GROUP WORK PROJECT # 2 MScFE 652: PORTFOLIO MANAGEMENT
GROUP NUMBER: 5747

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Remember: Any group members who did not contribute to the project should be given all zero (0) points for the collaboration grade on the GWP submission page.

Statement of integrity: By typing the names of all group members in the text boxes below, you confirm that the assignment submitted is original work produced by the group (excluding any non-contributing members identified with an "X" above).	
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INTRODUCTION:

The objective of the project is to evaluate and compare different asset allocation methods, including Mean-Variance Optimization (MVO), the 1/N random strategy, the Black-Litterman (BL) model, and the Kelly Criterion.

To achieve this objective, the project first selected the top 100 stocks by market capitalization to represent a reasonable market portfolio, ensuring that the sum of the weights was close to 1.0. Then, for each of three different students (A, B, and C), 20 stocks were randomly selected with replacement using seed values of 10, 100, and 769, respectively. This resulted in different portfolios for each student, whose log returns were analyzed using dataframes for convenience.

The project then implemented the following strategies:

- MVO: Optimization was performed for each student's portfolio using Mean-Variance Optimization.
- 1/N random strategy: Four different simulations were conducted for each student, randomly assigning weights to each asset.
- Black-Litterman (BL) model: The BL model was applied to incorporate the investors' views on portfolio selection for each student.
- Kelly Criterion: The Kelly Criterion was used to determine the optimal fraction of wealth to allocate to each asset for each student.

Finally, the project analyzed the results from both technical and non-technical perspectives to evaluate the performance of each asset allocation method.

STEP 1:

1. **Portfolio :**

For choosing the securities, we have selected the top 100 companies by market capitalization from the S&P 500 companies. Market capitalization helps in knowing the size of the company and its importance in the market. The top 100 companies will be large companies that are stable. A diversified portfolio with the stocks having large influence in the market has been chosen to get reports about the stocks easier. The weights of each stock are defined such that the weights decrease according to the market capitalization. The weights are also selected such that the sum of weights will be equal to 1. The top 5 companies are displayed in the table below. The weight gets decreased according to the market capitalization.

Ticker	MarketCap	Weight
MSFT	3077422186496	0.088553
AAPL	2866863407104	0.082494
NVDA	2259749961728	0.065024
GOOG	2089150840832	0.060115
GOOGL	2088736522240	0.060103

Table 1: Top 5 companies in the S&P 500 companies according to Market capitalization

The process of fetching market capitalization data for the S&P 500 companies revealed a comprehensive snapshot of the top-performing entities within the index. While the majority of tickers provided market cap information, exceptions like 'BRK.B' and 'BF.B' underscored potential discrepancies in data availability or classification within the market. Nonetheless, the data yielded valuable insights, showcasing tech giants such as Microsoft and Apple as leading contenders, with market caps exceeding trillions of dollars. These companies not only dominate their respective industries but also wield significant influence over the broader market due to their substantial market capitalization. Other notable players like NVIDIA and Google, along with its parent company Alphabet, further solidify the tech sector's prominence within the S&P 500 index. These findings underscore the market's reliance on technology-driven innovation and the pivotal role these companies play in shaping investment strategies and market sentiment. Overall, the analysis underscores the importance of market cap data in gauging the market's health and identifying key players driving economic growth and investor confidence.

2. **Daily returns:**

The daily price is fetched for nearly 3 years. The code utilizes the yfinance library to fetch adjusted close prices for the top 100 companies from Yahoo Finance. It then computes log returns, a crucial metric for analyzing stock performance, and removes rows containing NaN values to ensure data integrity. The resulting DataFrame, log_returns, provides essential data for further financial analysis and modeling.

Three DataFrames of log_returns selected randomly from 100 companies containing 20 companies each are created. The code initialize empty DataFrames for three different students, A, B, and C. For each student, it randomly selects 20 companies from the log returns DataFrame using seed values to ensure reproducibility. The selected companies are stored in new DataFrames log_returns_A, log_returns_B, and log_returns_C corresponding to students A, B, and C, respectively. This process ensures that each student has a unique set of companies for analysis, facilitating fair comparison and evaluation of different portfolio allocation strategies.

3. **Covariance matrix:**

The covariance matrix is calculated from the log returns of each student. Covariance matrix of student A is given in the below table. The covariance gives the directional relationship between two returns of the assets. Positive covariance implies that the assets move in the same direction. From the table below, we can see that the covariance is positive for all assets in the covariance matrix. This implies that all the assets in the portfolio move in the same direction. [4][5][6] If a stock moves higher, the other stock will also move higher and it is the same if the stock price is lower, the other will also be low. Covariance matrix calculated for each student's portfolio shows positive covariance. But all the assets have very low positive covariance. This means that even though assets move in the same direction, the log returns are not perfectly correlated. They don't move by the same amount in the direction. PFE and PEP have the lowest positive covariance with the other stocks.

