3. System dynamics of interest rate effects on aggregate demand

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INTRODUCTION

'Perverse' Interest Rate Effects on Aggregate Demand?

Heterodox economics has always been sceptical of the Fed's ability to 'fine-tune' the economy, in spite of the long-running Monetarist claims about the efficacy of monetary policy (even if orthodox wisdom is used to disdain discretion). The canonization of Chairman Greenspan over the past decade and a half has eliminated most orthodox squeamishness about a discretionary Fed, while currently fashionable theory based on the 'new monetary consensus' has pushed monetary policy front and centre. As Galbraith argues, lack of empirical support for such beliefs has not dampened enthusiasm. Like Galbraith, the followers of Keynes have always insisted that 'Business firms borrow when they can make money and not because interest rates are low' (Galbraith, 2004, p. 45). Even orthodox estimates of the interest rate elasticity of investment are so low that the typical rate adjustments used by the Fed cannot have much effect.

Conventional belief can still point to interest rate effects on consumption, with two main channels. Consumer durables consumption, and increasingly even consumption of services and non-durables, rely on credit and, thus, might be interest-sensitive. Second, falling mortgage rates lead to refinancing, freeing disposable income for additional consumption. Ultimately, however, whether falling interest rates might stimulate consumption must depend on different marginal propensities to consume (MPCs) between creditors and debtors. In reality, many consumers are simultaneously debtors and creditors, making analysis difficult because a reduction of rates lowers both debt payments and interest income. If we can assume that these do not have asymmetric effects (a highly implausible assumption), we can focus only on net debtors and creditors. The conventional wisdom has always been that net creditors have lower MPCs than do net debtors, so we can assume that lower rates stimulate consumption by redistributing after-interest income to debtors. Still, the consumer lives in the same business climate as firms, and if the central bank lowers rates in recession, the beneficial impacts can be overwhelmed by employment and wage and profit income effects. Further, as society ages and net financial wealth becomes increasingly concentrated in the hands of the elderly whose consumption is largely financed out of interest income, it becomes less reasonable to assume a low MPC for net creditors. Perhaps the MPC of creditors is not much different from that of debtors, which makes the impact of rising rates on consumption all the more ambiguous. This does not mean that lowering rates in recession (or raising them in expansion) is bad policy, but it could account for the observation made by Galbraith and others before him that monetary policy is ineffectual.

Most conventional and even unconventional explorations of interest rate effects have focused on demand-side effects. It is also possible that raising interest rates can have a perverse impact on prices coming from the supply side. Interest is a business cost much like energy costs that will be passed along if competitive pressures permit. The impact on aggregate demand arising from this might be minimal or ambiguous, but it is conceivable that tight monetary policy might add to cost-push inflationary pressures while easy policy would reduce them. For monetary policy to work in the conventional manner on constraining inflation, interest rate effects on aggregate demand would have to dominate such supply-side impacts. This seems ambiguous at best, given the uncertain impacts of interest rates on demand.

What has been largely ignored is the impact that interest rate changes have on government spending, and hence on aggregate demand through the 'multiplier' channel. In a somewhat different context to the issue to be pursued here, some Post Keynesian authors have recognized that rising rates tend to increase the size of government budget deficits. Indeed, in the late 1980s, a number of countries with large budget deficits would have had balanced budgets if not for interest payments on outstanding debt – Italy was a prime example, with government interest payments amounting to more than 10 per cent of GDP (Brazil was another). It was recognized at the time by a few analysts that lowering rates was probably the only way to reduce the government's deficit, a path later successfully followed. However, conventional wisdom holds that high deficits cause high interest rates, hence government can quickly find itself in an 'unsustainable' situation (rising rates increase deficits that cause markets to raise rates even higher) that can be remedied only through 'austerity': raising taxes and cutting non-interest spending. In truth, for countries on a floating exchange rate, the overnight interbank lending rate (Fed funds rate in the US) is set by the Central Bank, and government 'borrowing' rates are determined relative to this by arbitragers mostly in anticipation of future overnight rate targets. The heterodox literature on rate setting by the Central Bank is large and the arguments need not be repeated here. What is important is to recognize the government's ability to reduce its deficit spending by lowering interest rates.

What we want to investigate is the nearly ignored possibility that lowering/raising interest rates will lower/raise aggregate demand in a manner opposite to normal expectations. This would occur if lowering rates lowered government deficits by reducing interest payments, which are essentially the same as any other transfer payments from government to the private sector. Assume an economy in which private debt is small relative to GDP, and in which the interest elasticity of private investment (and other private spending) is small. By contrast, government debt is assumed to be large with holdings distributed across 'widows and orphans' with high spending propensities. Raising interest rates will have little direct effect on the private sector, which carries a low debt load and whose spending is not interest-sensitive, in any case. However, rising interest rates increase government interest payments: to the extent that the debt is short-term or at variable interest rates, the simulative impact on private sector incomes and spending is hastened.

