college and another college respectively.

Problem to test $H_0 : P_1 = P_2$

$H_1 : P_1 \neq P_2$

$$P = \frac{x_1 + x_2}{n_1 + n_2} = \frac{400 + 450}{600 + 900} = 0.566, \quad Q = 1 - P = 0.434$$

Test statistic $Z = \dfrac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$

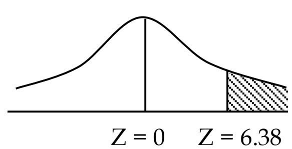
$$= \frac{0.666 - 0.5}{\sqrt{0.566 \times 0.434\left(\frac{1}{600} + \frac{1}{900}\right)}}$$

$$= \frac{0.166}{0.026} = 6.38$$

P = 2 Prob. $(Z \geq Z_{cal})$

$= 2\,P(Z \geq 6.38)$

$= 2 \times 0 = 0$



Critical value $\alpha = 1\% = 0.01$

Decision: $P = 0 < \alpha = 0.01$

Reject H_0 at 1% level of significance.

Exercise 2

39. Rainfall records of a

$n = 12, \bar{x} = 50, S = 30, \mu = 512, \alpha = 10\%$

Problem to test $H_0 : \mu = 512$

$H_1 : \mu < 512$

Test statistic $t = \dfrac{\bar{x} - \mu}{\dfrac{S}{\sqrt{n-1}}} = \dfrac{50 - 512}{\dfrac{30}{\sqrt{12-1}}} = -51.07$

Critical value

At $\alpha = 10\%$, critical value is t tabulated $= t_{\alpha\,(n-1)} = 1.363$

Decision

$|t| = 51.07 > t_{tabulated} = 1.363$

Reject H_0 at 10% level of significance.

Conclusion: Average rainfall of the place was less than 512 mm.

40. Ten patients are selected at

B.P.(x)	d = x – 147	d^2
125	– 22	484
147	0	0
118	– 29	841
145	– 2	4
140	– 7	49
128	– 19	361
155	8	64
150	3	9
160	13	169
149	2	4
	$\sum d = -53$	$\sum d^2 = 1985$

Population average BP(μ) = 135, $\alpha = 5\%$

Problem to test $H_0 : \mu = 135$

$H_1 : \mu \neq 135$

$\bar{x} = A + \dfrac{\sum d}{n} = 147 + \dfrac{-53}{10} = 141.7$

$S^2 = \dfrac{1}{n-1}\sum(d - \bar{d})$

$= \dfrac{1}{n-1}\left[\sum d^2 - n\bar{d}^2\right] = \dfrac{1}{10-1}\left[1985 - 10 \times \left(\dfrac{-53}{10}\right)^2\right] = 189.34$

$S = 13.76$

Test statistic $t = \dfrac{\bar{x} - \mu}{S/\sqrt{n}} = \dfrac{141.7 - 135}{\dfrac{13.76}{\sqrt{10}}} = 1.53$

Critical value

At $\alpha = 5\%$, critical value is t tabulated $= t_{\alpha/2\,(n-1)} = 2.262$

Decision: $t = 1.53 < t_{tabulated} = 2.262$

Accept H_0 at $\alpha = 5\%$

Conclusion: Average B.P. of patients is 135.

41. A fertilizer mixing

$n = 100, \mu = 12, n = 10$

Nitrate (x)	11	14	13	12	13	12	13	14	11	12	$\sum x = 125$
x^2	121	196	169	144	169	144	169	196	121	144	$\sum x^2 = 1573$

$\bar{x} = \dfrac{\sum x}{n} = \dfrac{125}{10} = 12.5$

$S^2 = \dfrac{1}{n-1}\sum(x - \bar{x})^2$

$= \dfrac{1}{10-1}[\sum x^2 - n\,\bar{x}^2]$

$= \dfrac{1}{9}\,[1573 - 10 \times (12.5)^2] = 1.166$

$\therefore$ $S = 1.08$

Problem to test $H_0 : \mu = 12$

$H_1 : \mu \neq 12$

Let $\alpha = 5\%$ then 95%

Confidence limit for $\mu = \bar{x} \pm t_{\alpha/2\,(n-1)} \dfrac{S}{\sqrt{n}}$

$$= 12.5 \pm 2.262 \times \frac{1.08}{\sqrt{10}}$$

$$= 12.5 \pm 0.77 = (11.72, 13.27)$$

Decision: $\mu = 12$ lies in 95% confidence interval.

Accept H_0 at 5% level of significance.

Conclusion: Machine is not defective.

42 A random sample of

$n = 16, \bar{x} = 53, \sum(x - \bar{x})^2 = 150, \mu = 56, (100 - \alpha)\% = 99\%, \alpha = 1\%$

$S^2 = \dfrac{1}{n-1}\sum(x - \bar{x})^2 = \dfrac{1}{16-1} \times 150 = 10$

$\therefore S = 3.162$

Problem to test $H_0 : \mu = 56$

$H_1 : \mu \neq 56$

Test statistics $t = \dfrac{\bar{x} - \mu}{\frac{S}{\sqrt{n}}} = \dfrac{53 - 56}{\frac{3.162}{\sqrt{16}}} = -3.79$

Critical value

At $\alpha = 1\%$ critical value is $t_{tabulated} = t_{\alpha/2\,(n-1)} = 2.947$

Decision

$|t| = 3.79 > t_{tabulated} = 2.947$

Reject H_0 at 1% level of significance.

Conclusion: Sample is not from population having mean 56.

43. In the past

$\mu = 0.05, n = 10, \bar{x} = 0.053, s = 0.003$

Problem to test $H_0 : \mu = 0.05$

$H_1 : \mu \neq 0.05$

Test statistic $t = \dfrac{\bar{x} - \mu}{\frac{s}{\sqrt{n-1}}} = \dfrac{0.053 - 0.05}{\frac{0.003}{\sqrt{9}}} = 3$

Critical value: Let $\alpha = 5\%$ then critical values $t_{tabulated} = t_{\alpha/2\,(n-1)} = 2.262$

Decision: $t = 3 > t_{tabulated} = 2.262$

Reject H_0 at 5% level of significance.

Conclusion: Machine is not working in proper order.

44. The time

$n = 10, \alpha = 5\%, \mu = 30$

Time spent (x)	x^2
35	1225
20	400
30	900
45	2025
60	3600
40	1600
65	4225
40	1600
25	625
50	2500
$\sum x = 410$	$\sum x^2 = 18700$

$\bar{x} = \dfrac{\sum x}{n} = \dfrac{410}{10} = 41$

$S^2 = \dfrac{1}{n}[\sum x^2 - n\bar{x}^2] = \dfrac{1}{10-1}[18700 - 10 \times 41^2] = 210$

$S = 14.49$

Problem to test $H_0 : \mu = 30$

$H_1 : \mu > 30$

Test statistic $t = \frac{\bar{x} - \mu}{\frac{S}{\sqrt{n}}} = \frac{41 - 30}{\frac{14.49}{\sqrt{10}}} = 2.4$

Critical value: At $\alpha = 5\%$, critical value is t tabulated = $t_{\alpha(n-1)} = 1.833$

Decision: $t = 2.4 > t_{tabulated} = 1.833$

Reject H_0 at 5% level of significance.

Conclusion: Average time spent by customers is more than 30 min.

45. A random sample of

$n = 10$, confidence level $(100 - \alpha)\% = 95\%$, $\alpha = 5\%$

Let population mean life of bulb = μ

Life (x)	d = x – 28	d^2
24	– 4	16
26	– 2	4
32	4	16
28	0	0
20	– 8	64
20	- 8	64
23	– 5	25
34	6	36
30	2	4
43`	15	225
	$\sum d = 0$	$\sum d^2 = 454$

95% confidence limit for $\mu = \bar{x} \pm t_{\alpha/2\,(n-1)} \dfrac{S}{\sqrt{n}}$

$$= 28 \pm 2.262 \times \frac{7.1}{\sqrt{10}}$$

$$= 28 \pm 5.078$$

$$= (22.9, 33.07)$$

$$\bar{x} = A + \frac{\sum d}{n} = 28 + \frac{0}{10} = 28$$

$$S^2 = \frac{1}{n-1}[\sum d^2 - n\bar{d}^2] = \frac{1}{10-1}\left[454 - 10 \times \left(\frac{0}{10}\right)^2\right] = \frac{1}{9} \times 454 = 50.44$$

$\therefore S = 7.1$

At $\alpha = 5\%$ critical value $t_{\alpha/2\,(n-1)} = 2.262$

46. A random sample of size

$n = 10$, $\bar{x} = 28$, $s = 7.1$

Let μ = Population mean life, $\alpha = 5\%$

Confidence limit for $\mu = \bar{x} \pm t_{\alpha/2\,(n-1)} \dfrac{s}{\sqrt{n-1}}$

$$= 28 \pm 2.262 \times \frac{7.1}{\sqrt{10-1}}$$

$$= 28 \pm 5.353 = (22.64, 33.35)$$

47. The average number

$\bar{x}_1 = 200$, $\bar{x}_2 = 250$, $s_1 = 20$, $s_2 = 25$, $n_1 = n_2 = 25$, $\alpha = 1\%$

Let μ_1 and μ_2 be population average by machine first and second respectively.

$$S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2} = \frac{25 \times 20^2 + 25 \times 25^2}{25 + 25 - 2} = 533.85$$

Problem to test $H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$

$$\text{Test statistic } t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{S^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{200 - 250}{\sqrt{533.85\left(\frac{1}{25} + \frac{1}{25}\right)}} = -7.65$$

Decision: $|t| = 7.65 > t_{tabulated} = 2.704$

Reject H_0 at 1% level of significance.

Conclusion: Both machines are not equally effective.

48. Two kinds of

Manure I (x_1)	Manure II (x_2)	x_1^2	x_2^2
18	29	324	841
20	28	400	784
36	26	1296	676
50	35	2500	1225
49	30	2401	900
36	44	1296	1936
34	46	1156	2116
49		2401	
41		1681	
$\sum x_1 = 333$	$\sum x_2 = 238$	$\sum x_1^2 = 13455$	$\sum x_2^2 = 8478$

$$\bar{x}_1 = \frac{\sum x_1}{n_1} = \frac{333}{9} = 37$$

$s_1^2 = \frac{1}{n_1}\sum(x_1 - \bar{x}_1)^2 = \frac{1}{n_1}[\sum x_1^2 - n_1 \bar{x}_1^2] = \frac{1}{9}[13455 - 9 \times 37^2] = 126$

$\bar{x}_2 = \frac{\sum x_2}{n_2} = \frac{238}{7} = 34$

$s_2^2 = \frac{1}{n_2}\sum(x_2 - \bar{x}_2)^2 = \frac{1}{n_2}[\sum x_2^2 - n_2 \bar{x}_2] = \frac{1}{7}[8478 - 7 \times 34^2] = 55.14$

$S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2} = \frac{9 \times 126 + 7 \times 55.14}{9 + 7 - 2} = 108.57$

$\alpha = 5\%$

Let μ_1 and μ_2 be population mean yield due to use of manure I and II respectively.

Problem to test $H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$

Test statistic $t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{S^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{37 - 34}{\sqrt{108.57\left(\frac{1}{9} + \frac{1}{7}\right)}} = 0.571$

Critical value

At $\alpha = 5\%$, Critical value is $t_{tabulated} = t_{\alpha/2\,(n_1 + n_2 - 2)} = 2.145$

Decision:

$t = 0.571 < t_{tabulated} = 2.145$

Accept H_0 at 5% level of significance.

Conclusion: There is no significant difference between mean yields on using manure I and II.

49. To test the effect of

$n_1 = n_2 = 12$

$\bar{x}_1 = 4.8$, $s_1 = 0.4$, $\bar{x}_2 = 5.1$, $s_2 = 0.36$, $\alpha = 5\%$

Let μ_1 and μ_2 be population mean yield with untreated and treated.

$S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2} = \frac{12 \times 0.4^2 + 12 \times 0.36^2}{12 + 12 - 2} = 0.157$

Problem to test $H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 < \mu_2$

Test statistic: $t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{S^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{4.8 - 5.1}{\sqrt{0.157\left(\frac{1}{12} + \frac{1}{12}\right)}}$

$= \frac{-0.3}{0.161} = -1.863$

Critical value: At $\alpha = 5\%$, critical value $t_{tabulated} = t_{\alpha(n_1+n_2-2)} = 1.71$

Decision: $|t| = 1.86 > t_{tabulated} = 1.717$

Reject H_0 at 5% level of significance.

Conclusion: There is significant improvement in rice production because of fertilizer.

50. Two new drugs A and B

Let μ_1 and μ_2 be population average of reduced in BP due to drugs A and B respectively.

Drug A (x_1)	Drug B (x_2)	x_1^2	x_2^2
7	10	49	100
16	12	256	144
14	16	196	256
9	14	81	196
10	11	100	121
11	12	121	144
6	13	36	169
8	8	64	64
10	12	100	144
9	15	81	225
	5		81
	12		144
$\sum x_1 = 100$	$\sum x_2 = 144$	$\sum x_1^2 = 1084$	$\sum x_2^2 = 1788$

$\bar{x}_1 = \frac{\sum x_1}{n_1} = \frac{100}{10} = 10$, $\bar{x}_2 = \frac{\sum x_2}{n_2} = \frac{144}{12} = 12$

$$s_1^2 = \frac{1}{n_1}\left[\sum x_1^2 - n\,\bar{x}_1^2\right] = \frac{1}{10}[1084 - 10 \times 10^2] = 8.4$$

$$s_2^2 = \frac{1}{n_2}\left[\sum x_2^2 - n\,\bar{x}_2\right] = \frac{1}{12}[1788 - 12 \times 12^2] = 5.0$$

$$S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2} = \frac{10 \times 8.4 + 12 \times 5}{10 + 12 - 2} = 7.2$$

Problem to test $H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 < \mu_2$

Test statistic $$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{S^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{10 - 12}{\sqrt{7.2\left(\frac{1}{10} + \frac{1}{12}\right)}} = -1.75$$

Critical value

At $\alpha = 5\%$ critical value $t_{tabulated} = t_{\alpha(n_1 + n_2 - 2)} = 1.725$

Decision:

$|t| = 1.75 > t_{tabulated} = 1.725$. Reject H_0 at $\alpha = 5\%$

Conclusion: Drug A is less effective than drug B in reducing BP.

51. The means of two random

$n_1 = 9$, $n_2 = 7$, $\bar{x}_1 = 196.42$, $\bar{x}_2 = 198.82$,

$\sum(x_1 - \bar{x}_1)^2 = 26.94$, $\sum(x_2 - \bar{x}_2)^2 = 18.73$

Let μ_1 and μ_2 be population mean of 1st and 2nd population respectively.

$$s_1^2 = \frac{1}{n_1}\sum(x_1 - \bar{x}_1)^2 = \frac{26.94}{9} = 2.993$$

$$s_2^2 = \frac{1}{n_2}\sum(x_2 - \bar{x}_2)^2 = \frac{18.73}{7} = 2.675$$

$$S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2} = \frac{9 \times 2.993 + 7 \times 2.675}{9 + 7 - 2} = 3.261$$

Problem to test $H_0 : \mu_1 = \mu_2$

$H_2 : \mu_1 \neq \mu_2$

Test statistics $$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{S^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{196.42 - 198.82}{\sqrt{3.261\left(\frac{1}{9} + \frac{1}{7}\right)}} = \frac{-2.4}{0.91} = -2.63$$

Critical value

Let $\alpha = 5\%$ then critical value is $t_{tabulated} = t_{\alpha/2\,(n_1 + n_2 - 2)} = 2.145$

Decision: $|t| = 2.63 > t_{tabulated} = 2.145$

Reject H_0 at $\alpha = 5\%$

Conclusion: Samples are drawn from different normal population.

52. A group of five

Let μ_1 and μ_2 be population average weight due to medicine A and B respectively.

Medicine A (x_1)	Medicine B (x_2)	x_1^2	x_2^2
42	38	1764	1444
39	42	1521	1764
48	56	2304	3136
60	64	3600	4096
41	68	1681	4624
	69		4761
	62		3844
$\sum x_1 = 230$	$\sum x_2 = 399$	$\sum x_1^2 = 10870$	$\sum x_2^2 = 23669$

$\bar{x}_1 = \frac{\sum x_1}{n_1} = \frac{230}{5} = 46, \quad \bar{x}_2 = \frac{\sum x_2}{n_2} = \frac{399}{7} = 57$

$s_1^2 = \frac{1}{n_1}[\sum x_1^2 - n_1 \bar{x}_1^2] = \frac{1}{5}[10870 - 5 \times 46^2] = 58$

$s_2^2 = \frac{1}{n_2}[\sum x_2^2 - n_2 \bar{x}_2^2] = \frac{1}{7}[23669 - 7 \times 57^2] = 132.28$

$S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2} = \frac{5 \times 58 + 7 \times 132.28}{5 + 7 - 2} = 121.59$

Problem to test $H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 < \mu_2$

Test statistic $\quad t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{S^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{46 - 57}{\sqrt{121.59\left(\frac{1}{5} + \frac{1}{7}\right)}} = -\frac{11}{656} = -1.7$

Critical value

Let $\alpha = 5\%$ then critical value is $t_{tabulated} = t_{\alpha/2\,(n_1 + n_2 - 2)} = 1.812$

Decision: $|t| = 1.7 < t_{tabulated} = 1.812$. Accept H_0 at $\alpha = 5\%$.

Conclusion: There is no significant difference in weight due to medicine A and B.

53. A drug was given

Increment in BP (d)	d^2
8	64
10	100
– 2	4
0	0
5	25
– 1	1
9	81
12	144
6	36
5	25
$\sum d = 52$	$\sum d^2 = 480$

$\bar{d} = \frac{\sum d}{n} = \frac{52}{10} = 5.2$

$S_d^2 = \frac{1}{n-1}[\sum d^2 - n\,\bar{d}^2] = \frac{1}{10-1}[480 - 10 \times 5.2^2] = 23.28$

$S_d = 4.82$

Problem to test $H_0 : \mu_d = 0$

$H_1 : \mu_d > 0$

Test statistic $t = \frac{\bar{d}}{\frac{S_d}{\sqrt{n}}} = \frac{5.2}{\frac{4.82}{\sqrt{10}}} = 3.411$

Critical value

Let α = 5% then critical value $t_{tabulated} = t_{\alpha/2\,(n_1+n_2-2)} = 1.833$.

Decision

$t = 3.411 > t_{tabulated} = 1.833$. Reject H_0 at $\alpha = 5\%$.

Conclusion: Drug has increased B. P.

54. The following data gave

Let μ_1 and μ_2 be mean yield in first year and second year then $\mu_d = \mu_1 - \mu_2$, $\alpha = 5\%$

Serial no.	Yield in 1st year (x)	Yield in 2nd year (y)	d = x – y	d^2
1	23	24	– 1	1
2	20	19	1	1
3	19	22	– 3	9
4	21	18	3	9
5	18	20	– 2	4
6	20	22	– 2	4
7	18	20	– 2	4
8	22	20	2	4
9	16	18	– 2	4
10	18	17	1	1
			$\sum d = -5$	$\sum d^2 = 41$

$\bar{d} = \frac{\sum d}{n} = \frac{-5}{10} = -0.5$

$S_d^2 = \frac{1}{n-1}[\sum d^2 - n\,\bar{d}^2] = \frac{1}{10-1}[41 - 10 \times (-0.5)^2] = 4.27$

$\therefore S_d = 2.06$

Problem to test $H_0 : \mu_d = 0$

$H_1 : \mu_d \neq 0$

Test statistic $t = \frac{\bar{d}}{\frac{S_d}{\sqrt{n}}} = \frac{-0.5}{\frac{2.06}{\sqrt{10}}} = -0.76$

Critical value

At α = 5% critical value in $t_{tabulated} = t_{\alpha/2\,(n_1+n_2-2)} = 2.262$

Decision: $|t| = 0.70 < t_{tabulated} = 2.262$

Accept H_0 at 5% level of significance.