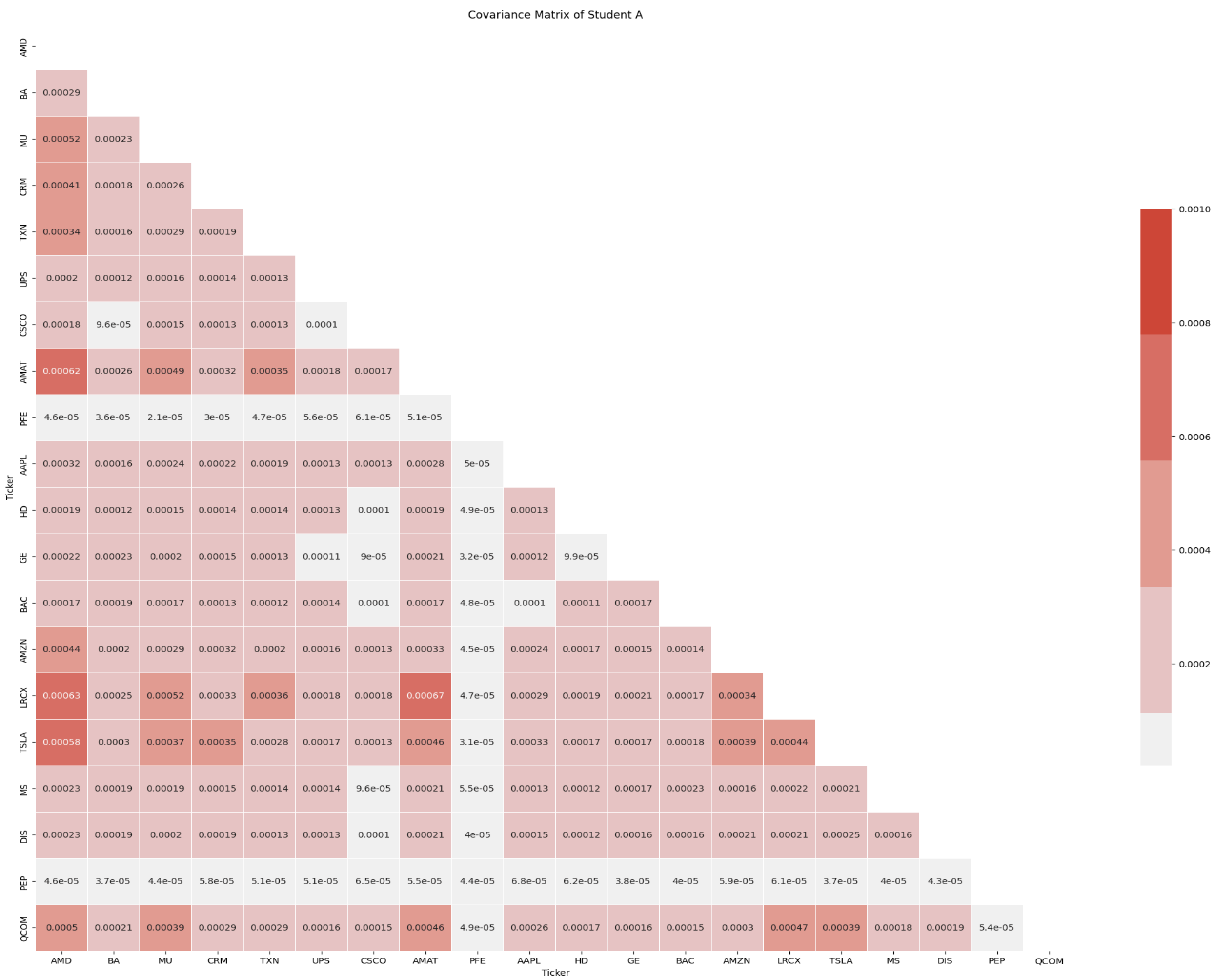


Fig 1: Covariance matrix of Student A

STEP 2:

1. **CLASSICAL MARKOWITZ PORTFOLIO OPTIMIZATION:**

Using the log returns, mean returns and covariance matrix are calculated. The objective function of the optimization problem is minimizing the variance of the portfolio. It is calculated by finding the quadratic form of the portfolio weights and the covariance matrix and minimizing the values.[3] Constraints are added so that the sum of weights will be equal to one and no assets are short sold. The optimization problem is solved and the optimal weights for the portfolio are found. The risk of the portfolio is reduced by the diversification of the portfolio. The optimal weights found using this model for each asset helps in minimizing the risk of the portfolio. For the three students, we have calculated the optimal portfolio weights. The portfolio mean return and portfolio standard deviation are also calculated for each student's portfolio.