This is, of course, the most favourable case. However, on not implausible assumptions about private and public debt ratios and interest rate elasticities, it is possible for the government interest payment channel to overcome the negative impact that rising rates have on private demand. We will proceed as follows. First, we will briefly introduce the methodology to be used, system dynamics modelling. We then set out the model and explain the variables and parameters. For the first part of our analysis, we will use historical US data to set most parameters. We will determine, given those parameters (debt ratios and interest elasticities), at what level of interest rates we can begin to obtain 'perverse' results – where further increases actually stimulate demand. For the final part of the analysis, we will determine values for government debt ratios at which 'perverse' results can be obtained, for different levels of interest rates. Throughout we will assume uniform MPCs, and so leave for further research the questions about distribution effects. Further, we will not include interest rate effects on consumer borrowing; this is equivalent to assuming that consumer borrowing is not interest-sensitive, or that any interest rate effects on net (private) debtors is exactly offset by effects on net (private) creditors. Finally, as we will briefly mention below, further research will be needed to explore implications arising from international debtor/creditor relations (Americans hold foreign liabilities, while foreigners hold American liabilities – both private and public – and, even if domestic MPCs are the same across debtors and creditors, the MPCs of foreigner creditors and debtors could be different).

System Dynamics Modelling

System Dynamics is a numerical method for creating dynamic models of systems. The origin of System Dynamics (SD) modelling is to be found in its development by MIT's Jay Wright Forrester in the 1950s, initially as a method for designing and understanding electronic feedback control systems known as 'servomechanisms'. Its later expansion from engineering into social applications came with Forrester's writing of *Industrial Dynamics* (Forrester, 1961) and later *Urban Dynamics* (Forrester, 1969) where SD was applied to city planning.[1] As SD modelling expanded its scope from engineering applications to business management to social systems, its underlying method remained consistent: 'the application of feedback control systems principles and techniques to managerial, organizational, and socioeconomic problems' (Roberts, 1978, 3).[2]

The use of SD allows a form of experimentation with the theoretical model under analysis, by allowing the observance of the effects of manipulation of relevant variables. This facility becomes most useful when dealing with complex models of social systems with many feedbacks loops that cannot be solved analytically by a closed system of equations (see Forrester, 1995).

In this study, interest rate effects on aggregate demand are observed in both 'static' and dynamic modes. A detailed discussion of SD is beyond the scope of this chapter. For additional information on SD modelling, particularly in a heterodox context, refer to (Radzicki, 1988, 1990).

METHOD

The Model

The model used in this study, represented in Figure 3.1, is the familiar Aggregate Demand/GDP model. The definition and description of model variables are in Table 3.1. Variables are set to 2000 National Income and Product Account (NIPA) values as listed in the Statistical Abstract of the United States (2001).

Model Structure

In the model (see Figure 3.1), there are three feedback loops to note.

1. The positive loop from GDP to NI (national income) to DI (disposable income) to C (consumption) to AD (aggregate demand) to dGDP (change of GDP) to GDP.
2. The positive loop from GDP to Dp (public debt) to DS (public debt service) to XP (transfer payments) to DI … back to GDP.
3. The negative loop from dGDP to GDP back to dGDP.

The first two loops drive the increase/decrease in GDP, as producers, with some delay, increase/decrease aggregate supply in response to the perceived increase/decrease in aggregate demand (AD). The third loop prevents the

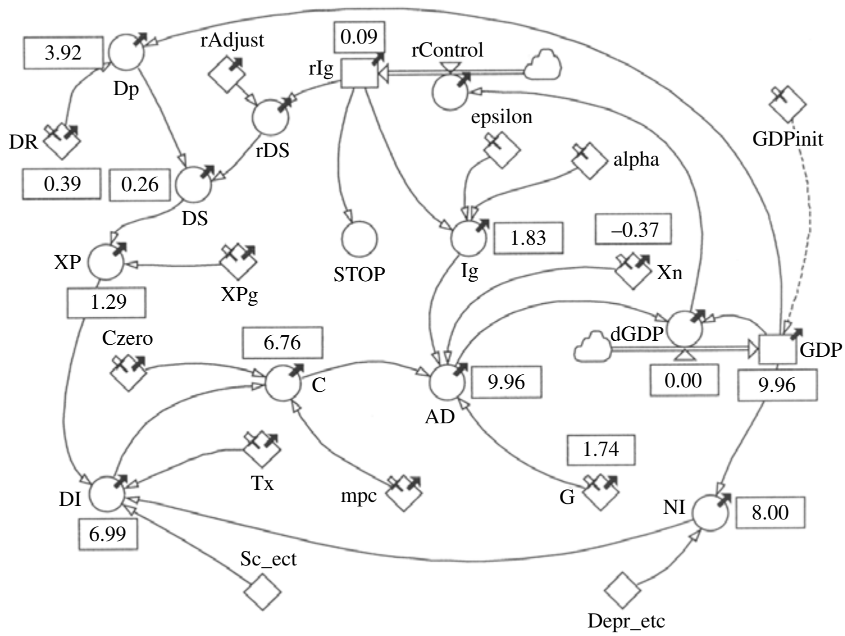

Figure 3.1 Aggregate demand/GDP model

positive loops from driving the system explosively and brings GDP into 'equilibrium' with aggregate demand, again with some delay, by decreasing the change in GDP (dGDP) as the AD–GDP gap decreases.

