

Theme: Statistical Related Risk: Volatility & Statistical

Related Risk: Correlation

Statistics

```
# Libraries
import pandas as pd
import numpy as np
import seaborn as sns
import matplotlib.pyplot as plt

import yfinance as yf
# from ydata_profiling import ProfileReport
# from IPython.display import IFrame

#statistical tests
import statsmodels.api as sm
from scipy import stats

#import yfinance as yf

btcusdt_ticker = "BTC-USD"
start_date = "2020-01-01"
end_date = "2023-04-25"

df = yf.download(btcusdt_ticker, start=start_date, end=end_date)
print(df.head())
```

```
[*****100%*****] 1 of 1 completed
```

| | Open | High | Low | Close | Adj Close | \ |
|------------|-------------|-------------|-------------|-------------|-------------|---|
| Date | | | | | | |
| 2020-01-01 | 7194.892090 | 7254.330566 | 7174.944336 | 7200.174316 | 7200.174316 | |
| 2020-01-02 | 7202.551270 | 7212.155273 | 6935.270020 | 6985.470215 | 6985.470215 | |
| 2020-01-03 | 6984.428711 | 7413.715332 | 6914.996094 | 7344.884277 | 7344.884277 | |
| 2020-01-04 | 7345.375488 | 7427.385742 | 7309.514160 | 7410.656738 | 7410.656738 | |
| 2020-01-05 | 7410.451660 | 7544.497070 | 7400.535645 | 7411.317383 | 7411.317383 | |

| | Volume |
|------------|-------------|
| Date | |
| 2020-01-01 | 18565664997 |
| 2020-01-02 | 20802083465 |
| 2020-01-03 | 28111481032 |
| 2020-01-04 | 18444271275 |

✓ 0s completed at 11:24 AM



Double-click (or enter) to edit

```
# installing additional package for desriptive statistics in html format
!pip install ydata_profiling
```

```
import ydata_profiling as ypr
from ydata_profiling import ProfileReport
```

```
Looking in indexes: https://pypi.org/simple, https://us-python.pkg.dev/colab-wheels/public/simple/
Collecting ydata_profiling
  Downloading ydata_profiling-4.1.2-py2.py3-none-any.whl (345 kB)
    _____ 345.9/345.9 kB 2.5 MB/s eta 0:00:00
Collecting matplotlib<3.7,>=3.2
  Downloading matplotlib-3.6.3-cp310-cp310-manylinux_2_17_x86_64.manylinux2014_x86_64.whl (11.8 MB)
    _____ 11.8/11.8 MB 79.4 MB/s eta 0:00:00
Requirement already satisfied: pydantic<1.11,>=1.8.1 in /usr/local/lib/python3.10/dist-packages (1.10.2)
Requirement already satisfied: PyYAML<6.1,>=5.0.0 in /usr/local/lib/python3.10/dist-packages (5.4.1)
Collecting tqdm<4.65,>=4.48.2
  Downloading tqdm-4.64.1-py2.py3-none-any.whl (78 kB)
    _____ 78.5/78.5 kB 9.6 MB/s eta 0:00:00
Requirement already satisfied: jinja2<3.2,>=2.11.1 in /usr/local/lib/python3.10/dist-packages (2.11.2)
Collecting scipy<1.10,>=1.4.1
  Downloading scipy-1.9.3-cp310-cp310-manylinux_2_17_x86_64.manylinux2014_x86_64.whl (33.7 MB)
    _____ 33.7/33.7 MB 16.4 MB/s eta 0:00:00
Requirement already satisfied: seaborn<0.13,>=0.10.1 in /usr/local/lib/python3.10/dist-packages (0.11.2)
Requirement already satisfied: numpy<1.24,>=1.16.0 in /usr/local/lib/python3.10/dist-packages (1.23.5)
Collecting phik<0.13,>=0.11.1
  Downloading phik-0.12.3-cp310-cp310-manylinux_2_17_x86_64.manylinux2014_x86_64.whl (679.5 kB)
    _____ 679.5/679.5 kB 49.0 MB/s eta 0:00:00
Collecting typeguard<2.14,>=2.13.2
  Downloading typeguard-2.13.3-py3-none-any.whl (17 kB)
Requirement already satisfied: statsmodels<0.14,>=0.13.2 in /usr/local/lib/python3.10/dist-packages (0.13.5)
Collecting multimethod<1.10,>=1.4
  Downloading multimethod-1.9.1-py3-none-any.whl (10 kB)
Requirement already satisfied: requests<2.29,>=2.24.0 in /usr/local/lib/python3.10/dist-packages (2.28.1)
Collecting htmlmin==0.1.12
  Downloading htmlmin-0.1.12.tar.gz (19 kB)
  Preparing metadata (setup.py) ... done
Requirement already satisfied: pandas!=1.4.0,<1.6,>1.1 in /usr/local/lib/python3.10/dist-packages (1.5.3)
Collecting visions[type_image_path]==0.7.5
  Downloading visions-0.7.5-py3-none-any.whl (102 kB)
    _____ 102.7/102.7 kB 12.2 MB/s eta 0:00:00
Collecting imagehash==4.3.1
  Downloading ImageHash-4.3.1-py2.py3-none-any.whl (296 kB)
    _____ 296.5/296.5 kB 12.6 MB/s eta 0:00:00
Requirement already satisfied: PyWavelets in /usr/local/lib/python3.10/dist-packages (1.4.1)
Requirement already satisfied: pillow in /usr/local/lib/python3.10/dist-packages (9.4.0)
Requirement already satisfied: networkx>=2.4 in /usr/local/lib/python3.10/dist-packages (2.8.8)
Requirement already satisfied: attrs>=19.3.0 in /usr/local/lib/python3.10/dist-packages (23.1.0)
Collecting tangled-up-in-unicode>=0.0.4
```

Collecting tangled-up-in-unicode>=0.2.0

Downloading tangled_up_in_unicode-0.2.0-py3-none-any.whl (4.7 MB)

4.7/4.7 MB 44.5 MB/s eta 0:00:00

Requirement already satisfied: MarkupSafe>=2.0 in /usr/local/lib/python3.10/dist-packages (from Flask==2.2.2) (2.1.2)
 Requirement already satisfied: kiwisolver>=1.0.1 in /usr/local/lib/python3.10/dist-packages (from Flask==2.2.2) (1.4.2)
 Requirement already satisfied: pyparsing>=2.2.1 in /usr/local/lib/python3.10/dist-packages (from Flask==2.2.2) (3.0.9)
 Requirement already satisfied: contourpy>=1.0.1 in /usr/local/lib/python3.10/dist-packages (from matplotlib==3.5.2) (1.0.7)
 Requirement already satisfied: fonttools>=4.22.0 in /usr/local/lib/python3.10/dist-packages (from matplotlib==3.5.2) (4.25.0)
 Requirement already satisfied: packaging>=20.0 in /usr/local/lib/python3.10/dist-packages (from matplotlib==3.5.2) (23.0)
 Requirement already satisfied: python-dateutil>=2.7 in /usr/local/lib/python3.10/dist-packages (from matplotlib==3.5.2) (2.8.2)
 Requirement already satisfied: cycler>=0.10 in /usr/local/lib/python3.10/dist-packages (from matplotlib==3.5.2) (0.11.0)
 Requirement already satisfied: pytz>=2020.1 in /usr/local/lib/python3.10/dist-packages (from matplotlib==3.5.2) (2022.7.1)
 Requirement already satisfied: joblib>=0.14.1 in /usr/local/lib/python3.10/dist-packages (from sklearn==1.2.1) (1.2.0)
 Requirement already satisfied: typing-extensions>=4.2.0 in /usr/local/lib/python3.10/dist-packages (from sklearn==1.2.1) (4.4.0)
 Requirement already satisfied: certifi>=2017.4.17 in /usr/local/lib/python3.10/dist-packages (from urllib3==1.26.12) (2022.9.24)
 Requirement already satisfied: idna<4, >=2.5 in /usr/local/lib/python3.10/dist-packages (from urllib3==1.26.12) (3.4)
 Requirement already satisfied: urllib3<1.27, >=1.21.1 in /usr/local/lib/python3.10/dist-packages (from urllib3==1.26.12) (1.26.12)
 Requirement already satisfied: charset-normalizer~=2.0.0 in /usr/local/lib/python3.10/dist-packages (from urllib3==1.26.12) (2.0.12)
 Requirement already satisfied: patsy>=0.5.2 in /usr/local/lib/python3.10/dist-packages (from statsmodels==0.13.2) (0.5.2)
 Requirement already satisfied: six in /usr/local/lib/python3.10/dist-packages (from statsmodels==0.13.2) (1.16.0)