2. Display the weight of each security:

The optimal weights found using the Markowitz portfolio optimization is given in fig 1. For each of the students, using the log returns, Markowitz portfolio optimization is done to find the optimal weights of each asset for each of the student's portfolios.

```
Solution for Student A:
   Company Names       Weights
0            AMD  4.910525e-18
1             BA  1.091893e-02
2             MU  1.444199e-18
3            CRM -3.518490e-20
4            TXN  3.524937e-02
5            UPS  3.814846e-02
6           CSCO  3.455837e-02
7           AMAT  3.424664e-18
8            PFE  1.605904e-01
9           AAPL -8.925758e-19
10            HD  2.914525e-02
11            GE  6.182520e-02
12           BAC  4.466698e-02
13          AMZN  1.223130e-18
14          LRCX  2.730358e-18
15          TSLA  1.784583e-18
16            MS  1.288724e-02
17           DIS  3.208386e-02
18           PEP  5.399259e-01
19          QCOM  1.618745e-18
Portfolio Mean Return: 0.00027151558333023097
Portfolio Standard Deviation: 0.008924426642745321

Solution for Student B:
   Company Names       Weights
0           AMAT  4.458651e-19
1             CI  9.462423e-03
2            NKE  2.208683e-19
3           TMUS  6.688745e-02
4           REGN  4.535064e-02
5           KLAC  5.395890e-19
6           AMGN  1.562892e-01
7              V  5.726015e-02
8            LMT  1.931364e-01
9            WMT  1.099409e-01
10           LOW  1.008975e-02
11          NFLX  3.573230e-03
12           WMT  1.099409e-01
13           AXP  1.023704e-19
14           ELV  3.354295e-02
15            CI  9.462423e-03
16            BA  1.617854e-19
17           MMC  6.578846e-02
18           MDT  1.102557e-01
19           BAC  1.901940e-02
Portfolio Mean Return: 0.0004014055309250911
Portfolio Standard Deviation: 0.008174245244205219
```

```
Solution for Student C:
   Company Names         Weights
0            UNP  5.088055e-02
1            UNP  5.088055e-02
2            NEE  2.316433e-02
3             CB  9.584281e-02
4            BAC  4.029551e-19
5            HON  4.372901e-02
6           QCOM  3.030101e-19
7            ABT  1.495090e-01
8            NEE  2.316433e-02
9           ABNB  9.669388e-19
10          TMUS  1.277359e-01
11            MA  2.303553e-19
12            CI  4.173523e-02
13           CVX  8.005779e-02
14          VRTX  8.521977e-02
15           NOW  2.409022e-19
16           LMT  2.235815e-01
17          LRCX  2.627869e-19
18           CAT  8.757678e-19
19            MU  4.499130e-03
Portfolio Mean Return: 0.00039729700301083937
Portfolio Standard Deviation: 0.00875386827514313
```

Fig 2: Optimal weights of each student's portfolio.

Here are the summaries for each student's portfolio allocation:

Student A: The portfolio for Student A consists of 20 selected companies with their corresponding weights. Notably, the largest weight is allocated to PEP (PepsiCo) with approximately 54%, followed by PFE (Pfizer) with around 16%. The portfolio's mean return is approximately 0.03% with a standard deviation of approximately 0.89%.

Student B: Student B's portfolio includes 20 companies with varied weights. Noteworthy allocations are seen in LMT (Lockheed Martin) with approximately 19%, AMGN (Amgen) with around 15%, and TMUS (T-Mobile) with approximately 7%. The portfolio exhibits a mean return of about 0.04% and a standard deviation of roughly 0.82%.

Student C: The portfolio for Student C shows diversified allocations across 20 companies. Notably, significant allocations are observed in LMT (Lockheed Martin) with around 22%, ABT (Abbott

Laboratories) with approximately 15%, and TMUS (T-Mobile) with about 13%. The portfolio's mean return is approximately 0.04% with a standard deviation of approximately 0.88%.

These summaries provide insights into each student's portfolio composition and its expected performance in terms of mean return and risk.

