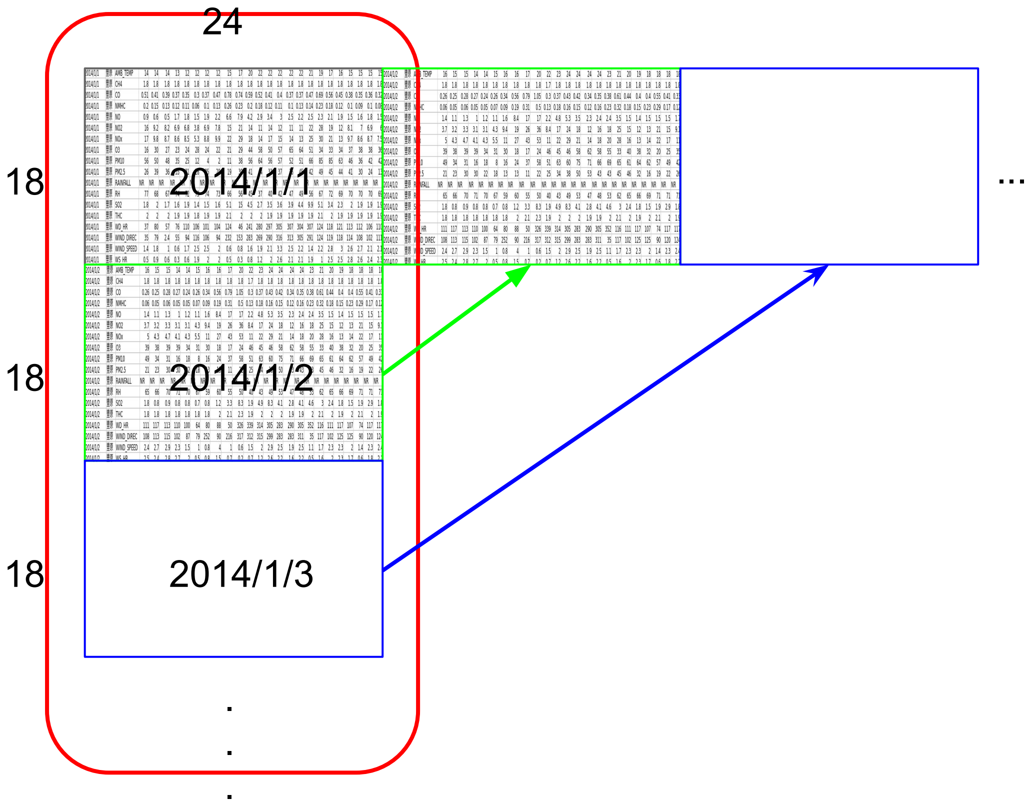
Machine Learning HW1
TA Hours

MLTAs
ntumlta2019@gmail.com

Outline

- Simple linear regression using gradient descent (with adagrad)
    - Extract features
    - Implement linear regression
    - Adagrad
    - Predict pm2.5

Extract Features



Extract Features

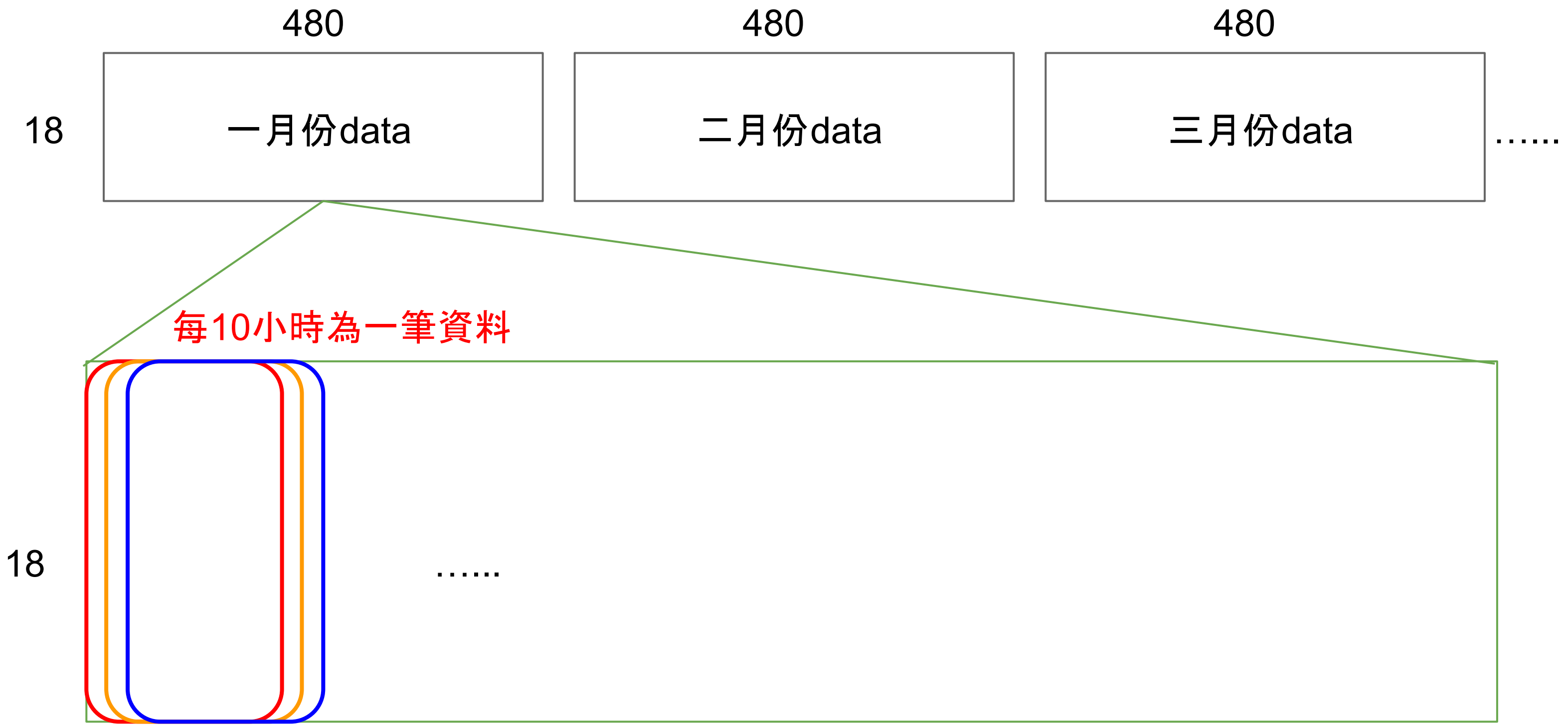
<u>Pseudo code</u>

```
Declare a 18-dim vector (Data)
for i_th row in training data :
    Data[i_th row%18].append(every element in i_th row)
```