Conclusion: There is no significant difference in mean yields of two successive years.

55. Ten students

Let μ_1 and μ_2 be mean scores of population in I and V respectively.

$\mu_d = \mu_1 - \mu_2$, $\alpha = 5\%$

S. No.	Marks in 1st test (x)	Marks in 2nd test (y)	d = x – y	d^2
1	50	65	– 15	225
2	52	55	– 3	9
3	53	65	– 12	144
4	60	65	– 5	25
5	65	60	5	25
6	67	67	0	0
7	48	49	– 1	1
8	69	82	– 13	169
9	72	74	– 2	4
10	80	86	– 6	36
			$\sum d = -52$	$\sum d^2 = 638$

$\bar{d} = \frac{\sum d}{n} = \frac{-52}{10} = -5.2$

$S_d^2 = \frac{1}{n-1}[\sum d^2 - n\,\bar{d}^2] = \frac{1}{10-1}[638 - 10 \times (-5.2)^2] = 40.844$

$S_d = 6.39$

Problem to test $H_0 : \mu_d = 0$

$H_1 : \mu_d < 0$

Test statistic $t = \frac{\bar{d}}{\frac{S_d}{\sqrt{n}}} = \frac{-5.2}{\frac{6.39}{\sqrt{10}}} = -2.57$

Critical value

At $\alpha = 5\%$ critical value is $t_{tabulated} = t_{\alpha/2\,(n_1 + n_2 - 2)} = 1.833$

Decision: $|t| = 2.57 > t_{tabulated} = 1.833$

Reject H_0 at $\alpha = 5\%$

Conclusion: There is improvement from test I to test V.

56. Memory capacity of

Let μ_1 and μ_2 be mean memory capacity before and after training $\mu_1 - \mu_2 = \mu_d$.

Roll no.	Before training (x)	After training (y)	d = x – y	d^2
1	12	15	– 3	9
2	14	16	– 2	4
3	11	10	1	1
4	8	7	1	1
5	7	5	2	4
6	10	12	– 2	4
7	3	10	– 7	49
8	0	2	– 2	4
9	5	3	2	4
10	6	8	– 2	4
			$\sum d = -12$	$\sum d^2 = 84$

$$\bar{d} = \frac{\sum d}{n} = \frac{-12}{10} = -1.2$$

$$S_d^2 = \frac{1}{n-1}[\sum d^2 - n\bar{d}^2] = \frac{1}{10-1}[84 - 10 \times (-1.2)^2] = 7.73$$

$S_d = 2.78$

Problem to test $H_0 : \mu_d = 0$

$H_1 : \mu_d < 0$

Test statistic $\quad t = \dfrac{\bar{d}}{S_d/\sqrt{n}} = \dfrac{-1.2}{\frac{2.78}{\sqrt{10}}} = -1.36$

Critical value

Let $\alpha = 5\%$ then critical value is $t_{tabulated} = t_{\alpha/2\,(n_1 + n_2 - 2)} = 1.833$

Decision

$|t| = 1.36 < t_{tabulated} = 1.833$. Accept H_0 at $\alpha = 5\%$.

Conclusion: Training was not effective.

Excises – 3(I)

9. In 30 toss of a coin

Here, no. of H (n_1) = 16, no of T (n_2)= 14, r = 22, α = 0.05

Problem to test H_0 : The sequence is in random order

H_1 : The sequence is not in random order

Test statistic r= 23

Critical value

At α = 5% n_1 = 16 and n_2 = 14, critical value in $\underline{r}$ = 10, $\bar{r}$ = 22

Decision

$r = 23 \notin (\underline{r} = 10, \bar{r} = 22)$

Reject H_0 at 5% level of significance.

Conclusion: The sequence is not in random order.

10. A random samples

n = 15, Number of N (n_1) = 10, Number of Y (n_2) = 5

Number of run (r) = 7

Problem to test H_0 : The sequence is in random order.

H_1 : The sequence is not in random order.

Test statistic r = 7

Critical value

Let α = 5% then critical value for x_1 = 10 and x_2 = 7 is $\underline{r}$ = 5, $\bar{r}$ = 14

Decision

$r = 7 \in (\underline{r} = 5, \bar{r} = 14)$

Accept H_0 at 5% level of significance.

Conclusion: The sequence is in random order.

11. In Production Line

No. of D(n_1) = 11 No. of N (n_2) = 17

No. of run (r) = 13, α = 0.05

Problem to test H_0 : The sequence is in random order

H_1 : The sequence is not in random order.

Test statistic r = 13

Critical value

At α = 0.05, n_1 = 11 and n_2 = 17, critical value is ($\underline{r}$ = 9, $\bar{r}$ = 20)

Decision

$r = 13 \in (\underline{r} = 9, \bar{r} = 20)$

accept H_0 at 5% level of significance.

Conclusion: The sequence is in random order.

12. The following are

n = 24, α = 0.01

Arranging data in ascending order

25, 26, 27, 28, 28, 28, 28, 29, 29, 30, 30, 30, 31, 31, 31, 31, 32, 33, 33, 33, 35, 35, 36, 38

$$M_d = \left(\frac{n+1}{2}\right)^{th} \text{ item}$$

$$= \frac{24+1}{2} = 12.5 = \frac{12^{th} \text{ item} + 13^{th} \text{ item}}{2} = \frac{30+31}{2} = 30.5$$

Assign A for no greater than 30.5

Assign B for no less than 30.5 in given sequence.

B B A B B B A A A B A A A B A B A B B B A A A B

Number of B (n_1) = 12 Number of A (n_2) = 12

Number of run (r) = 13

$$\mu_r = \frac{2\, n_1\, n_2}{n_1 + n_2} + 1 = \frac{2 \times 12 \times 12}{12 + 12} + 1 = 13$$

$$\sigma_r^2 = \frac{2\, n_1\, n_2\, (2n_1\, n_2 - n_1 - n_2)}{(n_1 + n_2)^2\, (n_1 + n_2 - 1)} = \frac{2 \times 12 \times 12\, (2 \times 12 \times 12 - 12 - 12)}{(12 + 12)^2\, (12 + 12 - 1)} = 5.73$$

σ_r = 2.39

Problem to test

H_0 : No. of students absent in consecutive days is in random order

H_1 : No. of students absent in consecutive days is not in random order.

Test statistic $Z = \frac{r - \mu_r}{\sigma_r} = \frac{13 - 13}{2.39} = 0$

Critical value

At $\alpha = 0.01$, critical value is $Z_{tabulated} = Z_{\alpha/2} = 2.576$

Decision

$Z = 0 < Z_{tabulated} = 2.576$

Accept H_0 at $\alpha = 0.01$

Conclusion:

No. of students is absent in consecutive days are in random order.

13. The height of

$n = 15, \alpha = 1\%$

Arranging data in ascending order

63.6, 64, 64.4, 65.3, 66.8, 67, 67.6, 67.7, 68.4, 69, 69.8, 70.2, 70.4, 71, 71.5

$M_d = \left(\frac{n+1}{2}\right)^{th}$ item $= \frac{15+1}{2} = 8^{th}$ item $= 67.7$

Assign A for values greater than 67.7.

Assign B for values less than 67.7 for given sequence.

B A A A B A A B B A B B B A A

Number of B $(n_1) = 7$, Number of A $(n_2) = 7$, Number of run $(r) = 8$

$\mu_r = \frac{2 n_1 n_2}{n_1 n_2} + 1 = \frac{2 \times 7 \times 7}{7 + 7} + 1 = 8$

$\sigma_r^2 = \frac{2 n_1 n_2 (2 n_1 n_2 - n_1 - n_2)}{(n_1 + n_2)^2 (n_1 + n_2 - 1)} = \frac{2 \times 7 \times 7 (2 \times 7 \times 7 - 7 - 7)}{(7 + 7)^2 (7 + 7 - 1)} = 3.23$

$\sigma_r = 1.79$

Problem to test H_0: Order of height is in random

H_1: Order of height is not in random.

Test statistic $Z = \frac{r - \mu_r}{\sigma_r} = \frac{8 - 8}{1.79} = 0$

Critical value

At $\alpha = 1\%$ critical value is $Z_{tabulated} = Z_{\alpha/2} = 2.567$

Decision

$Z = 0 < Z_{tabulated} = 2.567$

Accept H_0 at 1% level of significance.

Conclusion: Order of height of football player is random.

14. The following are

$n_1 (D) = 11$

$n_2(N) = 14$

Problem H_0 test

$H_0: P = \frac{1}{2}$

$H_1: P \neq \frac{1}{2}$

$= \min \{n_1, n_2\} = \min n \{11, 14\}$

$= 11$

$\alpha = 5\%$

$p = \sum_{x=0}^{x_0} (n, x) \left(\frac{1}{2}\right)^x$

$= \sum_{x=0}^{11} (n, x) \left(\frac{1}{2}\right)^x$

$= 0.345$

Decision

$2p = 2 \times 0.345$

$= 0.69 > \alpha = 0.05$

Accept H_0 at $\alpha = 5\%$

Conclusion

Defective and non defective are equally produced.

15. Hundred Workers

$n = 100$

$n_1 (N) = 40$

$n_2 (0) = 60$

Problem to H_0 test

$H_0 : P = \frac{1}{2}$

$H_1: P \neq \frac{1}{2}$

$x_0 = \min(n_1, n_2)$

$= \min(40, 60)$

$= 40$

$np = 100 \times \frac{1}{2} = 50$

$$Z = \frac{(x_0 + 0.5) - np}{\sqrt{npq}}$$

$$= \frac{(40 + 0.5) - 50}{\sqrt{100 \times \frac{1}{2} \times \frac{1}{2}}}$$

$$= \frac{-9.5}{5}$$

$= -1.9$

$|Z| = 1.9$

At $\alpha = 5\%$ critical value

$Z_{\alpha/2} = 1.96$

Decision

$|Z| = 1.9 < Z_{\alpha/2} = 1.96$

Accept H_0 at $\alpha = 5\%$

Conclusion

Both brand of cell phone are equally popular

16. In a certain computer

Machine	No of wire (fo)	cf_0	fe	cfe	F_0	F_e	$\|Fe - F_0\|$
1	2	2	5	5	2/30	5/30	3/30
2	0	2	5	10	2/30	10/30	8/30
3	4	6	5	15	6/30	15/30	9/30
4	8	14	5	20	14/30	20/30	6/30
5	5	19	5	25	19/30	25/30	6/30
6	11	30	5	30	30/30	30/30	0/30
	N = 30						

Expected frequency for each class $= \frac{\text{Sum of frequncy}}{\text{No. of class}}$

$= \frac{30}{6} = 5$

Problem to test

H_0 : machines equally cut wire

H_1 : machine do not equally cut wire

Test statistic $D_0 = \max |Fe - Fo|$

$= \frac{9}{30} = 0.3$

Critical value

At $\alpha = 5\%$ $D_{n,\alpha} = D_{30,0.05} = 0.242$

Decision

$D_0 = 0.3 > D_{n,\alpha} = 0.242$

Reject H_0 at $\alpha = 5\%$

Conclusion

Machines do not equally cut wire.

17 The numbers of virus

Capacity of hard disk (GB)	No of virus infected (fo)	cf_0	fe	cfe	F_0	F_e	$\|Fe - F_0\|$
500	11	11	11/50	10	10	10/50	1/50
320	15	26	26/50	10	20	20/50	6/50
1000	20	46	46/50	10	30	30/50	16/50
2000	20	46	46/50	10	30	30/50	16/50
4000	1	50	50/50	10	50	50/50	0/50

	N = 50						

Expected frequency for each class = $\frac{\text{Sum of frequency}}{\text{No. of class}}$

$= \frac{50}{5} = 10$

Problem to test

H_0 : Hard disk are uniformly infected

H_1 : Hard disk are not uniformly infected

Test statistic

$D_0 = \text{Max}\{|Fe - Fo|\} = 16/50 = 0.32$

Critical value

Let $\alpha = 5\%$

Then $D_{n,\alpha} = D_{50,\,0.05} = \frac{1.36}{\sqrt{n}}$

$= \frac{1.36}{\sqrt{50}}$

$= 0.192$

Decision

$D_0 = 0.32 > D_{n,\alpha} = 0.192$

Reject H_0 at $\alpha = 5\%$

Conclusion

Hard disks are not uniformly infected.

18. A game

Matched set	Frequency (fo)	cf_0	fe	cfe	F_0	F_e	$\lvert Fe - F_0\rvert$
0	1	1	1/20	4	4	4/20	3/20
1	0	1	1/20	4	8	8/20	7/20
2	5	6	6/20	4	12	12/30	6/20
3	7	13	13/20	4	16	16/20	3/20
4	7	20	20/20	4	20	20/20	0/20

Expected frequency for each class = $\frac{\text{sum of frequency}}{\text{No. of class}}$

$= \frac{20}{4} = 4$

Problem to test

H_0: chimpanzees recognize colors uniformly

H_1: Chimpanzees do not recognize colors uniformly

Test statistic

Test statistic

$D_0 = \text{Max} \times \{|Fe - Fo|\} = \frac{7}{20} = 0.35$

Critical value

A + α = 5% critical value

$D_{n,\alpha} = D_{(20,0.05)} = 0.294$

Decision

$D_0 = 0.35 > D_{n,\alpha} = 0.294$

Reject H_0 at a = 5%

Conclusion

Chimpanzees do not recognize colors uniformly.

Ex 3 (iii) Statistics

7. The following are

To find Md of combined group

Arrange data in ascending

30, 32, 34, 39, 40, 42, 43, 44, 45, 46, 47, 47, 48, 49, 50, 51, 55, 55, 59, 71

$n = 20$

$$Md = \left(\frac{n+1}{2}\right)^{th} \text{ item}$$

$$= \frac{20+1}{2} = 10.5^{th} = \frac{10^{th}\text{ item} + 11^{th}\text{ item}}{2} = \frac{46+47}{2} = 46.5$$

No. of obs in first sample less than or equal to 46.5 (a) = 6

$$n_1 = 10, n_2 = 10, k = \frac{n_1 + n_2}{2} = \frac{10+10}{2} = 10$$

Let Md_1 & Md_2 be median yield due to treatment x & y respectively.

Problem to test $H_0 : Md_1 = Md_2$

$H_1 : Md_1 = Md_2$

Test statistic

$$P(A = a) = \frac{C(n_1, a)\, C(n_2, k-a)}{C(n_1 + n_2, k)} = \frac{C(10, a)\, C(10, 10-a)}{C(20, 10)} \quad a = 0, 1, 2, \ldots 10$$

Critical value

$P = P(A \geq a) = P(A \geq 6)$

$$= \sum_{a=6}^{10} \frac{C(10, a)\, C(10, 10-a)}{C(20, 10)}$$

$$= \frac{C(10, 6)\, C(10, 4)}{C(20, 10)} + \frac{C(10, 7)\, C(10, 3)}{C(20, 10)} + \frac{C(10, 8)\, C(10, 2)}{C(20, 10)}$$

$$+ \frac{C(10, 9)\, C(10, 1)}{C(20, 10)} + \frac{C(10, 10)\, C(10, 0)}{C(20, 10)}$$

$$= \frac{210 \times 210}{184756} + \frac{120 \times 120}{184756} + \frac{45 \times 45}{184756} + \frac{10 \times 10}{184756} + \frac{1 \times 1}{184756}$$

$= 0.328$

Decision

$2P = 0.656 > \alpha = 0.05$

Accept H_0 at 5% level of significance.

Conclusion: Both treatments are equally effective.

8. The length of life

To find M_d of combined group arrange data in ascending order 7, 11, 17, 20, 24, 25, 25, 28, 30, 34, 39, 41, 45, 47, 59, 61, 68, 75, 76, 83, 96, 99, 101, 108, 125, 140, 165, 178, 200, 238

$$Md = \left(\frac{n+1}{2}\right)^{th} \text{ item} = \frac{30+1}{2} = 15.5$$

$$= \frac{15^{th}\text{ item} + 16^{th}\text{ item}}{2} = \frac{59+61}{2} = 60$$

No. of obs ≤ 60 in first sample (a) = 5

No. of obs ≤ 60 in second sample (b) = 10

No. of obs > 60 in first sample (c) = 9

No. of obs > 60 second sample (d) = 6

The 2×2 contingency table

	Neon	Helium	Total
No. obs ≤ Md	5 (a)	10 (b)	15
No. of obs > Md	9 (c)	6 (d)	15
	14	16	30

Let Md_1 and Md_2 be median life of bulb Neon and Helium respectively.

Problem to test $H_0 : Md_1 = Md_2$

$H_1 : Md_1 \neq Md_2$

Test statistic

$$\chi^2 = \frac{N(ad - bc)^2}{(a+c)(b+d)(a+b)(c+d)} = \frac{30(5 \times 6 - 10 \times 9)^2}{14 \times 16 \times 15 \times 15} = 2.14$$

Critical value

At α = 5% critical value is $\chi^2_{\alpha\,(1)} = 3.84$

Decision

$\chi^2 = 2.4 < \chi^2_{tabulated} = 3.84$

Accept H_0 at 5% level of significance.

Conclusion

There is no significant difference in median lives of two types of electric bulb.

9. The same C programming

To find Md of combined group arrange date in ascending order.

31, 36, 43, 45, 48, 60, 62, 66, 73, 73, 74, 76, 77, 77, 77, 78, 78, 80, 82, 85, 88, 89, 91, 93

$$Md = \left(\frac{n+1}{2}\right)^{th} \text{ item} = \frac{24+1}{2} = 12.5^{th} \text{ item}$$

$$= \frac{12^{th} \text{ item} + 13^{th} \text{ item}}{2} = \frac{76+77}{2} = 76.5$$

No. of obs ≤ 76.5 in first sample (a) = 7

No. of obs ≤ 76.5 in second sample (b) = 5

No. of obs > 76.5 in first sample (c) = 5

No. of obs > 76.5 second sample (d) = 7

	Teacher A	Teacher B	Total
No. of obs ≤ Md	7 (a)	5 (b)	12
No. of obs > Md	5 (c)	7 (d)	12
Total	12	12	24

Let Md_1 and Md_2 be median marks by teacher A and teacher B respectively.

Problem to test $H_0 : Md_1 = Md_2$

$H_1 : Md_1 = Md_2$

Test statistic $$\chi^2 = \frac{N\,(ad - bc)^2}{(a+c)\,(b+d)\,(a+b)\,(c+d)}$$

$$= \frac{24\,(7 \times 7 - 5 \times 5)^2}{12 \times 12 \times 12 \times 12} = 0.666$$

Critical value

At $\alpha = 5\%$ critical value is $\chi^2_{tabulated} = \chi^2_{\alpha(1)} = 3.84$

Decision

$\chi^2 = 0.666 < \chi^2_{tabulated} = 3.84$

Accept H_0 at 5% level of significance.

Conclusion: There is no significant difference in marks by two teachers.

10. A quality c

Arranging combined data in ascending order

16.7, 17.6, 17.8, 18.1, 18.9, 19.8, 19.8, 20.2, 22.8, 23.2, 24.0

Here, n = 11, $\alpha = 5\%$

$$\alpha = \left(\frac{n+1}{2}\right)^{th} \text{ item} = 19.8$$

No. of obs in first sample ≤ 19.8 (a) = 3

$$n_1 = 5,\ n_2 = 6,\ k = \frac{n_1 + n_2}{2} = \frac{5+6}{2} = 5.5 \approx 6$$

Let Md_1 and Md_2 be median yield of variety I and II respectively.