Building wheels for collected packages: htmlmin

Building wheel for htmlmin (setup.py) ... done

Created wheel for htmlmin: filename=htmlmin-0.1.12-py3-none-any.whl size=27096 sha256=...

Stored in directory: /root/.cache/pip/wheels/dd/91/29/a79cecb328d01739e64017b6f...

Successfully built htmlmin

Installing collected packages: htmlmin, typeguard, tqdm, tangled-up-in-unicode, sc

Attempting uninstall: tqdm

Found existing installation: tqdm 4.65.0

Uninstalling tqdm-4.65.0:

Successfully uninstalled tqdm-4.65.0

Attempting uninstall: scipy

Found existing installation: scipy 1.10.1

Uninstalling scipy-1.10.1:

Successfully uninstalled scipy-1.10.1

Attempting uninstall: matplotlib

Found existing installation: matplotlib 3.7.1

Uninstalling matplotlib-3.7.1:

Successfully uninstalled matplotlib-3.7.1

Successfully installed htmlmin-0.1.12 imagehash-4.3.1 matplotlib-3.6.3 multimethod

WARNING: The following packages were previously imported in this runtime:

[matplotlib,mpl_toolkits]

You must restart the runtime in order to use newly installed versions.

RESTART RUNTIME

df_btc = df

returns_btc = df

profile = ProfileReport(df_btc)

profile.to_file('df_btc.html')

Summarize dataset:

51/51 [00:10<00:00, 2.94it/s,

100%

Completed]

Generate report structure:

1/1 [00:06<00:00,

100%

6.96s/it]

Render HTML · 100%

1/1 [00:02<00:00 2.95s/it]

profile

Overview

Dataset statistics

| | |
|-------------------------------|----------|
| Number of variables | 6 |
| Number of observations | 1210 |
| Missing cells | 0 |
| Missing cells (%) | 0.0% |
| Duplicate rows | 0 |
| Duplicate rows (%) | 0.0% |
| Total size in memory | 66.2 KiB |
| Average record size in memory | 56.0 B |

Variable types

| | |
|---------|---|
| Numeric | 6 |
|---------|---|

Alerts

| | |
|--|------------------|
| Open is highly overall correlated with High and 3 other fields (High, Low, Close, Adj Close) | High correlation |
| High is highly overall correlated with Open and 3 other fields (Open, Low, Close, Adj Close) | High correlation |
| Low is highly overall correlated with Open and 3 other fields (Open, High, Close, Adj Close) | High correlation |
| Close is highly overall correlated with Open and 3 other fields (Open | High correlation |

I am interested only in the information in `df_btc['Adj Close']`, and I will use it for further analysis. So, in case you are interested in the other columns like: Open, High, Low, Close or Volume, the same type of analysis can be applied towards them as well. Let's do some statistical evaluations to prove whether the data is normally distributed. I will use the visual method called: "Q-Q Plot". Here you can see that the data is NOT normally distributed because otherwise it should be very close to the 45-degree line added to the plot (because normally distributed data do so).

```
returns_btc['returns'] = df['Adj Close'].pct_change()
returns_btc = returns_btc.dropna()
```

```
profile_returns = ProfileReport(returns_btc)
profile_returns
```

| | |
|----------------------------|-------------------------------|
| Summarize dataset: | 65/65 [00:19<00:00, 3.33it/s, |
| 100% | Completed] |
| Generate report structure: | 1/1 [00:09<00:00, |
| 100% | 9.93s/it] |
| Render HTML: 100% | 1/1 [00:03<00:00, 3.52s/it] |

Overview

Dataset statistics

| | |
|------------------------|------|
| Number of variables | 7 |
| Number of observations | 1209 |
| Missing cells | 0 |
| Missing cells (%) | 0.0% |
| Duplicate rows | 0 |
| Duplicate rows (%) | 0.0% |

| | |
|-------------------------------|----------|
| Duplicate rows (%) | 0.0% |
| Total size in memory | 75.6 KiB |
| Average record size in memory | 64.0 B |

Variable types

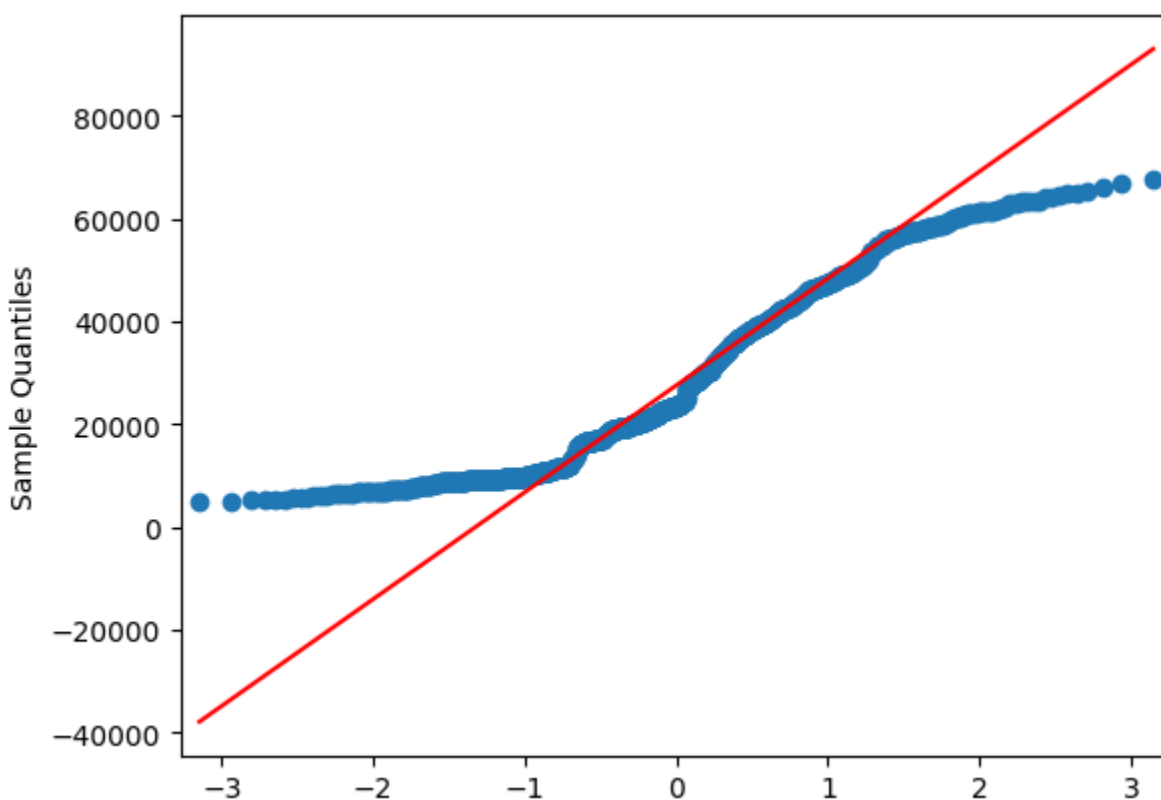
| | |
|---------|---|
| Numeric | 7 |
|---------|---|