In addition, although not strictly loops, two feedback effects are the basic effects analysed in this study:

4. The positive debt service 'loop' from rDS (average interest rate on public debt) to DS … interfacing with loop #2 back to GDP.
5. The negative Investment 'loop' from rIg (average prime rate) to Ig (investment) to AD … back to GDP.

The Investment Function

A constant elasticity investment function is derived as follows:

1. $\varepsilon = (dI_g/dr) * (r/I_g) = \overline{\varepsilon}$
2. $(dI_g/I_g) = \overline{\varepsilon} dr/r$
3. $\ln(I_g) = \overline{\varepsilon} \ln(r) + c$
4. $I_g = e^c r^{\overline{\varepsilon}}$
5. $let \ldots \alpha = e^c$

Table 3.1 Equations for aggregate demand/GDP model (values in trillions)

Variable	Definition	Description
AD	$C + G + I_g + Xn$	aggregate demand
alpha (α)	1.0	coefficient of investment function
C	mpc * (DI) + Czero	consumption function
Czero	0.473540	estimated autonomous consumption given C and mpc
Depr_etc.	–0.3706 + 0.3749 + 1.2571 + 0.7696 – 0.0279 – 4.042	NI adjustments (depr, factor income, etc.)
dGDP	(AD – GDP)	change in GDP per quarter (in static model dGDP = 0)
DI	NI – Tx + XP – Sc_etc.	disposable income
Dp	DR * GDP	public debt
DR	(3.4101 + 0.5114)/9.9631	debt ratio (public + FRB debt)/ GDP
DS	MAX(Dp * rDS, .01 * Dp)	debt service (max of Dp*rDS or 1%*Dp)
epsilon (ε)	–0.25	interest rate elasticity of investment
G	1.7437	government spending
GDP	GDPinit	(set to GDPinit at start of simulation)
GDPinit	9.9631	GDP initial value for simulation
I_g	$\text{alpha} * rI_g^{\text{epsilon}}$	investment function
mpc	0.90	assumed marginal propensity to consume
NI	GDP – Depr_etc.	national income
rDS	rI_g – rAdjust	average debt service rate (rounded)
rI_g	0.09	average prime rate (rounded)
rAdjust	0.09 – (0.2552/3.9215) = 0.024923	prime rate adjustment to debt service rate
rControl	IF((ABS(dGDP) < 0.01), 0.01,0)/Timestep	increases rI_g by 1% when simulation reaches temporary 'equilibrium'
Sc_etc.	0.01990	personal income adjustment from NI (includes corporate savings)
STOP	STOPIF(r > 0.2)	stops simulation when rI_g is > 20%
Tx	0.7056 + 0.286 + 1.2919	personal and corporate tax and social security contributions
Xn	–0.3707	net exports
XP	XPg + DS	transfer payments (includes federal debt service)
XPg	1.036	government transfer payments

6. $\therefore I_g = \alpha r^{\bar{\varepsilon}}$

A cursory review of the literature on interest rate elasticity found a fairly wide range of estimates, with most below –0.25, although a few were substantially larger. We also found that setting ε = –0.25, the investment function approximates the 2000 value for gross private domestic investment (Ig = 1,832.7) when α = 1 and rIg = 0.09 – (Ig = 1.8257). Hence we use –0.25 as the interest rate elasticity of investment for this analysis. The graph of the function for interest rates from 1 per cent to 20 per cent is shown in Figure 3.2.