Problem to test $H_0 : Md_1 = Md_2$

$H_1 : Md_1 \neq Md_2$

Test statistic

$$P\,(A = a) = \frac{C\,(n_1, a)\ C\,(n_2, k - a)}{C\,(n_1 + n_2, k)},\ a = 0, 1, 2, \ldots\ldots 5$$

Critical value

$P = P\,(A \geq a) = P(A \geq 3)$

$$= \sum_{a=3}^{5} \frac{C\,(5, a)\ C(6, 6-a)}{C\,(11, 6)}$$

$$= \frac{C(5,3)\ C\,(6,3)}{C\,(11,6)} + \frac{C\,(5,4)\ C\,(6,8)}{C\,(11,6)} + \frac{C\,(5,5)\ C\,(6,1)}{C\,(11,6)}$$

$$= \frac{10 \times 20}{462} + \frac{5 \times 15}{462} + \frac{6}{462} = \frac{281}{462} = 0.6$$

Decision

$2\,P = 1.2 > \alpha = 0.05$

Accept H_0 at 5% level of significance.

Conclusion

There is no significant difference in production of wheat using varieties of wheat.

11.

Motor M (fx)	Motor N (fy)	cfx	Fx	cfy	fy	|Fx – Fy|
3	4	3	3/40	4	4/40	1/40
6	3	9	9/40	7	7/40	2/40
7	5	16	16/40	12	12/40	4/40
4	6	20	20/40	18	18/40	2/40
3	3	23	23/40	21	21/40	2/40
5	2	28	28/40	23	23/40	5/40
5	4	33	33/40	27	27/40	6/40
3	7	36	36/40	34	34/40	2/40
4	6	40	40/40	40	40/40	0/40
40	40					

Problem to test

H_0: F(x) = F(y)

H_0: F(x) ≠ f(y)

Test statistic

D_0 = Max {|F(x) – F(y)|}

= 6/40 = 0.15

= 0.15

At α = 5%

Critical value

$$D_{(n_1,n_2,\alpha)} = 1.36\sqrt{\frac{n_1 + n_2}{n_1 \times n_2}}$$

$$= 1.36\sqrt{\frac{40 + 40}{40 \times 40}}$$

$$= 1.36 . \sqrt{0.05}$$

= 0.304

Decision

$D_0 = 0.15 < D_{(n_1,n_2,\alpha)} = 0.304$

Accept H_0 at α = 5%

Conclusion

There is no significant difference between ....of two types of motor.

12.

Time	No of website by A(fx)	No. of website designed by B (fy)	cfx	fx	cfy	fy	|Fx - Fy|
0 - 4	2	6	2	2/30	6	6/30	4/30
4 - 8	7	9	9	9/30	15	15/30	6/30
8- 12	12	4	26	21/30	23	23/30	2/30
12 - 16	5	4	26	26/30	27	27/30	1/30
16 - 20	4	3	30	30/30	30	30/30	0/30

Problem to test

H_0 : F(x) = F(y)

H_1 : F(x) > F(y)

Test statistic

D_0 = max {|Fx – Fy|} = 6/30 = 0.2

Critical value

At α = 5% critical value

$$D_{(n_1,n_2,\alpha)} = 1.22\sqrt{\frac{n_1 + n_2}{n_1 n_2}}$$

$$= 1.22\sqrt{\frac{30 + 30}{30 \times 30}}$$

= 0.314

Decision

$D_0 = 0.2 < D_{(n_1, n_{2,\alpha})} = 0.314$

Accept H_0 at $\alpha = 5\%$

Conclusion

There is no significant difference between website designed by A and B.

13.

No of	No of junior programmer (fx)	No of senior programmer c(fy)	cfx	fx	cfy	fy	|Fx - Fy|
0 - 2	7	5	7	7/26	5	5/25	0.069
2 - 4	6	4	13	13/26	9	9/25	0.14
4 - 6	4	6	17	17/26	15	15/25	0.053
6 - 8	2	3	19	19/26	18	18/25	0.0107
8 - 10	2	4	21	21/26	22	22/25	0.072
10 - 12	3	2	24	24/26	24	924/25	0.036
12 - 14	2	1	26	26/26	25	25/25	0
	26	25					

Problem to test

$H_0 : F(x) = F(y)$

$H_1: F(x) \neq F(y)$

Test statistic $D_0 = \max \{|F(x) - F(y)|\} = 0.14$

Critical value at $\alpha = 5\%$

$$D_\alpha = 1.36\sqrt{\frac{n_1 + n_2}{n_1 n_2}}$$

$$= 1.36\sqrt{\frac{26 + 25}{26 \times 25}}$$

$$= 1.36\sqrt{0.0784}$$

$= 0.38$

Decision

$D_0 = 0.14 < D_\alpha = 0.38$

Accept H_0 at $\alpha = 5\%$

Conclusion

There is no significant difference between junior senior programmer.

14. Two groups of rats

Let Md_1 and Md_2 be medians of trained rats and untrained rats.
Ranking combined data in ascending order.

Trained rat	Rank	Untrained rat	Rank
78	7	110	9
64	4	70	5
75	6	53	3
45	1	51	2
82	8		
	$R_1 = 26$		$R_2 = 19$

Here, $n_1 = 5$, $n_2 = 4$

$$U_1 = n_1 n_2 + \frac{n_1 (n_1 + 1)}{2} - R_1 = 5 \times 4 + \frac{5(5+1)}{2} - 26 = 9$$

$$U_2 = n_1 n_2 + \frac{n_2 (n_2 + 1)}{2} - R_2 = 5 \times 4 + \frac{4(4+1)}{2} - 19 = 11$$

$U_0 = \min(U_1, U_2) = \min(9, 11) = 9$

Problem to test $H_0 : Md_1 = Md_2$

$H_1 : Md_1 \neq Md_2$

Test statistic $U_0 = 9$

Critical value

Let $\alpha = 5\%$, then critical value is $U_{tabulated} = U_{\alpha(n1,\, n2)} = 1$

Decision

$U_0 = 9 > U_{tabulated} = 1$

Accept H_0 at 5% level of significance.

Conclusion

No significant difference between trails of trained and untrained rats.

15. The nicotine contents

Let Md_1 and Md_2 be median of brand A and brand B respectively.
Combine both group and rank in ascending order.

Brand A	Rank	Brand B	Rank

2.1	4	4.1	12
4.0	10.5	0.6	1
6.3	18	3.1	7
5.4	14.5	2.5	6
4.8	13	4.0	10.5
3.7	9	6.2	17
6.1	16	1.6	2
3.3	8	2.2	5
		1.9	3
		5.4	14.5
	$R_1 = 93$		$R_2 = 78$

Here, $n_1 = 8$, $n_2 = 10$

$$U_1 = n_1 n_2 + \frac{n_1 (n_1 + 1)}{2} - R_1 = 8 \times 10 + \frac{8 (8 + 1)}{2} - 93 = 23$$

$$U_2 = n_1 n_2 + \frac{n_2 (n_2 + 1)}{2} - R_2 = 8 \times 10 + \frac{10 (10 + 1)}{2} - 78 = 57$$

$U_0 = \min (U_1, U_2) = \min (23, 57) = 23$

Problem to test $H_0 : Md_1 = Md_2$

$H_1 : Md_1 \neq Md_2$

Test statistic $U_0 = \text{Min}(U_1, U_2) = 23$

Critical value Let α = 5% then critical value from Mann Whitney U table is P = 0.0729

Decision

2 P = 0.1458 > α = 0.05. Accept H_0 at 5% level of significance.

Conclusion

There is no significant difference between average no. of trails of trained and untrained rats.

16. A farmer wishes

Let Md_1 and Md_2 be medians of wheat I and Wheat II combine both group and rank

Wheat I	Rank	Wheat II	Rank
15.9	9	16.4	12.5
15.3	5	16.8	15
16.4	12.5	17.1	17
14.9	3	16.9	16
15.3	5	18.0	19
16.0	10.5	15.6	8
14.6	2	18.1	20
15.7	5	17.2	18
14.5	1	15.4	7
16.6	14		
16.0	10.5		
	$R_1 = 77.5$		$R_2 = 132.5$

Here, $n_1 = 11$, $n_2 = 9$, $n = n_1 + n_2 = 11 + 9 = 20$

$$U_1 = n_1 n_2 + \frac{n_1 (n_1 + 1)}{2} - R_1 = 11 \times 9 + \frac{11 (11 + 1)}{2} - 77.5 = 87.5$$

$$U_2 = n_1 n_2 + \frac{n_2 (n_2 + 1)}{2} - R_2 = 11 \times 9 + \frac{9 (9 + 1)}{2} - 132.5 = 11.5$$

$U_0 = \min (U_1, U_2) = \text{Min} \{11.5, 87.5\} = 11.5$

Test statistic

$U_0 = 11.5$

$$\mu_U = \frac{n_1 n_2}{2} = \frac{11 \times 9}{2} = 49.5$$

$$\sigma_U = \sqrt{\frac{n_1 n_2}{n (n - 1)} \left(\frac{n^3 - n}{12} - \frac{\sum t_i^3 - t_i}{12} \right)}$$

$$= \sqrt{\frac{11 \times 9}{20 (20 - 1)} \left[\frac{20^3 - 20}{12} - \left(\frac{3^3 - 3}{12} + \frac{2^3 - 2}{12} + \frac{2^3 - 2}{12} \right) \right]} = 13.13$$

$$Z = \frac{U_0 - \mu_U}{\sigma_U} = \frac{11.5 - 49.5}{13.13} = -2.89$$

Critical value

At $\alpha = 0.01$, critical value is $Z_{tabulated} = Z_{\alpha/2} = 2.576$

Decision

$|Z| = 2.89 > Z_{tabulated}$. Reject H_0 at $\alpha = 0.01$.

Conclusion: There is significant difference between variety I and II.

17. Two independent random samples

Let Md_1 and Md_2 be median of unemployed men and unemployed women.

Combine both group and rank.

Women	Rank	Men	Rank
60	8	53	7
63	9	39	5
36	4	22	1
44	6	23	2

		24	3
	$R_1 = 27$		$R_2 = 18$

Here, $n_1 = 4$, $n_2 = 5$, $\alpha = 5\%$

$$U_1 = n_1 n_2 + \frac{n_1 (n_1 + 1)}{2} - R_1 = 4 \times 5 + \frac{4 (4 + 1)}{2} - 27 = 3$$

$$U_2 = n_1 n_2 + \frac{n_2 (n_2 + 1)}{2} - R_2 = 4 \times 5 + \frac{5 (5 + 1)}{2} - 18 = 32$$

$U_0 = \min (U_1, U_2) = \text{Min} (3, 32) = 3$

Problem to test $H_0 : Md_1 = Md_2$

$H_1 : Md_1 \neq Md_2$

Test statistic $U_0 = 3$

Critical value

At $\alpha = 5\%$ for $n_1 = 4$, $n_2 = 5$ and $U_0 = 3$

From Mann Whitney U table P = 0.055.

Decision

$2 P = 0.11 > \alpha = 0.05$

Accept H_0 at 5% level of significance.

Conclusion: There is no significant difference between average age of unemployed men and women.

18. The following are the

Here, $n_1 = 13$, $n_2 = 12$, $\alpha = 0.05$

Let, Md_1 and Md_2 be median time for men and women.

Combine both groups and rank.

Men	Rank	Women	Rank
16.5	2	18.6	14
20.0	23	17.8	7
17.0	5	18.3	11
19.8	22	16.6	3
18.5	13	20.5	24
19.2	18	16.3	1
19.0	17	19.3	19
18.2	10	18.4	12
20.8	25	19.7	20
18.7	15	18.8	16
16.7	4	19.9	21
18.1	9	17.6	6
17.9	8		
	$R_1 = 171$		$R_2 = 154$

$$U_1 = n_1 n_2 + \frac{n_1 (n_1 + 1)}{2} - R_1 = 13 \times 12 + \frac{13 (13 + 1)}{2} - 171 = 76$$

$$U_2 = n_1 n_2 + \frac{n_2 (n_2 + 1)}{2} - R_2 = 13 \times 12 + \frac{12 (12 + 1)}{2} - 154 = 80$$

$U_0 = \min (U_1, U_2) = \min(76, 80) = 76$

$$\mu_U = \frac{n_1 n_2}{2} = \frac{13 \times 12}{2} = 78$$

$$\sigma_U = \sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}} = \sqrt{\frac{13 \times 12 (13 + 12 + 1)}{12}} = 18.384$$

Problem to test $H_0 : Md_1 = Md_2$

$H_1 : Md_1 \neq Md_2$

Test statistic $Z = \frac{V_0 - \mu_U}{\sigma_U} = \frac{76 - 78}{18.38} = -0.108$

Critical value

At $\alpha = 5\%$ critical value is $Z_{tabulated} = Z_{\alpha/2} = 1.96$

Decision

$|Z| = 0.108 < Z_{tabulated} = 1.96$. Accept H_0 at 5% level of significance.

Conclusion: Samples come from identical population.

19. A die was rolled

Side	No. of times observed (o_i)	P_i	Expected frequency $E_i = NP_i$	$\frac{(0_i - \epsilon_i)^2}{\epsilon_i}$

1	8	1/6	10	4/10
2	9	1/6	10	1/10
3	13	1/6	10	9/10
4	7	1/6	10	9/10
5	15	1/6	10	25/10
6	8	1/6	10	4/10
Total	N = 60			

$\alpha = 5\%, k = 6$

Problem to test $H_0 : O_i = \in_i$

$H_1 : O_i \neq \in_i$

Test statistic $\chi^2 = \sum \frac{(O_i - \in_i)^2}{\in_i} = 5.2$

Critical value

At $\alpha = 5\%$ critical value $\chi^2_{tabulated} = \chi^2_{\alpha\,(k-1)} = 11.07$

Decision

$\chi^2 = 5.2 < \chi^2_{tabulated} = 11.07$

Accept H_0 at 5% level of significance

Conclusion: The die is fair.

20. A certain chemical plant

Let O_i and $\in_i$ be observed and expected frequency respectively.

Component	Observed (O_i)	P_i	$E_i = NP_i$	$\frac{(O_i - E_i)^2}{E_i}$
Sodium chloride	130	62/100	124	36/24
Magnesium	6	4/100	8	4/8
Other components	64	34/100	68	16/68
Total	N = 200			$\sum \frac{(O_i - E_i)^2}{E_i} = 1.025$

$\alpha = 5\%, k = 3$

Problem to test $H_0 : O_i = \in_i$

$H_1 : O_i \neq \in_i$

Test statistic $\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} = 1.025$

Critical value

At $\alpha = 5\%$ and $k = 3$, critical value is $\chi^2_{tabulated} = \chi^2_{\alpha\,(k-1)} = 5.99$

Decision

$\chi^2 = 1.025 < \chi^2_{tabulated} = 5.99$

Accept H_0 at 5% level of significance.

Conclusion: The given data are consistent.

21. A group of

Let O_i and $\in_i$ be observed and expected frequencies

Actor	Frequencies (O_i)	P_i	$E_i = NP_i$	$\frac{(O_i - E_i)^2}{E_i}$
A	24	1/6	25	1/25
B	30	1/6	25	25/25
C	32	1/6	25	49/25
D	25	1/6	25	0
E	28	1/6	25	9/25
F	21	1/6	25	16/25
Total	N = 150			$\sum \frac{(O_i - E_i)^2}{E_i} = 4$

$\alpha = 5\%, k = 6$

Problem to test $H_0 : O_i = \in_i$

$H_1 : O_i \neq \in_i$

Test statistic $\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} = 4$

Critical value

At $\alpha = 5\%$ and $k = 6$, critical value is $\chi^2_{tabulated} = \chi^2_{\alpha\,(k-1)} = 11.07$

Decision

$\chi^2 = 4 < \chi^2_{tabulated} = 11.07$. Accept H_0 at 5% level of significance.

Conclusion: All actors are equally popular.

22. Genetic theory states that

Let O_i and E_i be observed and expected frequencies

Blood type	O_i	P_i	$E_i = NP_i$	$\frac{(O_i - E_i)^2}{E_i}$
A	90	1/4	75	225/75
AB	135	2/4	150	225/150
B	75	1/4	75	0/75
	N = 300			$\sum\frac{(O_i - E_i)^2}{E_i} = 4.5$

$\alpha = 5\%$, k = 3

Problem to test $H_0 : O_i = \epsilon_i$

$H_1 : O_i \neq \epsilon_i$

Test statistic $\chi^2 = \sum\frac{(O_i - E_i)^2}{E_i} = 4.5$

Critical value

At $\alpha = 5\%$ and k = 3, critical value is $\chi^2_{tabulated} = \chi^2_{\alpha\,(k-1)} = 5.99$

Decision

$\chi^2 = 4.5 < \chi^2_{tabulated} = 5.99$

Accept H_0 at 5% level of significance.

Conclusion

The proportion of blood type A, AB and B as 1 : 2 : 1 is correct.

23. A publishing house

Let O_i and ϵ_i be observed and expected frequencies

No. of misprint page	No. of page (f)	fx	O_i	P_i	$\epsilon_i = NP_i$	$\frac{(O_i - E_i)^2}{E_i}$
0	221	0	21	0.406	203	324/203
1	167	167	167	0.365	183	256/183
2	70	140	70	0.164	82	144/82
3	30	90	30	0.049	25	25/6
4 5	7 5	28 25	7 5 } 12	0.011 0.002	6 1 } 7	25/7
	N = 500	$\sum fx$ = 400				$\sum\frac{(O_i - E_i)^2}{E_i}$ = 13.322

$\bar{x} = \frac{\sum fx}{N} = \frac{450}{500} = 0.9$

$\therefore$ $\lambda = 0.9$ [Estimated parameter]

$P(x) = \frac{e^{-\lambda}\lambda^x}{x\,!} = \frac{e^{-0.9}(0.9)^x}{x\,!}$

Here, k = 6

Problem to test $H_0 : O_i = \epsilon_i$

$H_1 : O_i \neq \epsilon_i$

Test statistic $\chi^2 = \sum\frac{(O_i - E_i)^2}{E_i} = 13.322$

Critical value

Let $\alpha = 5\%$ then critical value is $\chi^2_{tabulated} = \chi^2_{\alpha\,(k-1-1-1)} = 7.815$

Decision

$\chi^2 = 13.322 > \chi^2_{tabulated} = 7.815$

Reject H_0 at 5% level of significance.

Conclusion: Pass on distribution does not fit the data.

24. In 50 random sample

No. of defective items	Freq (O_i)	P_i	$E_i = NP_i$	$\frac{(O_i - E_i)^2}{E_i}$
0 1	4 13 } 17	0.168 0.36	$8.4 \approx 8$ 18 } 26	81/26
2	17	0.308	$15.4 \approx 15$	4/15
3 4 	12 3 1 } 16	0.1323 0.028 0.0024	$6.615 \approx 7$ $1.4 \approx 1$ $0.12 \approx 0$ } 8	64/8
	N = 50			$\sum\frac{(O_i - E_i)^2}{E_i} = 11.11$

Here $n = 5, p = 0.3$

$P(x) = C(n, x)\, p^x q^{n-x} = C(5, x)\,(0.3)^x (0.7)^{5-x}$, $k = 6$

Problem to test $H_0 : O_i = \in_i$

$H_1 : O_i \neq \in_i$

Test statistic $\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} = 11.11$

Critical value

Let $\alpha = 5\%$ then critical value is $\chi^2_{tabulated} = \chi^2_{\alpha\,(k-1-2)} = 5.99$

Reject H_0 at 5% level of significance.

