GROUP WORK PROJECT # 1
GROUP NUMBER: 5747

MScFE 652: PORTFOLIO MANAGEMENT

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Remember: Any group members who did **not contribute to the project should be given all zero (0) points for the collaboration grade on the GWP submission page.*

Statement of integrity: By typing the names of all group members in the text boxes below, you confirm that the assignment submitted is original work produced by the group (excluding any non-contributing members identified with an "X" above).

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Note: You may be required to provide proof of your outreach to non-contributing members upon request.

N/A

Introduction and formulas:

In the world of economics and finance it is very important to have an understanding of the connection between risk and returns and it is even more important to have a way to measure this connection and this is where the Capital Asset Pricing Model (CAPM) model comes to help us. It is used to describe the relationship between systematic risk and expected return for a given asset. It helps with the pricing of risky securities, generating expected returns for assets given the risk of those assets and calculating costs of capital.

The next extensions and refinements of the model lead to the Fama_french three and later five models which includes market risk, size, and value factors, and incorporates profitability and investment factors respectively. Let's dig deeper into the parameters, formulas and the meaning of these models:

Capital Asset Pricing model formula [3],[4]:

$$R_{it} - R_{Ft} = a_i + b_i(R_{Mt} - R_{Ft}) + e_{it}$$

Where:

- R_{it} *is the return of asset* "i" *at time* "t"
- R_{Ft} *is the risk free rate of return at time* "t"
- R_{Mt} *is the return on the market portfolio*
- $R_{it} - R_{Ft}$ *and* $R_{Mt} - R_{Ft}$ *are excess returns on asset* "i" *and the market portfolio*
- $R_{it} - R_{Ft}$ *is the market premium* (*momentum factor*)
- e_{it} *is the error at* "i"

Fama – French Three – Factor Model formula [5], [6]:

$$R_{it} - R_{Ft} = a_i + b_i(R_{Mt} - R_{Ft}) + s_i SMB_i + h_i HML_i + e_{it}$$

Where:

- $s_i SMB_i$ *is the size premium* (*small minus big*):

This is the historical excess returns of small companies over large cap companies (*size factor*)

- $h_i HML_i$ *is the value premium* (*high minus low*):

This is the historical excess returns of value stocks vs. growth stocks (*value factor*)

Fama – French Five – Factor Model formula [1] ,[2]:

$$R_{it} - R_{Ft} = a_i + b_i(R_{Mt} - R_{Ft}) + s_i SMB_i + h_i HML_i + r_i RMW_i + c_i CMA_i + e_{it}$$

Where:

- $r_i RMW_i$ *is Profitability Factor* $(RMW - Robust\ Minus\ Weak)$:

It compares the returns of firms with high (robust) operating profitability and those with low (weak) operating profitability

- $c_i CMA_i$ *is the Investment Factor* $(CMA - Conservative\ Minus\ Aggressive)$:

This factor gauges the difference in returns between companies that invest conservatively and those that invest aggressively

STEP1:

A. The 5 factors in 5-factor Fama-French model are defined below:

- **Market:**

In the Capital Asset Pricing Model(CAPM), which is a single factor model, Market factor is the factor. In asset pricing theory, this factor is very important. It is defined as the excess return of the market over the risk-free rate. Let's assume it is defined as R_m-R_f. In the value-weight(VW) market portfolio, the return is the R_m. R_f is the risk-free return. This factor is also defined as the market risk premium.

The market factor gives the premium, the investors expect to return for holding this diversified portfolio. This portfolio has many risky assets. This factor gives the excess return compared to risk-free investments. Since the volatility and risk will be high for these assets, this market risk premium is defined as the compensation over the risk-free return.

The beta coefficient for the market factor is the sensitivity to market risk. It is multiplied by the market risk premium represented by the R_m-R_f to give the expected return of an asset. This implies that the expected return is proportional to systematic risk. The performance of individual assets is compared to the diversified portfolio. A positive beta indicates that the return of the asset is positively correlated with the market risk premium. If the market risk premium increases, the return of the asset also increases. It shows that market risk affects the asset. Negative beta coefficient means lower risk.

β	
>1	Higher risk, the asset return is more volatile than the market return
1	Average risk, the asset return moves along the market return
<1	Lower risk, the asset return is less volatile than the market return

Table 1: Beta values for CAPM (Sanchez, 2019)

- **SMB:**

Small Minus Big (SMB) is a size factor. This factor measures the excess return of small-cap companies over big-cap companies. In asset pricing theory, this factor is important. It is also measured as the return on a diversified portfolio of small-cap stocks minus the return on a diversified portfolio of big-cap stocks. This is the size premium. SMB represents the excess return of small-cap stocks over large-cap stocks. Since smaller companies have additional risk, it is said that small-cap stocks give higher returns compared to big-cap stocks. Due to the additional risk, investors might need a premium for holding small-cap stocks. This premium is represented by the SMB factor.

In the Fama-French three-factor model, this size factor is one of the factors added with the CAPM factor. The performance of small-cap stocks is compared with large-cap stocks. Over a long period, small-cap stocks have given high average returns compared to large-cap stocks. This is because small-cap companies have higher growth potential and high liquidity risk. Based on market capitalization, which is the size of the company, the stocks are ordered in portfolios. Then the returns are compared.

Positive beta indicates that the asset performs well when small-cap stocks perform better than the big-cap companies. Negative beta indicates that the stock belongs to a large-cap company and is less risky, thus the returns will be less.

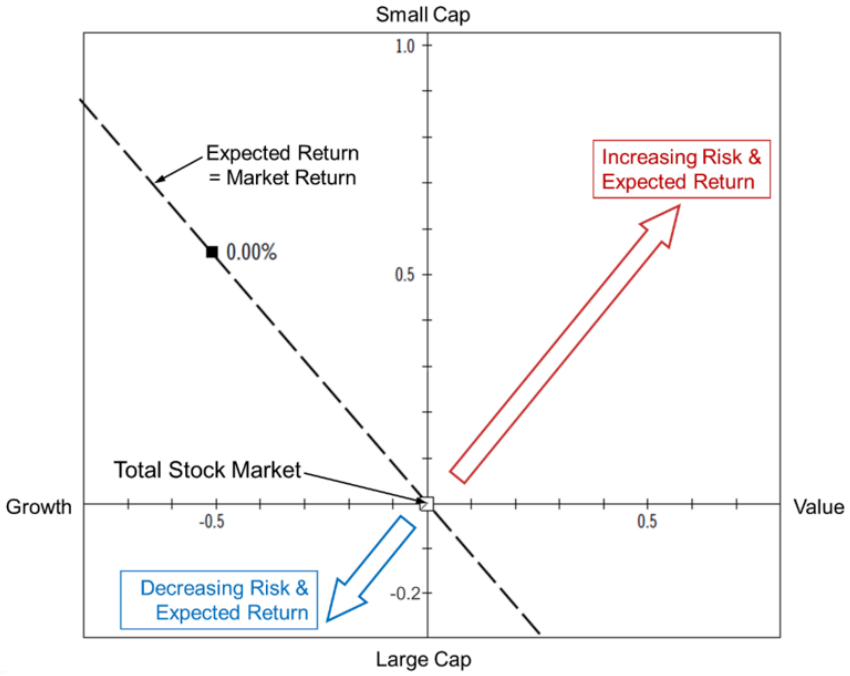


Fig 1: Three- factor model: risk exposure (Bogleheads, 2024)

This figure 1 illustrates the three-factor model where all 3 factors of the Fama-French three-factor model are displayed.