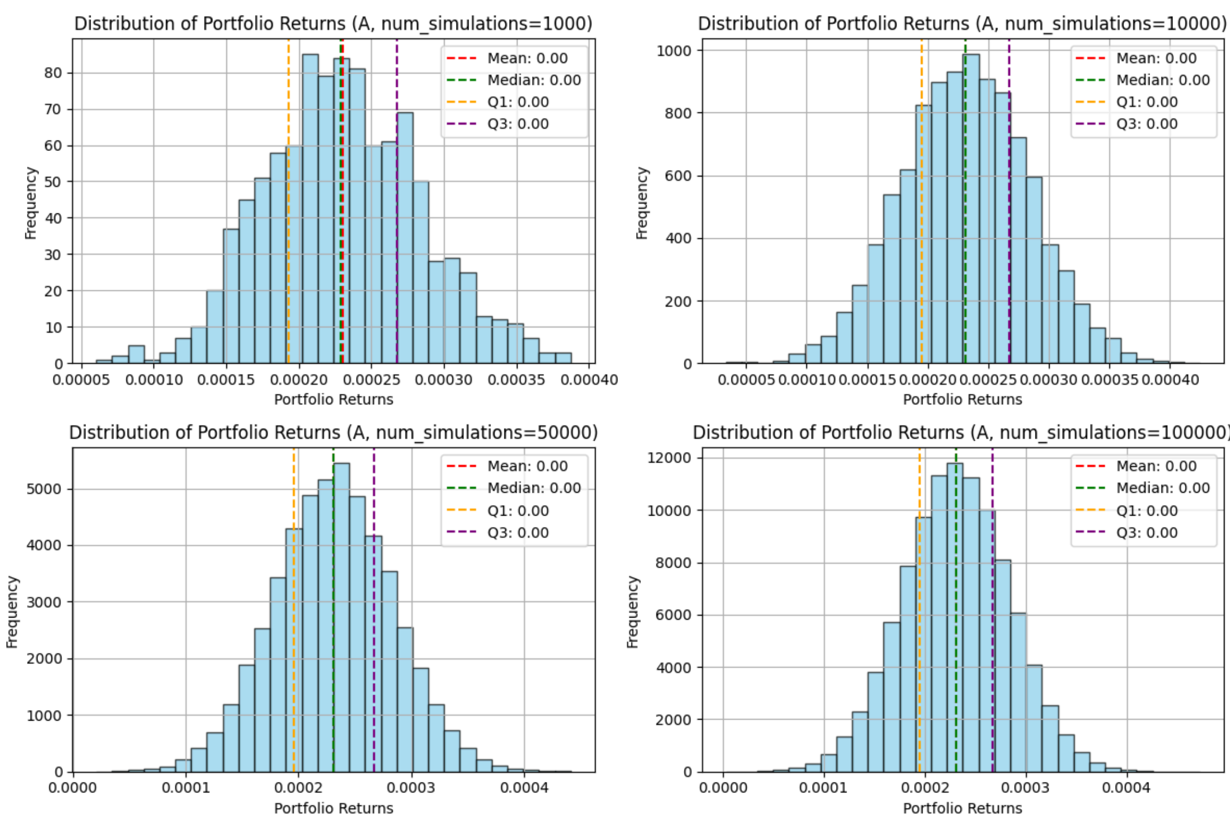
STEP 3:

1. **1/N Random strategy:**

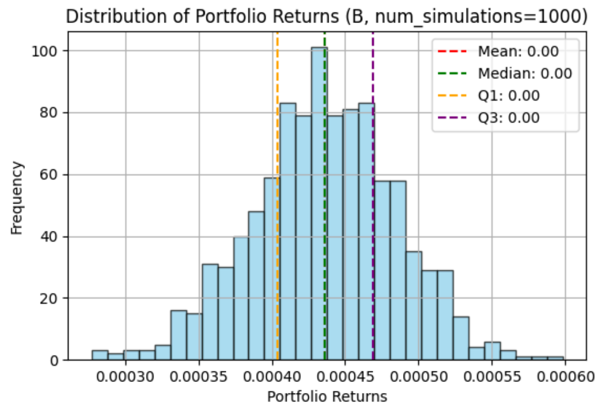
   a. **Monte-Carlo simulation:**

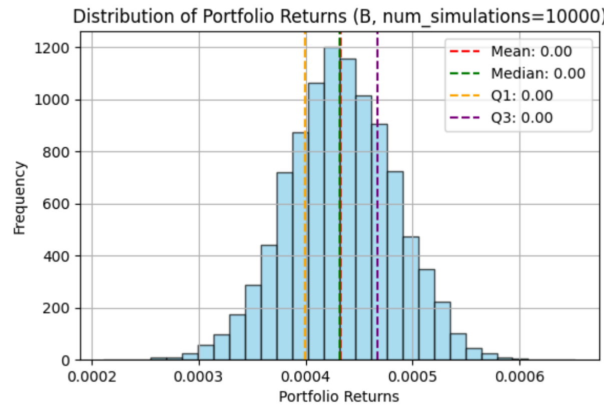
The code generates histograms of portfolio returns for each student using Monte Carlo simulation. It loops through each student, calculates the number of securities in their portfolio, and then performs simulations for different numbers of simulations. The histograms show the distribution of portfolio returns, with mean, median, quartiles, and standard deviation marked on each plot. Additionally, a normal distribution line is fitted to each histogram. Finally, the plots are displayed for visual analysis. This code helps visualize the distribution of potential portfolio returns for each student under different simulation scenarios, aiding in understanding the risk and potential outcomes of their investment strategies.