Conclusion: Binomial distribution is not good with p = 0.3.

25. The number of telephone

Number of calls (x)	No. of days (f)	fx	O_i	P_i	$E_i = NP_i$	$\frac{(O_i - E_i)^2}{E_i}$
0	45	0	45	0.386	39	36/39
1	25	25	25	0.367	37	144/37
2	20	40	20	0.174	17	9/17
3	10	30	10	0.055	6	16/6
	N = 100	$\sum fx = 95$				$\sum\frac{(O_i - E_i)^2}{E_i} = 8.011$

$\bar{x} = \frac{\sum fx}{N} = \frac{95}{100} = 0.95$

$\therefore$ $\lambda = 0.95$ (estimated)

$P(x) = \frac{e^{-\lambda} \lambda^x}{x!} = \frac{e^{-0.95} (0.95)^x}{x!}$

$k = 4$

Problem to test $H_0 : O_i = \in_i$

$H_1 : O_i \neq \in_i$

Test statistic $\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} = 8.011$

Critical value

At $\alpha = 5\%$, critical value is $\chi^2_{tabulated} = \chi^2_{tabulated} = 5.99$

Decision

$\chi^2 = 8.011 > \chi^2_{tabulated} = 5.99$. Reject H_0 at 5% level of significance.

Conclusion: The no. of calls received per day does not fit the Poisson distribution.

26. The distribution of persons

Let O_{ij} and E_{ij} be observed and expected frequencies in i^{th} row and j^{th} column.

Sex \ Blood group	O	A	B	AB	Total (O_i)
Male (m)	100	40	45	10	195
Female (f)	110	35	55	5	205
Total ($O_{.j}$)	210	75	100	15	400

Group	O_{ij}	E_{ij}	$(O_{ij} - E_{ij})$	$\frac{(O_{ij} - E_{ij})^2}{E_{ij}}$
MO	100	102	– 2	0.039
MA	40	37	3	0.243
MB	45	49	– 4	0.326
MAB	10	7	3	1.285
FO	110	108	2	0.037
FA	35	38	– 3	0.236
FB	55	51	4	0.313
FAB	5	8	– 3	1.125
				$\sum\sum\frac{(O_{ij} - E_{ij})^2}{E_{ij}} = 3.604$

Expected frequency for $\mu_O = \frac{O_{1.} \times O_{.1}}{N} = \frac{195 \times 210}{400} = 102$

Problem to test

H_0 : There is no association between blood group and sex.

H_1 : There is association between blood group and sex.

Test statistic $\chi^2 = \sum\sum \frac{(O_{ij} - E_{ij})^2}{E_{ij}} = 3.604$

Critical value

Let α = 5% then critical value is $\chi^2_{tabulated} = \chi^2_{\alpha\,(2-1)\,(3-1)} = 5.99$

Decision

$\chi^2 = 3.604 < \chi^2_{tabulated} = 5.99$, Accept H_0 at 5% level of significance.

Conclusion: There is no association between blood group and sex.

27. A random sample of

	0 – 1 (A)	2 – 3 (B)	Over 3 (c)	Total (O_i)
Elementary (E)	14	37	32	83
Secondary (S)	19	42	17	78
College (C)	12	17	10	39
Total ($O_{.j}$)	45	96	59	200

O_{ij} and $\in_{ij}$ be observed and expected frequencies in i^{th} row and j^{th} column.

Group	O_{ij}	$E_{ij} = \frac{O_{i.} \times O_{.j}}{N}$	$(O_{ij} - E_{ij})$	$\frac{(O_{ij} - E_{ij})^2}{E_{ij}}$
EA	14	19	– 5	1.315
EB	37	40	– 3	0.225
EC	32	24	8	2.666
SA	19	18	1	0.055
SB	42	37	5	0.675
SC	17	23	– 6	1.565
CA	12	9	3	1
CB	17	19	– 2	0.2105
CC	10	12	– 2	0.333
				$\sum\sum\frac{(O_{ij} - E_{ij})^2}{E_{ij}} = 8.04$

Here no. of rows (r) = 3, no. of columns (c) = 3, α = 0.05

Problem to test H_0 : Number of children is independent of education.

H_1 : Number of children is dependent of education.

Test statistic $\chi^2 = \sum\sum\frac{(O_{ij} - E_{ij})^2}{E_{ij}} = 8.04$

Critical value: At α = 5%, for r = 3 and c = 3, critical value is $\chi^2_{tabulated} = \chi^2_{\alpha\,(r-1)\,(c-1)} =$ 9.488

Decision

$\chi^2 = 8.04 < \chi^2_{tabulated} = 9.488$

Accept H_0 at 5% level of significance.

Conclusion: Number of children is independent of education.

28. Test whether the color

		Son's eye colour		
		No light	Light	Total
Father's eye color	No light	230 (a)	148 (b)	378
	Light	151 (c)	471 (d)	622
Total		381	619	1000

Problem to test

H_0 : There is no association between father's eye color and son's eye color

H_1 : There is association between father's eye color and son's eye color.

Test statistic $\chi^2 = \frac{N\,(ad - bc)^2}{(a + c)\,(b + d)\,(a + b)\,(c + d)}$

$= \frac{100\,(230 \times 471 - 148 \times 151)^2}{381 \times 619 \times 378 \times 622} = 133.32$

Critical value

At α = 5% critical value is $\chi^2_{tabulated} = \chi^2_{tabulated} = 3.84$

Reject H_0 at 5% level of significance.

Conclusion: There is association of son's eye color with father's eye color.

29. 88 residents of city

Sex / Drinking habit	Male	Female	Total
Drinking tea	40 (a)	33 (b)	73
No drinking tea	30 (c)	12 (d)	15
Total	43	45	88

Problem to test H_0: Drinking tea habit is not associated with sex.

H_1: Drinking tea habit is associated with sex

Test statistic

$$\chi^2 = \frac{N\left(|ad - bc| - \frac{N}{2}\right)^2}{(a+c)(b+d)(a+b)(c+d)} = \frac{88\,(|140 \times 12 - 33 \times 3| - 44)^2}{43 \times 45 \times 73 \times 15} = 4.71$$

Critical value

Let $\alpha = 5\%$ then critical value is $\chi^2_{tabulated} = \chi^2_{\alpha\,(1)} = 3.84$

Decision

$\chi^2 = 4.71 > \chi^2_{tabulated} = 3.84$. Reject H_0 at 5% level of significance.

Conclusion: Drinking tea habit is associated with gender.

30. Out of a sample of

Influenza	Drug		Total
	Administered	Not administered	
Attacked	24 (a)	34 (b)	58
Not attacked	50 (c)	12 (d)	62
Total	74	46	120

Problem to test H_0: No association between drug and influenza.

H_1: Association between drug and influenza.

Test statistics $\chi^2 = \frac{N\,(ad - bc)^2}{(a+c)(b+d)(a+b)(c+d)}$

$$= \frac{120\,(24 \times 12 - 34 \times 50)^2}{74 \times 46 \times 58 \times 62} = 19.54$$

Critical value: At $\alpha = 5\%$, critical value is $\chi^2_{tabulated} = \chi^2_{\alpha\,(1)} = 3.84$.

Decision

$\chi^2 = 19.54 > \chi^2_{tabulated} = 3.84$. Reject H_0 at 5% level of significance.

Conclusion: New drug is effective in controlling influenza.

31. The following table

Flower color	Leaf type		Total
	Flat	Curled	
Pink	3 (a)	22 (b)	25
Red	9 (c)	11 (d)	20
Total	12	33	45

Problem to test H_0 : Leaf and flower color are independent.

H_1 : Leaf and flower color are dependent.

Test statistic

$$\chi^2 = \frac{N\left(|ad - bc| - \frac{N}{2}\right)^2}{(a+c)(b+d)(a+b)(c+d)} = \frac{45\,(|3 \times 11 - 22 \times 9| - 22.5)^2}{12 \times 33 \times 25 \times 20} = 4.61$$

Critical value

Let $\alpha = 5\%$ then critical value is $\chi^2_{tabulated} = \chi^2_{\alpha\,(1)} = 3.84$

Decision

$\chi^2 = 4.61 > \chi^2_{tabulated} = 3.84$. Reject H_0 at 5% level of significance.

Conclusion: Flower color is not independent of flatness of leaf.

Excises - 3(iii)

7. The following table

Before (x_i)	After (y_i)	$d_i = y_i - x_i$	R_i
66	71	5	5
80	82	2	2.5
69	68	– 1	– 1
52	56	4	4
75	73	– 2	– 2.5

Here, n = 5

S (+) = 11.5 S (–) = 3.5

Let Md_A and Md_B be median after and before respectively

Probable to test

$H_0 : Md_B = Md_A$

$H_1 : Md_B > Md_A$

Test statistic T = min{S(+), S(–)} = 3.5

Critical value

At $\alpha = 0.05$, $T_{\alpha,n} = T_{0.055,}5 = 1$

Decision

$T = 3.5 > T_{\alpha,n} = 1$

Accept H_0 at $\alpha = 5\%$

Conclusion - smoking has no effect on persons weight.

8. The following table

Before (x_i)	After (y_i)	$d_i = y_i - x_i$	R_i
57	60	3	4
80	90	10	8
64	62	– 2	– 3
70	70	0	
90	95	- 1	- 1.5
59	58	– 1	– 1.5
76	80	4	5
98	99	1	
70	75	5	6.5
83	94	11	9

Here, n = 9

S (+) = 4 + 8 + 6.5 + 5 + 1.5 + 6.5 + 9

= 40.5

S (-) = 3 + 1.5 = 4.5

Let Md_B and Md_A be median before and after respectively.

Problem to test $H_0 : Md_A = Md_B$

$H_1: Md_A > Md_B$

Test statistic T = min { S(+), S (–)} = min {40.5, 4.5} = 4.5

Critical value

At $\alpha = 5\%$

$T_{\alpha,n} = T_{0.05,\,9} = 8$

Decision $T = 4.5 < T_{\alpha,\,n} = 8$

Reject H_0 at $\alpha = 5\%$

9. Following the table

First (x_i)	Second (y_i)	$d_i = y_i - x_i$	R_i
470	510	40	6
530	550	20	4
610	600	- 10	- 1
440	490	50	7
600	585	- 15	- 2
590	620	30	5
580	598	18	3

Here n = 7

S(+) = 6 + 4 + 7+ 5 + 3 = 25

S (-) = 1 + 2 = 3

Let Md_x and Md_y be median at First attempt respectively problem to test

$H_0: Md_x = Md_y$

$H_1 : Md_x \neq Md_y$

Test statistic T = min {S(+), S(-)} = min {3, 25} = 3

Critical value

At α = 0.05, $T_{\alpha, n} = T_{0.05, 7}$ = 2

Decision

T = 3 > $T_{\alpha, n}$ = 2

Accept H_0 at α = 5%

10. The following table

Arch x	Arch y	d = y – x	R_i
1	0	– 1	7
0	0	0	
2	1	– 1	– 7
3	0	– 3	- 22
1	2	1	7
0	0	0	
2	0	-2	- 17
2	1	- 1	- 7
3	1	- 2	- 17
0	2	2	17
1	0	- 1	- 7
1	1	0	
4	2	- 2	- 17
1	1	0	
2	1	- 1	- 7
1	0	- 1	- 7
3	2	- 1	- 7
5	2	- 3	-22
2	6	4	25
1	0	- 1	- 7
3	2	- 1	-7
2	3	1	7
4	0	-4	- 25
1	2	1	7
3	1	- 2	- 17
2	0	-2	- 17
0	1	1	7
2	0	-2	- 17
4	1	-3	- 22
1	5	4	25

Let Md_x and Md_y be median of Arch x and Arch y respectively

Problem to test H_0: Mdx = Mdy

H_1 : Mdx > Mdy

S (+) = 7 + 7 + 17 + 25 + 7 + 7 + 6 + 25 = 102

S (–) = 7 + 22 + 17 + 7 + 17 + 7 + 17 + 7 + 7 + 7 + 22 + 7 + 25 + 17 + 17 + 22 =

Test statistic

T = min {S (+), S (-)} = 102

$$\mu_T = \frac{n(n+1)}{4} = \frac{26(26+1)}{4} = 175.5$$

$$\sigma_T^2 = \frac{n(n+1)(2n+1)}{24}$$

$$= \frac{26\,(26+1)\,(2 \times 26 + 1)}{24}$$

= 1550.25

σ_T = 39.37

$$Z = \frac{T - \mu_T}{\sigma_T} = \frac{102 - 175.5}{39.37} = -1.86$$

|Z| = 1.86

Critical value

Let = 5% then critical value

Z_α = 1.645

Decision

|Z| = 1.86 > $Z\alpha$ = 1.645

Reject H_0 at α = 5%

Conclusion

Archaeologist x is better than y

11. following the table

Objective questions	1	2	3	4	5	R_i	R_i^2
Q_1	1	0	0	1	1	3	0

Q_2	0	1	1	0	1	3	9
Q_3	1	1	1	0	0	3	9
C_j	2	2	3	1	3	$\Sigma R_i = \Sigma C_j = 11$	$\Sigma R_i^2 = 31$
C_j^2	4	4	9	1	9	$\Sigma c_j^2 = 27$	

Here k = 4, N = 5

Problem to test H_0: There is no significant difference between objective questions.

H_1: There is significant difference between objective questions

Test Statistics

$$Q = \frac{(K-1)\,[K\Sigma R_i^2 - (\Sigma R_i)^2}{K\Sigma C_i - \Sigma c_i^2}$$

$$= \frac{(4-1)\,[4 \times 31 - (11)^2}{4 \times 11 - 27}$$

$$= \frac{9}{17}$$

= 0.529

Critical value

Let α = 5% then critical value $X^2\alpha\,(K-1) = X^2 0.05(3) = 7.81$

Decision = $Q = 0.52 < X^2\alpha\,(K-1) = 7.81$, accept α = 5%

Conclusion

Three is no significant difference between objective questions.

12. The following question

Buffaloes

Diet	1	2	3	4	5	6	7	8	9	R_i	R_i^2
P	I	D	I	D	I	I	D	D	I	5	25
Q	I	I	I	D	I	I	D	I	D	6	36
R	I	D	D	D	I	D	I	I	I	5	25
C_j	3	1	2	0	3	2	1	2	2	$\Sigma R_i = \Sigma C_j = 16$	$\Sigma R_i^2 = 86$
C_j^2	9	1	4	0	9	4	1	4	4	$\Sigma C_j^2 = 36$	

K = 3, n = 9

Problem to test H_0: Diets are equally effective

H_1: Diets are not equally effective

$$\text{Test statistic } Q = \frac{(k-1)\,[K\Sigma R_i^2 - (\Sigma R_i)^2]}{K\Sigma C_i - \Sigma C_i^2} = \frac{(3-1)\,[3 \times 86 - 16^2]}{3 \times 16 - 36} = 0.333$$

Critical value

At α = 5% Critical value $X^2_{\alpha\,(k-1)} = X^2_{0.05\,(2)} = 5.991$

Decision

$Q = 0.333 < X^2_{\alpha(k-1)} = 5.99$. Accept H_0 at α = 5%

Conclusion

There diets are equally effective

13. Following are

Method I	Rank	Method II	Rank	Method III	Rank
94	17	85	12	89	15
88	14	82	10	67	2
91	16	79	8	72	4.5
74	6	84	11	76	7
87	13	61		69	3
97	18	72	4.5		
		80	9		
R_i	84		55.5		31.5
$\frac{R_i^2}{n}$	1176		4440.035		198.45

Let Md_1 , Md_2 and Md_3 be median of I, II and III method respectively.

$n_1 = 6$, $n_2 = 7$, $n_3 = 5$, n = 18, α = 0.05, K = 3 $E_1 = 2$

Problem to test H_0: $Md_1 = Md_1 = Md_3$

H_1 : At least one Md_i is different, i = 1, 2, 3

Test Statistics

$$H = \frac{\frac{12}{n(n+1)}\Sigma\frac{R_i^2}{n_i} - 3(n+1)}{1 - \Sigma\frac{t_i^3 - t_i}{n^3 - n}}$$

$$= \frac{\frac{12}{18(18+1)} \times 1814.48 - 3 \times 19}{1 - \frac{(2^3 - 2)}{18^3 - 18}}$$

$$= \frac{6.665}{0.998}$$

$= 6.678$

Critical value

At $\alpha = 0.05$ critical value from χ^2 table is $\chi^2_{\alpha(k-1)} = 5.99$

Decision

$H = 6.678 > \chi^2_{\alpha(k-1)} = 5.99$

Reject H_0 at 5% level of significance.

Conclusion: Three methods are significantly different.

14. An agricultural experiment

Let, Md_1, Md_2 and Md_3 be median of N, P and K respectively.

$n_1 = n_2 = n_3 = 4$, $n = 12$, $k = 3$, $\alpha = 0.05$

Combine three groups and rank

	N	Rank	P	Rank	k	Rank	
	122	11	81	8	80	6	
	80	6	80	6	82	9	
	138	12	79	4	65	2.5	
	121	10	65	2.5	58	1	
R_i		39		20.5		18.5	$\sum R_i^2/n_i$ = 570.87
R_i^2/n_i		380.25		105.06		85.56	

Problem to test $H_0 : Md_1 = Md_2 = Md_3$

H_1 : At least one Md_i is different, $i = 1, 2, 3$

Test statistic

$$H = \frac{\frac{12}{n(n+1)}\sum\frac{R_i^2}{n_i} - 3(n+1)}{1 - \sum\frac{t_i^3 - t_i}{n^3 - n}} = \frac{\frac{12}{12(12+1)} \times 570.87 - 3(12+1)}{1 - \left[\frac{(2^3 - 2) + (3^3 - 3)}{12^3 - 12}\right]} = 5$$

Critical value

Critical value form Kruskal Wallis H table for $n_1 = 4$, $n_2 = 4$, $n_3 = 4$ and $H = 5$ is $P_0 = 0.054$.

Decision

$P_0 = 0.054 > \alpha = 0.05$

Accept H_0 at 5% level of significance.

Conclusion: Three fertilizers N, P and K are equally effective.

15. For the following scores

Let, Md_1, Md_2 and Md_3 be median of group A, B and C respectively. $n_1 = 5$, $n_2 = 5$, $n_3 = 4$, $k = 3$, $n = 14$ combine three groups and rank.

	Group A	Rank	Group B	Rank	Group C	Rank	
	96	4	82	2	115	7	
	128	9	124	8	149	13	
	83	2	132	10	166	14	
	61	1	135	11	147	12	
	101	5	109	6			
R_i		21		37		46	$\sum Ri^2/n_i$ = 391
Ri^2/n_i		88.2		273.8		529	

Problem to test $H_0 : Md_1 = Md_2 = Md_3$

H_1 : At least one Md_i is different, $i = 1, 2, 3$

Test statistic $\quad H = \frac{12}{n(n+1)}\sum\frac{R_i^2}{n_i} - 3(n+1)$

$$= \frac{12}{14(14+1)} \times 891 - 3(14+1) = 5.914$$

Critical value

From kruskal wallis H table critical value for $H = 5.914$, $n_1 = 5$, $n_2 = 5$, $n_3 = 4$ is $P = 0.01$

Let $\alpha = 5\%$

Decision

$P = 0.01 < \alpha = 0.05$

Reject H_0 at 5% level of significance.

Conclusion

Three groups A, B and C are significantly different.

16. An experiment designed

Let Md_1, Md_2 and Md_3 be median of method I, II and III respectively.