- **HML:**

High Minus Low (HML) is a value factor. This factor is the excess returns of value stocks over growth stocks. Low book-to-market ratios represent value stocks. High book-to-market ratios represent growth stocks. In asset pricing models, this factor explains the performance of value stocks compared to growth stocks. This factor calculates the risk and return by focusing on stocks with low book-to-market ratios (low B/M) over the high book-to-market ratios (high B/M). This means that this factor is the return spread of the low expensive stocks minus the high expensive stocks. The excess returns of value portfolios over growth portfolios represents the premium.

Depending on the book-to-market ratio, the stocks can be divided into three additional groups. They are Low (L), Medium (M) and High (H). Based on the ratio, this division is done. In the B/M ratio, the lowest 30% are the low group. If the B/M ratio is above 70%, they belong in the high group. The remaining 40% in the middle of the B/M ratio belongs to the medium group.

In the Fama-French three-factor model, this value factor is added with the size factor from above and the market risk factor from the CAPM model. In the long-term, value stocks give higher returns than growth stocks. The value stocks would be undervalued by the market. When the price of the stock adjusts to intrinsic value, it gives higher returns. Value stock is a stock that trades at a lower price, it trades at a discount. For holding value stocks, the investors get the High Minus Low premium to offset the risk. On the diversified portfolio, this value factor measures the difference between returns of stocks with high price-to-book ratios and low book-to-market ratios.

Positive beta indicates that stock has low book-to-market ratios. It implies that value stocks perform better than growth stocks. Negative beta implies the stock is a growth stock. The returns will be low compared to value stocks.

		Size - capitalization	
		Small	**Big**
Value - B/M ratio	**Low**	S/L	B/L
	Medium	S/M	B/M
	High	S/H	B/H

Table 2: SMB and HML factor (2 x 3) (Sanchez, 2019)

This table 2 (Sanchez, 2019) is understood from (Fama and French, 1992). It explains the 2 x 3 factors, where size factor has 2 groups and value factor (B/M ratio) has 3 groups.

- **RMW**:

Robust Minus Weak (RMW) is a profitability factor. The factor measures the excess returns of companies with high profitability and companies with low profitability. The companies with high and steady profitability are robust and companies with low profitability are weak. In asset pricing models, this is an important factor that explains the performance difference between companies with high and low profitability. In a diversified portfolio, this profitability factor is the difference between the returns on stocks with robust and weak profitability.

In the Fama-French five-factor model, this profitability factor is added along with the above 3 factors - market factor, size factor and value factor, from the Fama-French three-factor model. The companies with robust performance will have high returns compared to companies with weak performance. Historically, over a long period, stocks belonging to companies with high profitability perform better than the stocks of companies with low profitability. Investors prefer high profitability stocks because it gives high returns. These high profitability stocks have low risk. Thus, the premium value must be paid by investors for investing in these high profitability stocks. The above three factors are mentioned as a premium given to the investors because of the high risk. But here, due to the low risk of high profitability stocks, the investors pay the premium. This premium is calculated by ordering stocks based on profitability in the portfolios. Then the returns of portfolios with high profitability and low profitability are compared. The excess returns of high profitability over low profitability is this profitability factor, which is the premium needed for investing.

Positive beta indicates that the stock has high profitability. A negative beta implies that the stock is a low profitability stock.

- **CMA:**

Conservative Minus Aggressive (CMA) is the investment factor. This factor represents the excess return of companies with conservative investment over companies with aggressive investment strategies. In asset pricing models, this factor is an important factor, which explains the performance between companies with conservative investment over companies with aggressive investment strategies.

Conservative investment companies have better stability, low volatility and risk management. Aggressive investment strategies have high risk. In the long term, companies with conservative investments perform better than aggressive investment strategies. Conservative investment companies also have better cash flow, thus, stocks of conservative investment companies give high returns than companies with aggressive investment strategies. This Conservative Minus Aggressive factor is calculated by ordering stocks into portfolios based on investment strategies. In the diversified portfolio, the returns of stocks with conservative investment is compared with aggressive investment. The CMA factor is the excess return of conservative portfolios over aggressive portfolios. This premium is needed to be paid by the investor to reduce risk and give high returns. In this, the aggressive investment strategies are mostly underweighted to reduce risk for the investor. In the Fama-French five-factor model, this investment factor along with the above 4 factors form the factors. In the Fama-French three-factor model, the premium is given to the investors, for the Fama-French five-factor model, the additional 2 factors are the premium paid by the investors to have high returns.

A positive beta suggests that stock follows a conservative investment strategy. It gives high returns and low risk. A negative beta indicates that stock follows an aggressive investment strategy. It has high risk.

B. Non-technical:

The investment's rate of return can be calculated based on the following factors:

- **Market factor:**

The Market factor explains returns by finding the systematic risk. For holding a diversified risky portfolio, the investors are given market premium. This is the market factor. The market factor represents the excess return of the market portfolio over the risk-free rate. The expected return is proportional to market risk.

- **Size factor:**

The Small Minus Big (SMB - Size factor) explains returns by finding the historical excess returns of small-cap stocks over large-cap stocks. Small-cap stocks have higher average returns compared to large-cap stocks in a long period [10]. This is due to the additional risk present in smaller companies. Due to this risk, investors are given a premium for holding stocks. This premium is the SMB factor.

- **Value Factor:**

The High Minus Low (HML - Value Factor) explains returns by finding the historical excess returns of value stocks over growth stocks. Low book-to-market ratios have performed better than the growth stocks in the long term [9]. Investors receive a premium for holding value stocks, because of the capital appreciation. This premium is the HML factor. This gives high average returns. B/M (book-to-market ratio) is positively correlated to return [11].

- **Profitability Factor :**

The Robust Minus Weak (Profitability Factor) explains returns by finding the historical excess returns of stocks of companies with high profitability over those with low profitability. Companies with higher profitability have long-term better performance [11] . High profitable companies are less risky and give constant cash flows. This factor is the premium paid by the investors. Profitability factor is negatively correlated to B/M.