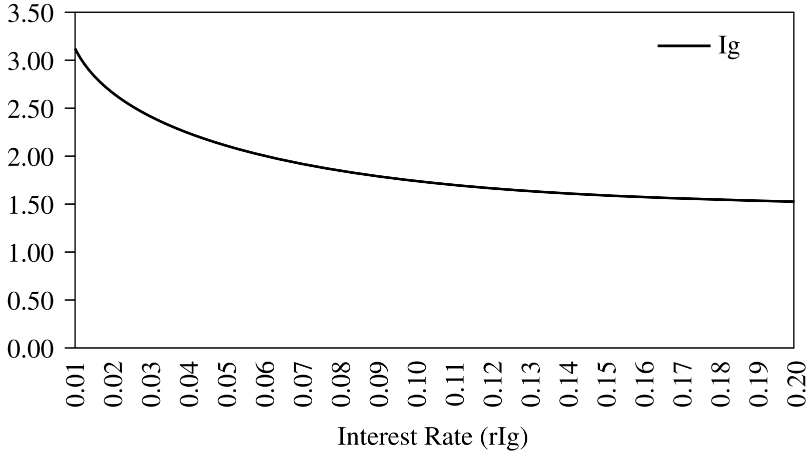

Figure 3.2 Constant elasticity investment function

The Debt Service Function and the Debt Ratio

Government debt service (DS) is a linear function of the prime rate (rIg) and public debt (Dp). In 2000 this debt was 3.9215 (3.4101 owed to the public and 0.5114 owed to the Fed). Payment on this debt equalled 0.2552. This corresponds to an effective average interest rate of 6.5077 per cent. This is, of course lower than the prime rate used in the investment function. The difference is used as an adjustment (rAdjust) to the prime rate for computing rDS, the debt service rate, and equals approximately 2.4923 per cent. This value is used throughout as the spread between rIg and the DS rate (rDS).[3]

The public debt (Dp) to GDP debt ratio (DR) is calculated from 2000 values (Dp=3.9215/GDP=9.9631) and equals 0.3936. This ratio is held constant for the first parts of this study.

Static Analysis

The model is initialized in 'equilibrium' at 2000 values with GDP = 9.9631 and other variables set accordingly. In order to obtain 'static' results from an essentially dynamic model, the linkage between AD and GDP is broken. This linkage is modelled by the flow 'dGDP' that functions to gradually increase or decrease the level of GDP if the gap between AD and GDP (AD–GDP) is positive or negative, respectively. To perform a 'static' analysis, dGDP is forced to zero (0). Thus GDP (as well as NI and Dp) will remain constant regardless of changes in AD (see Figure 3.5).

Dynamic Analysis

For the dynamic analysis, the model is also initialized in 'equilibrium' at 2000 values with GDP = 9.9631, other variables set accordingly. However, the linkage between AD and GDP is fully active since dGDP is set to AD–GDP. The functioning of a 'flow' variable such as dGDP in an SD model is that it does not respond to its value immediately, but over some adjustment time period. In this simulation, the dGDP adjustment time is one year, which is four times the simulation time step of one quarter. This causes a gradual increase or decrease in the level of GDP as the gap between AD and GDP (AD–GDP) is positive or negative, respectively.

RESULTS

Static Analysis

Under the control of the variable rControl, the value of rIg is increased from 9 per cent to 20 per cent and then stops. The results are shown in Table 3.2, Figure 3.3 and Figure 3.4.

As the value of rIg increases from 9 per cent to 20 per cent, Ig decreases while DS increases. Ig decreases in accordance with it functional form. Since there is no change in Dp, DS increases linearly as rDS (rIg – rAdjust) increases (see Figure 3.3 and Table 3.2).

AD changes value in response to the change in Ig (ΔIg) plus the change in DS (ΔDS), both resulting from the change in rIg alone. The total effect on ΔAD, with mpc = 0.9, is ΔIg + 0.9*ΔDS, but since ΔDS is constant all change in ΔAD is attributable to ΔIg. As can be seen in Figure 3.4 and Table 3.2 the combined effect on ΔAD is initially negative but becomes positive between an rIg of 12 per cent and 13 per cent (remember that this represents the investment function interest rate; the debt service function interest rate is lower by approximately

Table 3.2 Static mode interest rate effects on key variables

AD (s)	GDP	rIg	Ig	DS (s)	ΔIg	ΔDS	ΔAD=ΔIg+ 0.9*ΔDS	Dp
9.9631	9.9631	0.09	1.8257	0.2552	na	na	na	3.9215
9.9509	9.9631	0.10	1.7783	0.2944	–0.0475	0.0392	–0.0122	3.9215
9.9444	9.9631	0.11	1.7364	0.3336	–0.0419	0.0392	–0.0066	3.9215
9.9423	9.9631	0.12	1.6990	0.3728	–0.0374	0.0392	–0.0021	3.9215
9.9439	9.9631	0.13	1.6654	0.4121	–0.0337	0.0392	**0.0016**	3.9215
9.9486	9.9631	0.14	1.6348	0.4513	–0.0306	0.0392	0.0047	3.9215
9.9560	9.9631	0.15	1.6069	0.4905	–0.0280	0.0392	0.0073	3.9215
9.9656	**9.9631**	0.16	1.5811	0.5297	–0.0257	0.0392	0.0096	3.9215
9.9771	9.9631	0.17	1.5574	0.5689	–0.0238	0.0392	0.0115	3.9215
9.9903	9.9631	0.18	1.5353	0.6081	–0.0221	0.0392	0.0132	3.9215
10.0049	9.9631	0.19	1.5146	0.6474	–0.0206	0.0392	0.0147	3.9215
10.0209	9.9631	0.20	1.4953	0.6866	–0.0193	0.0392	0.0160	3.9215

Note: Bold type in columns AD(s) and GDP identifies the point of return to (near) initial values. Bold type in the ΔAD column identifies the point where ΔAD changes from negative (AD declining) to positive.

