A Confidence Interval for the Median Survival Time

Source: Ron Brookmeyer and John Crowley (1982),
Biometrics 38, pages 29-41

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Fall 2000

The following tables are an example of how to compute a confidence interval for median survival time. These pages are an attempt to clarify the computations.

SIGN TEST FOR CENSORED DATA OF Ho: = 1/2

	X_i	$X_i > M$	$X_i \leq M$	δ_i				
1	20^+	No	Yes	0	.26667	1.0000	.26667/1	.26667
2	21	No	Yes	1	.26667	.80000	0	0
3	26^+	No	Yes	0	.26667	.80000	.26667/.8	.33334
4	27	No	Yes	1	.26667	.53333	0	0
5	34	No	Yes	1	.26667	.26667	0	0
6	35^+	Yes	No	0	.26667	.26667	.26667/.26667	1.0000

Sum = 1.6000075

TEST STATISTIC U

GREENWOOD'S CONSISTENT ESTIMATE OF THE VARIANCE of .

i	X_i			n	d_i	$N_x(X_i)$	$N_x(X_i)\{N_x(X_i)+ d_i\}$		
1	20^+	.26667	.071113	6	0	5	5(5+0)=25	0	.000000
2	21	.26667	.071113	6	1	4	4(4+1)=20	1/20 = .05000	.050000
3	26^+	.26667	.071113	6	0	3	3(3+0)=09	0	.050000
4	27	.26667	.071113	6	1	2	2(2+1)=06	1/6 = .16667	.216667
5	34	.26667	.071113	6	1	1	1(1+1)=02	1/2 = .50000	.716667
6	35^+	.26667	.071113	6	0	0	0(0+0)=00	0	.716667

APPROXIMATE ALPHA LEVEL TEST OF Ho: The median = M or S°(M)= 1/2 using K_M Estimator

Decision Rule under the null hypothesis is not to Reject Ho when

Thus for a **test** that the survival of a lifetime = 34 is equal to 1/2 is computed as

$(.26667-.5)^2 \leq 3.8415(.07113)(.716667)$
$(.23333)^2 \leq 3.8415(.07113)(.716667)$
$.0544 \leq .19583$ is True

Decision: Do not reject Ho:

95% CONFIDENCE INTERVAL FOR THE MEDIAN
USING EQUATIONS IN BROOKMEYER AND CROWLEY

AN ASYMPTOTIC 1-α CONFIDENCE REGION FOR THE MEDIAN IS THE SET OF ALL PARAMETER VALUES NOT REJECTED BY THE SIGN TEST AT LEVEL α. THAT IS,

, m=time, $S^{O}(m)$=survival

m	$S^{O}(m)$	$(S^{O}(m)-.5)^{2}$	C_{α}	$(S^{O}(m))^{2}$			
21	0.8000	0.0900	3.8415	.6400	1/20=.05000	.050000	3.8415 x .03200 = .12295
27	0.5333	0.0011	3.8415	.2844	1/6 =.16667	.216667	3.8415 x .06163 = .23684
34	0.2667	0.0544	3.8415	.0711	1/2 =.50000	.716667	3.8415 x .05096 = .19569

SAS OUTPUT:

Percent	Point Estimate	95% Confidence Interval [Lower	Upper)
75	.	27.0000	.
50	34.0000	21.0000	.
25	27.0000	21.0000	34.0000

A CONFIDENCE INTERVAL FOR THE MEDIAN

AN ASYMPTOTIC 1-α CONFIDENCE REGION FOR THE MEDIAN IS THE SET OF ALL PARAMETER VALUES NOT REJECTED BY THE SIGN TEST AT LEVEL α. THAT IS,

$$\left\{ m : (S^o(m) - 1/2)^2 \le c_\alpha \, \sigma^2(S^o(m)) \right\}$$

, m=time, S^O(m)=survival

CONFIDENCE INTERVAL FOR THE MEDIAN USING EQUATIONS IN SAS
95% Confidence Interval for the Median

Time m	Survival $S^o(m)$	$(S^o(m)-,5)^2$	C_α	$StdErr(S^o(m))^2$	$Var(S^o(m))$	$C_\alpha\ Var(S^o(m))$
21.0000	0.8000	0.0900	3.8415	.1789	.03200	.12295
27.0000	0.5333	0.0011	3.8415	.2483	.06163	.23684
34.0000	0.2667	0.0544	3.8415	.2257	.05096	.19569

SAS OUTPUT:

Percent	Point Estimate	95% Confidence Interval [Lower	Upper)
75	.	27.0000	.
50	34.0000	21.0000	.
25	27.0000	21.0000	34.0000