GROUP WORK PROJECT # 1 MScFE 620: Derivative Pricing
GROUP NUMBER: 3892

FULL LEGAL NAME	LOCATION (COUNTRY)	EMAIL ADDRESS	MARK X FOR ANY NON-CONTRIBUTING MEMBER
CHI CUONG NGUYEN	Australia	anthony.nguyen.dn5@gmail.com	
Marin Stoyanov	Bulgaria	azonealerts@gmx.com	
Eliezer NIYITEGEKA	Rwanda	niyitegekaeliezer@outlook.com	

Statement of integrity: By typing the names of all group members in the text boxes below, you confirm that the assignment submitted is original work produced by the group (excluding any non-contributing members identified with an "X" above).	
Team member 1	CHI CUONG NGUYEN
Team member 2	Marin Stoyanov
Team member 3	Eliezer NIYITEGEKA

Use the box below to explain any attempts to reach out to a non-contributing member. Type (N/A) if all members contributed. **Note:** You may be required to provide proof of your outreach to non-contributing members upon request.

STEP 1

Q.1. YES. Put-Call parity applies for European options (both puts and calls). It is a mechanism (system of equations) of relationship between the prices of puts and calls with the same expiry date (European style only), with the same strike and over the same underlying asset (mostly stocks but it is applicable to other instruments as well) that can define the prices of the puts or calls in order the party to be complied. The reason that this can happen is exactly the fact that the European option can be exercised only at the expiration date [1]. The put-call parity applied to European options is represented in the following equation:

$$C_0 + Ke^{-rT} = S_0 + P_0$$

Q.2. For call price:

$$C_0 + Ke^{-rT} = S_0 + P_0$$

$$=> C_0 = S_0 + P_0 - Ke^{-rT}$$

$$=> C_0 = S_0 + P_0 - \frac{K}{e^{rT}}$$

Q.3. And for Put price:

$$C_0 + Ke^{-rT} = S_0 + P_0$$

$$=> P_0 = C_0 + Ke^{-rT} - S_0$$

$$=> P_0 = C_0 + \frac{K}{e^{rT}} - S_0$$

Where:

C_0 is the price of the European call option at time 0

P_0 is the price of the European put option at time 0

K is the strike of the options

S_0 is the price of the stock (the underlying asset) at time 0

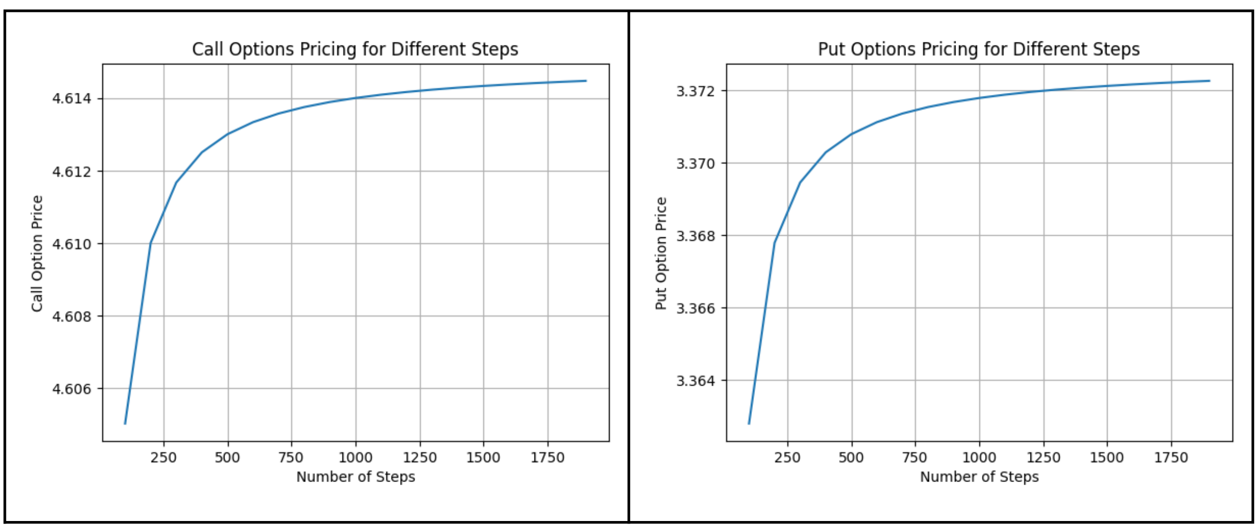
e^{-rT} is the discounted value on the expiration date at the risk-free rate

Q.4. NO. Put-Call parity does not apply for American options because they have the possibility of an early exercise and this feature distorts the put-call parity calculations because they rely on an exactly the same expiry date when the option can be exercised.

Q.5 a-b.

We choose N = 400 steps as we estimate the duration of 3 months could be approximately 66-68 trading (transactions day) or about 400 H4 trading periods in total (H4 trading frame consists of 4 hours of trading or 6 periods = 24/4 per day). Moreover, another reason behind the decision of number of steps in the tree is the tradeoff between the pricing accuracy and the computing time. In fact, the overall pricing process is based on a step-envolving binomial tree with two possible price change for each stage (up and down scenario). During the tree-path evolution calculation, expected return and volatility of the price will be measured based on the probability of going up or down. Finally, after computing all nodes' underlying asset prices and payoffs, a discounted process will be applied to measure the price of the option. When simulating the number of steps required for the option pricing accuracy, we found that the more number of steps implemented would be associated with the higher accuracy option price. However, the pricing curve starts to flatten after 400-step-trial even when a quick rate of accuracy increase was witnessed for the range of step number from 1 to 400 before that, implying the diminishing effects when the number of steps reach to a certain level. Therefore, here, we conclude that the number of step 400 is the optimal choice to balance the tradeoff between pricing accuracy and computing duration and resource consumption. Please refer the figure 1 for the details of pricing results in comparison to the number of steps utilized.