2.4923 per cent). The effect on AD can be seen in Figure 3.5 as the slope of the AD curve becomes positive at this point.

Figure 3.3 Static (S) v. dynamic (D) mode debt service function

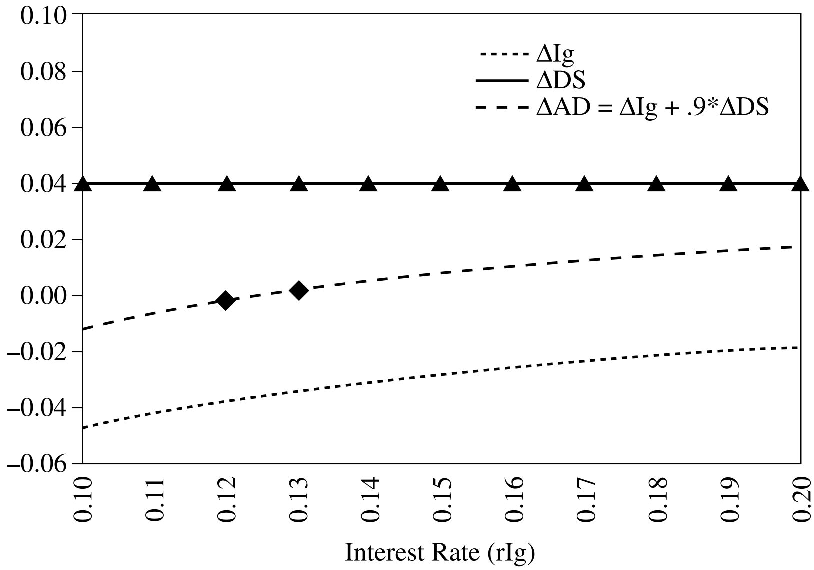

Figure 3.4 Static mode interest rate effects on investment and debt service and their combined effect on aggregate demand

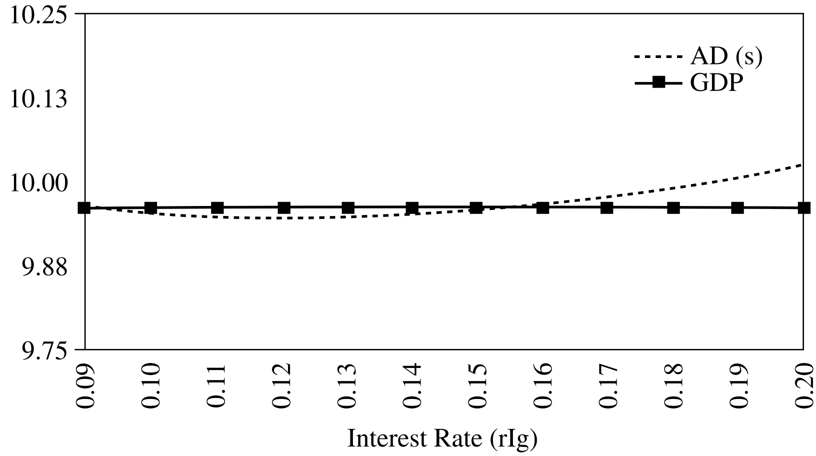

Figure 3.5 Static mode interest rate effects on aggregate demand and GDP

Dynamic Analysis

As in the 'static' case, under the control of the variable rControl the value of rIg is increased from 9 per cent to 20 per cent and then stops. The results are shown in Table 3.3, Figure 3.6, and Figure 3.7. The results are those of the relevant variables at the time step in which the simulation reaches a temporary 'equilibrium' and before rIg is increased by 1 per cent.[4]

As the value of rIg increases from 9 per cent to 20 per cent, Ig decreases while DS increases. Again, this results in a decrease in Ig in accordance with its functional form, and an increase in DS. However, since Dp increases/decreases as GDP increases/decreases the DS function no longer yields linear results with respect to rIg (see Figure 3.3 and Table 3.3).

The AD to GDP interface is fully active through dGDP, and AD now changes value not only in response to the change in Ig (ΔIg), which as in the 'static' mode depends only on the change in rIg, but also in response to the change in DS (ΔDS) which varies as both Dp and rIg vary.[5] The total effect on AD from ΔIg and ΔDS is as before, ΔIg + 0.9*ΔDS. However, there is also an effect on DI (and therefore C and therefore AD) through NI that is an effect of the change in GDP. GDP changes non-linearly in the direction of AD, as seen in Figure 3.7. Thus the total effect on AD is the sum of all changes in GDP, Ig and DS as the simulation moves from 'equilibrium' to 'equilibrium' and equals $\sum$(ΔIg + 0.9*(Δ_GDP + ΔDS)) (see the note for Table 3.3).