- **Investment Factor:**

The Conservative Minus Aggressive (Investment Factor) explains returns by finding the excess returns of stocks of companies with conservative investment over those with aggressive investment policies. Companies with conservative investment strategies have better stability and low volatility. So, Investors mostly choose conservative investment policies.. The CMA factor is the premium paid for holding shares of companies with conservative investment strategies. High B/M value stocks tend to have low profitability and investment [11].

STEP 2:

A 3-year time period is selected and daily factor returns from the Fama-French five-factor model are going to be calculated here.

a. The daily factor returns data is imported for the Fama-French five-factor model. It contains data for all 5 factors. A 3-year time period is selected with the end date as '2024-02-29'. Then the factor returns are plotted over the time period.

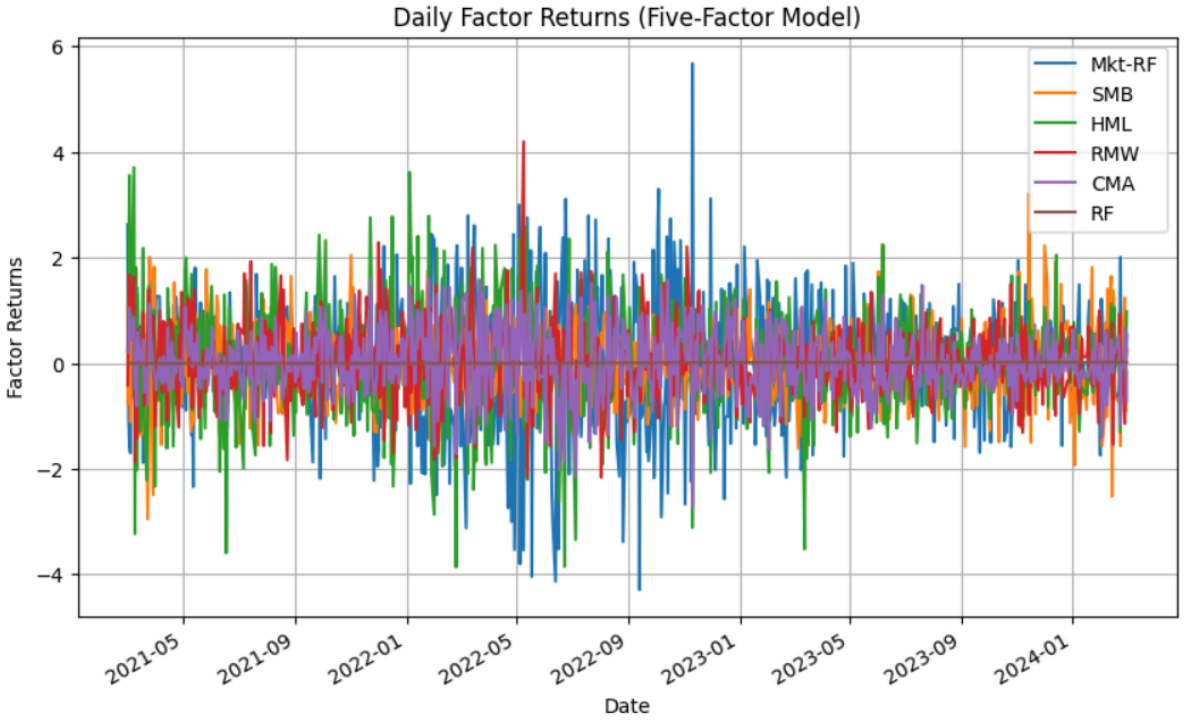


Fig 2: Daily factor returns

b. The difference between observations is calculated to find the changes in factor returns. The correlation between the changes in factor returns is calculated. The correlation between the changes in 5 factors returns is given in Fig 3.

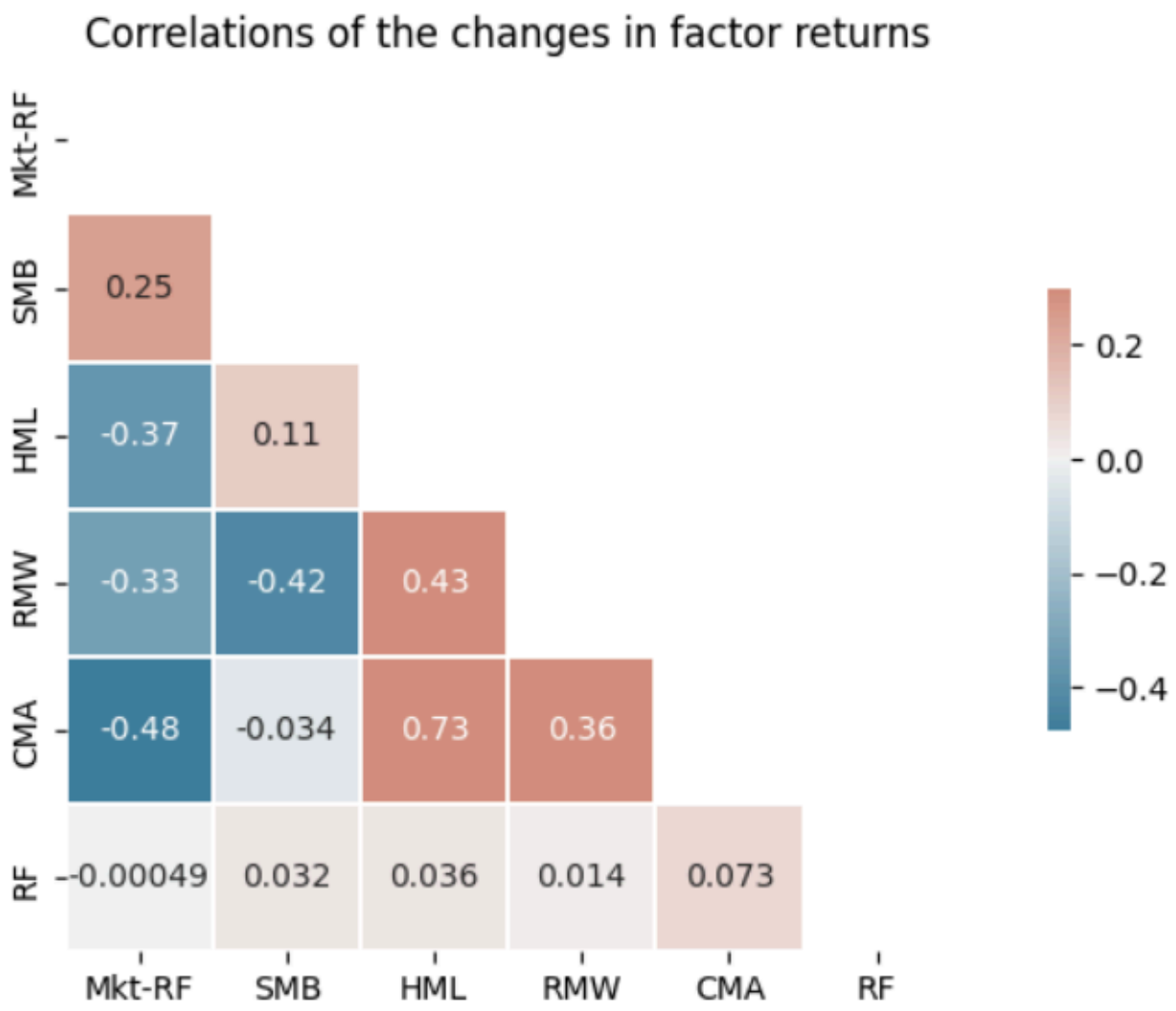


Fig 3: Correlations of the changes in factor returns

From the figure, we can see that Market factor and CMA factor is the most negatively correlated among the 5 factors and HML factor and CMA factor are the most positively correlated.