Alerts

| | |
|--|------------------|
| Open is highly overall correlated with High and 3 other fields (High, Low, Close, Adj Close) | High correlation |
| High is highly overall correlated with Open and 3 other fields (Open, Low, Close, Adj Close) | High correlation |
| Low is highly overall correlated with Open and 3 other fields (Open, Close, Adj Close, High) | High correlation |

```
# import statsmodels.api as sm

# create Q-Q plot with 45-degree line added to the plot
"""
this plot can be used to see the qqplot of:
- returns_btc['Adj Close']
- returns_btc['returns']
"""
fig = sm.qqplot(returns_btc['Adj Close'], line='q')
plt.show()
```



Theoretical Quantiles

Visualizations are not exactly scientific proof for anything because every human being has his own interpretations and abstract thinking, and it is possible for someone not to be able to see it as it is. In this case I will use the very formal statistical test: Shapiro-Wilk test and it will mathematically (statistically) prove or not whether there is a normal distribution. If the value of the Shapiro test's p-value is less than 0.05. This means that with 95% confidence the data is NOT normally distributed

```
#from scipy import stats

# perform Shapiro-Wilk test
shapiro_test = stats.shapiro(returns_btc['returns'])
print(shapiro_test)

ShapiroResult(statistic=0.9171086549758911, pvalue=3.349220646237701e-25)
```

So, what can we do in this case?

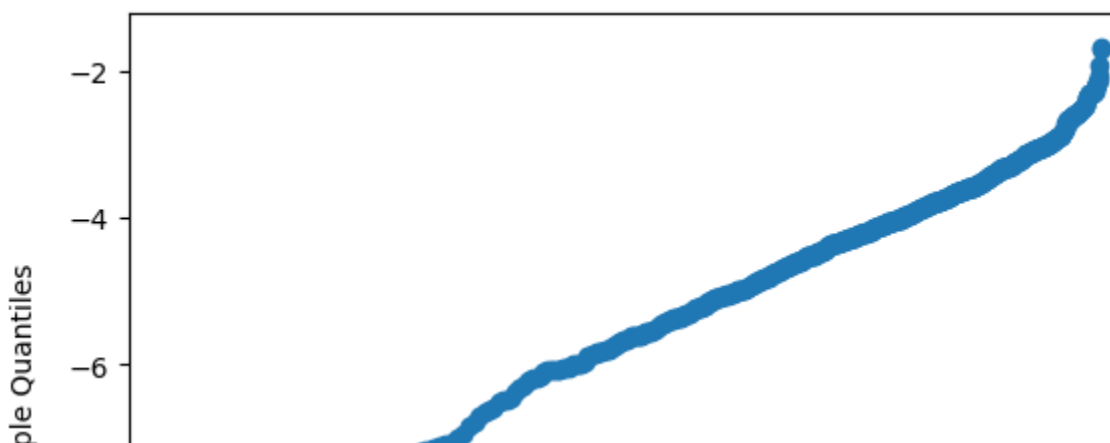
I will try to normalize the data be using the so called `numpy.log()` method which is a simple logarithm transformation over all the values in the data.

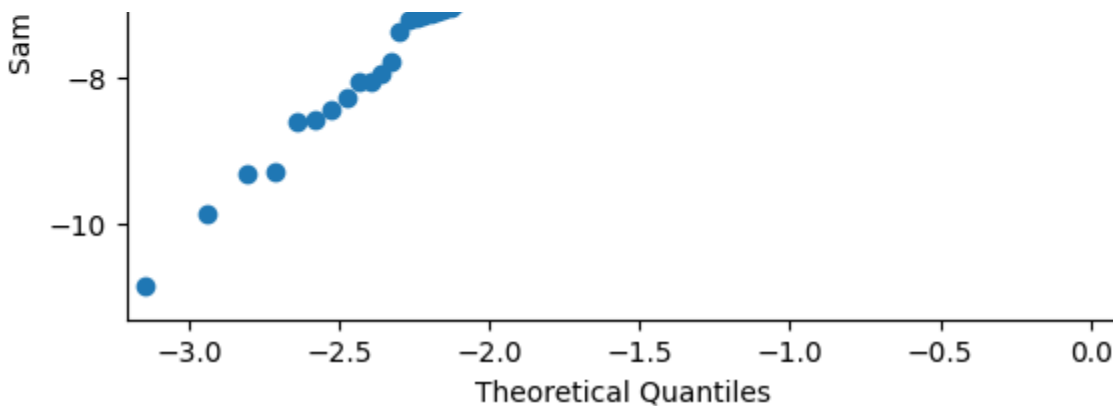
And the I will use once more the previous methods to check for normality

```
logged_btc = np.log(returns_btc['returns'])

fig = sm.qqplot(logged_btc)
plt.show()
```

```
/usr/local/lib/python3.10/dist-packages/pandas/core/arraylike.py:402: RuntimeWarni
result = getattr(ufunc, method)(*inputs, **kwargs)
```





```
# perform Shapiro-Wilk test
shapiro_test = stats.shapiro(logged_btc)
print(shapiro_test)

ShapiroResult(statistic=nan, pvalue=1.0)
```

And now with a p-value of 1 from the Shapiro Wilk test for normality we can say that the logged data of Bitcoin returns is normally distributed with very few outliers in the tails.