Figure 1: European Call and Put Option Prices with different steps (range from 100 top 2000)



Q.6 a-b.

Table 1: Greek Delta for the European call and put at t=0

	Call Option Price	Put Option Price
Greek Delta at t=0	0.57	-0.43

Technically, delta is defined as a sensitivity measure of the option prices changed relative to the movement of the underlying asset prices. Delta might range from 0 to +1 for a call option while this range is limited from -1 to 0 for a put [2]. Regarding the delta values in this question, the call option displays a positive Greek Delta (**Δ** = 0.57) and the put option reports a negative Geek Delta (**Δ** = -0.43). This makes sense since the delta would proxy the option price change in relation to the change in the underlying asset prices. In case of the call option, a positive delta means an increase in the price of the underlying stocks (or assets) will be associated with a proportional increase in the call option price. On the contrary, a negative delta of the put option represents the fact that when the underlying asset witnesses a rise in prices, the put option price will drop.

Q.7 a-b.

Table 2: Greek Delta for the European call and put at t=0

	Call Price Change	Put Price Change
σ = 20%	4.61	3.37
σ = 25%	5.60	4.35
Vega	19.66	19.66

When there is a 5% raise in the volatility **σ** (from 20% to 25%) both the call and put will display a increase in price. In particular, the 5% increase in **σ** results in the relatively similar appreciation nearly $1 for both types of options, while the call price changes for 0.99$ or the vegas is 19.66, the put price changes for $0.98 or the vegas is 19.66. This phenomenon could be attributed to the payoff change direction of the 2 option contracts. In fact, as the higher volatility will manifest more impacts at the high underlying asset price level, a call option, which pegs more on the price appreciation, will get a larger call price change. In contrast, a put option, which is contracted based on the belief of the underlying asset price depreciation, will get less impact at the high stock price, also reducing the impacts of higher rate of volatility.

Q.8 a and b:

Initially we chose the 500-th step because the line is starting to get flatten around the 500-th step but in this section we have agreed to follow the logic of the conclusions from above so we agreed upon choosing 400 steps as our final choice. About me commenting on section 5: well I really liked the idea of incorporating the time (as trading days/hours) in the final choice of steps because there could be some invisible patterns like the last day of the month or the last trading hour in the trading session (as we know in such moments volumes, activity and trading as a whole spikes at such moments) so it is a good thing to follow such patterns.

Q.9 a and b:

Analogously the logic is the same as in Q.6 but here with the American options we can see that on absolute terms we need more to hedge the delta of an American put than an European put. This is because of the feature that the American options can be exercised earlier than the expiration date and thus they are more expensive thanks to that uncertainty unlike the European options.

Table 3: Greek Delta for the Americancall and put at t=0

	Call Option Price	Put Option Price
Greek Delta at t=0	0.57	-0.45

As we can see by comparing the data for European and American call and put options (visible in table 1 and table 3), the only difference is in the price of the put.

Q.10 a and b:

Table 4: Greek Delta for the American call and put at t=0

	Call Price Change	Put Price Change
σ = 20%	4.61	3.48
σ = 25%	5.60	4.46
Vega	19.66	19.59

It is visible in table 4 how change in underlying asset's volatility affects the price of American calls and puts and it is not surprising that the American puts are more expensive. With table 2 and 4 we are proving that both European and American options are sensitive towards stock's volatility.

Q11. We saw that for the European option the Put-Call parity holds.