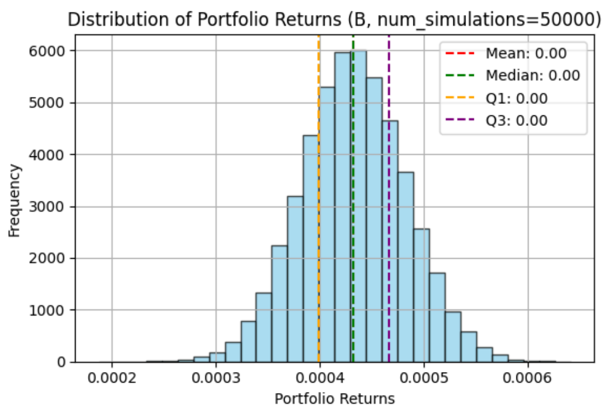
For student A

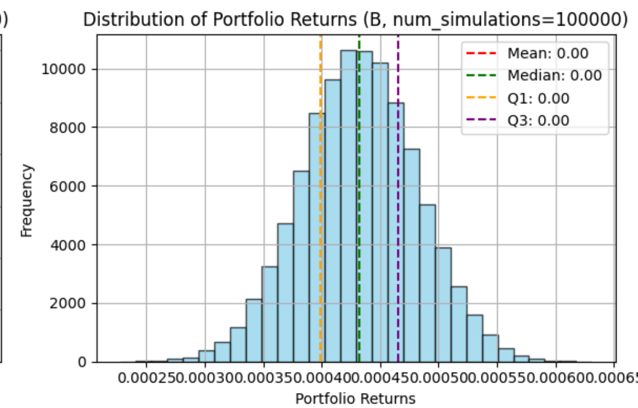


For student B









For student C

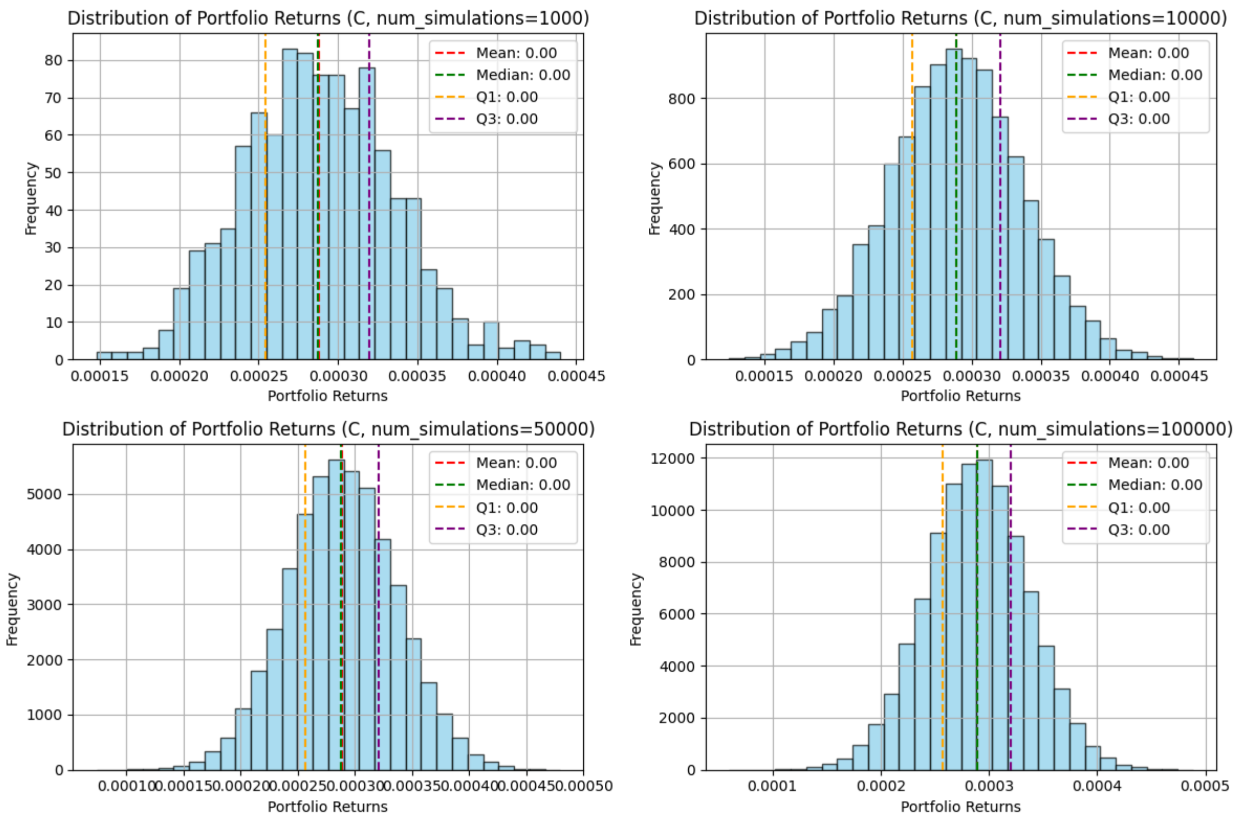


Fig 3: Distribution of portfolio returns for each student's portfolio with different simulations