As can be seen in Figure 3.6 (right axis) the combined effect on ΔAD is initially negative but becomes positive between an rIg of 13 per cent and 14 per

Table 3.3 Dynamic mode interest rate effects on key variable

Qtrs	AD (d)	GDP	rIg	∑Δ_GDP	DS (d)	∑ΔIg	∑ΔDS	ΔAD=∑(ΔIg+ 0.9*(Δ_GDP+ΔDS))
0	9.9631	9.9631	0.09	0.00000	0.2552	0.0000	0.0000	0.0000
12	9.9226	9.9325	0.10	–0.03055	0.2935	–0.0475	0.0383	–0.0405
42	9.8261	9.8359	0.11	–0.09661	0.3294	–0.0419	0.0359	–0.0965
55	9.7900	9.8000	0.12	–0.03596	0.3667	**–0.0374**	**0.0374**	–0.0361
56	9.7887	9.7975	0.13	–0.00250	0.4052	–0.0337	0.0385	–0.0013
57	9.7908	9.7953	0.14	–0.00220	0.4437	–0.0306	0.0385	**0.0021**
58	**9.7964**	**9.7942**	0.15	–0.00113	0.4822	–0.0280	0.0385	0.0057
68	9.8287	9.8187	0.16	**0.02457**	0.5220	–0.0257	0.0399	0.0323
130	10.0567	10.0468	0.17	0.22808	0.5737	–0.0238	0.0517	0.2280
206	10.3549	10.3450	0.18	0.29818	0.6314	–0.0221	0.0577	0.2982
299	10.7420	10.7321	0.19	0.38715	0.6973	–0.0206	0.0659	0.3871
409	11.2329	11.2229	0.20	0.49073	0.7734	–0.0193	0.0761	0.4908

Note: Δ_GDP is the **actual** change in GDP per time step rather than the AD/GDP gap (dGDP=AD–GDP) which is roughly four times Δ_GDP per time step. Bold type in columns ∑Δlg and ∑ΔDS identifies the point where the cumulative negative effect on Δlg (because of increases in rlg) equals the cumulative positive effect on ΔDS. Bold type in the ΔAD column identifies the point where ΔAD changes from negative (AD declining) to positive. Bold type in the ∑Δ_GDP column identifies the point where ∑Δ_GDP changes from negative (GDP declining) to positive.

cent. This is later than in the 'static' case and will be explained below. The effect on AD can be seen in Figure 3.7 as the slope of the AD curve becomes positive

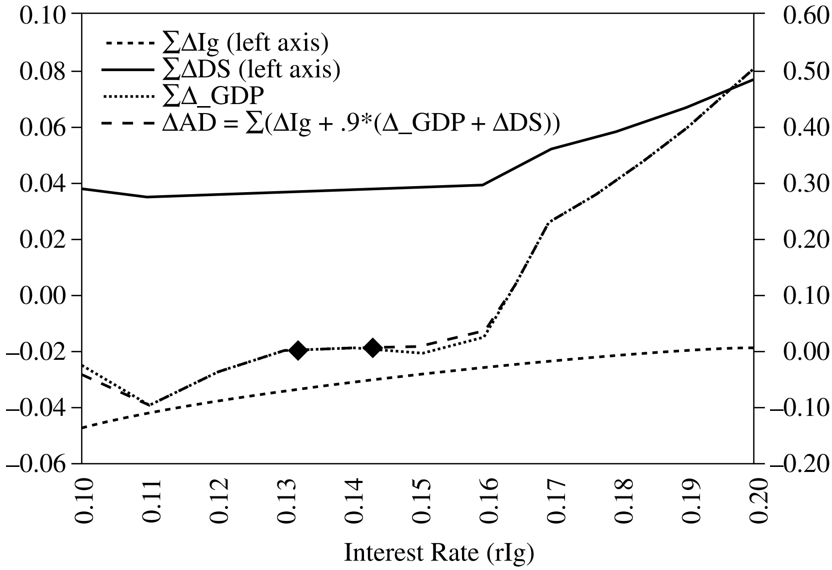

Figure 3.6 *Dynamic mode interest rate effects on investment, debt service and GDP and their combined effect on aggregate demand*

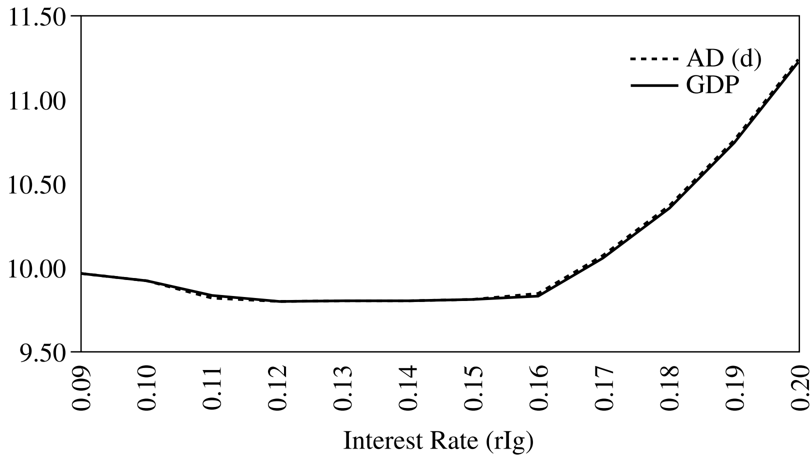

Figure 3.7 *Dynamic mode interest rate effects on aggregate demand and GDP*

at this point with AD overtaking GDP between an rIg of 14 per cent and 15 per cent.