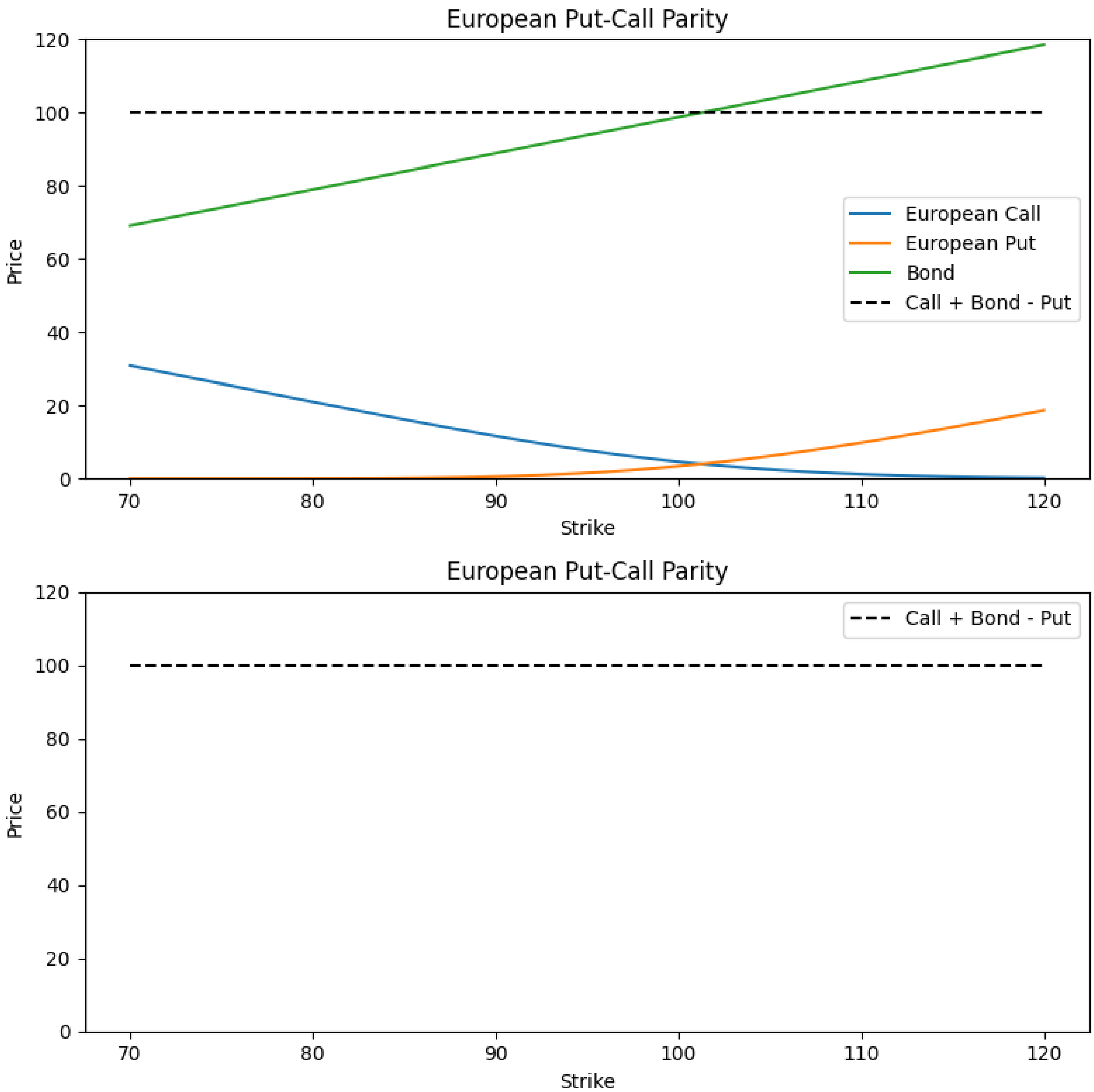
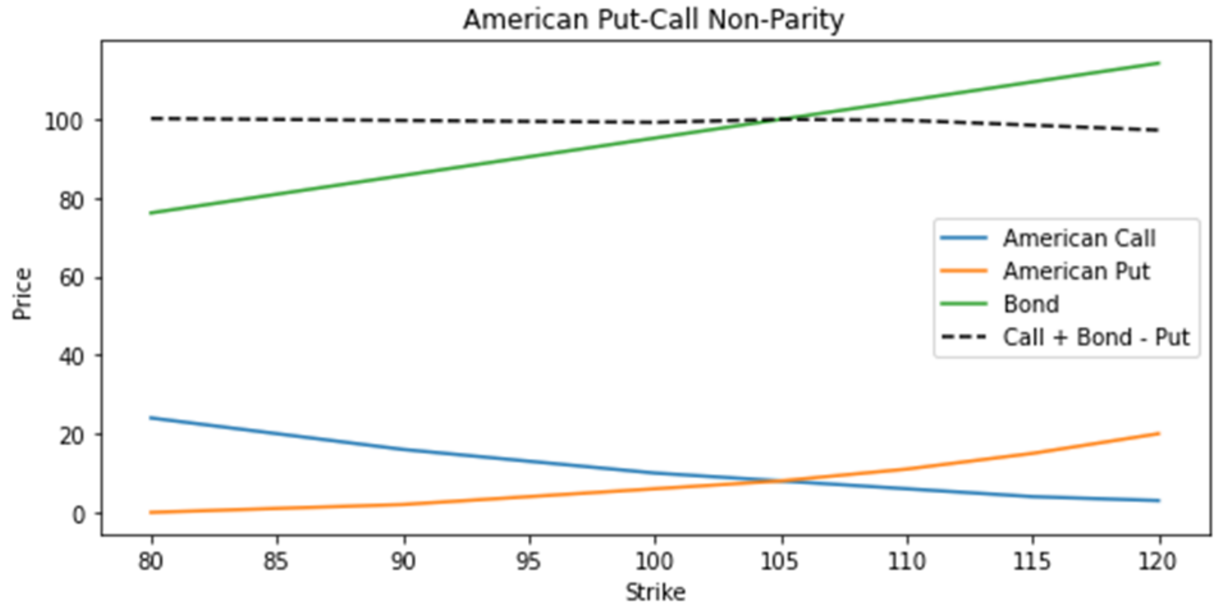


Fig.2. European put-call parity chart

On very rare occasions the put-call parity does not hold for European options with the same strike, same underlying asset and the same date of expiration because if it doesn't then there an arbitrage situation will be available where one can buy/sell one side of the put-call parity equation and immediately sell/buy the other side of the equation leading to riskless trade and profits [4]. This thing happens very rarely and disappears almost immediately because "there is no free lunch". It is usually an exception rather than a pattern.

Q12. The American options does not hold the put-call parity because it can be exercised before the expiry date.