Correlations

```
!pip install statsmodels
!pip install pmdarima
!pip install vectorbt
```

```
Looking in indexes: https://pypi.org/simple, https://us-python.pkg.dev/colab-wheels/public/simple/
Requirement already satisfied: statsmodels in /usr/local/lib/python3.10/dist-packages (0.13.2)
Requirement already satisfied: numpy>=1.17 in /usr/local/lib/python3.10/dist-packages (1.21.2)
Requirement already satisfied: pandas>=0.25 in /usr/local/lib/python3.10/dist-packages (1.3.2)
Requirement already satisfied: packaging>=21.3 in /usr/local/lib/python3.10/dist-packages (21.3)
Requirement already satisfied: patsy>=0.5.2 in /usr/local/lib/python3.10/dist-packages (0.5.2)
Requirement already satisfied: scipy>=1.3 in /usr/local/lib/python3.10/dist-packages (1.3.2)
Requirement already satisfied: pytz>=2020.1 in /usr/local/lib/python3.10/dist-packages (2020.1)
Requirement already satisfied: python-dateutil>=2.8.1 in /usr/local/lib/python3.10/dist-packages (2.8.1)
Requirement already satisfied: six in /usr/local/lib/python3.10/dist-packages (1.16.0)
Looking in indexes: https://pypi.org/simple, https://us-python.pkg.dev/colab-wheels/public/simple/
Collecting pmdarima
  Downloading pmdarima-2.0.3-cp310-cp310-manylinux_2_17_x86_64.manylinux2014_x86_64.whl (1.8 MB)
    1.8/1.8 MB 18.4 MB/s eta 0:00:00
Requirement already satisfied: numpy>=1.21.2 in /usr/local/lib/python3.10/dist-packages (1.21.2)
Requirement already satisfied: pandas>=0.19 in /usr/local/lib/python3.10/dist-packages (1.3.2)
Requirement already satisfied: scikit-learn>=0.22 in /usr/local/lib/python3.10/dist-packages (0.22)
Requirement already satisfied: scipy>=1.3.2 in /usr/local/lib/python3.10/dist-packages (1.3.2)
Requirement already satisfied: setuptools!=50.0.0, >=38.6.0 in /usr/local/lib/python3.10/dist-packages (57.0.0)
Requirement already satisfied: Cython!=0.29.18, !=0.29.31, >=0.29 in /usr/local/lib/python3.10/dist-packages (0.29.32)
Requirement already satisfied: statsmodels<0.13.2 in /usr/local/lib/python3.10/dist-packages (0.13.2)
```



```

Requirement already satisfied: statsmodels<0.13.2 in /usr/local/lib/python3.10/dist-packages
Requirement already satisfied: urllib3 in /usr/local/lib/python3.10/dist-packages
Requirement already satisfied: joblib>=0.11 in /usr/local/lib/python3.10/dist-packages
Requirement already satisfied: pytz>=2020.1 in /usr/local/lib/python3.10/dist-packages
Requirement already satisfied: python-dateutil>=2.8.1 in /usr/local/lib/python3.10/dist-packages
Requirement already satisfied: threadpoolctl>=2.0.0 in /usr/local/lib/python3.10/dist-packages
Requirement already satisfied: patsy>=0.5.2 in /usr/local/lib/python3.10/dist-packages
Requirement already satisfied: packaging>=21.3 in /usr/local/lib/python3.10/dist-packages
Requirement already satisfied: six in /usr/local/lib/python3.10/dist-packages (from
Installing collected packages: pmdarima
Successfully installed pmdarima-2.0.3
Looking in indexes: https://pypi.org/simple, https://us-python.pkg.dev/colab-wheels/public/simple/
Collecting vectorbt
  Downloading vectorbt-0.25.0-py3-none-any.whl (526 kB)
    

---

526.7/526.7 kB 7.7 MB/s eta 0:00:00
Collecting dateparser
  Downloading dateparser-1.1.8-py2.py3-none-any.whl (293 kB)
    

---

293.8/293.8 kB 26.5 MB/s eta 0:00:00
Requirement already satisfied: pytz in /usr/local/lib/python3.10/dist-packages (fr
Requirement already satisfied: pandas in /usr/local/lib/python3.10/dist-packages (
Requirement already satisfied: matplotlib in /usr/local/lib/python3.10/dist-packag
Collecting dill
  Downloading dill-0.3.6-py3-none-any.whl (110 kB)
    

---

110.5/110.5 kB 12.0 MB/s eta 0:00:00
Requirement already satisfied: requests in /usr/local/lib/python3.10/dist-packages
Requirement already satisfied: tqdm in /usr/local/lib/python3.10/dist-packages (fr
Requirement already satisfied: imageio in /usr/local/lib/python3.10/dist-packages
Collecting schedule
  Downloading schedule-1.2.0-py2.py3-none-any.whl (11 kB)
Requirement already satisfied: numba>=0.56.0 in /usr/local/lib/python3.10/dist-pac
Requirement already satisfied: numpy>=1.16.5 in /usr/local/lib/python3.10/dist-pac
Requirement already satisfied: scikit-learn in /usr/local/lib/python3.10/dist-pack
Requirement already satisfied: plotly>=4.12.0 in /usr/local/lib/python3.10/dist-pa
Requirement already satisfied: scipy in /usr/local/lib/python3.10/dist-packages (f
Requirement already satisfied: ipywidgets>=7.0.0 in /usr/local/lib/python3.10/dist
Collecting mypy_extensions
  Downloading mypy_extensions-1.0.0-py3-none-any.whl (4.7 kB)
Requirement already satisfied: ipykernel>=4.5.1 in /usr/local/lib/python3.10/dist-

```

```

from statsmodels.tsa.stattools import adfuller, grangercausalitytests
from statsmodels.tools.tools import add_constant
from statsmodels.tsa.stattools import lagmat2ds
from statsmodels.regression.linear_model import OLS
from statsmodels.tsa.arima.model import ARIMA
import statsmodels.api as sm
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from statsmodels.tsa.stattools import acf, pacf, ccf
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
import yfinance as yf
import vectorbt as vbt
import pmdarima as pmd
from scipy.ndimage import shift

```

```
import plotly.graph_objects as go
```

```
df = yf.download("BTC-USD", start="2020-01-01", end="2023-04-25")
```

```
df_10yt = yf.download("^TNX", start="2020-01-01", end="2023-04-25")[['Adj Close']].reindex(df.index)
```

```
df_10yt['US_10_Year_Treasury_Yield'] *= 0.01
```

```
df = df.join(df_10yt, how='left')
```

```
df = df.dropna(subset=['US_10_Year_Treasury_Yield'])
```

```
df['Return'] = df['Adj Close'].pct_change()
```

```
df.index = pd.to_datetime(df.index)
```

```
df = df[['Return', 'US_10_Year_Treasury_Yield']].iloc[1:].copy()
```

```
df['Return'].describe()
```

```
[*****100%*****] 1 of 1 completed
```

```
[*****100%*****] 1 of 1 completed
```

```
count      832.000000
```

```
mean         0.002676
```

```
std          0.044690
```

```
min         -0.371695
```

```
25%         -0.016921
```

```
50%          0.001386
```

```
75%          0.022990
```

```
max          0.211097
```

```
Name: Return, dtype: float64
```

```
df
```

| | Return | US_10_Year_Treasury_Yield |
|------------|-----------|---------------------------|
| Date | | |
| 2020-01-03 | 0.051452 | 0.01788 |
| 2020-01-06 | 0.057773 | 0.01811 |
| 2020-01-07 | 0.050774 | 0.01827 |
| 2020-01-08 | -0.010269 | 0.01874 |
| 2020-01-09 | -0.024851 | 0.01858 |
| ... | ... | ... |
| 2023-04-18 | 0.032349 | 0.03572 |
| 2023-04-19 | -0.051809 | 0.03602 |
| 2023-04-20 | -0.020008 | 0.03545 |
| 2023-04-21 | -0.034309 | 0.03570 |
| 2023-04-24 | 0.009108 | 0.03515 |

832 rows × 2 columns

Augmented Dickey–Fuller test: The testing procedure for the ADF test is the same as for the Dickey–Fuller test [16] but it is applied to the model

$$\Delta y_t = \alpha + \beta t + \gamma y_{t-1} + \sum_{i=1}^{p-1} \delta_i \Delta y_{t-i} + \epsilon_t$$

The unit root test is then carried out under the null hypothesis $\gamma = 0$ [17]