$n_1 = 6$, $n_2 = 7$, $n_3 = 5$, $n = 18$, $k = 3$

Combined three groups and rank

	Method I	Rank	Method II	Rank	Method III	Rank	
	77	18	60	9	49	3	
	54	6	41	1	52	4.5	
	67	14	59	8	69	15	
	74	17	65	12	47	2	
	71	16	62	10	56	7	
	66	13	64	11			
			52	4.5			
R_i		84		55.5		31.5	$\sum Ri^2/n_i = 1814.48$
Ri^2/n_i		1176		440.03		198.45	

$t_1 = 2$, $\alpha = 0.05$, $k = 3$

Problem to test $H_0 : Md_1 = Md_2 = Md_3$

H_1 : At least one Md_i is different, $i = 1, 2, 3$

Test statistic

$$H = \frac{\frac{12}{n(n+1)}\sum\frac{R_i^2}{n_i} - 3(n+1)}{1 - \sum\frac{t_i^3 - t_i}{n^3 - n}} = \frac{\frac{12}{18(18+1)} \times 1814.48 - 3(18+1)}{1 - \frac{(2^3 - 2)}{18^3 - 18}} = 6.67$$

Critical value

At $\alpha = 0.05$, critical value from $\chi^2_{tabulated}$ is $\chi^2_{\alpha(k-1)} = 5.99$

Decision

$H = 6.67 > \chi^2_{\alpha(k-1)} = 5.99$

Reject H_0 at 5% level of significance.

Conclusion: Samples are not from identical population.

17. The following data

Let Md_1, Md_2 and Md_3 be median of calculator A, B and C respectively.

$n_1 = 5$, $n_2 = 7$, $n_3 = 6$, $n = 18$

combine three groups and rank.

	Calculator A	Rank	Calculator B	Rank	Calculator C	Rank	
	4.9	4	5.5	8.5	6.4	15	
	6.1	12	5.4	7	6.8	18	
	4.3	1	6.2	13	5.6	10	
	4.6	2	5.8	11	6.5	16	
	5.3	6	5.5	8.5	6.3	14	
			5.2	5	6.6	17	
			4.8	3			
R_i		25		56		90	$\sum R_i^2/n_i = 1923$
Ri^2/n_i		125		448		1350	

$t_1 = 2$, $\alpha = 0.05$, $k = 3$

Problem to test $H_0 : Md_1 = Md_2 = Md_3$

H_1 : At least one Md_i is different, $i = 1, 2, 3$

Test statistic

$$H = \frac{\frac{12}{n(n+1)}\sum\frac{R_i^2}{n_i} - 3(n+1)}{1 - \sum\frac{t_i^3 - t_i}{n^3 - n}} = \frac{\frac{12}{18(18+1)} \times 1923 - 3(18+1)}{1 - \frac{(2^3 - 2)}{18^3 - 18}} = 10.49$$

Critical value

At $\alpha = 0.05$, critical value from $\chi^2_{tabulated}$ is $\chi^2_{\alpha(k-1)} = 5.99$

Decision

$H = 10.49 > \chi^2_{\alpha\,(k-1)} = 5.99$

Reject H_0 at 5% level of significance.

Conclusion

Operating time of three calculators in significantly different.

18 A research of wants

Let Md_1, Md_2, Md_3 b median of Alpha, sigma and Gamma school.

No. of School (k) = 3, No. of grade (n) = 4

School	I	Rank	II	Rank	III	Rank	IV	Rank	R_i	R_i^2
Alpha	89	3	98	3	70	3	80	3	12	144
Sigma	45	2	76	2	40	2	55	1	7	49
Gamma	20	1	58	1	35	1	67	2	5	25
										ΣR_i^2 =218

Problem to that

H_0: $Md_1 = Md_2 = Md_3$

H_1 : At least are Md_i different = 1, 2, 3

Test statistic

$$F_r = \frac{12}{nk(k+1)} \Sigma R_i^2 - 3n\,(k+1)$$

$$= \frac{12}{4 \times 3(3+1)} \times 218 - 3 \times 4 \times (3+1)$$

$$= \frac{12 \times 218}{48} - 48 = 6.5$$

Critical value

Let $\alpha = 5\%$ then $p = P(Fr > 6.5) = 0.042$

Decision

$P = 0.042 < \alpha = 0.05$

Reject H_0 at $\alpha = 5\%$

Conclusion

There is significant difference on performance of schools.

19 A survey of conducted

HB Births	Winter	Spring	Summer	Fall
A	92	112	94	77
Rank	2	4	3	1
B	9	11	2	4
Rank	1	3	2	4
C	58	71	51	62
Rank	2	4	1	3
D	19	26	19	18
Rank	2.5	4	2.5	1
R_i	7.5	15	8.5	9
R^2	56.25	225	72.25	81

$ER_i^2 = 434.5$

Here K = 4, n = 4, E = 2

Let Md_1, Md_2, Md_3 and Md_y be median of winter, spring, summer and fall.

Problem to test

H_0: $Md_1 = Md_2 = Md_3 = Md_4$

H_1: At least are Md_i is different

i = 1, 2, 3

Test statistic

$$F_r = \frac{12}{nk(k+1)} \Sigma R_i^2 - 3n\,(k+1) \;/\; 1 - \frac{t^3 - t}{n(k^3 - K)}$$

$$= \frac{\frac{12}{4 \times 4 \times 5} \times 434.5 - 3 \times 4 \times 4}{1 - \frac{(2^3 - 2)}{4(4^3 - 4)}}$$

$$= \frac{5.175}{1 - 0.025}$$

$= 5.3$

Critical value

Let = 5% them

$P = P(fr > 5.3) = 0.928$

Decision

$P = 0.928 > \alpha = 0.05$

Accept H_0 at $\alpha = 5\%$

Conclusion

Birth rates constant are all four seasons.

20 Scores of

Group	I	Rank	II	Rank	III	Rank	Rank	IV	Rank	R_i	R_i^2
Alpha	89	3	98	3	70	3	80	3	3	12	144
Sigma	45	2	76	2	40	2	55	1	1	7	49
Gamma	20	1	58	1	35	1	67	2	2	5	25
											$\Sigma R_i^2 = 218$

Here $K = 3$, $n = 6$, $t_1 = 2$, $t_2 = 3$, $t_3 = 2$

Let Md_A, Md_B and Md_C be median of graph A, B of C respectively

Problem to test

H_0: Md A = Md B = Md C

H_1: At least are Md_i is different

$$\text{Test Statistic } Fr = \frac{\frac{12}{nk(k+1)} \Sigma R_i^2 - 3n(k + i)}{1 - \frac{\Sigma ti3 - ti}{n(k^3 - K)}}$$

$$= \frac{\frac{12}{6 \times 3 \times 4} 450 - 3 \times 6 \times 4}{1 - \frac{(2^3 - 2) + (3^3 - 3) + (2^3 - 2)}{6(3^3 - 3)}}$$

$$= \frac{3}{1 - \frac{(6 + 24 + 6)}{144}}$$

=

$$= \frac{3}{1 - \frac{(6 + 36)}{144}}$$

$$= \frac{3}{0.75}$$

$= 4$

Critical value

At $\alpha = 5\%$

$P = P(F_r > 4) = 0.184$

Decision

$P = 0.184 > \alpha = 0.05$

Accept H_0 at $\alpha = 5\%$

Conclusion

There is no significant difference between matched graph.

Exercise 4

9. If

$r_{12} = 0.5$ $r_{23} = 0.1$, $r_{13} = 0.4$

$$r_{12.3} = \frac{r_{12} - r_{13}\, r_{23}}{\sqrt{1 - r_{13}^2}\sqrt{1 - r_{23}^2}} = \frac{0.5 - 0.4 \times 0.1}{\sqrt{1 - 0.4^2}\sqrt{1 - 0.1^2}} = \frac{0.46}{0.916 \times 0.994} = 0.505$$

$$r_{13.2} = \frac{r_{13} - r_{12}\, r_{32}}{\sqrt{1 - r_{12}^2}\sqrt{1 - r_{32}^2}} = \frac{0.4 - 0.5 \times 0.1}{\sqrt{1 - 0.5^2}\sqrt{1 - 0.1^2}} = \frac{0.4 - 0.05}{\sqrt{0.75}\sqrt{0.99}} = 0.406$$

10. For a

$r_{12} = 0.4$, $r_{23} = 0.5$, $r_{13} = 0.6$

$$R_{1.23} = \sqrt{\frac{r_{12}^2 + r_{13}^2 - 2\, r_{12}\, r_{13}\, r_{23}}{1 - r_{23}^2}}$$

$$= \sqrt{\frac{0.4^2 + 0.6^2 - 2 \times 0.4 \times 0.6 \times 0.5}{1 - 0.5^2}} = 0.611$$

$$r_{23.1} = \frac{r_{23} - r_{21}\, r_{31}}{\sqrt{1 - r_{21}^2}\ \sqrt{1 - r_{31}^2}} = \frac{0.5 - 0.4 \times 0.6}{\sqrt{1 - 0.4^2}\sqrt{1 - 0.6^2}} = \frac{0.26}{\sqrt{0.86}\sqrt{0.64}} = 0.35$$

$R_{1.23}^2 = (0.611)^2 = 0.373$

$r_{23.1} = (0.35)^2 = 0.1225$

11 .Are the

$r_{23} = 0.8$, $r_{31} = -0.5$, $r_{12} = 0.6$

$$R_{1.23} = \sqrt{\frac{r_{12}^2 + r_{13}^2 - 2\, r_{12}\, r_{13}\, r_{23}}{1 - r_{23}^2}}$$

$$= \sqrt{\frac{0.6^2 + (-0.5)^2 - 2 \times 0.6 \times (-0.5) \times 0.8}{1 - 0.8^2}} = 1.74$$

It is impossible. Hence, given data is inconsistent.

12. From the data

$r_{12} = 0.77$, $r_{13} = 0.72$, $r_{23} = 0.52$

$$r_{12.3} = \frac{r_{12} - r_{13}\, r_{23}}{\sqrt{1 - r_{13}^2}\sqrt{1 - r_{23}^2}} = \frac{0.77 - 0.72 \times 0.52}{\sqrt{1 - 0.72^2}\ \sqrt{1 - (0.52)^2}} = \frac{0.3956}{0.693 \times 0.894} = 0.638$$

$$r_{13.2} = \frac{r_{13} - r_{12}\, r_{32}}{\sqrt{1 - r_{12}^2}\sqrt{1 - r_{32}^2}} = \frac{0.72 - 0.77 \times 0.52}{\sqrt{1 - 0.77^2}\sqrt{1 - 0.52^2}} = 0.85$$

$$r_{23.1} = \frac{r_{23} - r_{21}\, r_{31}}{\sqrt{1 - r_{21}^2}\ \sqrt{1 - r_{31}^2}} = \frac{0.52 - 0.77 \times 0.72}{\sqrt{1 - 0.77^2}\sqrt{1 - 0.72^2}} = -0.077$$

13. Suppose a computer

$r_{12} = 0.91$, $r_{13} = 0.33$, $r_{23} = 0.81$

$$r_{12.3} = \frac{r_{12} - r_{13}\, r_{23}}{\sqrt{1 - r_{13}^2}\sqrt{1 - r_{23}^2}} = \frac{0.91 - 0.33 \times 81}{\sqrt{1 - 0.33^2}\sqrt{1 - 0.81^2}} = 1.16$$

It is not possible. It contains error.

14. The following

$r_{12} = 0.8$, $r_{13} = 0.44$, $r_{23} = 0.54$

$$r_{13.2} = \frac{r_{13} - r_{12}\, r_{32}}{\sqrt{1 - r_{12}^2}\sqrt{1 - r_{32}^2}} = \frac{0.44 - 0.8 \times 0.54}{\sqrt{1 - 0.8^2}\sqrt{1 - 0.54^2}} = 0.0158$$

15. Consider the

$n = 10$, $\Sigma X_1 = 10$, $\Sigma X_2 = 20$, $\Sigma X_3 = 30$, $\Sigma X_1^2 = 20$, $\Sigma X_2^2 = 68$, $\Sigma X_3^2 = 170$, $\Sigma X_1 X_2 = 10$, $\Sigma X_1 X_3 = 15$, $\Sigma X_2 X_3 = 64$

$$r_{12} = \frac{n \sum X_1X_2 - \sum X_1 \sum X_2}{\sqrt{n \sum X_1^2 - (\sum X_1)^2}\sqrt{n \sum X_2^2 - (\sum X_2)^2}}$$

$$= \frac{10 \times 10 - 10 \times 20}{\sqrt{10 \times 20 - 10^2}\sqrt{10 \times 68 - 20^2}} = \frac{-100}{\sqrt{100}\sqrt{280}} = -0.59$$

$$r_{13} = \frac{n \sum X_1X_3 - \sum X_1 \sum X_3}{\sqrt{n \sum X_1^2 - (\sum X_1)^2}\sqrt{n \sum X_3^2 - (\sum X_3)^2}}$$

$$= \frac{10 \times 15 - 10 \times 30}{\sqrt{10 \times 20 - 10^2}\sqrt{10 \times 170 - 30^2}} = \frac{-150}{\sqrt{100}\sqrt{800}} = -0.53$$

$$r_{23} = \frac{n \sum X_2X_3 - \sum X_2 \sum X_3}{\sqrt{n \sum X_2^2 - (\sum X_2)^2}\sqrt{n \sum X_3^2 - (\sum X_3)^2}}$$

$$= \frac{10 \times 64 - 20 \times 30}{\sqrt{10 \times 68 - 20^2}\sqrt{10 \times 170 - 30^2}} = \frac{40}{\sqrt{280}\sqrt{800}} = 0.084$$

Now,

$$r_{12.3} = \frac{r_{12} - r_{13}\, r_{23}}{\sqrt{1 - r_{13}^2}\sqrt{1 - r_{23}^2}} = \frac{-0.59 - (-0.53) \times 0.084}{\sqrt{1 - (-0.53)^2}\sqrt{1 - (0.084)^2}} = -0.645$$

$$R_{1.23} = \sqrt{\frac{r_{12}^2 + r_{13}^2 - 2\, r_{12}\, r_{13}\, r_{23}}{1 - r_{23}^2}}$$

$$= \sqrt{\frac{(-0.59)^2 + (-0.53)^2 - 2 \times (-0.59) \times (-0.53) \times 0.084}{1 - (0.084)^2}} = 0.76$$

16. From the information

X_1	X_2	X_3	X_1X_2	X_1X_3	X_2X_3	X_1^2	X_2^2	X_3^2
6	14	21	84	126	294	36	196	441
8	16	22	128	176	352	64	256	484

9	17	27	153	243	459	81	289	729
11	18	29	198	319	522	121	324	841
12	20	31	240	372	620	144	400	964
14	23	32	322	448	736	196	529	1024
$\sum X_1 = 60$	$\sum X_2 = 108$	$\sum X_3 = 162$	$\sum X_1X_2 = 1125$	$\sum X_1X_3 = 1684$	$\sum X_2X_3 = 2983$	$\sum X_1^2 = 642$	$\sum X_2^2 = 1994$	$\sum X_3^2 = 4480$

$$r_{12} = \frac{n\sum X_1X_2 - \sum X_1 \sum X_2}{\sqrt{n\sum X_1^2 - (\sum X_1)^2}\sqrt{n\sum X_2^2 - (\sum X_2)^2}}$$

$$= \frac{6 \times 1125 - 60 \times 108}{\sqrt{6 \times 642 - (60)^2}\sqrt{6 \times 1994 - (108)^2}} = 0.98$$

$$r_{13} = \frac{n\sum X_1X_3 - \sum X_1 \sum X_3}{\sqrt{n\sum X_1^2 - (\sum X_1)^2}\sqrt{n\sum X_3^2 - (\sum X_3)^2}}$$

$$= \frac{6 \times 1684 - 60 \times 162}{\sqrt{6 \times 642 - (60)^2}\sqrt{6 \times 4480 - (162)^2}} = 0.95$$

$$r_{23} = \frac{n\sum X_2X_3 - \sum X_2 \sum X_3}{\sqrt{n\sum X_2^2 - (\sum X_2)^2}\sqrt{n\sum X_3^2 - (\sum X_3)^2}}$$

$$= \frac{6 \times 2983 - 108 \times 162}{\sqrt{6 \times 1994 - (108)^2}\sqrt{6 \times 4480 - (162)^2}} = 0.92$$

Now,

$$r_{12.3} = \frac{r_{12} - r_{13}\, r_{23}}{\sqrt{1 - r_{13}^2}\sqrt{1 - r_{23}^2}} = \frac{0.98 - 0.95 \times 0.92}{\sqrt{1 - (0.95)^2}\sqrt{1 - (0.92)^2}} = 0.86$$

$$r_{13.2} = \frac{r_{13} - r_{12}\, r_{32}}{\sqrt{1 - r_{12}^2}\sqrt{1 - r_{32}^2}} = \frac{0.95 - 0.98 \times 0.92}{\sqrt{1 - (0.98)^2}\sqrt{1 - (0.92)^2}}$$

$$= \frac{0.0484}{0.198 \times 0.391} = 0.62$$

$$r_{23.1} = \frac{r_{23} - r_{21}\, r_{31}}{\sqrt{1 - r_{21}^2}\;\sqrt{1 - r_{31}^2}} = \frac{0.92 - 0.98 \times 0.95}{\sqrt{1 - 0.98^2}\sqrt{1 - 0.95^2}} = -0.177$$

17. Given the following..........

Let β_1 and β_2 be population regression

Coefficient of Y on X_1 keeping X_2

Constant and Y on X_2 keeping X_1

Constant

Problem of test

H_0: $\beta_1 = 0$

H_1: $\beta_1 \neq 0$

Test statistic

$$t = \frac{b_1}{sb_1} = \frac{4}{1.2} = 3.33$$

Critical value

let α = 5% then critical value

$t_{\alpha/2(n-k-1)} - t_{0.05/2}(17) = 2.11$

Decision

$t = 3.33 > t_{\alpha/2}(n - k - 1) = 2.11$

Reject H_0 at α = 5%

Again

Problem H_0 test

$H_0 : \beta_1 = 0$

$H_1 : \beta_2 \neq 0$

Test statistic

$$t = \frac{b_2}{sb_2} = \frac{3}{0.8} = 3.75$$

Critical value $t_{\alpha/2}(n - k - 1) = 2\%$

Decision

$t = 3.75 > t_{\alpha/2(n-k-1)} = 2.11$

Reject H_0 at α = 5%

Conclusion

Each explanary variable makes a significant contribution to the regression model.