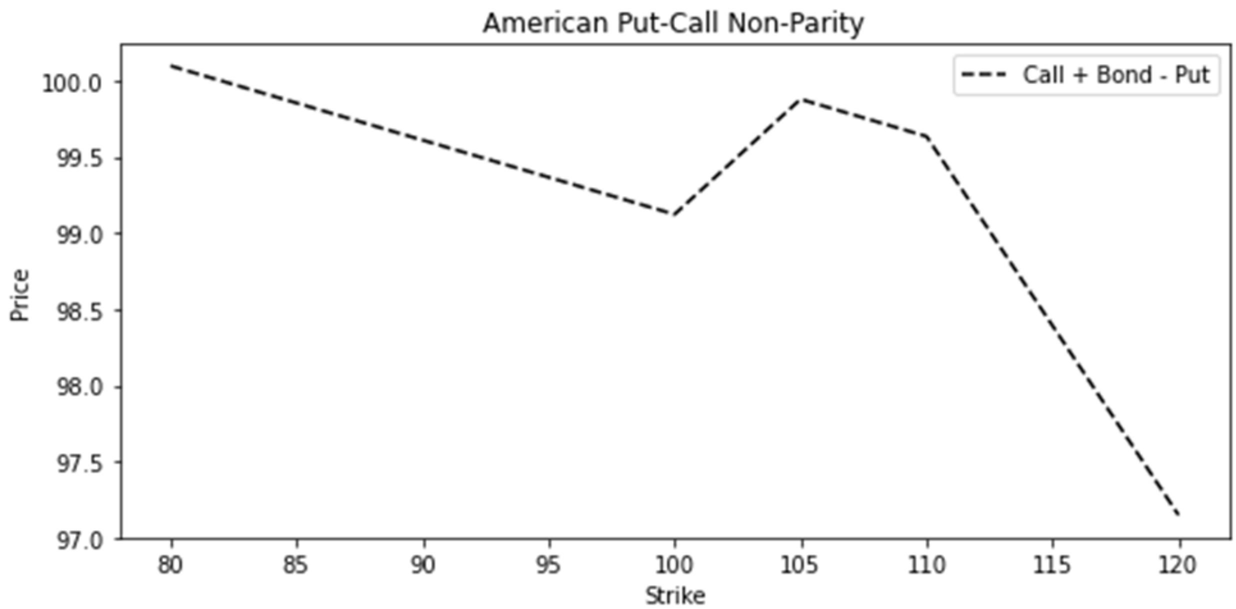


Fig.3. American put-call parity chart

The case with American options is totally different because they have the feature of an early exercise and in this case when a suitable situation occurs and an option is exercised before expiration, this will destroy the put-call parity equation and it will no longer be adequate and useful, just because one side of the equation would be exercised and no longer existing. [5]

Q13. Both American and European call options produced the same output. As American options didn't gain any advantages from the early exercise, they kept acting just like the European options. This will not be the case forever because if an opportunity for an early exercise of the American options occurs, then it will be taken and the payoff will be different.

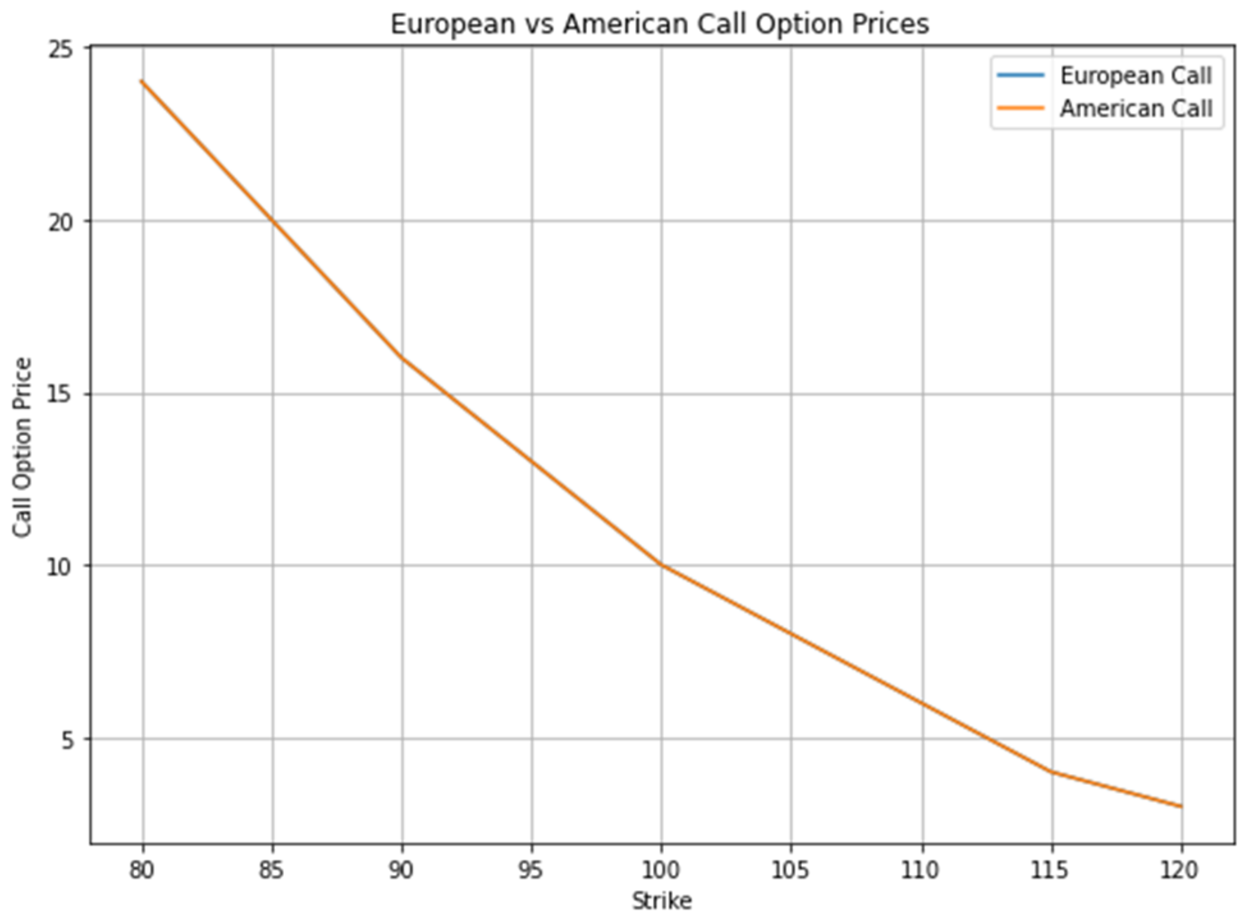


Fig.4. European vs American call options chart

Q14. The American put benefited from early exercise more than the European put.
As a rule American options are more expensive than the European ones because thanks to their feature of an early exercise, they bring more uncertainty and thus their hedging cost more than the European. Also American options are more in demand exactly thanks to that feature and thus they are more expensive as well [6].