To ensure the stability of the results, it is essential to verify that both time series are stationary before conducting a Granger causality analysis. We will employ the Augmented Dickey-Fuller (ADF) test to confirm stationarity, with the p-value being used as a threshold, where a value exceeding 0.05 indicates non-stationarity.

```
for col in ['US_10_Year_Treasury_Yield', 'Return']:
    p_value = adfuller(df[col])[1]
    if p_value > 0.05:
        print(f'{col} is nonstationary with p-value {p_value}, so we need to do differe
    else:
        print(f'{col} is stationary with p-value {p_value}')
```

```
US_10_Year_Treasury_Yield is nonstationary with p-value 0.9419752839401622, so we
Return is stationary with p-value 1.797948926210907e-17
```

```
df['US_10_Year_Treasury_Yield'] = df['US_10_Year_Treasury_Yield'].diff()
df = df.rename(columns={'US_10_Year_Treasury_Yield': f'US_10_Year_Treasury_Yield_diff_1'})
df = df.dropna(subset=['US_10_Year_Treasury_Yield_diff_1'])
p_value = adfuller(df['US_10_Year_Treasury_Yield_diff_1'])[1]
print(p_value)
```

```
6.883099792070528e-13
```

```
split_point = int(0.2*len(df))
df_test = df.iloc[-split_point:]
df = df.iloc[:-split_point]
print(f"df length: {len(df)}, df_test length: {len(df_test)}")
```

```
df length: 665, df_test length: 166
```

```
def create_corr_plot(df, factor='', plot_pacf=False):
    """
    Code source: https://community.plotly.com/t/plot-pacf-plot-acf-autocorrelation-plot
    """
    series = df[factor]
    corr_array = pacf(series.dropna(), nlags=100, alpha=0.05) if plot_pacf else acf(series)
    corr_array = (corr_array[0][1:], corr_array[1][1:])
```

```

lower_y = corr_array[1][:,0] - corr_array[0]
upper_y = corr_array[1][:,1] - corr_array[0]

fig = go.Figure()
[fig.add_scatter(x=(x,x), y=(0,corr_array[0][x]), mode='lines',line_color='#3f3f3f'
  for x in range(len(corr_array[0])))
fig.add_scatter(x=np.arange(len(corr_array[0])), y=corr_array[0], mode='markers', m
  marker_size=12)
fig.add_scatter(x=np.arange(len(corr_array[0])), y=upper_y, mode='lines', line_color=
fig.add_scatter(x=np.arange(len(corr_array[0])), y=lower_y, mode='lines', fillcolor
  fill='tonexty', line_color='rgba(255,255,255,0)')
fig.update_traces(showlegend=False)
fig.update_xaxes(range=[-1,42])
fig.update_yaxes(zeroLineColor='#000000')

title=f'Partial Autocorrelation (PACF): {factor}' if plot_pacf else f'Autocorrelati
fig.update_layout(title=title, width=600, height=400)
fig.show()

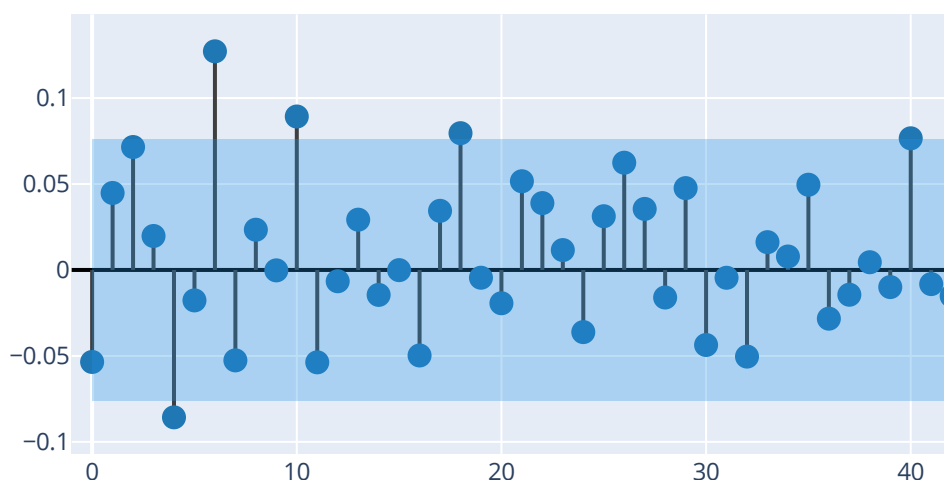
```

```

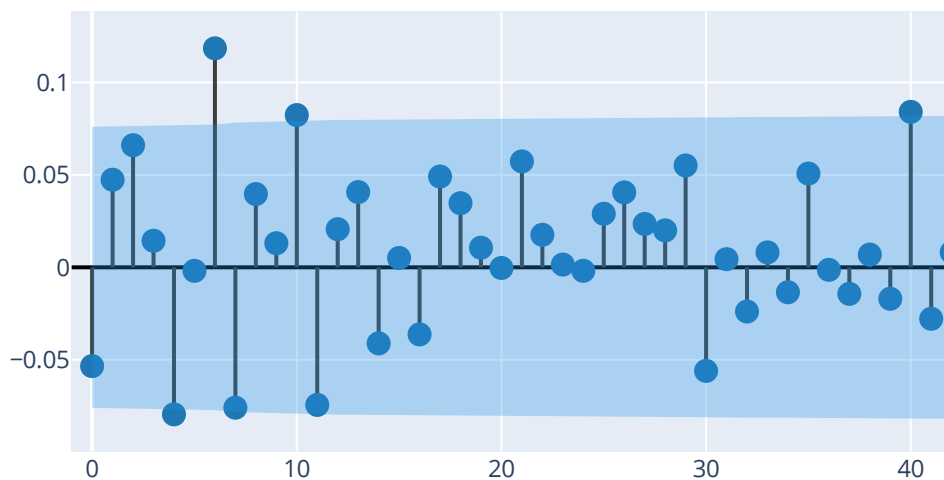
create_corr_plot(df, 'Return', True)
create_corr_plot(df, 'Return', False)
create_corr_plot(df, 'US_10_Year_Treasury_Yield_diff_1', True)
create_corr_plot(df, 'US_10_Year_Treasury_Yield_diff_1', False)