18. In order to

n = 10

TSS = 357.6

SSE = 23.5

SSR = TSS – E = 397.6 – 23.3 = 374.1

Let β_1 of β_2 be regression coefficient of y on X_1 keeping X_2 constant and Y on X_2 keeping X_1 Constant

Problem to test

H_0: $\beta_1 = \beta_2 = 0$

H_1: At least are β_j is different j = 1,2

$$\text{Test statistic } F = \frac{MSR}{MSE} = \frac{\frac{SSR}{K}}{\frac{SSE}{n - K - 1}}$$

$$= \frac{\frac{374.1}{2}}{\frac{23.5}{10-2-1}}$$

$$= \frac{187.05}{3.35}$$

$$= 55.83$$

Critical value

At $\alpha = 5\%$ Critical value

$F_{\alpha\{k, n-k-1\}} = F_{0.05(2,7)} = 4.74$

Decision

$F = 55.83 > F_{\alpha(k, v-\kappa-1)} = 4.74$

Reject $H_0 = \alpha = 5\%$

Conclusion

Regression equation is significance

19. Suppose you are

$y = 5 + 18x_1 + 20x_1$

$x = 28$

TSS = 250

SSE = 100

$Sb_1 = 3.2$

$Sb_2 = 5.5$

SSR = TSS – SSE = 250 - 100 = 150

Let β_1 and β_2 be regression coefficient of Y on X_1 keeping X_2 constant and Y on X_2 keeping X_1 Constant respectively

Problem to test

H_0: $\beta_2 = 0$

H_1: $\beta_2 \neq 0$

Test statistic

$$t = \frac{b_2}{sb_2} = \frac{20}{5.5} = 3.63$$

Critical value

At $\alpha = 1\%$ critical value

$t\alpha_{/2(n-k-1)} = t_{0.01/2}\ (28 - 2 - 1) = 2.787$

Decision

$t = 3.63 > t_{\alpha/2(n-K-1)} = 2.787$

Reject H_0 at $\alpha = 1\%$

Conclusion

Regression coefficient s significant

Again

Problem to test

H_0: $\beta_1 = \beta_2 = 0$

H_1: At least are $\beta_j \neq 0$, j = (1, 2)

Test statistic $F = \frac{MSR}{MSE}$

$$= \frac{\frac{150}{2}}{\frac{100}{28-2-1}}$$

$$= \frac{75}{4} = 18.75$$

Critical value

At $\alpha = 5\%$ critical value is $F_{\alpha(k, n-k-1)}$

F0.05(2,25) = 3.39

Decision

$F = 18.75 > F_{\alpha\ (k, n-k-1)} = 3.39$

Reject H_0 at $\alpha = 5\%$

Conclusion

On all reprehension coefficients are significant

20. From the following

Here, $\sum x_1 = 13$, $\sum x_2 = 11$, $\sum x_3 = 51$, $\sum x_1^2 = 63$, $\sum x_2^2 = 95$, $\sum x_1x_3 = 136$, $\sum x_1x_2 = -24$, $\sum x_3^2 = 450$, n = 10

To find regression equation of x_3 on x_1 and x_2

$x_3 = a + b_1x_1 + b_2x_2$

Now, $\sum x_3 = na + b_1 \sum x_1 + b_2 \sum x_2$

Or, $51 = 10a + 13b_1 + 11b_2$ ………. (i)

$\sum x_1x_3 = a \sum x_1 + b_1 \sum x_1^2 + b_2 \sum x_1x_2$

Or, $77 = 13a + 63b_1 + (-24)\ b_2$

Or, $77 = 13a + 63b_1 - 24b_2$ ……… (ii)

$\sum x_2x_3 = a\sum x_2 + b_1\sum x_1x_2 + b_2\sum x_2^2$

Or, $136 = 11a + (-24)\,b_1 + 95b_2$

Or, $136 = 11a - 24b_1 + 95b_2$ ……... (iii)

Solving (i), (ii) and (iii)

$a = 1.008,\ b_1 = 1.676,\ b_2 = 1.738$

Hence, regression equations

$x_3 = 1.008 + 1.676\,x_1 + 1.738\,x_2$

When, $x_1 = 1$ and $x_2 = 4$

$x_3 = 1.008 + 1.676\,x_1 + 1.738 \times 4 = 9.636$

$TSS = \sum(x_3 - \bar{x}_3)^2 = \sum x_3^2 - n\,\bar{x}_3^{\,2} = 450 - 10 \times \left(\frac{51}{10}\right)^2 = 189.9$

$SSE = \sum x_3^2 - a\sum x_3 - b_1\sum x_1x_3 - b_2\sum x_2x_3$

$= 450 - 1.008 \times 51 - 1.676 \times 77 - 1.738 \times 136 = 33.17$

$SSR = TSS - SSE = 189.9 - 33.17 = 156.728$

$S_{3.12} = \sqrt{\frac{SSE}{n-k-1}} = \sqrt{\frac{33.17}{10-2-1}} = 2.17$

$R^2 = \frac{SSR}{TSS} = \frac{156.728}{189.9} = 0.82$

21. From the following

$\Sigma(Y - \bar{y})^2 = 3450$, $\Sigma(Y - \hat{Y})^2 = 365.7$, $\Sigma X_1X_2 = 5779$, $\Sigma YX_2 = 6796$, $\Sigma YX_1 = 40830$, $\Sigma Y^2 = 48139$, $\Sigma X_1^2 = 3483$, $\Sigma X_2^2 = 976$, $\Sigma Y = 753$, $\Sigma X_1 = 643$, $\Sigma X_2 = 106$, $n = 12$

To find regression equation of y on x_1 and x_2

i.e. $Y = a + b_1X_1 + b_2X_2$

$\sum Y = na + b_1\sum X_1 + b_2\sum X_2$

Or, $753 = 12a + 643\,b_1 + 106\,b_2$ ……... (i)

$\sum YX_1 = a\sum X_1 + b_1\sum X_1^2 + b_2\sum X_1X_2$

Or, $40830 = 643\,a + 3483\,b_1 + 5779\,b_2$ …….. (ii)

$\sum YX_2 = a\sum X_2 + b_1\sum X_1X_2 + b_2\sum X_2^2$

Or, $6796 = 106\,a + 5779\,b_1 + 976\,b_2$ ……… (iii)

Solving (i), (ii) and (iii)

$a = 30.69,\ b_1 = -0.0038,\ b_2 = 3.652$

Hence regression equation is

$Y = 30.69 - 0.0038\,X_1 + 3.652\,X_2$

$SSE = \sum(Y - \hat{Y})^2 = 365.7$

$SE_{Y.X_1X_2} = \sqrt{\frac{SSE}{n-k-1}} = \sqrt{\frac{365.7}{12-1-1}} = 6.37$

$TSS = \sum(Y - \bar{y})^2 = 3450$

$SSR = TSS - SSE = 3450 - 365.7 = 3084.3$

Coefficient of determination $R^2 = \frac{SSR}{TSS} = \frac{3084.3}{3450} = 0.89$

22. The table shows

X_1	X_2	X_3	X_1X_2	X_1X_3	X_2X_3	X_2^2	X_3^2
5	12	51	60	255	612	144	2601
7	20	55	140	385	1100	400	3025
8	30	58	240	464	1740	900	3364
6	40	60	240	360	2400	1600	3600
10	33	70	330	700	2310	1089	4500
9	25	66	225	594	1650	625	4356
$\sum X_1 = 45$	$\sum X_2 = 160$	$\sum X_3 = 360$	$\sum X_1X_2 = 1235$	$\sum X_1X_3 = 2758$	$\sum X_2X_3 = 9812$	$\sum X_2^2 = 4758$	$\sum X_3^2 = 21846$

To find regression equation of X_1 on X_2 and X_3

$X_1 = a + b_2X_2 + b_3X_3$

$\sum X_1 = n\,a + b_2\sum X_2 + b_3\sum X_3$

Or, $45 = 6a + 160\,b_2 + 360\,b_3$ …………. (i)

$\sum X_1X_2 = a\sum X_2 + b_2\sum X_2^2 + b_3\sum X_2X_3$

Or, $1235 = 160\,a + 4758\,b_2 + 9812\,b_3$ ………… (ii)

$\sum X_1X_3 = a\sum X_3 + b_2\sum X_2X_3 + b_3\sum X_3^2$

Or, $2758 = 360\,a + 9812\,b_2 + 21846\,b_3$ ……….. (iii)

Solving (i), (ii) and (iii)

$a = -7.862,\ b_2 = -0.048,\ b_3 = 0.277$

Hence, regression equation is

$X_1 = -7.862 - 0.048X_2 + 0.277X_3$

When, $X_2 = 50$, $X_3 = 100$

$X_1 = -7.862 - 0.048 \times 50 + 0.277 \times 100 = 19.78$

23. From the following set

Y	X_1	X_2	YX_1	YX_2	X_1X_2	X_1^2	X_2^2
6	1	3	6	18	3	1	9
10	3	– 1	30	– 10	– 3	9	1

9	2	4	18	36	8	4	16
14	–2	7	– 28	98	– 14	4	49
7	3	2	21	14	6	9	4
5	6	– 4	30	– 20	– 24	36	16
$\sum Y$ = 51	$\sum X_1$ = 13	$\sum X_2$ = 11	$\sum YX_1$ = 77	$\sum YX_2$ = 136	$\sum X_1X_2$ = – 24	$\sum X_1^2$ = 63	$\sum X_2^2$ = 95

To find regression of Y on X_1 and X_2

$Y = a + b_1 X_1 + b_2 X_2$

$\sum Y = na + b_1 \sum X_1 + b_2 \sum X_2$

Or, $51 = 6a + 13b_1 + 11b_2$ …….. (i)

$\sum YX_1 = a \sum X_1 + b_1 \sum X_1^2 + b_2 \sum X_1X_2$

Or, $77 = 13a + 63b_1 + (-24) b_2$ ……… (ii)

$\sum YX_2 = a \sum X_2 + b_1 \sum X_1X_2 + b_2 \sum X_2^2$

$136 = 11a + (-24) b_1 + 95 b_2$ ………… (iii)

Solving (i), (ii) and (iii)

$a = 12.424$, $b_1 = -1.487$, $b_2 = -0.382$

Hence regression equation is

$Y = 12.424 - 1.487 X_1 - 0.382 X_2$

$b_1 = -1.487$ mean when X_1 changes by 1 the Y decreases by 1.487 keeping X_2 constants $b_2 = -0.382$ mean when X_2 changes by 1 then Y decreases by 0.382 keeping X_1 constant.

When $X_1 = -10$, $X_2 = 4$

$Y = 12.424 - 1.487 \times (-10) - 0.382 \times 4 = 25.766$

24. A developer of food

Piglet number	Initial weight X_1	Initial age X_2	Weight gain Y	Y^2	YX_1	YX_2	X_1X_2	X_1^2	X_2^2
1	39	8	7	49	273	56	312	1521	64
2	52	6	6	36	312	36	312	2304	36
3	49	7	8	64	392	56	343	2401	49
4	46	12	10	100	460	120	552	2116	144
5	61	9	9	81	549	81	549	3721	81
6	35	6	5	25	175	30	210	225	36
7	25	7	3	9	75	21	175	625	49
8	55	4	4	16	220	16	220	3025	16
	362	59	52	380	2456	416	2673	17338	475

$\sum X_1 = 362$, $\sum X_2 = 59$, $\sum Y = 52$, $\sum Y^2 = 280$, $\sum YX_1 = 2456$, $\sum X_1X_2 = 2673$, $\sum X_1^2 = 17338$, $\sum X_2^2 = 475$

To find regression equation of Y on X_1 and X_2 as

$Y = a + b_1 X_1 + b_2 X_2$

$\sum Y = n a + b_1 \sum X_1 + b_2 \sum X_2$

Or, $52 = 8a + 362 b_1 + 59 b_2$ ……….. (i)

$\sum X_1Y = a \sum X_1 + b_1 \sum X_1^2 + b_2 \sum X_1 X_2$

$2456 = 362a + 17338 b_1 + 2673 b_2$ ……… (ii)

$\sum X_2Y = a \sum X_2 + b_1 \sum X_1X_2 + b_2 \sum X_2^2$

$416 = 59a + 2673 b_1 + 479 b_2$ …….. (iii)

Solving (i), (ii) and (iii)

$a = -3.66$, $b_1 = 0.105$, $b_2 = 0.732$

Hence regression equation is

$Y = -3.66 + 0.105 X_1 + 0.732 X_2$

When $X_1 = 48$, $X_2 = 9$

$Y = -3.66 + 0.105 \times 48 + 0.732 \times 9 = 7.968 \approx 8$

$SSE = \sum Y^2 - a \sum Y - b_1 \sum X_1Y - b_2 \sum X_2Y$

$= 380 - (-3.66) \times 52 - 0.105 \times 2456 - 0.732 \times 416 = 7.928$

$$S_e = \sqrt{\frac{SSE}{n - k - 1}} = \sqrt{\frac{7.928}{8 - 2 - 1}} = 1.259$$

11. Carry out ANOVA of following output of wheat per field obtained as a result of 3 varieties of wheat A, B and C.

A 10	B 5	A 20	C 15
B 6	A 15	C 11	B 10
C 22	B 12	C 18	A 16

Solution

Problem to test

H_0: The difference in efficiency among 3 varieties of wheat are not significant.

H_1: The difference in efficiency among 3 varieties of wheat are significant.

Now,

A	10	15	20	16
B	6	5	12	10
C	22	11	18	15

calculations

Text entry technique					$T_{i.}$
A	10	15	20	16	61
B	6	5	12	10	33
C	22	11	18	15	66
					$\Sigma T_{i.}$ =160

Here,

$G = \Sigma T_{i.} = 160$, $N = 12$

$$C.F. = \frac{G^2}{N} = \frac{(160)^2}{12} = 2133.33$$

$$\sum_{i,j} u_{ij}^2 = 100+225+400+256+36+25+144+100+484+121+324+225=2240$$

$$TSS = \sum_{i,j} u_{ij}^2 - C.F. = 2240 - 2133.33 = 306.67$$

$$SST = \frac{\Sigma_i T_{i.}^2}{r} - C.F.$$

$$= \frac{1}{4}\{3721+1089+4356\} - 2133.33 = 2291.5 - 2133.33 = 158.167$$

$SSE = TSS - SST$

$= 306.67 - 158.167 = 148.5$

ANOVA table

S.V.	d.f.	S.S.	M.S.	F_{Cal}	F_{Tab}
Treatment	2	158.167	79.083	4.793	$F_{0.05(2,9)}$ =4.26
Error	9	148.5	16.5		
Total	11	306.67			

Decision

$F_T = 4.79 > F_{0.05(2,9)} = 4.26$, Reject H_0 at 5% level of significance.

Conclusion

The output of wheat per field of 3 three types of wheats are significant.

12. Carry out ANOVA for following design.

A 8	C 10	A 6	B 10
C 12	B 8	B 9	A 8
B 10	A 8	C 10	C 9

Also calculate the relative efficiency of the design with respect to CRD.

Solution

Here, treatments are replicated along row wise as only. Hence this is the case of RBD.

Problem to test

H_{0T}: There is no significant difference between treatments.

H_{1R}: There is significant difference between treatments.

H_{0B}: There is no significant difference between block

H_{1B}: There is significant difference between block.

Here,P

Treatments	Block				$T_{i.}$
	I	II	III	IV	
A	8	8	6	8	30
B	10	8	9	10	37
C	12	10	10	9	41
$T_{.j}$	30	26	25	27	108

$N = 3 \times 4 = 12$

$$G = \sum_i T_{i.} = \sum_j T_{j.} = 108$$

$$C.F. = \frac{G^2}{N} = \frac{(108)^2}{12} = 972$$

$$\sum_i \sum_j y_{ij}^2 = 64+100+144+64+64+100+36+81+100+64+100+81 = 998$$

$$TSS = \sum_i \sum_j y_{ij}^2 - C.F. = 998 - 972 = 26$$

$$SST = \frac{1}{r}\sum_i T_{i.}^2 - C.F.$$

$$= \frac{1}{4}[900+1369+1681] - 972 = 987.5 - 972 = 15.5$$

$$SSB = \frac{1}{t}\sum_i T_{.j}^2 - C.F.$$

$$= \frac{1}{3}[900+676+625+729] - 972 = 976.7 - 972 = 4.67$$

$SSE = TSS - SST - SSB$

$= 48.25 - 6.5 - 16.25 = 25.5$

ANOVA table

S.V.	d.f.	S.S.	M.S.	F_{Cal}	F_{Tab}
Treatments	2	15.5	7.75	$F_T = 7.97$	$F_{0.05(2,6)} = 5.14$
Blocks	3	4.67	1.556	$F_R = 1.6$	$F_{0.05(3,6)} = 4.76$
Error	6	5.83	0.972		
Total	11	26			

Decision

$F_T = 7.97 > F_{0.05(2,6)} = 5.14$, accept H_{0T} at 5% level of significance.

$F_B = 1.6 < F_{0.05(3,6)} = 4.76$, accept H_{0B} at 5% level of significance.

Conclusion

There is significant difference between treatments and there is no significant difference between blocks.

Now, efficiency of RBD with respect to CRD is

$$\frac{s_e'^2}{s_e^2} = \frac{r(t-1)\,MSE + (r-1)\,MSB}{(rt-1)\,MSE} = \frac{4\times2\times0.972 + 3\times1.556}{11\times0.972} = 1.163$$

RBD is 16.3% efficient than CRD.

13. Set up the analysis of variance for the following results of a design.

A 10	B 15	C 20
B 25	C 10	A 15
C 25	A 20	B 15

Also calculate the efficiency of the design over i) RBD ii) CRD.

Ans: $F_R = F_C = 0.241$, $F_T = 0.142$, Accept H_{0R}, H_{0C}, H_{0T}, Efficiency = 0.74, 0.74, 0.62

Solution

Problem to test

H_{0R}: There is no significant difference between rows

H_{1R}: There is significant difference between rows.

H_{1C}: There is no significant difference between columns

H_{1C}: There is significant difference between columns.

H_{0T}: There is no significant difference between treatments

H_{1T}: there is significant difference between treatments.

Row	Column			
	I	**II**	**III**	$T_{i..}$
I	A 10	B 15	C 20	45
II	B 25	C 10	A 15	50
III	C 25	A 20	B 15	60
$T_{.j.}$	60	45	50	155

$T_{..A} = 10+15+20 = 45$

$T_{..B} = 15+25+15 = 55$

$T_{..C} = 20+10+25= 55$

G = 1154.3, N =9

$$C.F. = \frac{G^2}{N} = \frac{(155)^2}{9} = 2669.44$$

$$\sum_{(i,j,k)} y_{ijk}^2 = (10)^2 + (15)^2 + (20)^2 + (25)^2 + (10)^2 + \ldots\ldots\ldots + (15)^2 = 2925$$

$$TSS = \sum_{(i,j,k)} y_{ijk}^2 - C.F.$$

$$= 2925 - 2669.44 = 255.56$$

$$SSR = \frac{\Sigma_i T_{i..}^2}{m} - C.F.$$

$$= \frac{1}{3}\{(45)^2+(50)^2+(60)^2\} - 2669.44$$

$$=2708.33-2669.44$$

$$= 38.89$$

$$SSC = \frac{\Sigma_j T_{.j.}^2}{m} - C.F.$$

$$= \frac{1}{3}\{(60)^2+(45)^2+(50)^2\} - 2669.44$$

$$=2708.33-2669.44$$

$$= 38.89$$

$$SST = \Sigma \frac{T_{..k}^2}{m} - C.F.$$

$$= \frac{1}{3}\{(45)^2+(55)^2+(55)^2\} - 2669.44$$

$$=2691.66-2669.44$$

$$= 22.22$$

SSE = TSS –SSR –SSC – SST

= 255.56 – 38.89 – 38.89 – 22.22 = 155.56

ANOVA table

S.V.	d.f.	S.S.	M.S.	F_{Cal}	F_{Tab}
Row	2	38.89	19.445	$F_R = 0.25$	
Column	2	38.89	19.445	$F_C = 0.25$	
Treatment	2	22.22	11.11	$F_T = 0.1429$	$F_{0.05(2,3)} = 9.55$
Error	2	155.56	77.778		
Total	8	255.56			

Decision

$F_R = 0.25 < F_{0.05(2,3)} = 9.55$, accept H_{0T} at 5% level of significance.

$F_C = 0.25 < F_{0.05(2,3)} = 9.55$, accept H_{0C} at 5% level of significance.

$F_T = 0.1429 < F_{0.05(2,3)} = 9.55$, accept H_{0T} at 5% level of significance.