The slope of GDP increases dramatically around an rIg of 16 per cent (Δ_GDP becomes positive between an rIg of 15 per cent and 16 per cent, Table 3.3) causing accelerated feedback effects through DS by way of Dp, and AD by way of DI (Figure 3.6).

Static v. Dynamic Comparison

As seen in Figure 3.8, the feedback effects of GDP on AD causes dynamic AD initially to drop below static AD, but eventually to overcome static AD at an rIg between 16 per cent and 17 per cent. The rates of change are dramatically different before an rIg of 12 per cent and after an rIg of 16 per cent.

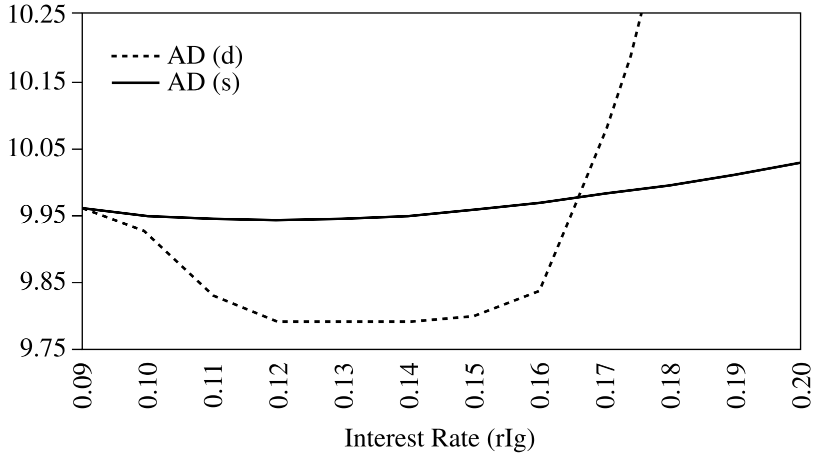

Figure 3.8 Comparison of static v. dynamic aggregate demand

This outcome depends on the functional form of the investment function (see Figure 3.2). Before an rIg of 12 per cent the negative change in Ig (ΔIg) dominates the positive change in DS (ΔDS) becoming approximately equal at an rIg of 12 per cent. After an rIg of 16 per cent the combined positive effects of ΔDS and Δ_GDP dominate the increasing smaller negative ΔIg effect.[6]

The shift to positive ΔAD occurs later for the dynamic case than for the static case (between 13 per cent–14 per cent rather than between 12 per cent–13 per cent; see Figure 3.9) because of the 'inertia' caused by the lower GDP level which takes longer for dGDP (AD-GDP) to overcome.

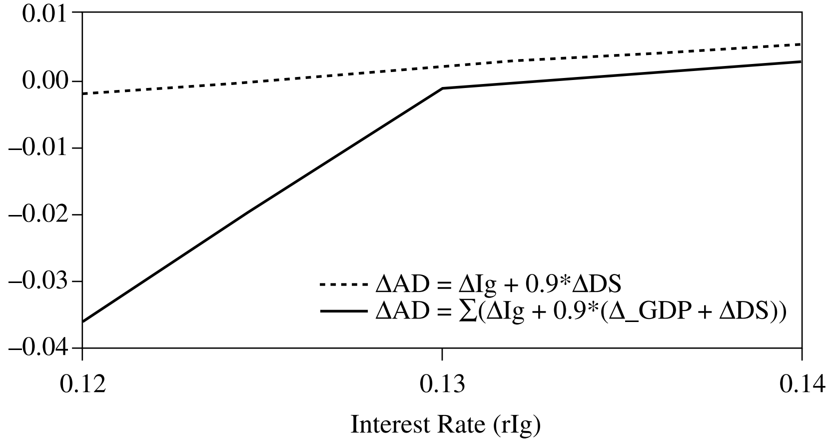

Figure 3.9 Static v. dynamic change in aggregate demand by interest rate

Static Model with Variable Debt Ratio

In this section we make the government's debt ratio variable to find the 'knife edge' or 'tipping point' at which an increase of the interest rate will actually stimulate the economy. We derive the debt ratio (DR) where the interest rate change effect on AD from a change in Ig and change in DS equals zero, that is ΔIg (r) + ΔDS(r) = ΔAD(r) = 0.