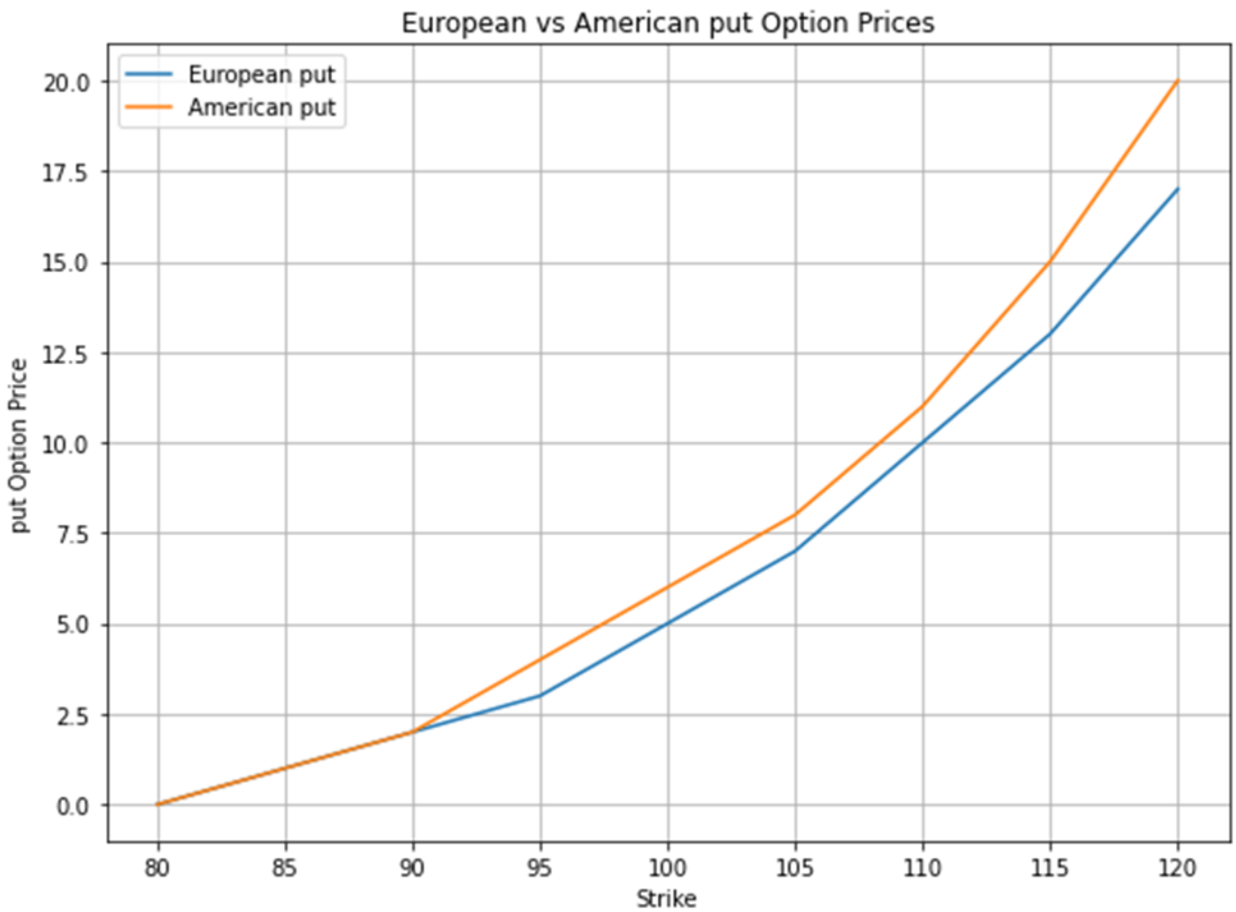


Fig.5. European vs American put options chart

STEP 2

Q.15 a-b.

Table 5: Call Option Prices with 5 Strike Price at moneyness ranging from 90% to 110%

	American Call Option Price
K = 90	11.71
K = 95	7.78
K = 100	4.49
K = 105	2.55
K = 110	1.22

It is normal to see the price of call option decrease when the strike K is getting away (upward) from the current stock price. This is the case that as if it is pricing in that the more far away (upward) is the strike K from the stock price, the harder it is for the option to get in-the-money and thus to be exercised.

Q.16 a-b.

Table 6: Put Option Prices with 5 Strike Price at moneyness ranging from 90% to 110%

	American Put Option Price
K = 90	0.60
K = 95	1.63
K = 100	3.37
K = 105	6.46
K = 110	10.28

On the other side when the strike is going away upwards from the stock's price it is normal to see the price of the put increasing because it is getting more profitable and suitable for exercise and thus getting a better payoff.

Q.17 a-b.

Table 7: Call Option Prices with 5 Strike Price at moneyness ranging from 90% to 110%

	European Call Option Price
K = 90	11.71
K = 95	7.78
K = 100	4.49
K = 105	2.55
K = 110	1.22

Regarding the call options, the prices will decrease when the strike prices are increased into higher level. This correlation could be explained by the rate of the strike and its relation to the possibility of being out-the-money (OTM) or in-the money (ITM). In details, for a given strike, a call option will become more valuable if the underlying asset prices increase following the formula of payoff C = max(St - K, 0). Therefore, if we raise up the strike price high, the payoff of the call will decrease, leading to a lower call price at the end.

Q.18 a-b.

Table 8: Put Option Prices with 5 Strike Price at moneyness ranging from 90% to 110%

	European Put Option Price
K = 90	0.59
K = 95	1.60
K = 100	3.25
K = 105	6.25
K = 110	9.86

Regarding the put options, the prices will increase when the strike prices are increased into higher level. This correlation could be explained by the rate of the strike and its relation to the possibility of being out-the-money (OTM) or in-the money (ITM). In details, for a given strike, a put option will become less valuable if the underlying asset prices increase following the formula of payoff C = max(K - St, 0). Therefore, if we raise up the strike price high, the payoff of the call will increase, leading to a higher put price at the end.