c. We have collected economic data for a 2-year time period. The data we have chosen is from the S & P 500 Index (Standard and Poor's 500). This shows the performance of 500 large-cap companies in the U.S. the log returns of S&P 500 in fig 4.

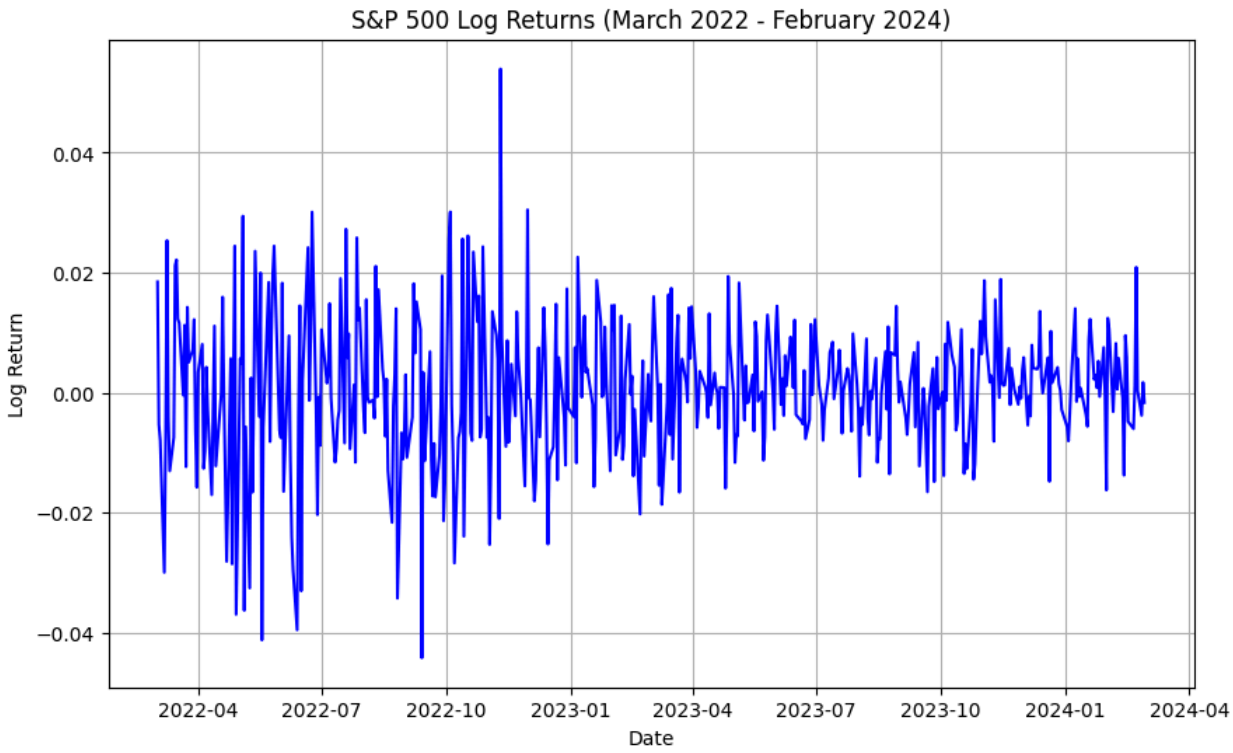


Fig 4: S&P 500 Log returns

STEP 3:

a. FF3 betas:

Amazon stock is selected and daily stock price for the same three year period is collected. Daily log returns of Amazon stock price is calculated and this is merged with the Fama-french three-factor returns collected. This data is used for finding the Fama-French three-factor betas.

The dataset is divided using a 80-20 split into training and testing datasets. Testing set is used for model evaluation.

Daily log returns of Amazon stock is taken as a dependent variable and the three factors - Mkt-RF, SMB and HML are taken as independent variables. Least square regression and Robust regression are done for calculating the FF3 betas.

b. Coefficients and model metrics:

The LS regression model gives coefficients for all 3 factors. These coefficients are the beta values for each factor for Amazon stock. When all three factors are zero, the expected daily log returns of Amazon stock is -0.0005. The betas for each factor is given below.

FACTOR	BETA
Mkt-RF	0.0146
SMB	-0.0044
HML	-0.0049

Table 3: FF3 betas (LS regression)

LS regression model gives several model metrics for evaluation. They are R-squared, adjusted R-squared, F-statistic and AIC.

R-squared: 0.612
Adj. R-squared: 0.610
F-statistic: 314.9
AIC : -3334

Approximately, 61% of variance in daily log returns is explained by the factors. F-statistic tells the significance of the regression. Lower AIC implies it is a good model.

Robust regression model also gives coefficients for all 3 factors. This regression can handle outliers. This model gives different coefficients. These coefficients are the beta values for each factor for Amazon stock. When all three factors are zero, the expected daily log returns of Amazon stock is -0.0005. The betas for each factor is given below.

FACTOR	BETA
Mkt-RF	0.0139
SMB	-0.0032
HML	-0.0048

Table 4: FF3 betas (Robust regression)

Robust regression model gives p-values and 95% confidence interval for evaluation. In the below table [0.025] is the lower bound and [0.975] is the upper bound. With 95% confidence, we can say that these coefficients belong to these bounds.

FACTOR	[0.025]	[0.975]
Mkt-RF	0.013	0.015
SMB	-0.004	-0.002
HML	-0.006	-0.004

Table 5: 95% confidence interval (FF3 betas - Robust regression)

From the beta values, we have found using both regression models, we can see that Mkt-Rf is positive but it is less than 1. This implies that the risk is low. SMB and HML factors have negative betas. It shows that the Amazon stock chosen is a growth stock and it belongs to large-cap stocks.
For both regression models, we can see that the p-values for all three factors are 0, which implies that these coefficients are statistically significant. These factors are highly significant in these models.