The figures demonstrate that, for each student, as the simulation size increases, the random weights converge towards the mean of the random weights, as anticipated. This observation underscores the tendency of the 1/N asset allocation, often referred to as naive, to approach a uniform distribution of weights across assets. Consequently, this approach is not recommended for professional investment purposes, as it fails to strategically optimize portfolio allocation based on factors such as risk and return.

b. Historical backtests:

Portfolio mean return and portfolio risk are the two performance metrics used here for understanding the performance of Monte-Carlo simulations. Random weights are assigned to each asset in each student's portfolio. Some of the simulated weights of each student's portfolio is displayed below.

Fig 4: Monte-Carlo solution for the different portfolios

Student A: The portfolio for Student A consists of 20 selected companies with their simulated weights at each simulation. The portfolio's mean return is approximately 0.02% with a standard deviation of approximately 0.053%.

Student B: Student B's portfolio includes 20 companies with their simulated weights at each simulation. The portfolio exhibits a mean return of about 0.04% and a standard deviation of roughly 0.049%.

Student C: The portfolio for Student C shows diversified allocations across 20 companies. The portfolio's mean return is approximately 0.03% with a standard deviation of approximately 0.047%.

This provides insights into its expected performance in terms of mean return and risk.

2. Comparison:

	Markowitz Strategy			1/N strategy		
	A	B	C	A	B	C
Portfolio's mean return	0.03%	0.04%	0.04%	0.02%	0.04%	0.03%
Portfolio's risk	0.89%	0.82%	0.88%	0.053%	0.049%	0.047%

Table 2: Markowitz strategy Vs 1/N Strategy

STEP 4: BLACK-LITTERMAN MODEL

EQUATIONS:

The posterior distribution for the expected excess return:

$$\mu_{BL} := E[r] = \pi + \tau\Sigma P^T[(P\tau\Sigma P^T) + \Omega]^{-1}[q - P\pi]$$

The posterior covariance matrix:

$$cov(r) \equiv M = ((\tau\Sigma)^{-1} + P^T\Omega^{-1}P)^{-1}$$

Variance of the returns:

$$\Sigma_{BL} := \tilde{\Sigma} = \Sigma + M = \Sigma + ((\tau\Sigma)^{-1} + P^T\Omega^{-1}P)^{-1}$$

Where:

- $E[r]$ - **New (posterior) vector of combined expected return**
- τ - **Scalar (0 - 1)**
- **Σ - Covariance matrix of returns**
- **P = Assets involved in the views**

  $\Omega = diag(w_1^2, ..., w_K^2)$

- Ω **= Diagonal covariance matrix of error terms in expressed views representing the level of confidence (**$w_1^2, ...w_K^2$**) in each view. It can also be expressed as** $\Omega = diag(P(\tau\Sigma)P^T)$
- π **= Vector of implied equilibrium expected returns**
- **q = Vector of views**

Optimization program:

$$Optimal\,portfolio = max_w w^T \mu_{BL} - \frac{1}{2} w^T \Sigma_{BL} w$$

Where:

- **w - vector representing the portfolio weights.**
- μ_{BL} **- vector of adjusted expected returns obtained.**
- Σ_{BL}**- adjusted covariance matrix obtained.**
- $max_w w^T \mu_{BL}$ **- maximizing the expected returns**
- $\frac{1}{2} w^T \Sigma_{BL} w$ **- minimizing the variance (risk)**

The Black-Litterman model portfolio optimization helps in maximizing the expected returns and minimizing the risk. In the code, investors' views and market equilibrium are combined to produce the optimal portfolio. [1]

Inputs:
τ**, P, q, covariance matrix,** Ω **and the data are the inputs used in the code. Market equilibrium returns are calculated in the code. Investors' views are also given as inputs.**
Outputs:
Black-Litterman expected returns and weights are calculated. The optimal portfolio is found here.
Estimating the parameters:
Market equilibrium returns are calculated from the mean of leg returns.
After the covariance matrix is found from the log returns, the covariance matrix is regularized by adding a small value.