Conclusion

There is no significant difference between rows, there is no significant difference between columns and there is no significant difference between treatments.

Now,

Efficiency of LSD with respect to CRD is

$$\frac{\sigma_e'^2}{\sigma_e^2} = \frac{(m-1)\,MSE + MSR + MSC}{(m+1)\,MSE}$$

$$= \frac{2 \times 77.78 + 19.445 + 19.445}{4 \times 77.78} = 0.625 = 62.5\%$$

LSD is 37.5% less efficient than CRD.

m = 4, MSR = 0.711, MSC = 0.734, MST = 3.554, MSE = 1.177

When row is taken as block

$$\frac{\sigma'^2_e}{\sigma_e^2} = \frac{(m-1)\,MSE + MSC}{mMSE} = \frac{2 \times 77.78 + 19.445}{3 \times 77.78} = 0.75 = 75\%$$

Hence, LSD is 25% less efficient than RBD when row is taken as block.

When column is taken as block

$$\frac{\sigma'^2_e}{\sigma_e^2} = \frac{(m-1)\,MSE + MSR}{mMSE} = \frac{2 \times 77.78 + 19.445}{3 \times 77.78} = 0.75 = 75\%$$

Hence LSD is 25% less efficient than RBD when column is taken as block.

14. The table given below are yields of 3 varieties in a 4 replicate experiment for which one observation is missing. Estimate the missing value and then analyze the data.

P 19	R 29	P 23	Q 33
Q 26	P ?	Q 27	R 26
R 21	Q 28	R 22	P 26

Solution

Treatment	Block				$T_{i.}$
	I	II	III	IV	
P	19	? (x)	23	26	68+x
Q	26	28	27	33	114
R	21	29	22	26	98
$T_{.j}$	66	57+x	72	85	280+x

Here T' = 68, B' = 57, G' = 280, t = 3, r = 4

$$\text{Now, missing value } x = \frac{T't + B'r - G'}{(t-1)(r-1)} = \frac{68 \times 3 + 57 \times 4 - 280}{2 \times 3} = 25.33$$

Problem to test

H_{0T} : There is no significant difference between treatments.

H_{1T} : There is significant difference between treatments.

H_{0B} : There is no significant difference between blocks.

H_{1B} : There is significant difference between blocks.

G = G' +x = 280+25.33 = 305.33, N = 4 × 3 = 12

$$C.F. = \frac{G^2}{N} = \frac{(305.33)^2}{12} = 7768.867$$

$$\sum_i \sum_j y_{ij}^2 = (19)^2+(25.33)^2+(23)^2+(26)^2+(26)^2+\ldots\ldots+(26)^2 = 7927.609$$

$$TSS = \sum_i \sum_j y_{ij}^2 - C.F.$$

$$= 7927.609 - 7768.867 = 158.742$$

$$SST = \frac{1}{r}\sum_i T_{i.}^2 - C.F.$$

$$= \frac{1}{4}\{93.33^2+114^2+98^2\} - 7768.867 = 7827.622 - 7768.867 = 58.75$$

$$SSB = \frac{1}{t}\sum_j T_{.j}^2 - C.F.$$

$$= \frac{1}{3}\{66^2+82.33^2+72^2+85^2\} - 7768.867 = 7847.743 - 7768.867 = 78.875$$

$$\text{Adjustment factor (k)} = \frac{(B' + tT' - G')^2}{t(t-1)(r-1)^2} = \frac{(57 + 3 \times 68 - 280)^2}{3 \times 2 \times 9} = 6.68$$

$$SST_A = SST - k$$

$$= 58.75 - 6.68 = 52.07$$

$$SSE = TSS - SST_A - SSB$$

$$= 158.742 - 52.07 - 78.875$$

= 27.797

S.V.	d.f.	S.S.	M.S.S	F_{Cal}	F_{Tab}
Treatments	2	52.07	26.035	4.683	$F_{0.05(2,5)}$ =5.79
Blocks	3	78.875	26.292	4.729	$F_{0.05(3,5)}$ = 5.41
Error	5	27.797	5.559		
Total	10	158.742			

Decision

$F_T = 4.683 < F_{0.05(2,5)} = 5.79$, accept H_{0T} at 5% level of significance.

$F_B = 4.729 < F_{0.05(3,5)} = 5.41$, accept H_{0B} at 5% level of significance.

Conclusion

There is no significant difference between treatments.

There is no significant difference between blocks.

15. The table given below represents the yields of 4 varieties in a 4 replicate experiment for which one observation is missing. Estimate the missing value and then carry out the ANOVA

A 12	C 19	B 10	D 8
C 18	B 12	D 6	A ?
B 22	D 10	A 5	C 21
D 12	A 7	C 27	B 17

Solution

					$T_{i..}$
	A 12	C 19	B 10	D 8	49
	C 18	B 12	D 6	A ?	36
	B 22	D 10	A 5	C 21	58+x
	D 12	A 7	C 27	B 17	63
$T_{.j.}$	64	48	48	46+x	206+x

Here,

m = 4, R' = 58, C' = 46, T' = 12+7+5 = 24, G' = 206

$$\text{Missing value } (x) = \frac{m(R' + C' + T') - 2G'}{(m - 1)(m - 2)}$$

$$= \frac{4(58 + 46 + 24) - 2 \times 206}{3 \times 2} = \frac{512 - 412}{6} = 16.67$$

Problem to test

H_{0R}: There is no significant difference between rows

H_{1R}: There is significant difference between rows.

H_{1C}: There is no significant difference between columns

H_{1C}: There is significant difference between columns.

H_{0T}: There is no significant difference between treatments

H_{1T}: there is significant difference between treatments.

Now,

G = G' +x = 206 + 16.67 = 222.67

N = $m^2 = 4^2 = 16$

$$C.F. = \frac{G^2}{N} = \frac{(222.67)^2}{16} = 3098.87$$

$$\sum_{i,j,k} y_{ijk}^2 = 12^2 + 19^2 + 10^2 + 8^2 + 18^2 + \ldots\ldots + 12^2 = 3711.889$$

$$TSS = \sum_{i,j,k} y_{ijk}^2 - C.F.$$

$$= 3711.889 - 3098.87 = 613.02$$

$$SSR = \frac{1}{m}\sum_{i} T_{i..}^2 - C.F.$$

$$= \frac{1}{4}\{49^2 + 52.67^2 + 58^2 + 63^2\} - 3098.87 = 3127.03 - 3098.87 = 28.16$$

$$SSC = \frac{1}{m} \sum_{j} T._j.^2 - C.F.$$

$$= \frac{1}{4} \{64^2 + 48^2 + 48^2 + 62.67^2\} - 3098.87 = 3157.882 - 3098.87 = 59.012$$

$T.._A$ = 12+7+5+16.67 = 40.67

$T.._B$ = 22+12+10+17 = 61

$T.._C$ = 18+19+27+21 = 85

$T.._D$ = 12+10+6+8 = 36

$$SST = \frac{1}{m} \sum_{i} T.._k^2 - C.F.$$

$$= \frac{1}{4} \{40.67^2 + 61^2 + 85^2 + 36^2\} - 3098.87 = 375.14$$

$$\text{Adjustment factor (k)} = \frac{\{(m-1)T' + R' + C' - G'\}^2}{\{(m-1)(m-2)\}^2}$$

$$= \frac{\{3 \times 24 + 58 + 46 - 206\}^2}{\{3 \times 2\}^2} = 25$$

$SST_A = SST - k$

$= 375.14 - 25 = 350.14$

$SSE = TSS - SSR - SSC - SST_A$

$= 613.02 - 28.16 - 59.012 - 350.14$

$= 175.708$

ANOVA table

S.V.	d.f.	S.S.	M.S.S.	F_{Cal}	F_{Tab}
Rows	3	28.16	9.387	0.267	$F_{0.05(3,5)} = 5.41$
Columns	3	59.012	19.67	0.559	$F_{0.05(3,5)} = 5.41$
Treatments	3	350.14	116.713	3.321	$F_{0.05(3,5)} = 5.41$
Error	5	613.02	35.141		
Total	14				

Decision

$F_R = 0.267 < F_{0.05(3,5)} = 5.41$, accept H_{0R} at 5% level of significance.

$F_C = 0.559 < F_{0.05(3,5)} = 5.41$, accept H_{0C} at 5% level of significance.

$F_T = 3.321 < F_{0.05(3,5)} = 5.41$, accept H_{0T} at 5% level of significance.

Conclusion

There is no significant difference between rows. There is no significant difference between columns. There is no significant difference between treatments.

16. Complete the following table for the analysis of variance of a design.

S.V.	d.f.	S.S.	M.S.S.	F
Blocks	4	26.8	?	?
Treatment	3	?	?	?
Error	?	?	2.5	
Total	?	85.3		

Solution

S.V.	d.f.	S.S.	M.S.S.	F
Blocks	4	26.8	?	?
Treatment	3	?	?	?
Error	?	?	2.5	
Total	19	85.3		

Since total number of observation = (df of block+1)*(df of treatment+1) = 5*4=20

df of total =20-1 =19

MSE = 2.5

SSE= MSE * df of Error = 2.5*12 =30

SST= TSS-SSB-SSE = 85-3-26.8-30 = 28.5

MSB = SSB/df = 26.8/4 = 6.7

MST = SST/df = 28.5/3 = 9.5

F_b = MSB/MSE = 6.7/2.5 = 2.68

F_t = MST/MSE = 9.5/2.5 = 3.8

S.V.	d.f.	S.S.	M.S.S.	F
Blocks	4	26.8	6.7	2.68
Treatment	3	28.5	9.5	3.8
Error	12	30	2.5	
Total	19	85.3		

17. Fill in the blanks in the following analysis of variance table of a design.

Source of Variation	d.f.	S.S.	M.S.S.	F
Rows	?	72	?	2
Columns	?	?	36	?
Treatments	?	180	?	?
Error	6	?	12	
Total	?	?		

Solution: In the same way as in 16 we can get the values

Source of Variation	d.f.	S.S.	M.S.S.	F
Rows	3	72	24	2
Columns	3	108	36	3
Treatments	3	180	60	5
Error	6	72	12	
Total	15	432		

18. Complete the following table for analysis of variance of a design.

Source of variation	Degree of freedom	Sum of squares	Mean square
Columns	5	?	?
Rows	?	4.2	?
Treatments	?	?	2.43
Error	?	?	0.65
Total	?	39.65	

The columns as representing schools, the rows as classes, the treatments as methods of teaching and the observations as grades based on 100 points. Test the hypothesis that the treatment effects are equal to zero.

Solution: In the same way as in 16 we can get the values

Source of variation	Degree of freedom	Sum of squares	Mean square	
Columns	5	10.3	2.06	3.1
Rows	5	4.2	0.84	1.2
Treatments	5	12.15	2.43	3.7
Error	20	13	0.65	
Total	35	39.65		

Hypothesis to be test

H_{0T}: There is no significant difference between treatments

H_{1T}: there is significant difference between treatments.

F tabulated value $F_{0.05(5,20)} = 2.71$

$F_T = 3.78 > F_{0.05(3,5)} = 2.71$, reject H_{0T} at 5% level of significance.

19. Consider the partially completed ANOVA table below. Complete the ANOVA table and answer the followings. What design was employed? How many treatments were compared? How many observations were analyzed? At 0.05 level of significance, can one conclude that the treatments have different effects? Why?

Source of variation	Sum of Square	Degree of freedom	Mean Square	F
Treatments	231.5	2	?	?

Blocks	?	7	?	?
Error	573.75	?	?	
Total	903.75	23		

Solution

Source of variation	Sum of Square	Degree of freedom	Mean Square	F
Treatments	231.5	2	115.75	2.82
Blocks	98.5	7	14.07	0.34
Error	573.75	14	40.98	
Total	903.75	23		

Hypothesis to be test

H_{0T}: There is no significant difference between treatments

H_{1T}: there is significant difference between treatments.

F tabulated value $F_{0.05(2,14)} = 3.74$

$F_T = 2.82 > F_{0.05(2,14)} = 3.74$, accept H_{0T} at 5% level of significance.

20. From the following ANOVA table of RBD, determine it's efficiency with respect to CRD.

Source	D.F.	S.S.	M.S.S.
Between Blocks	5	21.55	4.31
Between Treatments	3	15.66	5.22
Error	15	12.3	0.82
Total	23	49.51	

Ans: 1.925

Solution

Here, t = 4, r = 6, MSE = 0.82, MST = 5.22, MSB = 4.31

Now, efficiency of RBD with respect to CRD is

$$\frac{s_e'^2}{s_e^2} = \frac{r(t-1)\,MSE + (r-1)\,MSB}{(rt-1)\,MSE} = \frac{6\times3\times0.82 + 5\times4.31}{23\times0.82} = 1.925$$

RBD is 92.5% efficient than CRD.

21. From the following ANOVA table of LSD , determine it's efficiency i)with respect to CRD ii) with respect to RBD when columns are taken as blocks iii) with respect to RBD when rows are taken as blocks.

Source of variation	Degree of freedom	Sum of squares	Mean sum of squa
Rows	3	259.5375	86.4375
Columns	3	155.2725	51.7575
Treatments	3	1372.1225	457.3742
Error	6	156.3700	26.0616
Total	15	1943.0775	

Ans: 1.6605, 1.2464, 1.5791

Solution

m = 4, MSR = 86.4375, MSC = 51.7575, MST = 457.3742, MSE = 26.0616

Now,

Efficiency of LSD with respect to CRD is

$$\frac{\sigma_e'^2}{\sigma_e^2} = \frac{(m-1)\,MSE + MSR + MSC}{(m+1)\,MSE}$$

$$= \frac{3\times26.0616 + 86.4375 + 51.7575}{5\times26.0616} = 1.6605 = 166.05\%$$

LSD is 66.05% efficient than CRD.

When column is taken as block

$$\frac{\sigma_e'^2}{\sigma_e^2} = \frac{(m-1)\,MSE + MSR}{mMSE} = \frac{3\times26.0616 + 89.4375}{4\times26.0616} = 1.6605 = 166.05\%$$

Hence LSD is 66.05% efficient than RBD when column is taken as block.

When row is taken as block

$$\frac{\sigma_e'^2}{\sigma_e^2} = \frac{(m-1)\,MSE + MSC}{mMSE} = \frac{3\times26.0616 + 51.7575}{4\times26.0616} = 1.2465 = 124.65\%$$

Hence, LSD is 24.65% efficient than RBD when row is taken as block.

Excises - 6

8. A computer

$$P = \begin{bmatrix} 0.3 & 0.7 \\ 0.4 & 0.6 \end{bmatrix}$$

2 Step Transition probability matrix

$$P^{(2)} = pp = \begin{bmatrix} 0.3 & 0.7 \\ 0.4 & 0.6 \end{bmatrix}\begin{bmatrix} 0.3 & 0.7 \\ 0.4 & 0.6 \end{bmatrix}$$

$$= \begin{bmatrix} 0.09 + 0.28 & 0.21 + 0.42 \\ 0.12 + 0.24 & 0.28 + 0.36 \end{bmatrix}$$

$$= \begin{bmatrix} 0.37 & 0.63 \\ 0.36 & 0.64 \end{bmatrix}$$

$P_{11}(3) = P_{11}P_{11}P_{11} + P_{11}\,P_{12}P_{21} + P_{12}P_{22}P_{21} + P_{12}P_{21}P_{11}$

$= 0.3 \times 0.3 \times 0.3 + 0.3 \times 0.7 \times 0.4 + 0.7 \times 0.6 \times 0.4 + 0.7 \times 0.4 \times 0.3$

$= 0.0027 + 0.084 + 0.168 + 0.084$

$= 0.363$

9. An off spring

figure

Transition probability matrix $\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$

$$= \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix}$$

$P_{21}^{(2)} = P_{21}P_{11} + P_{22}P_{21}$

$= 0.4 \times 0.7 + 0.6 \times 0.4$

$= 0.28 + 0.24 = 0.52$

10 A markov chain

Transition probability matrix

$$\text{Let } P = \begin{bmatrix} 0.4 & 0.6 & 0 \\ 0 & 0.7 & 0.3 \\ 0.2 & 0.2 & 0.6 \end{bmatrix}$$

$\pi = [\pi_1\ \pi_2\ \pi_3]$

For steady static

$\pi p = \pi$

$$\text{or } [\pi_1\ \pi_2\ \pi_3]\begin{bmatrix} 0.4 & 0.6 & 0 \\ 0 & 0.7 & 0.3 \\ 0.2 & 0.2 & 0.6 \end{bmatrix} = [\pi_1\ \pi_2\ \pi_3]$$

or $[0.4\pi_1 + 0.2\pi_3 \quad 0.6\pi_1 + 0.7\pi_2 + 0.2\pi_3 \quad 0.3\pi_2 + 0.6\pi_3] = [\pi_1\ \pi_2\ \pi_3]$

or, $0.4\pi_1 + 0.2\pi_3 = \pi_1$ ...(i)

$0.6\pi_1 + 0.7\pi_2 + 0.2\pi_3 = \pi_2$ ...(ii)

$0.3\pi_2 + 0.6\pi_3 = \pi_3$ ...(iii)

From (i) $\pi_1 - 0.4\pi_1 = 0.2\pi_2$

or $0.6\pi_1 = 0.2\pi_2$

or $0.6\pi_1 = 0.2\pi_2$

or $\pi_1 = \frac{1}{3}\pi_2$

From (iii) $\pi_3 - 0.6\pi_3 = 0.3\pi_2$

or, $0.4\pi_3 = 0.3\pi_2$

or $\pi_3 = \frac{3}{4}\pi_2$

Since $\pi_1 + \pi_2 + \pi_3 = 1$

or $\frac{1}{3}\pi_2 + \pi_2 + \frac{3}{4}\pi_2 = 1$

or $\frac{4\pi_2 + 12\pi_2 + 9\pi_2}{12} = 1$

or $25\pi_2 = 12$

or $\pi_2 = \frac{12}{25} = 0.48$

$\pi_1 = \frac{1}{3}\pi_2 = \frac{0.48}{3} = 0.16$

$\pi_3 = \frac{3}{4}\pi_2 = \frac{3}{4} \times 0.48$

$= 0.36$

11. A markov chain

Transition probability matrix

$$P = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

$\pi = (\pi_1\ \pi_2\ \pi_3)$

For steady static

$$[\pi_1\ \pi_2\ \pi_3]\begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix} = [\pi_1\ \pi_2\ \pi_3]$$

$$\text{or} \left[\pi_2 + \frac{\pi_3}{2} \quad \frac{\pi_1}{2} + \frac{\pi_3}{2} \quad \frac{\pi_1}{2}\right] = [\pi_1\ \pi_2\ \pi_3]$$

$$\text{or } \pi_2 + \frac{\pi_3}{2} = \pi_1 \quad \text{...(i)}$$

$$\frac{\pi_1}{2} + \frac{\pi_3}{2} = \pi_2 \quad \text{...(ii)}$$

$$\frac{\pi_1}{2} = \pi_3 \quad \text{...(iii)}$$

Use (iii) or (i)

$$\pi_2 + \frac{\pi_1}{2 \times 2} = \pi_1$$

$$\text{or } \pi_2 = \pi_1 - \frac{\pi_1}{4}$$

$$\text{or } \pi_2 = \frac{3\pi_1}{4}$$

Since $\pi_1 + \pi_2 + \pi_3 = 1$

$$\text{or } \pi_1 + \frac{3\pi_1}{4} + \frac{\pi_1}{2} = 1$$

$$\text{or } \frac{4\pi_1 + 3\pi_1 + 2\pi_1}{4} = 1$$

$$\text{or } 9\pi_1 = 4$$

$$\text{or } \pi_1 = \frac{4}{9}$$

$$\pi_2 = \frac{3}{4}\pi_1 = \frac{3}{4} \times \frac{4}{9} = \frac{3}{9}$$

$$\pi_3 = \frac{1}{2}\pi_1 = \frac{1}{2} \times \frac{4}{9} = \frac{2}{9}$$

12. Obtain transition

$$p = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{3} & 0 & \frac{2}{3} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

13. Task are send

$\lambda = 2$ per min

$$\Delta = 2 \text{ sec} = \frac{2}{60} \text{ min}$$

$$P = \Delta\lambda$$

$$= \frac{2}{60} \times 2 = \frac{4}{60} = \frac{1}{15}$$

for t = 8 sec. $n = \frac{t}{\Delta} = \frac{8/60}{\frac{2}{60}} = 4$

$P(x(n)>3) = 1 - p\{x(n) \le 4\}$

$= {}^4C_4 \left(\frac{1}{15}\right)^4 \left(\frac{14}{15}\right)^0$

$= \left(\frac{1}{15}\right)^4$

= 0.0000197

For t = 50 sec.