Step 4:

a. FF5 betas:

Amazon stock is the selected stock. Daily log returns of Amazon stock price is calculated and this is merged with the Fama-french five-factor returns collected. This data is used for finding the Fama-French five-factor betas.
The dataset is again divided using a 80-20 split into training and testing datasets. Testing set is used for model evaluation.
Daily log returns of Amazon stock is taken as a dependent variable and the five factors - Mkt-RF, SMB, HML, RMW and CMA are taken as independent variables. Least square regression and Robust regression are done for calculating the FF5 betas.

b. Coefficients and model metrics:

The LS regression model gives coefficients for all 5 factors. These coefficients are the beta values for each factor for Amazon stock. When all five factors are zero, the expected daily log returns of Amazon stock is -0.0003. The betas for each factor is given below.

FACTOR	BETA
Mkt-RF	0.0129
SMB	-0.0056
HML	-0.004
RMW	-0.0016
CMA	-0.0119

Table 6: FF5 betas (LS regression)

LS regression model gives several model metrics for evaluation. They are R-squared, adjusted R-squared and F-statistic.

R-squared: 0.648
Adj. R-squared: 0.645
F-statistic: 219.3
AIC : -3388

Approximately, 64% of variance in daily log returns is explained by the factors. F-statistic tells the significance of the regression. Lower AIC implies it is a good model.
For LS regression, we can see that the p-values of the HML coefficient is 0.707 and the RMW coefficient is 0.134 (>0.05), this implies that these factors are not statistically significant. Other factors have p-value less than 0.05, which implies these are statistically significant. They have greater significance and have a high impact on Amazon stock returns.

Robust regression model also gives coefficients for all 5 factors. This regression can handle outliers. This model gives different coefficients. These coefficients are the beta values for each factor for Amazon stock. When all five factors are zero, the expected daily log returns of Amazon stock is -0.0003. The betas for each factor is given below.

FACTOR	BETA
Mkt-RF	0.0129
SMB	-0.0037
HML	-0.0017
RMW	-0.0005
CMA	-0.0077

Table 7: FF5 betas (Robust regression)

Robust regression model gives p-values and 95% confidence interval for evaluation. In the below table [0.025] is the lower bound and [0.975] is the upper bound. With 95% confidence, we can say that these coefficients belong to these bounds.

	[0.025]	**[0.975]**
Mkt-RF	0.012	0.014
SMB	-0.005	-0.002
HML	-0.003	-0.000
RMW	-0.002	-0.001
CMA	-0.010	-0.005

Table 8: 95% confidence interval (FF5 betas - Robust regression)

For robust regression, we can see that the p-values of the RMW coefficient is 0.566 (>0.05), this implies that it is not statistically significant. Other factors have p-value less than 0.05, which implies these factors have higher significance in this model and have a high impact on Amazon stock returns.

From the beta values, we have found using both regression models, we can see that Mkt-Rf is positive but it is less than 1. This implies that the risk is low. SMB, HML, RMW and CMA factors have negative betas. From SMB and HML betas, we can see that the Amazon stock chosen is a growth stock and it belongs to large-cap stocks. From RMW and CMA beta values, we can see that it is a low profitability stock and it follows aggressive investment strategies.

Step 5:

c. Correlation matrix:

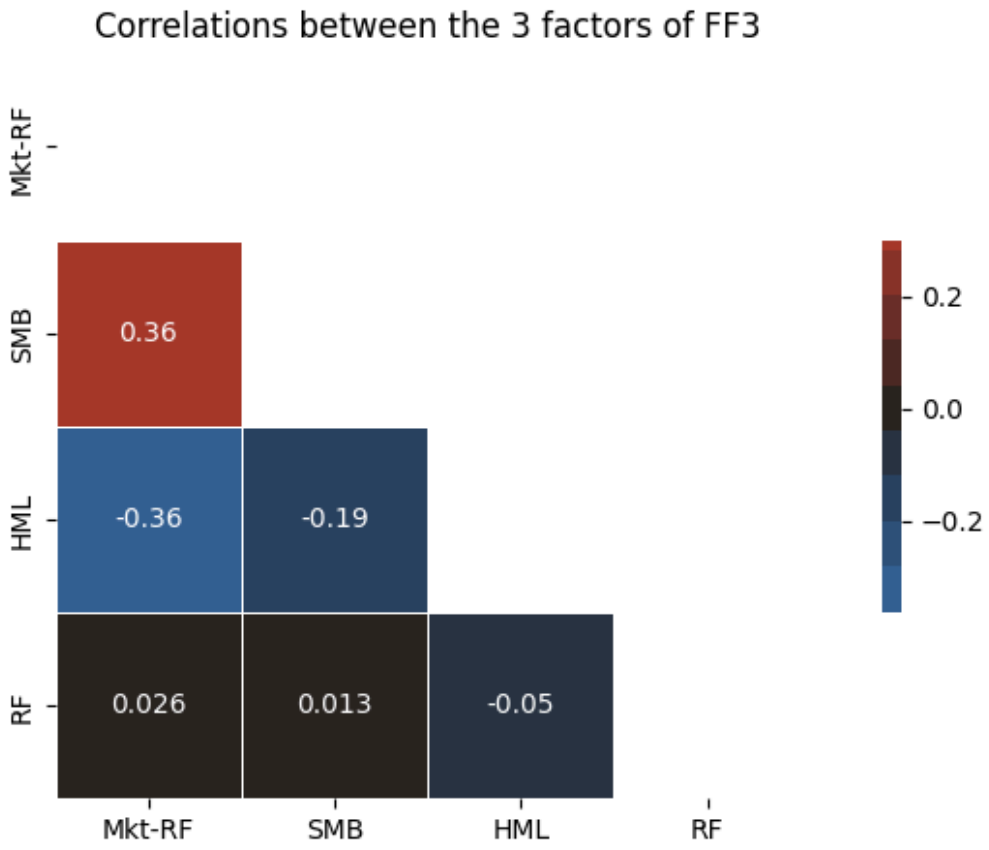


Fig 5: correlations between the 3 factors

d. Covariance matrix:

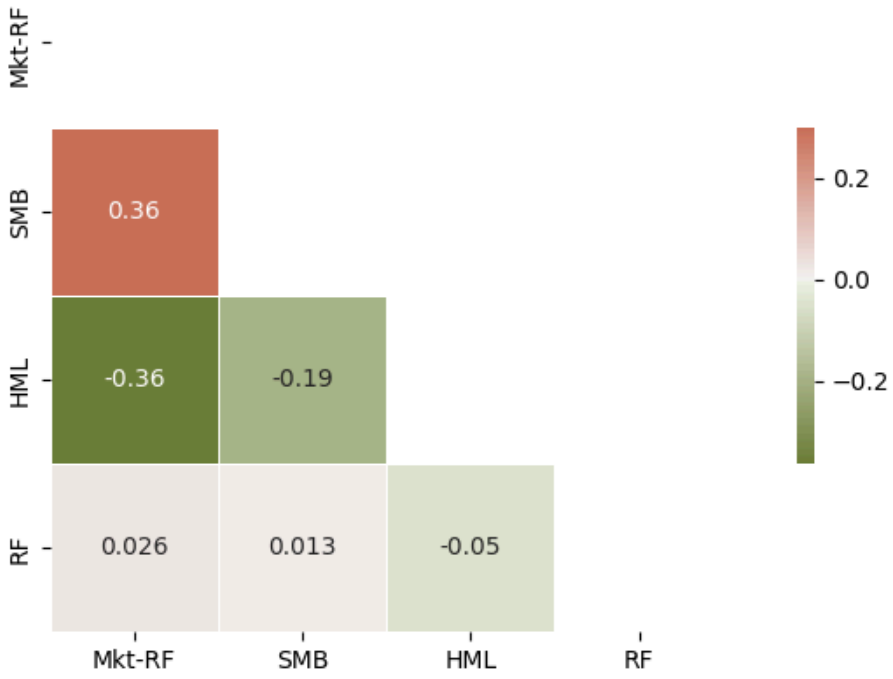


Fig 6: Covariance of the 3 factors

e. Compare and contrast:

The correlation matrix represents the linear relationship between variables and provides values ranging from -1 to 1, where 1 indicates a perfect positive correlation, -1 indicates a perfect negative correlation, and 0 indicates no correlation. By standardizing the values through division by the product of the standard deviations of the variables, it facilitates understanding the strength and direction of the linear relationship between variables, regardless of their scale. Moreover, the correlation matrix aids in identifying multicollinearity and assessing the strength of association between variables in terms of their relative variability.

In contrast, the covariance matrix depicts the joint variability between variables, with values being unbounded and dependent on the scale of the variables. It directly measures the extent to which two variables change together, offering insights into the direction and magnitude of the relationship between variables. However, unlike the correlation matrix, the covariance matrix does not standardize the values, making it challenging to compare relationships across variables with different scales. As a result, covariances can be difficult to interpret due to differences in the scale of variables.

Step 6.

With the additional 2 more factors in FF5 compared to FF3 we are getting more information that is used for predicting the returns of the portfolio so overall this is making the R^2, Adj. R^2 and F-statistics are getting higher which is telling us that the model is getting better by providing a more comprehensive view of the sources of risk and return in a portfolio. It can explain a larger portion of the variation in stock returns compared to the 3-factor model, making it a more accurate tool for evaluating portfolio performance and guiding investment decisions.

The CMA factor is incorporating the company's behavior and attitude towards their politics about investing (aggressive or conservative) and this is how additional view over the company's behavior is provided.

The RMW factor brings to the table the view of how companies control their investment process (not only how much assets and how fast they are buying). Here we can classify how companies are controlling their investments and thus we can categorize them as robust and weak companies. This factor is bringing another field of comparison so it is no wonder that the FF5 model is better than FF3.

metrics	FF3	FF5
R^2	0.612	0.648
Adj R^2	0.61	0.645
F-statistic	314.9	219.3

Table 9. FF3 vs. FF5 model comparison

- $R^2 = 0.612\ for\ FF3\ model\ and\ for\ FF5\colon\ R^2 = 0.648$

$The\ FF5\ model\ explains\ more\ of\ the\ variance\ in\ the\ dependent\ variable\ than\ the\ FF3\ model$

- $Adj.\ R^2 = 0.61\ for\ FF3\ model\ and\ for\ FF5\colon\ Adj.\ R^2 = 0.45$

$The\ FF5\ model\ explains\ the\ variance\ better\ than\ the\ FF3\ model$

- $F - statistic = 314.9\ for\ the\ FF3\ model\ and\ for\ FF5\colon\ F - statistic = 219.3$

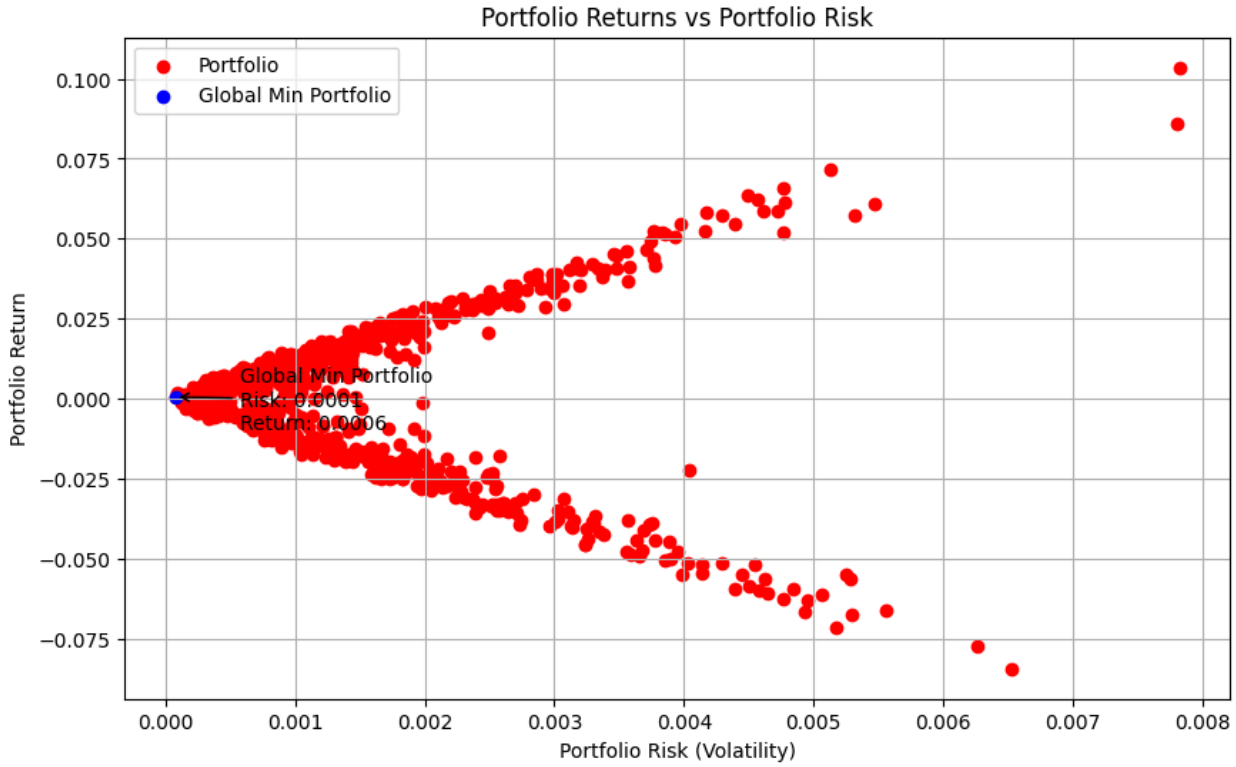
$We\ can\ say\ that\ the\ fit\ of\ the\ FF3\ model\ is\ better.$

The FF5 model explains more of the variance in the dependent variable, but the FF3 model has a better overall fit (as the F-statistic metric shows).