```

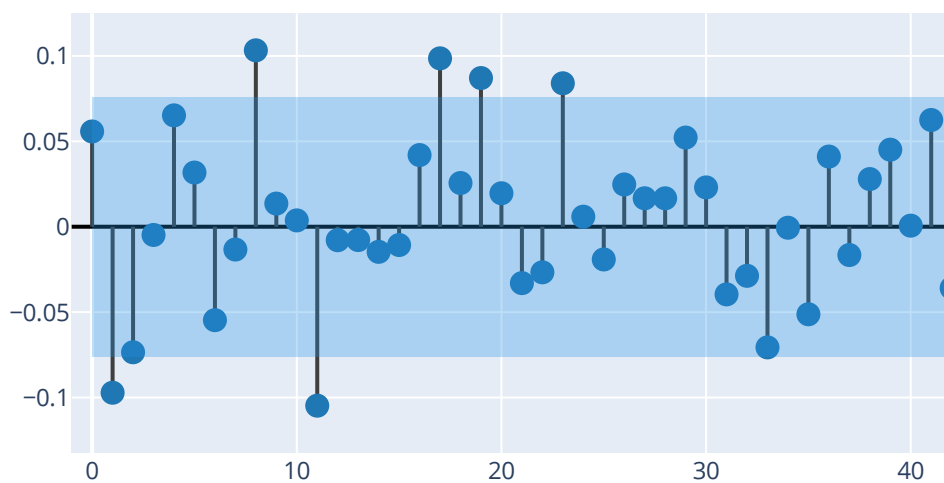
Partial Autocorrelation (PACF): Return



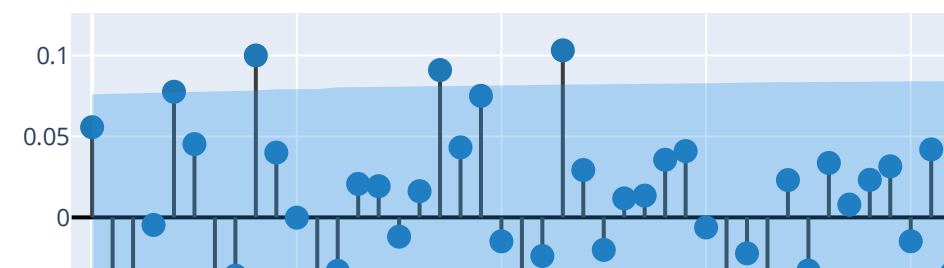
Autocorrelation (ACF): Return

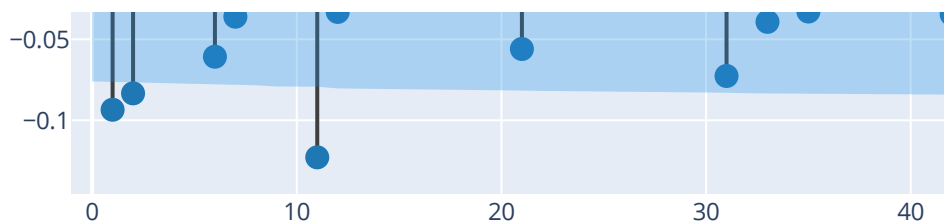


Partial Autocorrelation (PACF): US_10_Year_Treasury_Yield_diff_1



Autocorrelation (ACF): US_10_Year_Treasury_Yield_diff_1





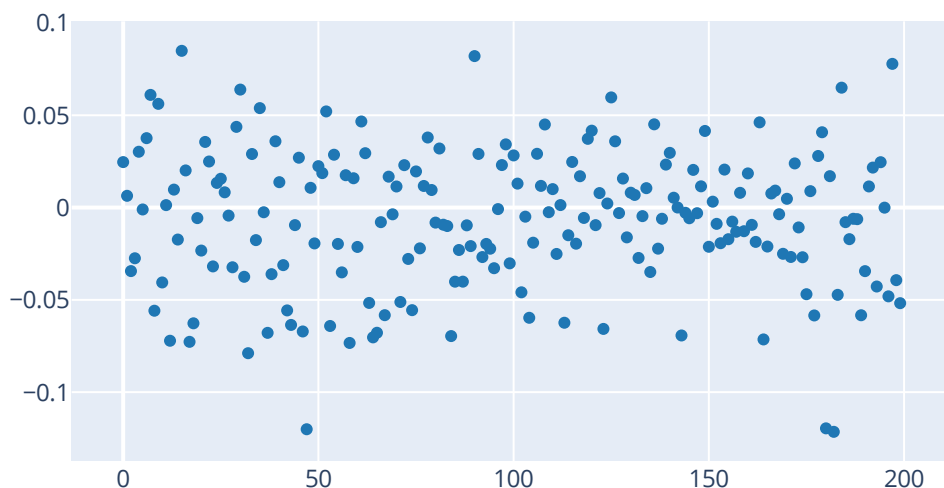
We can observe that there were no outliers in the earlier periods, but a persistent pattern of non-decay and frequent outliers outside of the range appeared in the later periods.

If we observe the correlation between US Treasury bond yields and Bitcoin we can see that there is no obvious relationship. However, we should do granger causality after we consider each lags of themself

```
corr_array = ccf(df['Return'], df['US_10_Year_Treasury_Yield_diff_1'])[:200]
fig = go.Figure()
fig.add_scatter(x=np.arange(len(corr_array)), y=corr_array, mode='markers', marker_color=
                marker_size=6)
fig.update_traces(showlegend=False)

title=f"Cocorrelation of {' and '.join(df.columns)}"
fig.update_layout(title=title, width=600, height=400)
fig.show()
```

Cocorrelation of Return and US_10_Year_Treasury_Yield_diff_1



X Granger-causes Y if the addition of the lagged values of X to the prediction of Y improves the prediction of Y, where the prediction is made using a linear regression model of the form:

$$Y_t = \alpha + \sum_{i=1}^p \beta_i Y_{t-i} + \sum_{i=1}^q \gamma_i X_{t-i} + \epsilon_t$$

Granger causality is a statistical technique used to determine whether a time series X can predict future values of another time series Y. Specifically, X is said to Granger-cause Y if past values of X, along with past values of Y, are statistically significant predictors of future values of Y. This is typically established using t-tests and F-tests on lagged values of X and Y.[15]

```
MAX_LAG = 100
result = grangercausalitytests(df, MAX_LAG, verbose=False)

for i in range(1, MAX_LAG + 1):
    if all(pvalue < 0.05 for pvalue in [r[1] for r in result[i][0].values()]):
        break
print(f"The lag number we consider to form the AR model with max lags = {i}")
print(result[i])

The lag number we consider to form the AR model with max lags = 47
({'ssr_ftest': (1.4640301942385112, 0.027184323824048208, 523.0, 47), 'ssr_chi2tes
  [0., 0., 0., ..., 0., 0., 0.],
  [0., 0., 0., ..., 0., 0., 0.],
  ...,
  [0., 0., 0., ..., 0., 0., 0.],
  [0., 0., 0., ..., 1., 0., 0.],
  [0., 0., 0., ..., 0., 1., 0.]])])

train_data = lagmat2ds(df.values, i, trim="both", dropex=1)

model = OLS(train_data[:, 0], add_constant(train_data[:, 1:]), prepend=False).fit()
model.summary()
```

```

                OLS Regression Results
Dep. Variable:  y                R-squared:    0.201
Model:          OLS              Adj. R-squared: 0.058
Method:         Least Squares    F-statistic:  1.404
Date:           Wed, 03 May 2023  Prob (F-statistic): 0.0119
Time:           08:23:31         Log-Likelihood: 1109.3
No. Observations: 618            AIC:         -2029.
Df Residuals:    523             BIC:         -1608.
Df Model:        94
Covariance Type: nonrobust

   coef  std err   t    P>|t| [0.025   0.975]
x1 -0.0560  0.041  -1.364  0.173 -0.137    0.025