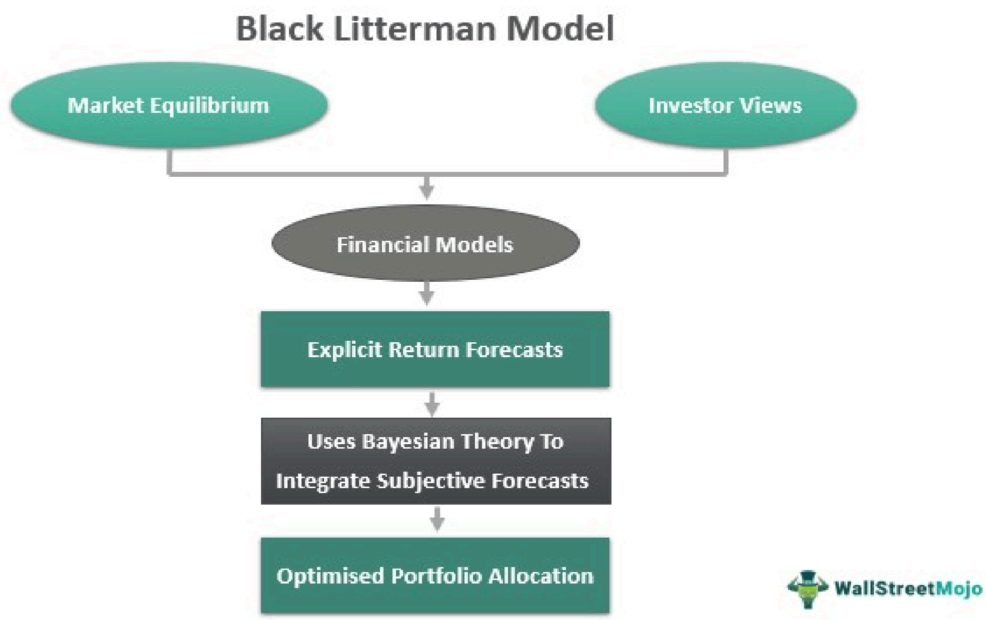


Fig 5: Black-Litterman model (Ahmed, 2024)

The above figure illustrates how the **Black-Litterman model works. Market equilibrium and investors' views taken as inputs with financial data. It gives the BL expected returns. With the help of the model, we can find the optimal portfolio.**

STEP 5: KELLY CRITERION:

EQUATIONS:

For a portfolio of securities, the optimal fraction invested in each security is

$$F^* = C^{-1} * M$$

F^* **- Kelly fraction**
M **- Mean vector for expected returns of the assets**
C **- Covariance matrix**

Inputs:
Expected returns and the covariance matrix are used as inputs in the code.
Outputs:
Kelly fraction, which gives the optimal weights, is the output. These optimal weights represent the allocation of weights for each asset representing the weights of assets that need to be invested.