Step 7

Classical Markowitz portfolio optimization

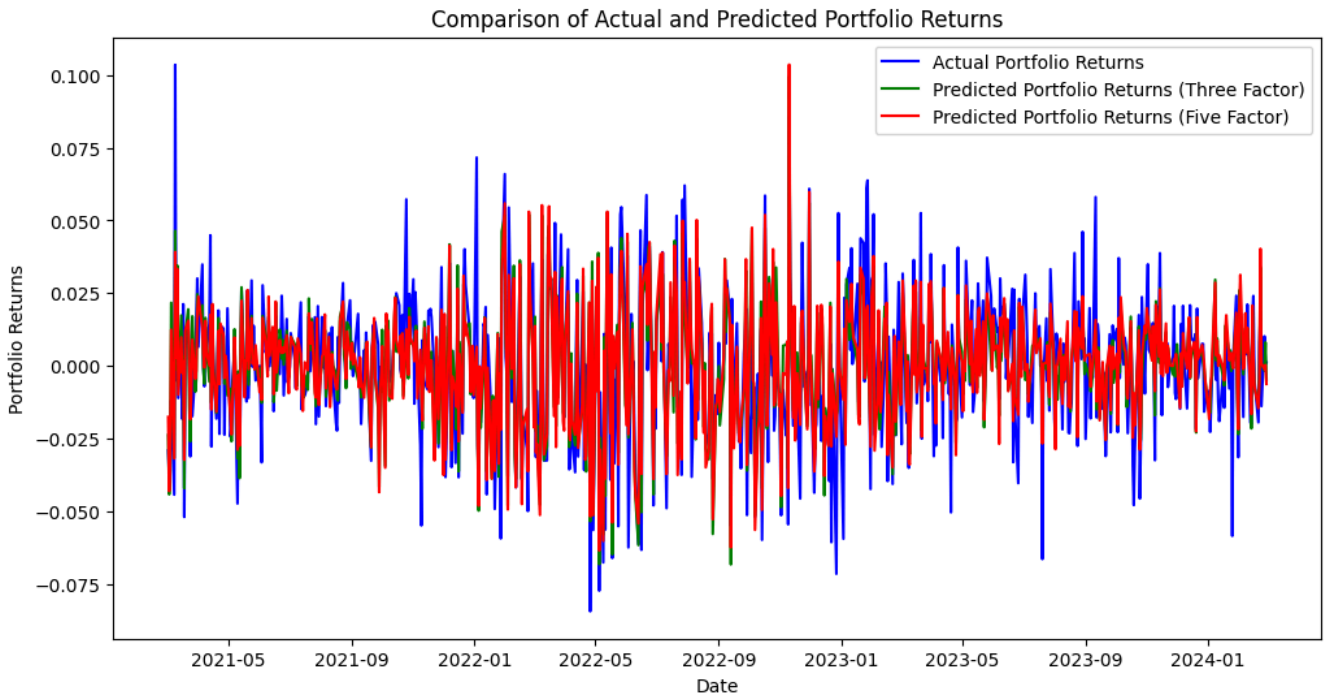
The optimal weights were determined using classical Markowitz portfolio optimization for the companies Apple, Amazon, Google, Microsoft, and Tesla, spanning from March 1, 2021, to February 29, 2024. The resulting optimal weights were as follows: 0.0986 for Apple, 0.2960 for Amazon, 0.1638 for Google, -0.0476 for Microsoft, and 0.4891 for Tesla. These weights were used to construct the efficient frontier, depicted below.



The global minimum portfolio is indicated on the plot with a risk level of 0.0001 and a return of 0.0006, given that short selling is permitted.

The FF3 and FF5 estimating the portfolio returns

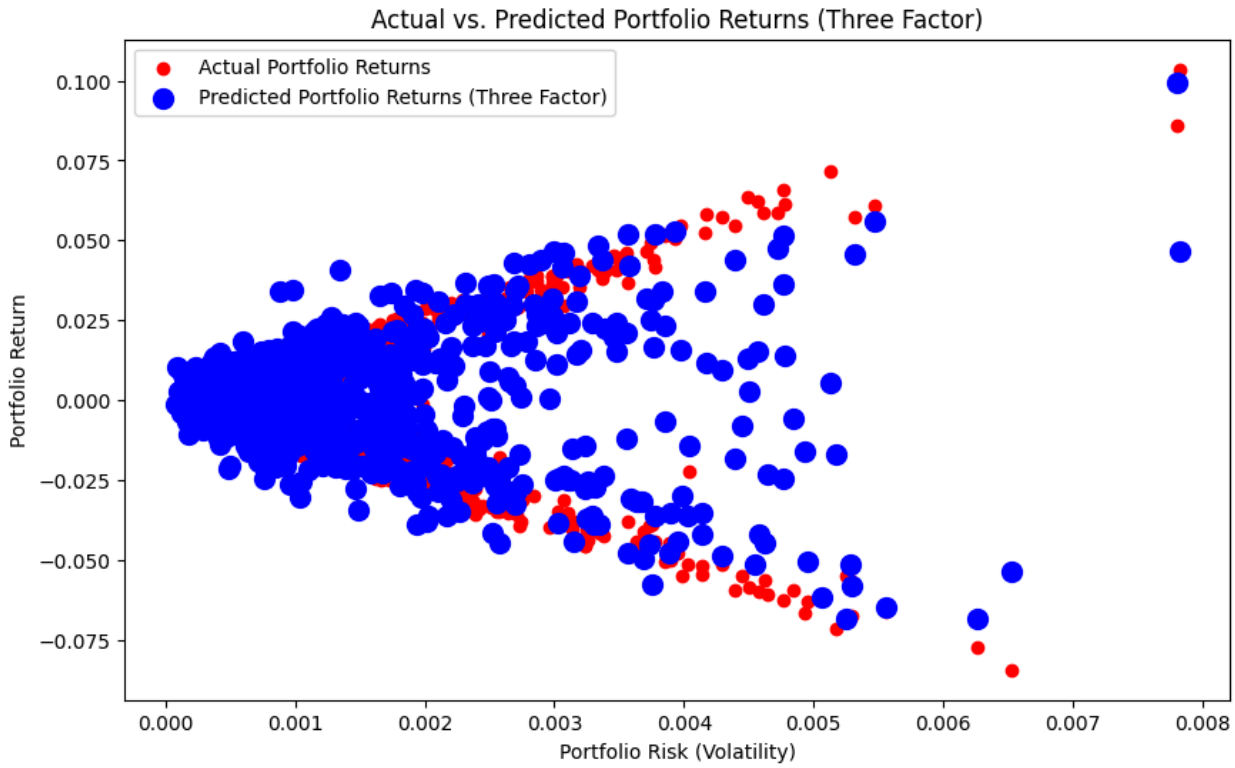
The aim is to illustrate how the three-factor and five-factor models explain portfolio returns by plotting the predicted portfolio returns based on FF3 and FF5 against dates, as depicted below.