```

| | | | | | | |
|------------|---------|-------|--------|-------|--------|--------|
| x2 | 0.0745 | 0.041 | 1.821 | 0.069 | -0.006 | 0.155 |
| x3 | 0.0513 | 0.041 | 1.250 | 0.212 | -0.029 | 0.132 |
| x4 | -0.0222 | 0.041 | -0.542 | 0.588 | -0.103 | 0.058 |
| x5 | -0.0815 | 0.041 | -1.989 | 0.047 | -0.162 | -0.001 |
| x6 | -0.0223 | 0.041 | -0.542 | 0.588 | -0.103 | 0.059 |
| x7 | 0.1346 | 0.041 | 3.268 | 0.001 | 0.054 | 0.216 |
| x8 | -0.0637 | 0.042 | -1.530 | 0.127 | -0.146 | 0.018 |
| x9 | -0.0047 | 0.042 | -0.112 | 0.911 | -0.087 | 0.077 |
| x10 | 0.0174 | 0.042 | 0.414 | 0.679 | -0.065 | 0.100 |
| x11 | 0.0728 | 0.042 | 1.732 | 0.084 | -0.010 | 0.155 |
| x12 | -0.0754 | 0.042 | -1.790 | 0.074 | -0.158 | 0.007 |
| x13 | 0.0268 | 0.042 | 0.634 | 0.526 | -0.056 | 0.110 |
| x14 | 0.0350 | 0.042 | 0.827 | 0.409 | -0.048 | 0.118 |
| x15 | -0.0417 | 0.042 | -0.989 | 0.323 | -0.124 | 0.041 |
| x16 | -0.0378 | 0.042 | -0.899 | 0.369 | -0.121 | 0.045 |
| x17 | -0.0295 | 0.042 | -0.699 | 0.485 | -0.112 | 0.053 |
| x18 | 0.0459 | 0.042 | 1.094 | 0.274 | -0.036 | 0.128 |
| x19 | 0.0735 | 0.042 | 1.753 | 0.080 | -0.009 | 0.156 |
| x20 | -0.0324 | 0.042 | -0.771 | 0.441 | -0.115 | 0.050 |
| x21 | -0.0036 | 0.042 | -0.085 | 0.933 | -0.086 | 0.079 |
| x22 | 0.1019 | 0.042 | 2.433 | 0.015 | 0.020 | 0.184 |
| x23 | 0.0362 | 0.042 | 0.860 | 0.390 | -0.047 | 0.119 |
| x24 | -0.0106 | 0.042 | -0.251 | 0.802 | -0.093 | 0.072 |
| x25 | -0.0101 | 0.042 | -0.239 | 0.811 | -0.093 | 0.073 |
| x26 | 0.0413 | 0.042 | 0.984 | 0.326 | -0.041 | 0.124 |
| x27 | 0.0470 | 0.042 | 1.113 | 0.266 | -0.036 | 0.130 |
| x28 | 0.0245 | 0.042 | 0.578 | 0.564 | -0.059 | 0.108 |
| x29 | -0.0107 | 0.042 | -0.254 | 0.800 | -0.094 | 0.072 |
| x30 | 0.0538 | 0.042 | 1.278 | 0.202 | -0.029 | 0.137 |
| x31 | -0.0117 | 0.042 | -0.278 | 0.781 | -0.094 | 0.071 |
| x32 | -0.0094 | 0.042 | -0.223 | 0.824 | -0.092 | 0.073 |
| x33 | -0.0653 | 0.042 | -1.549 | 0.122 | -0.148 | 0.018 |
| x34 | 0.0098 | 0.042 | 0.232 | 0.817 | -0.073 | 0.093 |
| x35 | -0.0324 | 0.042 | -0.764 | 0.445 | -0.116 | 0.051 |
| x36 | 0.0386 | 0.042 | 0.917 | 0.359 | -0.044 | 0.121 |
| x37 | 0.0076 | 0.042 | 0.183 | 0.855 | -0.075 | 0.090 |
| x38 | -0.0011 | 0.042 | -0.026 | 0.979 | -0.084 | 0.081 |
| x39 | 0.0143 | 0.042 | 0.342 | 0.733 | -0.068 | 0.097 |
| x40 | 0.0188 | 0.042 | 0.452 | 0.651 | -0.063 | 0.101 |
| x41 | 0.0758 | 0.041 | 1.833 | 0.067 | -0.005 | 0.157 |
| x42 | 0.0216 | 0.041 | 0.523 | 0.601 | -0.060 | 0.103 |
| x43 | -0.0049 | 0.041 | -0.118 | 0.906 | -0.086 | 0.076 |
| x44 | -0.0191 | 0.042 | -0.458 | 0.647 | -0.101 | 0.063 |
| x45 | -0.0276 | 0.041 | -0.666 | 0.505 | -0.109 | 0.054 |
| x46 | 0.0690 | 0.041 | 1.667 | 0.096 | -0.012 | 0.150 |

| | | | | | | |
|-----|---------|-------|--------|-------|---------|--------|
| x47 | 0.0593 | 0.041 | 1.439 | 0.151 | -0.022 | 0.140 |
| x48 | -0.1405 | 3.526 | -0.040 | 0.968 | -7.066 | 6.785 |
| x49 | -1.4077 | 3.520 | -0.400 | 0.689 | -8.323 | 5.507 |
| x50 | -3.6921 | 3.507 | -1.053 | 0.293 | -10.581 | 3.197 |
| x51 | -2.3706 | 3.482 | -0.681 | 0.496 | -9.212 | 4.471 |
| x52 | -2.1286 | 3.439 | -0.619 | 0.536 | -8.885 | 4.628 |
| x53 | -0.8455 | 3.432 | -0.246 | 0.805 | -7.587 | 5.896 |
| x54 | 4.6089 | 3.432 | 1.343 | 0.180 | -2.134 | 11.352 |
| x55 | -5.6301 | 3.418 | -1.647 | 0.100 | -12.345 | 1.085 |
| x56 | 1.9945 | 3.427 | 0.582 | 0.561 | -4.739 | 8.728 |
| x57 | -5.2990 | 3.437 | -1.542 | 0.124 | -12.051 | 1.453 |
| x58 | -2.7518 | 3.409 | -0.807 | 0.420 | -9.449 | 3.945 |
| x59 | -6.9279 | 3.392 | -2.042 | 0.042 | -13.591 | -0.264 |
| x60 | -1.2917 | 3.419 | -0.378 | 0.706 | -8.009 | 5.425 |
| x61 | -4.6094 | 3.450 | -1.336 | 0.182 | -11.387 | 2.168 |
| x62 | 4.9181 | 3.457 | 1.422 | 0.155 | -1.874 | 11.710 |
| x63 | -1.4209 | 3.444 | -0.413 | 0.680 | -8.187 | 5.345 |
| x64 | -6.9054 | 3.455 | -1.998 | 0.046 | -13.694 | -0.117 |
| x65 | -6.2151 | 3.464 | -1.794 | 0.073 | -13.019 | 0.589 |
| x66 | 2.6280 | 3.484 | 0.754 | 0.451 | -4.216 | 9.471 |
| x67 | -6.5963 | 3.479 | -1.896 | 0.059 | -13.431 | 0.238 |
| x68 | 3.4986 | 3.492 | 1.002 | 0.317 | -3.362 | 10.359 |
| x69 | 3.4777 | 3.477 | 1.000 | 0.318 | -3.352 | 10.307 |
| x70 | 0.9705 | 3.474 | 0.279 | 0.780 | -5.854 | 7.795 |
| x71 | 0.5608 | 3.481 | 0.161 | 0.872 | -6.278 | 7.399 |
| x72 | 0.6609 | 3.513 | 0.188 | 0.851 | -6.241 | 7.563 |
| x73 | 4.2869 | 3.508 | 1.222 | 0.222 | -2.605 | 11.179 |
| x74 | 2.2273 | 3.509 | 0.635 | 0.526 | -4.666 | 9.120 |
| x75 | -1.6180 | 3.508 | -0.461 | 0.645 | -8.509 | 5.273 |
| x76 | 1.9980 | 3.509 | 0.569 | 0.569 | -4.895 | 8.891 |
| x77 | 3.7962 | 3.500 | 1.085 | 0.279 | -3.079 | 10.672 |
| x78 | -2.1174 | 3.490 | -0.607 | 0.544 | -8.973 | 4.738 |
| x79 | -8.2332 | 3.489 | -2.360 | 0.019 | -15.087 | -1.379 |
| x80 | 1.5128 | 3.497 | 0.433 | 0.665 | -5.357 | 8.383 |
| x81 | -3.7095 | 3.510 | -1.057 | 0.291 | -10.604 | 3.185 |
| x82 | 3.0143 | 3.518 | 0.857 | 0.392 | -3.898 | 9.926 |
| x83 | -1.9300 | 3.498 | -0.552 | 0.581 | -8.801 | 4.941 |
| x84 | -3.5731 | 3.512 | -1.017 | 0.310 | -10.473 | 3.327 |
| x85 | 1.5465 | 3.517 | 0.440 | 0.660 | -5.363 | 8.456 |
| x86 | 1.0002 | 3.488 | 0.287 | 0.774 | -5.853 | 7.853 |
| x87 | -1.5921 | 3.497 | -0.455 | 0.649 | -8.463 | 5.279 |
| x88 | -0.0594 | 3.492 | -0.017 | 0.986 | -6.919 | 6.800 |
| x89 | -2.8451 | 3.498 | -0.813 | 0.416 | -9.717 | 4.027 |
| x90 | -5.9672 | 3.489 | -1.710 | 0.088 | -12.821 | 0.887 |
| x91 | -3.5421 | 3.500 | -1.012 | 0.312 | -10.418 | 3.334 |

```

x92 2.0373 3.531 0.577 0.564 -4.900 8.975
x93 -6.3790 3.532 -1.806 0.071 -13.318 0.560
x94 -9.9218 3.543 -2.800 0.005 -16.882 -2.962
const 0.0036 0.002 1.926 0.055 -7.24e-05 0.007
Omnibus: 28.105 Durbin-Watson: 1.943
Prob(Omnibus): 0.000 Jarque-Bera (JB): 81.417
Skew: -0.014 Prob(JB): 2.09e-18
Kurtosis: 4.778 Cond. No. 2.59e+03