$n = \frac{t}{\Delta} = \frac{\frac{50}{60}}{\frac{2}{60}} = \frac{50}{2} = 25$

$P\{x(n) > 10\} = 1 - p\{x(n) \le 10\}$

$= 1 - \sum_{x=0}^{10} {}^{25}C_x \left(\frac{1}{15}\right)^x \left(\frac{14}{15}\right)^{25-x}$

14. Printing jobs

$\lambda = 3$ her min

$\Delta = ?$

$p = 0.2$

$\Delta = \frac{P}{\lambda} = \frac{0.2}{3} = \frac{1}{15}$ min

$E(t) = \frac{1}{\lambda} = \frac{1}{3}$ min

$SD(t) = \sqrt{\frac{1-p}{\lambda^2}}$

$= \sqrt{\frac{1-0.2}{9}}$

$= \sqrt{\frac{0.8}{9}}$

= 0.29 min

15. For a binomial

Δ =2 sec. $= \frac{2}{60} = \frac{1}{30}$ min

$\lambda = 10$ par hr. $= \frac{10}{6} = \frac{1}{6}$ par min

t = 15 min

$n = \frac{t}{\Delta} = \frac{15}{1/30} = 450$

$P(x(n) \ge 3) = ?$

$P = \Delta\lambda = \frac{1}{30} \times \frac{1}{6} = \frac{1}{180}$

Using continuity correction

$$p\left(\frac{x(n) - rp}{\sqrt{rpq}} \ge \frac{3.5 - 450 \times \frac{1}{180}}{\sqrt{450 \times \frac{1}{180} \times \frac{179}{180}}}\right)$$

$= P\left(z > \frac{1}{1.576}\right)$

$= P(z > 0.634)$

$= 0.5 - P(0 < Z < 0.6\,34)$

= 0.5 – 0.2357 = 0.2643

16. Customers of certain

$\lambda = 10$ per min $= 10 \times 60 = 600$ per sec

$p = 0.1$

$\Delta = ?$

$\Delta = \frac{p}{\lambda} = \frac{0.1}{600} = \frac{1}{6000}$

$E(T) = \frac{\Delta}{p} = \frac{\frac{1}{600}}{0.1} = \frac{1}{6000} \times \frac{1}{0.1} = \frac{1}{600}$

$SD(T) = \frac{\Delta}{p}\sqrt{1-p} = \frac{1}{600}\sqrt{1-0.1}$

$= 0.0015$

17 Messages arrived

$\Delta = 2$ sec

$E(T) = 10$ sec

$E(T) = \frac{1}{\lambda}$

or $10 = \frac{1}{\lambda}$

$\lambda = \frac{1}{10}$

for $t = 100$ min $= 100 \times 60 = 6000$ sec

$n = \frac{t}{\Delta} = \frac{6000}{2} = 3000$

$p = \Delta\lambda = 2 \times \frac{1}{10} = \frac{1}{5} = 0.2$

$q = 0.8$

$np = 3000 \times 0.2 = 600$

Using continually correction

$= p\left(\frac{x(n) - np}{\sqrt{npq}} \le \frac{500.5 - 600}{\sqrt{600 \times 0.8}}\right)$

$= P(Z \le -\frac{99.5}{21.9})$

$= P(Z \le -4.5)$

$= 0$

18. Messages arrived

$\lambda = 5$ per 10 min

$= 5 \times 6$ per 60 min

$= 30$ per hr.

$P = 0.15$

$\Delta = ?$

(i) $\Delta = \frac{p}{\lambda} = \frac{0.15}{30}$

$= \frac{0.15}{30} \times 60$

$= 0.3$ min

(ii) $\lambda = 20$ per hr.

Them arrival rate is decreased.

19. Telephone calls

$\lambda = 2$ calls per 5 min.

$= 2 \times 3$ per 15 min

$= 6$ per 15 min

Let x = no of calls

$P(x > 10) = 1 - p(x \le 10)$

$= 1 - \sum_{x=0}^{10} \frac{e^{-15} 15^x}{x!} =$

20 The number of base ball

$\lambda = 5$ per 30 days

$\lambda = \frac{5}{2}$ per 15 days

$\lambda = \frac{35}{30}$ per 7 days

Let x = no. of rained out games

(a) $P(x > 5) = 1 - P(x \le 5)$

$= 1 - \sum_{x=0}^{5} \frac{e^{-5/2}(5/2)^x}{x!}$

$= 1 - 0.082\left[1 + \frac{5}{2} + \frac{25}{8} + \frac{125}{48} + \frac{625}{284} + \frac{3125}{3840}\right]$

$= 0.043$

(b) $P(x = 0) = \frac{e^{-35/30}\left(\frac{35}{30}\right)^0}{0!}$

$= e^{-35/30} = 0.311$

21. An internet services
$\lambda = 2$ per min
$t = 3$ min
Let x = No. offer
$P(x = 0) = \frac{e^{-\lambda t}(\lambda t)^x}{x!}$
$= \frac{e^{-2} \times 3(2 \times 3)^0}{0!}$
$= 0.0024$
$E(x) = \lambda t = 2 \times 3 = 6$
$V(t) = \lambda t = 2 \times 3 = 6$

22 Customer arrived
$\lambda = 9$ per hr.
$\mu = 1$ per 3 min
= 20 per 60 min
= 20 per hr.
$\Delta = 15 \text{ per sec.} = \frac{15}{60 \times 60} = \frac{1}{240} \text{ hr.}$
$P_A = \Delta\lambda = \frac{1}{240} \times 9 = \frac{9}{240} = 0.0375$
$P_S = \Delta\mu = \frac{1}{240} \times 20 = \frac{20}{240} = 0.0833$
$P_{00} = 1 - P_A = 1 - 0.0375 = 0.9625$
$P_{01} = P_A = 0.0375$
$P_{i(i-1)} = (1 - P_A) P_s = 0.9625 \times 0.0833 = 0.08$
$P_{ii}\ (1 - P_A)(1 - P_s) + P_A P_S$
$= 0.9625 \times (1 - 0.0833) + 0.0375 \times 0.0833$
$= 0.8823 + 0.0031 = 0.8854$
$P_{i(i+1)} = P_A(1 - Ps) = 0.0375 \times 0.9167 = 90.0343$
Transition probability matrix

$$\begin{bmatrix} P_{00} & P_{01} & P_{02} & P_{03} & P_{04} & - \\ P_{10} & P_{11} & P_{12} & P_{13} & P_{14} & - \\ P_{20} & P_{21} & P_{22} & P_{23} & P_{24} & - \\ P_{30} & P_{31} & P_{32} & P_{33} & P_{33} & - \\ P_{40} & P_{41} & P_{42} & P_{43} & P_{44} & - \end{bmatrix}$$

$$\begin{bmatrix} 0.9625 & 0.0375 & 0 & 0 & 0 & 0 \\ 0.08 & 0.8854 & 0.034 & 0 & 0 & 0 \\ 0 & 0.08 & 0.8854 & 0.0343 & 0 & 0 \\ 0 & 0 & 0.08 & 0.8854 & 0.0343 & 0 \\ 0 & 0 & 0 & 0.08 & 0.8854 & 0.0343 \end{bmatrix}$$

23 A printer
$\lambda = 20$ per hr
$\mu = 1$ per 40 sec.
= 90 per 3600 sec.
= 90 per hr.
$\Delta = 20 \text{ SEC} = \frac{20}{60 \times 60} = \frac{1}{180} \text{ hr.}$
$P_A = \Delta\lambda = \frac{1}{180} \times 20 = \frac{1}{9}$
$P_5 = \Delta\mu = \frac{1}{180} \times 90 = \frac{1}{2}$
$P_{00} = 1 - P_A = 1 - \frac{1}{9} = \frac{8}{9}$
$P_{01} = P_A = \frac{1}{9}$
$P_i(i - 1) = (1 - P_A)P_S = \frac{8}{9} + \frac{1}{2} = \frac{4}{9}$
$P_{ii} = (1 - P_A)(1 - P_S) + P_A P_s$

$= \frac{8}{9} + \frac{1}{2} + \frac{1}{9} \times \frac{1}{2} = \frac{9}{18}$

$P_i(i + 1) = P_A(1 - P_s) = \frac{1}{9} \times \frac{1}{2} = \frac{1}{18}$

Transition probability matrix

$$= \begin{bmatrix} P_{00} & P_{01} & P_{02} & P_{03} & P_{04} & - \\ P_{10} & P_{11} & P_{12} & P_{13} & P_{14} & - \\ P_{20} & P_{21} & P_{22} & P_{23} & P_{24} & - \\ P_{30} & P_{31} & P_{32} & P_{33} & P_{33} & - \\ P_{40} & P_{41} & P_{42} & P_{43} & P_{44} & - \end{bmatrix}$$

$$= \begin{bmatrix} 8/9 & 1/9 & 0 & 0 & 0 & - \\ 4/9 & 9/18 & 1/18 & 0 & 0 & - \\ 0 & 4/9 & 9/18 & 1/18 & 0 & - \\ 0 & 0 & 4/9 & 9/18 & 1/18 & - \end{bmatrix}$$

24. Jobs arrived

$\lambda = 8$ per hr.

$\mu = 1$ per 3 min

$= 20$ per 60 min

$= 20$ per hr.

$\Delta = 5 \text{ sec.} = \frac{5}{60 \times 60} = \frac{1}{720}$ hr.

$c = 3$

$P_A = \Delta\lambda = \frac{1}{720} \times 8 = \frac{1}{90}$

$Ps = \Delta\mu = \frac{1}{720} \times 20 = \frac{1}{36}$

$P_{00} = 1 - P_A = \frac{89}{90} = 0.9888$

$P_{01} = P_A = \frac{1}{90} = 0.012$

$P_{i(i-1)} = (1 - P_A)P_S = \frac{89}{90} \times \frac{1}{36} = 0.027$

$P_{ii} = (1 - P_A)(1 - P_S) + P_nP_s \quad = \frac{89}{90} \times \frac{35}{36} + \frac{1}{90} \times \frac{1}{36}$

$= 0.962$

$P_i (i + 1) = P_A(1 - P_s) = \frac{1}{90} \times \frac{35}{36} = 0.011$

Transition probability matrix

$$\begin{bmatrix} 0.988 & 0.012 & 0 & 0 \\ 0.027 & 0.962 & 0.011 & 0 \\ 0 & 0.027 & 0.962 & 0.011 \\ 0 & 0 & 0.027 & 0.973 \end{bmatrix}$$

25. A customer services

$\lambda = 10$ per hr

$\mu = 1$ per 4 min

$= 15$ per 60 min

$= 15$ per hr.

$\Delta = 1 \text{ min} = \frac{1}{60}$ hr

$C = 2$

$P_A = \lambda\Delta = 10 \times \frac{1}{60} = \frac{1}{6}$

$P_S = \mu\Delta = 15 \times \frac{1}{60} = \frac{1}{4}$

$P_{00} = 1 - P_A = 1 - \frac{1}{6} = \frac{5}{6}$

$P_{01} = P_A = \frac{1}{6}$

$P_i(i - 1) = (1 - P_A)p_s = \frac{5}{6} \times \frac{1}{4} = \frac{5}{24}$

$P_{ii} = (1 - P_A)(1 - P_S) + P_AP_S$

$= \frac{5}{6} \times \frac{3}{4} + \frac{1}{6} \times \frac{1}{4} = \frac{16}{24}$

$P_i(i + 1) = P_A (1 - P_s) = \frac{1}{6} \times \frac{3}{4} = \frac{3}{24}$

$$P = \begin{bmatrix} 5/6 & 1/6 & 0 \\ 5/24 & 16/24 & 3/24 \\ 0 & 5/24 & 19/24 \end{bmatrix}$$

For steady state

$\pi p = \pi$

or $[\pi_0\ \pi_1\ \pi_2] \begin{bmatrix} 5/6 & 1/6 & 0 \\ 5/24 & 16/24 & 3/24 \\ 0 & 5/24 & 19/24 \end{bmatrix} = [\pi_0\ \pi_1\ \pi_2]$

or $[5\frac{\pi_0}{6} + \frac{5\pi_1}{24}\ \frac{\pi_0}{6} + \frac{16\pi_1}{24} + \frac{5\pi_2}{24}\ \frac{3\pi_1}{24} + \frac{19\pi_2}{24}] = [\pi_0\ \ \pi_1\ \pi_2]$

$5\frac{\pi_0}{6} + \frac{5\pi_1}{24} = \pi_0$ ...(i)

$\frac{\pi_0}{6} + \frac{16\pi_1}{24} + \frac{5\pi_2}{24} = \pi_1$ ...(ii)

$\frac{3\pi_1}{24} + \frac{19\pi_2}{24} = \pi_2$ ...(iii)

From (ii)

$\frac{3\pi_1}{24} = \frac{5\pi_2}{24}$

or, $\pi_2 = \frac{3}{5}\pi_1$

From (i)

$\frac{5\pi_1}{24} = \frac{\pi_0}{6}$

or $\pi_0 = \frac{30}{24}\pi_1$

Since

$\pi_0 + \pi_1 + \pi_1 = 1$

or $\frac{5}{4}\pi_1 + \pi_1 + \frac{3}{5}\pi_1 = 1$

or, $\frac{25\ \pi_1 + 20\ \pi_1 + 12\ \pi_1}{20} = 1$

or $\frac{57}{20}\pi_1 = 1$

$\pi_1 = \frac{20}{57} = 0.35$

or $\pi_0 = \frac{5}{4}\pi_1$

or $\frac{5}{4} \times 0.35 = 0.43$

or $\pi_2 = \frac{3}{5}\pi_1$

or $\frac{3}{5} \times 0.35 = 0.21$

26. In queuing system

$\lambda = 1$ per 6 min

= 10 – per 6 min

= 10 per hr

$\mu = 1$ per 4 min

= 15 per 60 min

= 15 per hr

(ii) $\rho = \frac{\lambda}{\mu} = \frac{10}{15} = \frac{2}{3}$

$Ls = \frac{p}{1 - p} = \frac{\frac{2}{3}}{1 - \frac{2}{3}} = 2$

(i) $Ws = \frac{Ls}{\lambda} = \frac{2}{10} = \frac{1}{5}\text{ hr} = \frac{60}{5} = 12$ min

27. For an

$\lambda = 1$ per 5 min

$= 12$ per 60 min

$= 12$ per hr.

$\mu = 1$ per 3 min

$= 20$ per 60 min

$= 20$ per hr.

$$\rho = \frac{\lambda}{\mu} = \frac{12}{20} = \frac{3}{5} = 0.6$$

$$LS = \frac{\rho}{1-\rho} = \frac{0.6}{1-0.6} = \frac{3}{2}$$

(a) $Ws = \frac{Ls}{\lambda} = \frac{3}{2} \times 12 = \frac{1}{8}$ hr

$= \frac{60}{8} = 7.5$ min

(b) $P(n < 2)$

$= P(0) + \rho(1)$

$= 1 - \rho + \rho(1 - p)$

$= 1 - \rho^2 = 1 - (0.6)^2$

$= 0.64$

(c)

$$Lq = \frac{\rho^2}{1-\rho} = \frac{(0.6)^2}{1-0.6} = \frac{0.36}{0.4} = 0.9$$

28. Cars arrived

$\lambda = 1$ per 10 min

$= 6$ per 60 min

$= 6$ per hr.

$\mu = 1$ per 3 min

$= 20$ per 60 min

$= 2$ per hr.

$$\rho = \frac{\lambda}{\mu} = \frac{6}{20} = \frac{3}{10}$$

(a) $$L_q = \frac{\rho^2}{1-\rho} = \frac{\left(\frac{3}{10}\right)^2}{1-\frac{3}{10}} = \frac{9}{100} \times \frac{10}{7}$$

(b) Idle $= 1 - \rho = 1 - \frac{3}{10} = \frac{7}{10} = 0.7$

$$Ls = \frac{\rho}{1-\rho} = \frac{\frac{3}{10}}{1-\frac{3}{10}} = 0.7$$

(c) $Ws = \frac{Ls}{\lambda}$

$$= \frac{\frac{3}{7}}{6} = \frac{3}{7} + \frac{1}{6} = \frac{1}{14} \text{ hr} = \frac{60}{14} \text{ min}$$

$= 4.28$

29. Jobs are sent

$\lambda = 4$ per min

$\mu = 1$ per 10 sec.

$= 6$ per 60 sec.

$= 6$ per min

$$\rho = \frac{\lambda}{\mu} = \frac{4}{6} = \frac{2}{3}$$

$$Ls = \frac{\rho}{1-\rho} = \frac{\frac{2}{3}}{1-\frac{2}{3}} = 2$$

$$V(n) = \frac{\rho}{(1-\rho)^2} = \frac{\frac{2}{3}}{\left(1-\frac{2}{3}\right)^2} = \frac{\frac{2}{3}}{1-\frac{1}{9}} = 6$$

$SD = \sqrt{v(n)} = \sqrt{6} = 2.445$

$1 - \rho = 1 - \frac{2}{3} = \frac{1}{3} = 0.33$

30. In A bank cheques

$\lambda = 30$ per hr

$\mu = 1$ per 1.5 min

= 40 per 60 min

= 40 per hr.

$$\rho = \frac{\lambda}{\mu} = \frac{30}{40} = \frac{3}{4} = 0.75$$

= 0.1444

$$L_S = \frac{\rho}{1-\rho} = \frac{0.75}{1 - 0.75} = 3$$

$$W_S = \frac{Ls}{\lambda} = \frac{3}{30} = \frac{1}{10}\text{ hr} = 6\text{ min}$$

31. In a service department

$\lambda = 1$ per 10 min

= 6 per 60 min

= 6 per hr.

$\mu = 1$ per 6 min

10 per 60 min

= 10 per hr.

$$\rho = \frac{\lambda}{\mu} = \frac{6}{10} = 0.6$$

$$\text{(i) } Lq = \frac{\rho^2}{1 - \rho} = \frac{(0.6)^2}{1 - 0.6}$$

$$= \frac{0.36}{0.4} = 0.9$$

$$Ls = \frac{\rho^2}{1 - \rho} = \frac{0.6}{1 - 0.6} = \frac{3}{2} = 1.5$$

$$\text{(ii) } W_S = \frac{LS}{\lambda} = \frac{1.5}{6} = \frac{1}{4}\text{ hr}$$

= 15 min

(iii) $P_2 = \rho^2 (1 - p)$

$= (0.6)^2 [1 - 0.6]$

$= 0.36 \times 0.4$

$= 0.144$