**GROUP WORK PROJECT #** 2 MScFE 620: Derivative Pricing
**GROUP NUMBER:** 3892

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| **Statement of integrity:** By typing the names of all group members in the text boxes below, you confirm that the assignment submitted is original work produced by the group (excluding any non-contributing members identified with an "X" above). | |
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**Q1:**

For this part of the task we are using the closed-form of Black-Scholes formula which looks like this:
For a call option:

$$c = S_0 N(d_1) - Ke^{-rT} N(d_2)$$

For a put option:

$$p = Ke^{-rT} N(d_2) - S_0 N(d_1)$$

Where $N()$ represents the CDF of the standard normal distribution and the values for $d_1$ and $d_2$ are calculated using the following equations [4]:

$$d_1 = \frac{ln(\frac{S_0}{K}) + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}$$

$$d_1 = \frac{ln(\frac{S_0}{K}) + (r - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

Applying the stated equations, the results of the Black-Scholes options pricing calculation can be seen in table 1 and 2.

| | Black-Scholes Call | Black-Scholes Put |
|---|---|---|
| **Option Price** | 4.61 | 3.37 |
| **Greek Delta** | 0.55 | -0.45 |

Table 1. Comparison of Black-Scholes closed-form analytical derivation results for European call and put options comparison and their corresponding Delta.

In particular, delta is measured to represent the changes in option prices relative to changes in the underlying stock. Delta could be computed by the following formulas (for call and put options):

$$Call\ Options\ \Delta = \frac{\partial C}{\partial S} = N(d_1)$$

$$Put\ Options\ \Delta = \frac{\partial P}{\partial S} = N(d_1) - 1$$

On the other hand, vega is implemented to measure the change in the option price due to volatility changes as the following question:

$$v = \frac{\partial^2 V}{\partial \sigma} = SN'(d_1)\sqrt{T - t}$$

Measures of vega in call and put options could be found in the following table.

| | Black-Scholes Call | Black-Scholes Put |
|---|---|---|
| **Base Option Price (Sigma = 0.20)** | 4.61 | 3.37 |
| **Price at Higher Volatility (Sigma = 0.25)** | 5.59 | 4.35 |
| **Vega** | 19.76 | 19.76 |

Table 2. Comparison of Black-Scholes closed-form analytical derivation results for European call and put options stressed with volatility increase of 0.05% and their corresponding Vega.

**Q2:**

For the Monte Carlo simulations that helped us derive the European option we choose the number of 1 million simulations. We are following the theory that suggests that Monte Carlo estimates will converge to an analytical solution as the number of simulations increases [1]. In other words: the bigger the number of Monte Carlo simulations, the better the result will be. So we chose 1 million to be a good count of simulations because it is not little but also is not that much. There is another trade off with enormous count of simulation and it is that with more complex projects with many simulations comes the part that you will need a lot of computational power and thus a lot of processor time. So it is wise not to choose too many simulations.

The overall process looks like this: calculating a lot of simulations' results, then averaging in a final result and depending on the central limit theorem and the law of large numbers which suggest that the bigger the sample size is the better the sample average will tend to the real population average[2][3]. The same logic goes for every kind of calculation like options pricing and greeks. All our findings for question 2 are located in the following tables 3 and 4 which are located below.

| | Monte Carlo Black-Scholes Call | Monte Carlo Black-Scholes Put |
|---|---|---|
| **MC Option Price** | 4.61 | 3.37 |
| **Greek Delta** | 0.57 | -0.43 |

Table 3. Comparison between European Call and Put prices derived by Monte Carlo simulations and their corresponding Delta.

| | Monte Carlo Black-Scholes Call | Monte Carlo Black-Scholes Put |
|---|---|---|
| **Base Option Price (Sigma = 0.20)** | 4.61 | 3.37 |
| **Price at Higher Volatility (Sigma = 0.25)** | 5.58 | 4.38 |
| **Vega** | 19.37 | 20.21 |

Table 4. Comparison between European Call and Put prices derived by Monte Carlo simulations and stressed by 0.05 increase in volatility and their corresponding Vega.

**Q4:**
The following contents describe the results of the American call pricing under the Monte Carlo approach. The idea is the same as described above so it won't be repeated again here to avoid similarity issues. The same goes for question 5 to 7, so there will be just the table naming and brief information without much additional explanations.

| | Monte Carlo Amercian Call |
|---|---|
| **Option Price** | 4.64 |
| **Greek Delta** | 0.56 |

Table. 5. American call priced by Monte Carlo method and its Delta

| | American Call |
|---|---|
| **Base Option Price (Sigma = 0.20)** | 4.64 |
| **Price at Higher Volatility (Sigma = 0.25)** | 5.56 |
| **Vega** | 20.20 |

Table. 6. A comparison of American call priced by Monte Carlo with a spike of volatility from 20% to 25% and the according Vega

**Question 5:**
The following contents describe the results of the American put pricing under the Monte Carlo approach. The idea is the same as described above so it won't be repeated again here to avoid similarity issues. The same goes for question 5 to 7, so there will be just the table naming and brief information without much additional explanations.

| Monte Carlo Amercian Put | |
|---|---|
| **Option Price** | 3.40 |
| **Greek Delta** | -0.44 |

Table. 7. American put priced by Monte Carlo method and its Delta

| | American Put |
|---|---|
| **Base Option Price (Sigma = 0.20)** | 3.40 |
| **Price at Higher Volatility (Sigma = 0.25)** | 4.41 |
| **Vega** | 20.00 |

Table. 8. A comparison of American put priced by Monte Carlo with a spike of volatility from 20% to 25% and the according Vega