Data will become a vector like

2014/1/1	2014/1/2	2014/1/3	...

Extract Features



Extract Features

<u>Pseudo code</u>

```
Declare train_x for previous 9-hr data, and train_y for 10th-hr pm2.5
for i in all the given data:
    sample every10 hrs：
        train_x.append(previous 9-hr data)
        train_y.append(the value of 10th-hr pm2.5)
add a bias term to every data in train_x
```

Implement linear regression

Pseudo code

```
Declare weight vector, initial lr ,and # of iteration
for i_th iteration :
    y’ = the product of train_x  and weight vector
    Loss = y’ - train_y
    gradient = 2*np.dot((train_x)’, L )
    weight vector -= learning rate * gradient
```

$$\mathbf{y} = \mathbf{X}\mathbf{w} + \mathbf{b}$$

$$\mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}$$

$$\mathbf{X} = \begin{pmatrix} x_1^T \\ x_2^T \\ \vdots \\ x_n^T \end{pmatrix} = \begin{pmatrix} x_{11} & \cdots & x_{1p} \\ x_{21} & \cdots & x_{2p} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{np} \end{pmatrix}$$

$$\mathbf{w} = \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_p \end{pmatrix}, \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_p \end{pmatrix}$$

Implement linear regression

<u>Pseudo code</u>

```
1 | Declare weight vector, initial lr ,and # of iteration
2 | for i_th iteration :
3 |     y' = the inner product of train_x  and weight vector
4 |     Loss = y' - train_y
5 |     gradient = 2*np.dot((train_x)', L )
6 |     weight vector -= learning rate * gradient
```

3.

$$\begin{pmatrix} y_1' \\ y_2' \\ \vdots \\ y_n' \end{pmatrix} = \begin{pmatrix} x_{11} & \cdots & x_{1p} \\ x_{21} & \cdots & x_{2p} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{np} \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_p \end{pmatrix}$$

4.

$$L = \begin{pmatrix} y_1' \\ y_2' \\ \vdots \\ y_n' \end{pmatrix} - \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}$$

5.

gradient = 2 x

$$\begin{pmatrix} x_{11} & x_{21} & \cdots & x_{n1} \\ x_{12} & x_{22} & \cdots & x_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ x_{1p} & x_{2p} & \cdots & x_{np} \end{pmatrix} \begin{pmatrix} y_1' - y_1 \\ y_2' - y_2 \\ \vdots \\ y_n' - y_n \end{pmatrix}$$

p-dim vector

Adagrad

$\theta_{t+1,i} = \theta_{t,i} - \eta \cdot g_{t,i}.$

In its update rule, Adagrad modifies the general learning rate η at each time step t for every parameter θ_i based on the past gradients that have been computed for θ_i:

$$\theta_{t+1,i} = \theta_{t,i} - \frac{\eta}{\sqrt{G_{t,ii} + \epsilon}} \cdot g_{t,i}.$$

$G_t \in \mathbb{R}^{d \times d}$ here is a diagonal matrix where each diagonal element i, i is the sum of the squares of the gradients w.r.t. θ_i up to time step t 24 , while ϵ is a smoothing term that avoids division by zero (usually on the order of $1e-8$). Interestingly, without the square root operation, the algorithm performs much worse.

As G_t contains the sum of the squares of the past gradients w.r.t. to all parameters θ along its diagonal, we can now vectorize our implementation by performing an element-wise matrix-vector multiplication $\odot$ between G_t and g_t:

$$\theta_{t+1} = \theta_t - \frac{\eta}{\sqrt{G_t + \epsilon}} \odot g_t.$$

One of Adagrad's main benefits is that it eliminates the need to manually tune the learning rate. Most implementations use a default value of 0.01 and leave it at that.

Adagrad

Pseudo code

```
Declare weight vector, initial lr ,and # of iteration
Declare prev_gra storing gradients in every previous iterations
for i_th iteration :
    y’ = the inner product of train_x  and weight vector
    Loss = y’ - train_y
    gradient = 2*np.dot((train_x)’, L )
    prev_gra += gra**2
    ada = np.sqrt(prev_gra)
    weight vector -= learning rate * gradient / ada
```

Predict PM2.5

<u>**Pseudo code**</u>

```
read test_x.csv file
for every 18 rows :
    test_x.append([1])
    test_x.append(9-hr data)
    test_y = np.dot(weight vector, test_x)
```

Reference

1. Adagrad : https://youtu.be/yKKNr-QKz2Q?list=PLJV_el3uVTsPy9oCRY30oBPNLCo89yu49&t=705
2. RMSprop https://www.youtube.com/watch?v=5Yt-obwvMHI
3. Adam https://www.youtube.com/watch?v=JXQT_vxqwIs