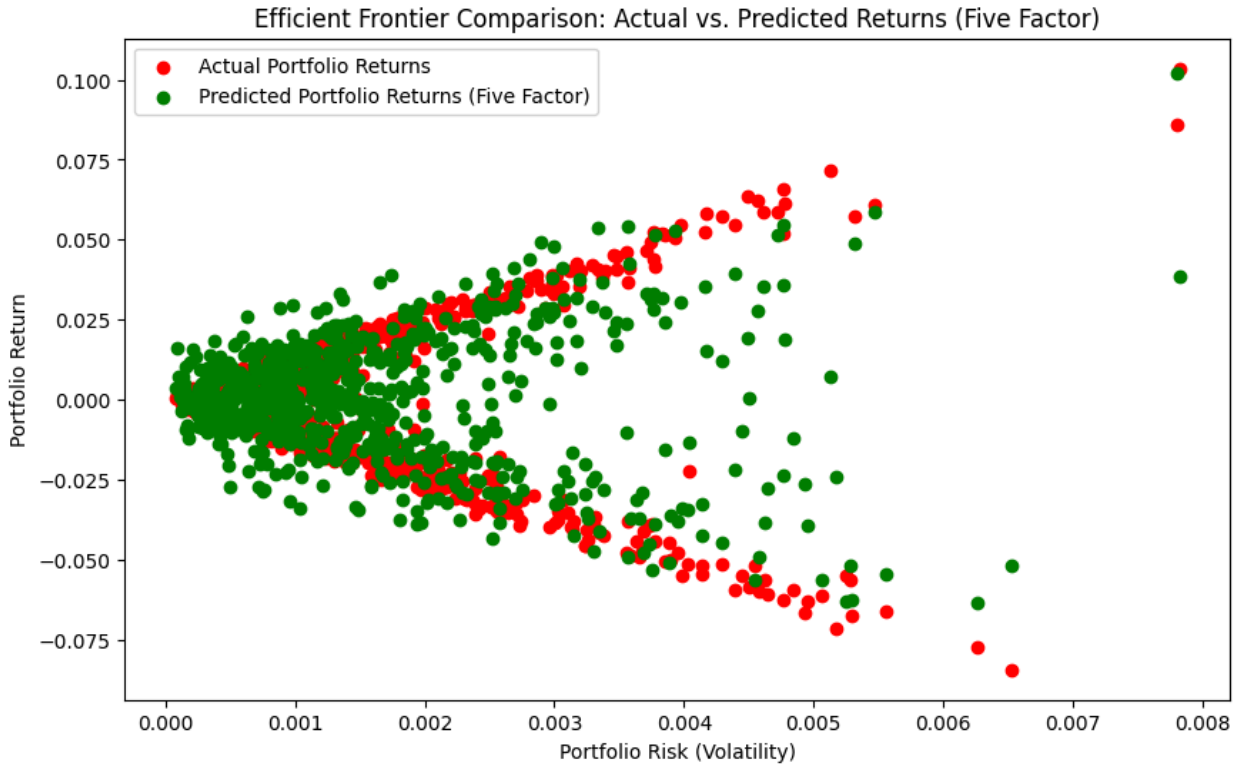
From the plot above, it appears that the predicted portfolio returns based on the five-factor model correspond closely to the actual portfolio returns derived from the optimization solution weights. However, the three-factor model performs less favorably in comparison. Additionally, the prediction of returns seems to exhibit more volatility compared to the actual portfolio returns. This discrepancy may be attributed to the fact that the optimization process prioritized minimum risks when determining the optimal weights.

The efficient frontier comparisons

The following plots depict comparisons between the efficient frontiers derived from classical Markowitz solutions and the predicted portfolio returns based on FF3 and FF5 models. For simplicity, it is assumed that the predicted portfolio returns have the same risk as the actual portfolio returns. However, this assumption may not hold true, as evidenced by the plot above, indicating that the predicted returns exhibit higher volatility.



The comparison suggests that the portfolio returns estimated using FF3 tend to overestimate returns compared to those estimated using FF5, as observed against the actual portfolio returns. Nevertheless, the predicted portfolio returns generally align well with the actual portfolio returns.



The plot illustrates that the five-factor model (FF5) outperforms the three-factor model (FF3) in estimating portfolio returns. This indicates that the inclusion of two additional factors enhances the predictive capabilities of the five-factor model compared to the three-factor model.

Contribution of each factor under FF3 and FF5

The extent to which each factor contributes to the portfolio is assessed by examining the portfolio returns as the response variable in the classical Markowitz framework and evaluating the significance of each coefficient. Under the FF3 model, both the Least Squares and Robust models show overall significance, with an adjusted R-squared value of 64.6%. However, the SMB variable is not deemed significant as its p-value exceeds 0.05. Similar outcomes are observed under the FF5 model, where both the Least Squares and Robust models exhibit overall significance. In the Least Squares model, SMB and HML are not significant, while all variables are significant in the Robust model, leading to an improvement in the adjusted R-squared value.

Step 8:

In this project, we have compared the performance of the Fama-French three-factor model and the Fama-French five-factor model. The results we have obtained are given here. From the beta coefficients we have found, for the Amazon stock, the Market factor is positive and the beta coefficients for all the other factors are negative. This implies that the market risk is low for the Amazon stock. The beta coefficient of size factor (SMB) implies that the Amazon stock is a big-cap stock and the beta coefficient of value factor (HML) implies that the Amazon stock is a growth stock. The HML factor and CMA factor have the most positive correlation for this stock. From the beat coefficients, we know that this stock is a growth stock, which implies that the profitability will be low. Negative beta coefficient for CMA factor implies that the Amazon stock over this time period follows aggressive investment strategies. This also shows that the profitability will be low compared to value stocks and conservative investments. Overall, the 5-factor model can explain a larger portion of the variation in stock returns compared to the 3-factor model, making the 5-factor model a better choice for evaluating portfolio performance and investment decisions.

Based on the technical analysis, it is concluded that the FF5 model surpasses FF3 in accurately estimating portfolio returns, specifically in the case of Amazon log returns, as evidenced by its overall significance in both the least squares and robust regression results. The FF5 model exhibits an enhancement in the adjusted R-squared value, indicating an overall improvement attributed to the inclusion of two additional factors that significantly contribute to the regression model. Thus, the objective of demonstrating the superiority of FF5 over FF3 in estimating security returns has been achieved.

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