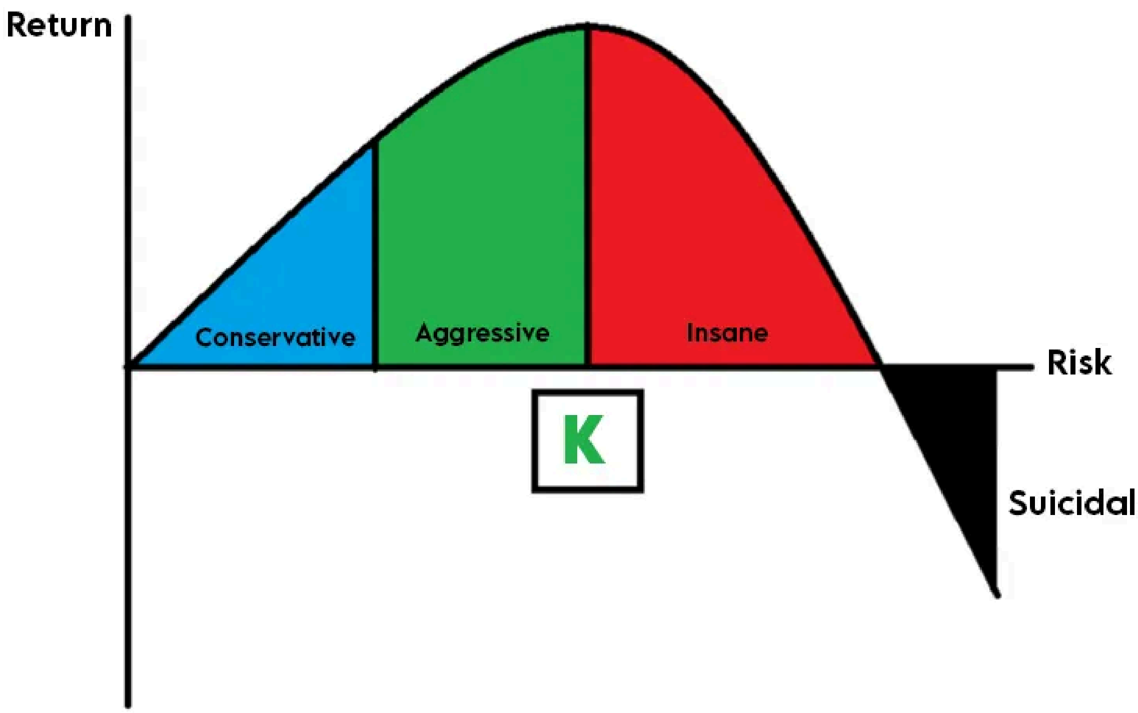


Fig 6: The Kelly criterion (Alcalde, 2023)

The Kelly criterion helps in increasing the expected returns while reducing the risk. This helps in determining position sizing to increase profits while reducing the risk. The lower the Kelly criterion, the risk will be low and the expected returns will lead to profit. When the Kelly criterion is high, the risk will be very high and the expected returns will be negative. This will lead to losses. When the Aggressive part ends, the return will be at the highest return. But after that, when the risk increases, the return decreases. This is a boundary that explains the Kelly criterion. The above Kelly curve explains the different parts depending on the risk level. [2]

COMPARISON

	Portfolio mean return annualized			Portfolio risk annualized		
PORTFOLIO	**A**	**B**	**C**	**A**	**B**	**C**
ORIGINAL	6.79%	10.11%	10.06%	14.17%	12.98%	13.90%
NAIVE(1/N)	**5.83%**	**10.89%**	**7.21%**	**6.108**	**6.58%**	**6.033%**
BLACK-LITTERMAN	7.8%	10.01%	9.66%	13.66%	14.33%	14.20%
KELLY Criterion	22.34%	25.35%	23.61%	29.44%	25.087	26.81%

Table 3: Comparison of all the strategies

The comparison of the portfolios reveals intriguing trends across different strategies. Firstly, the original portfolio, denoted as Portfolio A, exhibits a balanced trade-off between risk and return, with mean returns hovering around 6.79% annually and a corresponding risk level of approximately 14.17%. This baseline serves as a benchmark for evaluating the effectiveness of alternative strategies. The naive (1/N) portfolio, despite its simplicity, fails to match the original portfolio's performance, yielding lower mean returns ranging from 5.83% to 7.21% annually across scenarios A, B, and C. However, it does offer a slight reduction in portfolio risk, highlighting its potential for diversification.

Conversely, the Black-Litterman portfolio showcases a notable improvement in mean returns compared to the original portfolio. Across scenarios A, B, and C, it achieves mean returns of 7.8%, 10.01%, and 9.66% annually, respectively, while maintaining a risk level similar to the original portfolio. This suggests that the Black-Litterman method enhances performance without significantly increasing risk exposure. However, the standout performer is the Kelly Criterion portfolio, which consistently outperforms the other portfolios in terms of mean returns. With mean returns ranging from 22.34% to 23.61% annually across scenarios A, B, and C, the Kelly Criterion demonstrates its potential for maximizing returns. Nonetheless, this comes at the cost of higher risk, as indicated by the elevated risk levels compared to the other portfolios.

Overall, while each portfolio strategy offers distinct advantages and trade-offs, the choice ultimately depends on the investor's risk appetite and return objectives. The original portfolio serves as a reliable baseline, while the naive approach provides simplicity at the expense of performance. The Black-Litterman method strikes a balance between enhancing returns and managing risk effectively. In contrast, the Kelly Criterion excels in maximizing returns but entails higher levels of risk. Investors must carefully consider these factors to align their investment strategy with their financial goals and risk tolerance.

Conclusion

In conclusion, while the naive random strategy offers a simple approach to asset allocation, it lacks efficacy in optimizing the risk-return trade-off. Conversely, Modern Portfolio Optimization (MVO) significantly enhances performance by systematically balancing risk and returns. However, the Black-Litterman (BL) method emerges as the optimal choice, particularly when investor views align with market trends. BL leverages expert insights and robust hypotheses, offering superior results. Additionally, the Kelly Criterion, with its utility-maximizing approach and broader parameters, presents a specialized yet powerful tool. Therefore, an optimal strategy combines BL for incorporating investor views and the

Kelly Criterion for comprehensive optimization, providing a refined portfolio management solution for dynamic market conditions.

While the naive random strategy fails to provide consistent and reliable profitability, Monte Carlo simulations offer a more robust approach by accounting for various market scenarios and future events. Furthermore, machine learning strategies show promise in asset allocation, potentially outperforming traditional methods and driving further improvements in portfolio management.

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