Q19. European Call and Put prices versus the stock prices

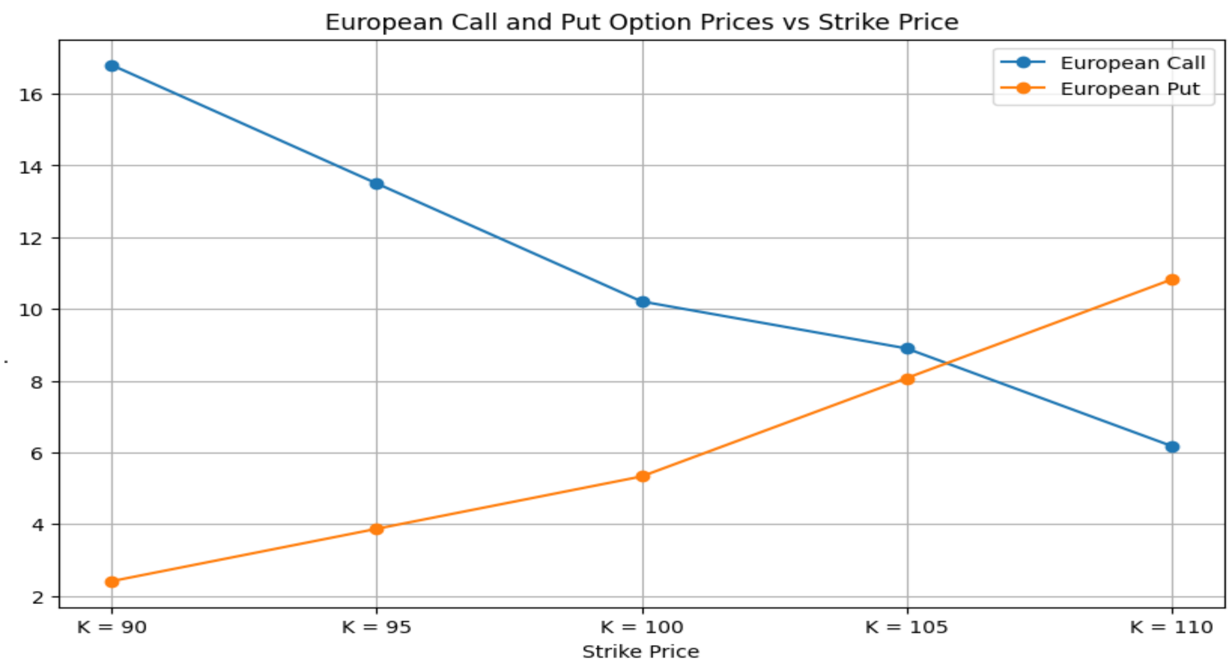


Fig.6. European call and put options chart

Q20. The American call and put prices versus stock prices

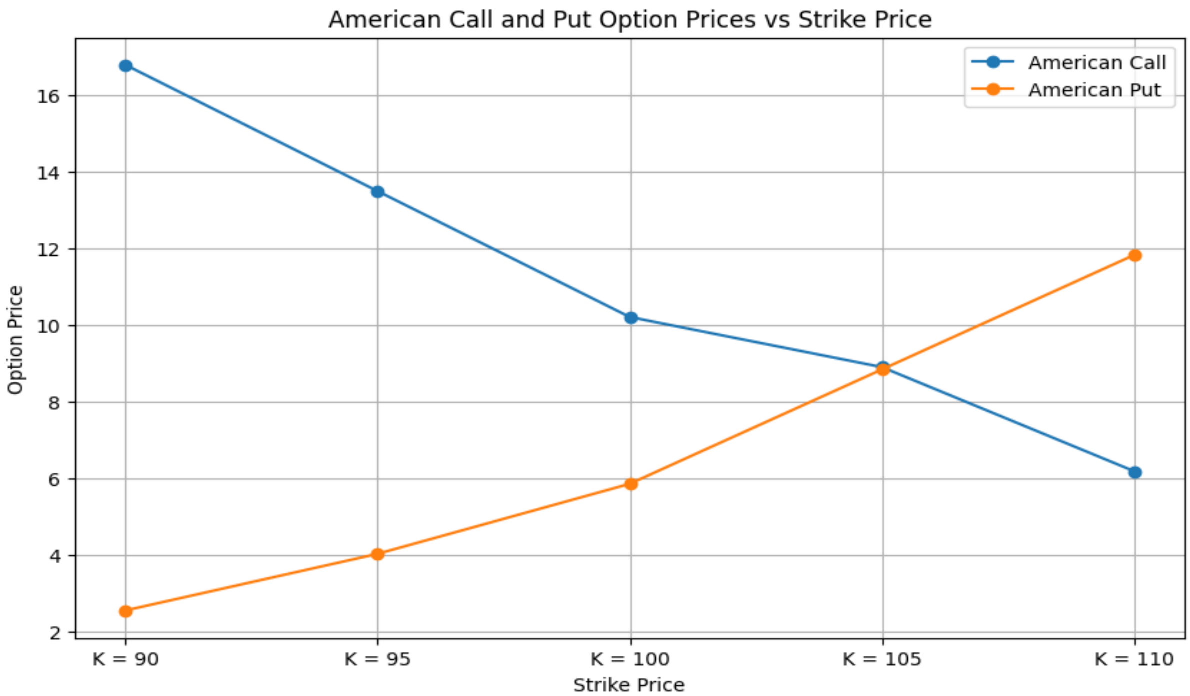
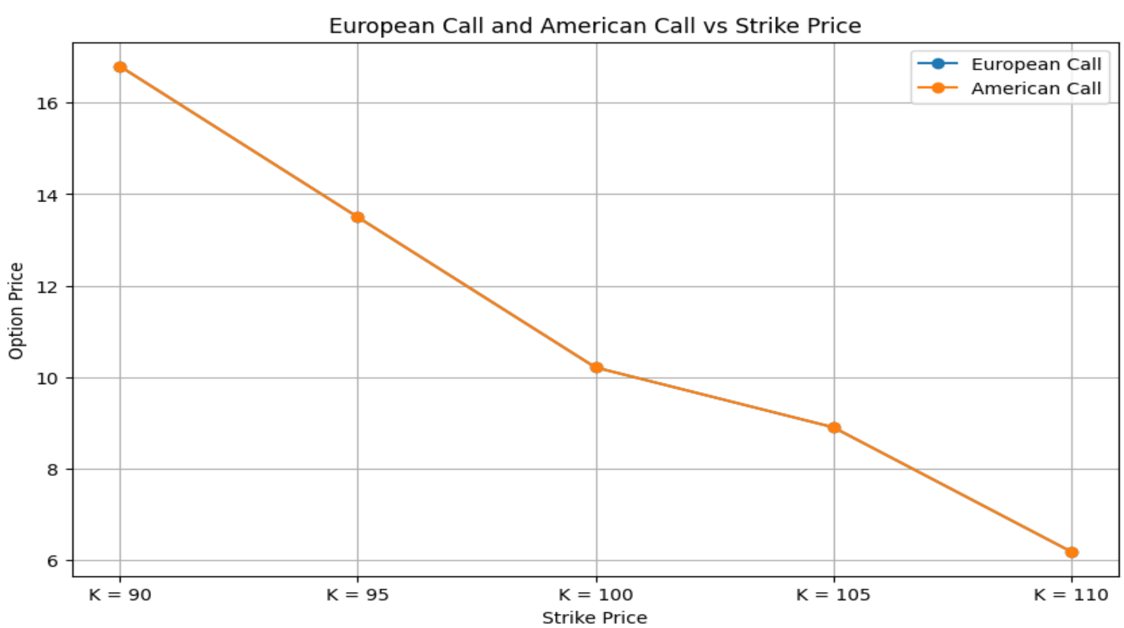
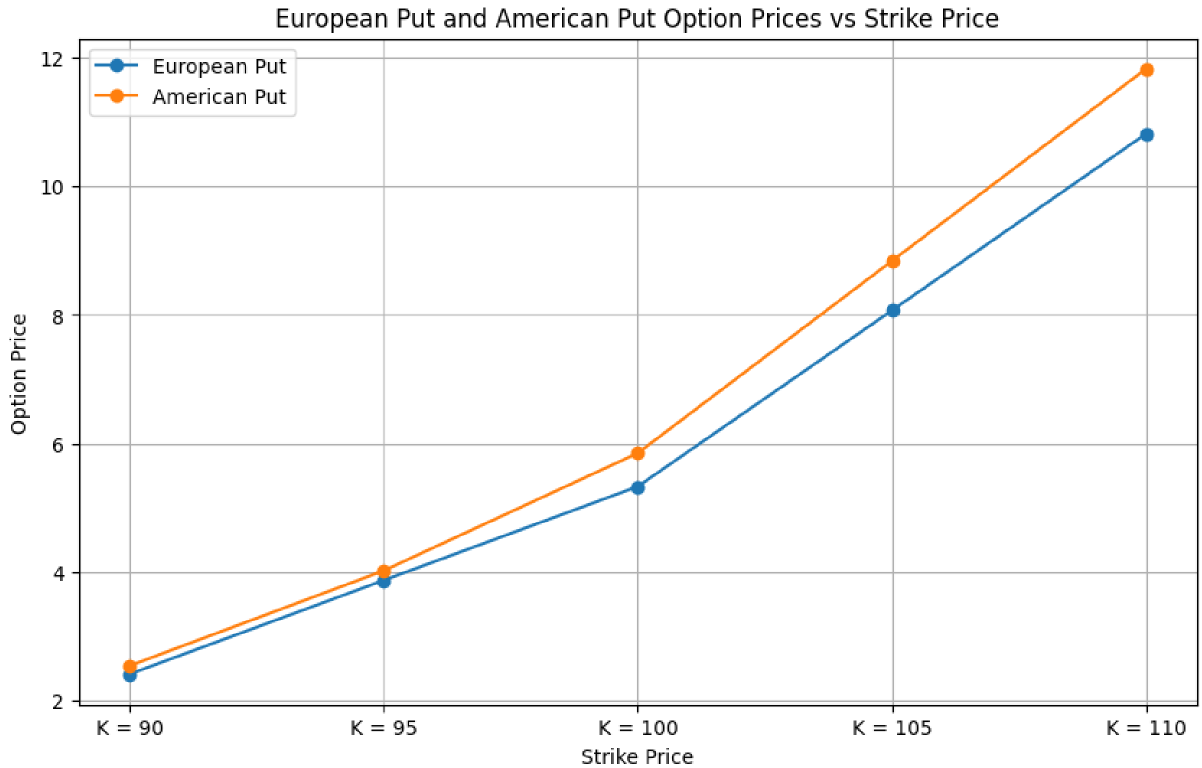


Fig.7. American call and put options chart

Q21. For trinomial model both European and American call produce same prices.



Q22. For trinomial models both European and American put produce the same prices.



Q23. The European option does hold the put-call parity for the trinomial model.
Q24. The American option does not hold the put-call parity for the trinomial model.

STEP 3

Q.25

a. Based on our computation, the price of the European Put Option is $13.82
b. The measurement of the values in the delta hedging process of the path "u,d,d" could be found in the table 9.