**Question 7:**

a) Prices of the European Call and Put options at the strikes, respectively 110% and 95% moneyness is presented in table 9.

| | European Call | European Put |
|---|---|---|
| **Strike Price** | 110.00 | 95.00 |
| **Option Price** | 1.19 | 1.53 |
| **Greek Delta** | 0.21 | -0.26 |

Table. 9. A comparison of American call and put options priced by Monte Carlo and the associated delta.

b) The portfolio aggregated by one long position of a call and one long position of a put will result in a portfolio delta of -0.05 ($\Delta$ = -0.05), implying $1 increase in the underlying stock will lead to $0.05 decrease in the portfolio value. In this case, the portfolio is more sensitive with the asset uptrend and might be associated with a loss when the stock price goes up. Therefore, to offset the upside risks, the portfolio could be delta hedged by buying 5 (=0.05 * 100) more underlying stocks.

c) The portfolio aggregated by one buy position of a call and one sell position of a put will result in a portfolio delta of +0.47 ($\Delta$ = 0.47), implying $1 increase in the underlying stock will lead to $0.47 increase in the portfolio value. In this case, the portfolio is more sensitive with the asset downtrend and might be associated with a loss when the stock prices drop. Therefore, to offset the upside risks, the portfolio could be delta hedged by having a short of 45 (=0.47 * 100) underlying stocks.

**Question 8:**

The barrier options are derivative products that rely on underlying asset price level, which is pre-set with the limit to a barrier/ knock-out level [5]. Specifically, a up-and-out option will cease to exist only when the underlying stock price passes and increases above a price barrier. Price of the Up-and-Out European Call is measured as the following table.

Up-and-Out European Call

| | |
|---|---|
| **Option Price** | 0.71 |

Table. 10. Up-and-Out barrier option pricing.

**Summary Table**

| Q# | Type | Exer | GWP 1 Method | GWP 2 Method | GWP 1 Price | GWP 2 Price | % of Difference |
|---|---|---|---|---|---|---|---|
| 5 | ATM Call | European | Binomial-Tree Method | Black-Scholes Approach | 4.61 | 4.61 | 0% |
| 5 | ATM Put | European | Binomial-Tree Method | Black-Scholes Approach | 3.47 | 3.37 | 2.88% |
| 5 | ATM Call | European | Binomial-Tree Method | Monte Carlo Simulations Method | 4.61 | 4.62 | -0.22% |
| 5 | ATM Put | European | Binomial-Tree Method | Monte Carlo Simulations Method | 3.37 | 3.37 | 0% |
| 8 | ATM Call | American | Binomial-Tree Method | Monte Carlo | 4.61 | 4.62 | -0.22% |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | Simulations Method | | | |
| 8 | ATM Put | American | Binomial-Tree Method | Monte Carlo Simulations Method | 3.48 | 3.41 | 2.01% |
| 16 | OTM Put | European | Trinomial Tree | Black-Scholes Approach | 1.6 | 1.53 | 4.38% |
| 17 | DOTM Call | American | Trinomial Tree | Monte Carlo Simulations Method | 1.22 | 1.19 | 2.46% |
| 17 | OTM Call | American | Trinomial Tree | Monte Carlo Simulations Method | 2.55 | 2.48 | 2.75% |
| 17 | ATM Call | American | Trinomial Tree | Monte Carlo Simulations Method | 4.49 | 4.62 | -2.90% |
| 17 | ITM Call | American | Trinomial Tree | Monte Carlo Simulations Method | 7.78 | 7.71 | 0.90% |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **17** | **DITM Call** | **American** | **Trinomial Tree** | **Monte Carlo Simulations Method** | **11.71** | **11.68** | **-0.26%** |
| **18** | **DOTM Put** | **American** | **Trinomial Tree** | **Monte Carlo Simulations Method** | **0.6** | **0.59** | **-1.67%** |
| **18** | **OTM Put** | **American** | **Trinomial Tree** | **Monte Carlo Simulations Method** | **1.63** | **1.55** | **4.91%** |
| **18** | **ATM Put** | **American** | **Trinomial Tree** | **Monte Carlo Simulations Method** | **3.37** | **3.41** | **-1.19%** |
| **18** | **ITM Put** | **American** | **Trinomial Tree** | **Monte Carlo Simulations Method** | **6.46** | **6.25** | **3.25%** |
| **18** | **DITM Put** | **American** | **Trinomial Tree** | **Monte Carlo Simulations Method** | **10.28** | **9.94** | **3.31%** |

Table. 11. Comparison results between the findings from GWP1 and GWP2

As we can see the absolute difference is lower than 5% so the findings from GWP1 and GWP2 are very close to each other. If, for some reason, there is a need for a better evaluation with smaller difference, we could higher up to number of simulation in the Monte Carlo function and thus the estimates will get closer to the real value because as we all know the result form the Monte Carlo simulations converge to the analytical solution (the actual and real value of the Black-Scholes equation solution) as the number of simulations increases [1].

# References:

1. Tompaidis, Stathis; Pricing American-style options by Monte Carlo simulation: alternatives to ordinary least squares; The Journal of Computational Finance, 18(1), pp. 121-143
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4. Schumacher, M. Johannes; Introduction to Financial Derivatives: Modeling, Pricing and Hedging; ISBN: 978-94-6240-612-4; pp. 123
5. Tian, M., Yang, X. and Zhang, Y., 2019. Barrier option pricing of mean-reverting stock model in uncertain environment. *Mathematics and Computers in Simulation*, *166*, pp.126-143.