```

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

[2] The condition number is large, 2.59e+03. This might indicate that there are strong multicollinearity or other numerical problems.

```

test_data = lagmat2ds(df_test.values, i, trim="both", dropex=1)

y = pd.Series(test_data[:, 0])
yhats = pd.Series(model.predict(add_constant(test_data[:, 1:], prepend=False)))

```

```

train_return_describe = pd.Series(model.predict()).describe()
train_return_describe

```

```

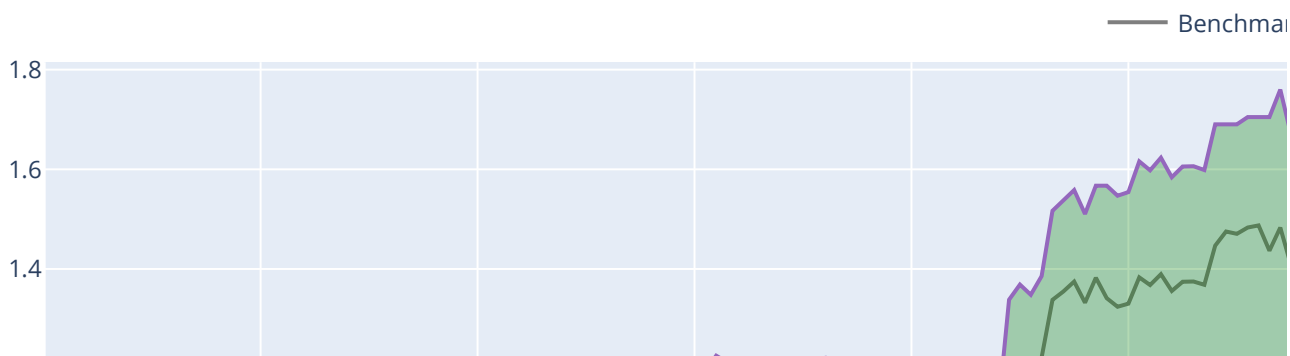
count    618.000000
mean      0.003379
std       0.020208
min      -0.077180
25%      -0.008667
50%       0.002950
75%       0.014720
max       0.096807
dtype: float64

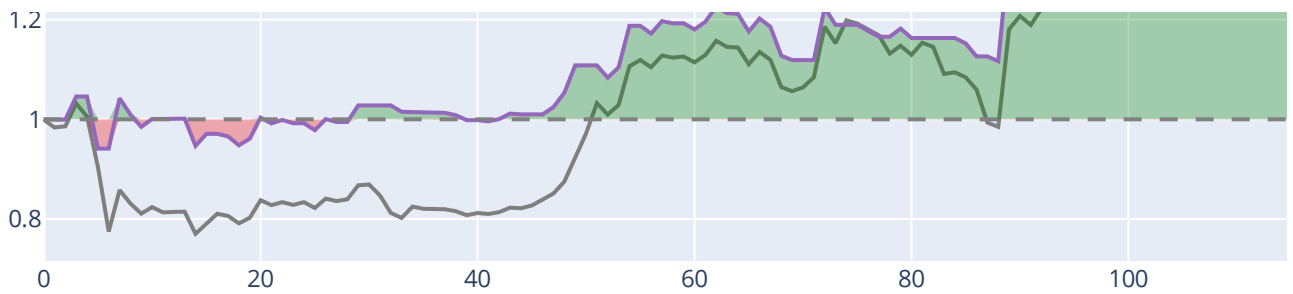
```

```

y_model_adjusted = y.copy()
y_model_adjusted[yhats < train_return_describe['25%']] = 0.
y_model_adjusted.vbt.returns(freq='d').plot_cumulative(benchmark_rets=y).show()

```





```
y_model_adjusted.vbt.returns(freq='d').stats(settings=dict(benchmark_rets=y))
```

```

Start                                0
End                                  118
Period                             119 days 00:00:00
Total Return [%]                    57.927419
Benchmark Return [%]                34.297639
Annualized Return [%]              306.176036
Annualized Volatility [%]          61.704038
Max Drawdown [%]                   10.266098
Max Drawdown Duration              43 days 00:00:00
Sharpe Ratio                        2.568081
Calmar Ratio                        29.823993
Omega Ratio                        1.66534
Sortino Ratio                       5.378637
Skew                                2.242645
Kurtosis                           12.329672
Tail Ratio                          1.704727
Common Sense Ratio                  6.924192
Value at Risk                       -0.030879
Alpha                              1.088425
Beta                                0.730401
dtype: object

```

REFERENCES:

1. Harsch Katara and Dheeraj Vaidya, Quartile Formula, WallStreetMojo, Quartile Formula | How to Calculate Quartile in Statistics | Example (wallstreetmojo.com)
2. Pritha Bhandari, How to find the Median, Scribbr, 02/19/2023, How to Find the Median | Definition, Examples & Calculator (scribbr.com)
3. Median - Formula, Meaning, Example | How to Find Median? (cuemath.com)
4. Quantile: Definition and How to Find Them in Easy Steps - Statistics How To
5. Avijeet Biswal, SimpleLearn, 04/03/2023, Percentile in Statistics: Overview & How to Calculate | Simplilearn
6. Jim Frost, Statistics by Jim, Interquartile Range (IQR): How to Find and Use It - Statistics By

Jim

7. Khan Academy, Calculating standard deviation step by step, Standard deviation: calculating step by step (article) | Khan Academy
8. Adam Hayes, Somer Anderson, Amanda Bellucco-Chatham, Co-efficient of Variation Meaning and How to Use It, Investopedia 09/16/2022, Co-efficient of Variation Meaning and How to Use It (investopedia.com)
9. Khan Academy; Mean, median and mode; Mean, median, and mode review (article) | Khan Academy
10. Statistics How To, Median Absolute Deviation, Median Absolute Deviation - Statistics How To
11. Ashish Kumar Srivastav, Dheeraj Vaidya, Skewness Formula, WallStreetMojo, Skewness Formula | How to Calculate Skewness? (with Examples) (wallstreetmojo.com)
12. Diksha Keni, Dheeraj Vaidya, Population Variance Formula, WallStreetMojo, Population Variance Formula | Step by Step Calculation | Examples (wallstreetmojo.com)
13. Dybvig, Philip H., and William J. Marshall. "The New Risk Management: The Good, the Bad, and the Ugly." Economic Research Federal Reserve Bank of St. Louis, vol. 79, no. 6, 1997, pp.9-21, <https://research.stlouisfed.org/publications/review/1997/11/01/the-new-risk-management-the-good-the-bad-and-the-ugly>. Accessed 30 April 2023.
14. Partial Auto Correlation Function Formula
15. Granger_causality
16. Dickey–Fuller test
17. Augmented Dickey–Fuller test
18. Leslie, J. R., Stephens, M. A., and Fotopoulos, S. (1986). Asymptotic distribution of the Shapiro–Wilk W for testing for normality. Ann.Statist. 14, 1497–1506

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