Table 9: European Put Option Delta Hedging Process

	t=0	t=1	t=2	t=3	Total
underlying_stock_price	180.00	199.34	180.00	162.54	NaN
put_option	13.82	5.01	9.88	19.46	NaN
delta_hedge	-0.47	-0.24	-0.53	NaN	NaN
stock_portfolio_value	-85.06	-39.17	-90.74	0.00	NaN
buy/sell	-0.47	0.23	-0.29	0.53	NaN
cash_account	85.06	-45.89	51.57	-19.46	71.27

In details, at t=0, we sell approximately 0.47 shares of the underlying, resulting a cash inflow if 0.47*180 ≅ $85.06.

At t=1, we buy 0.23 shares of the underlying stocks, to achieve the **Δ** = -0.24. By that way, we will have to spend 0.23 * 199.34 ≅ $45.89. The value of the stock portfolio at t=1 will be -$39.17 (=- 85.06 + 45.89) and we now need 0.24 shares in the future to cover.

At t=2, we continue a short-sell of 0.29 shares of the underlying stocks, to achieve the **Δ** = -0.53. By that way, we will get 0.29 * 180 ≅ $51.57. The value of the stock portfolio at t=2 will be -$90.74 (=-39.17 - 51.57) and we need 0.53 shares in the future to cover.

At t=3, our short portfolio 0.53 of underlying stock will worths -$86.15 (=-0.53*162.54). At t=3 (the maturity of the contract), we will need to spend 86.15$ to buy the stock to cover the contract. At the same time, we will also have to pay $19.46 to the put buyer for the put payoff.

Q. 26

Table 10: American Put Option Delta Hedging Process

	t=0	t=1	t=2	t=3	t=4	t=5	t=6	t=7	t=8	t=9	...	t=17	t=18	t=19	t=20	t=21	t=22	t=23	t=24	t=25	Total
underlying_stock_price	180.00	173.75	180.00	173.75	167.71	161.89	156.26	150.83	145.59	140.54	...	105.91	102.23	98.68	95.26	91.95	88.75	85.67	82.69	79.82	NaN
put_option	13.04	16.05	12.59	15.62	19.09	22.98	27.23	31.79	36.56	41.46	...	76.09	79.77	83.32	86.74	90.05	93.25	96.33	99.31	102.18	NaN
delta_hedge	-0.48	-0.56	-0.48	-0.57	-0.66	-0.75	-0.83	-0.90	-0.96	-0.99	...	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	NaN	NaN
stock_portfolio_value	-85.60	-100.41	-85.62	-101.07	-116.28	-130.56	-143.26	-153.89	-162.21	-167.35	...	-168.45	-168.45	-168.45	-168.45	-168.45	-168.45	-168.45	-168.45	0.00	NaN
buy/sell	-0.48	-0.09	0.08	-0.09	-0.09	-0.09	-0.08	-0.07	-0.06	-0.04	...	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	NaN
cash_account	85.60	14.81	-14.79	15.45	15.21	14.28	12.70	10.63	8.32	5.14	...	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-102.18	66.27

As the major difference between an American Put and its European counterpart is the expiration date exercise option; in which, an American put buyers have the right to execute the option any time before the maturity of the contract, therefore, an American put hedging process might require more active hedging actions in different steps of the path tree. The price of the Amercian Put ($13.04) is relatively lower than that of the European Put Option ($13.82) due to the higher frequency of early execution possibility.

Q. 27.

Table 11: Asian Put Option Delta Hedging Process

	t=0	t=1	t=2	t=3	t=4	t=5	t=6	t=7	t=8	t=9	...	t=17	t=18	t=19	t=20	t=21	t=22	t=23	t=24	t=25	Total
underlying_stock_price	180.00	173.75	167.71	161.89	156.26	150.83	145.59	140.54	135.65	130.94	...	98.68	95.26	91.95	88.75	85.67	82.69	79.82	77.05	79.82	NaN
put_option	13.04	16.05	19.48	23.30	27.48	31.96	36.64	41.47	46.35	51.06	...	83.32	86.74	90.05	93.25	96.33	99.31	102.18	104.95	102.18	NaN
delta_hedge	-0.48	-0.56	-0.65	-0.73	-0.81	-0.88	-0.94	-0.98	-1.00	-1.00	...	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	NaN	NaN
stock_portfolio_value	-85.60	-100.41	-115.02	-128.80	-141.20	-151.74	-160.15	-166.38	-168.50	-168.50	...	-168.50	-168.50	-168.50	-168.50	-168.50	-168.50	-168.50	-168.50	-79.82	NaN
buy/sell	-0.48	-0.09	-0.09	-0.09	-0.08	-0.07	-0.06	-0.04	-0.02	0.00	...	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	NaN
cash_account	85.60	14.81	-14.79	15.45	15.21	14.28	12.70	10.63	8.32	5.14	...	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-182.00	-13.55

6 rows × 27 columns

The Asian put option is a path-dependent option for the derivative pricing process, which requires more frequent computation in their average price and delta measures at each stage of the path tree [3]. The Monte Carlo simulation price of the Asian Put is reported at $6.71, representing about one half in prices compared to the put option of European Put in question 25 and American Put in question 26.

REFERENCES

1. James Chen, Gordon Scott, Pete Rathburn; Put-Call Parity: Definition, Formula, How it Works, and Examples; Investopedia
2. Hull, John, and Alan White. "Optimal delta hedging for options." *Journal of Banking & Finance* 82 (2017): 180-190.
3. Zhang, Z. Q., and W. Q. Liu. "Geometric average Asian option pricing for uncertain financial market." *Journal of Uncertain Systems* 8, no. 4 (2014): 317-320.
4. Stoll, Hans R. "The Relationship Between Put and Call Option Prices". Journal of Finance. 24 (5): 801–824
5. Mary Nisbet, Put-call parity theory and an empirical test of the efficiency of the London Traded Options Market, Journal of Banking & Finance, Volume 16, Issue 2, April 1992, Pages 381-403
6. Abhilash Ramachandran, Dheeraj Vaidya, CFA, FRM, European vs American Option, WallStreetMojo