Let:

1. $I_g = \alpha r^{\bar{\varepsilon}}$,
2. $let \ldots \alpha = 1$,
3. $\Delta I_g = (r + \Delta r)^{\bar{\varepsilon}} - r^{\bar{\varepsilon}}$,
4. $DS = mpc * GDP * DR * r$,
5. $\Delta DS = mpc * GDP * DR * [(r + \Delta r) - r] = mpc * GDP * DR * \Delta r$,
6. $let \ldots \Delta I_g + \Delta DS = \Delta AD = 0$,
7. $-[(r + \Delta r)^{\bar{\varepsilon}} - r^{\bar{\varepsilon}}] = mpc * GDP * DR * \Delta r$,
8. $\therefore DR = \dfrac{-[(r + \Delta r)^{\bar{\varepsilon}} - r^{\bar{\varepsilon}}]}{mpc * GDP * \Delta r}$.

We obtain the following results for DR with mpc = 0.9, ε = –0.25, Δr = 0.01 and various values for GDP, for investment interest rates from 3 per cent to 20 per cent. For the change in prime rate the change in the debt service interest rate is the same:

$$[(r + \Delta r) - r] = \Delta r = [(r - rAdjust) + \Delta r] - (r - rAdjust).$$

Table 3.4 'Equilibrium' debt ratios for various GDP/interest rate combinations

Investment Interest Rate r	Investment Ig=r^e	–Change in Investment –dlg = –[((r+Δr)^e)–r^e]	'Equilibirum' Debt Ratio at GDP1 DR1 = –dlg/ (mpc*GDP1*Δr)	'Equilibrium' Debt Ratio at GDP2 DR2 = –dgl/ (mpc*GDP2*Δr)	'Equilibrium' Debt Ratio at GDP3 DR3 = –dlg/ (mpc*GDP3*Δr)
0.03	2.4028	0.1667	3.7191	1.8596	0.9298
0.04	2.2361	0.1213	2.7061	1.3531	0.6765
0.05	2.1147	0.0942	2.1017	1.0508	0.5254
0.06	2.0205	0.0764	1.7037	0.8519	0.4259
0.07	1.9441	0.0638	1.4237	0.7118	0.3559
0.08	1.8803	0.0546	1.2169	0.6085	0.3042
0.09	1.8257	0.0475	1.0586	0.5293	0.2647
0.10	1.7783	0.0419	0.9339	0.4670	0.2335
0.11	1.7364	0.0374	0.8334	0.4167	0.2083
0.12	1.6990	0.0337	0.7508	0.3754	0.1877
0.13	1.6654	0.0306	0.6819	0.3409	0.1705
0.14	1.6348	0.0280	0.6235	0.3118	0.1559
0.15	1.6069	0.0257	0.5736	0.2868	0.1434
0.16	1.5811	0.0238	0.5305	0.2652	0.1326
0.17	1.5574	0.0221	0.4928	0.2464	0.1232
0.18	1.5353	0.0206	0.4597	0.2299	0.1149
0.19	1.5146	0.0193	0.4305	0.2152	0.1076
0.20	1.4953	0.0181	0.4044	0.2022	0.1011

As can be seen in Table 3.4 and Figure 3.10 below, for plausible 'real world' prime rates in the range of 10 per cent to 12 per cent, and for a 'real world' GDP of just under \$10 trillion, government debt ratios well under 50 per cent of GDP are sufficient to obtain perverse monetary policy results.

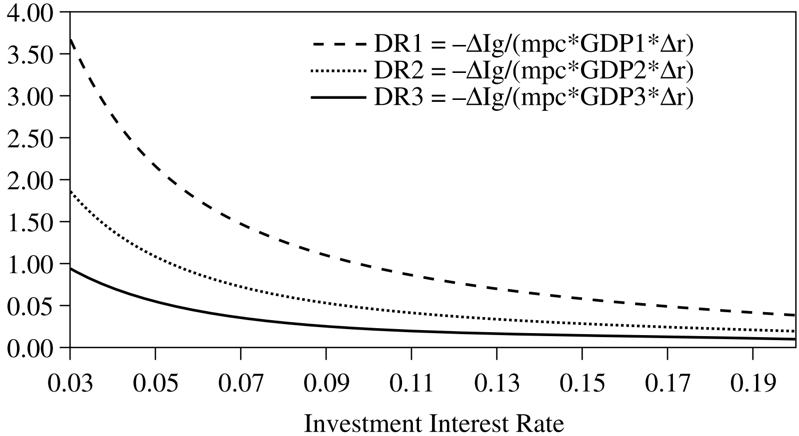

Figure 3.10 *'Equilibrium' debt ratios for various GDP/interest rate combinations*

Static Model with Variable Level of Public Debt

It is also useful to find the level of outstanding government debt at which the debt service impact of raising interest rates overwhelms the negative impact on aggregate demand resulting from effects of interest rates on investment. We might call this the 'equilibrium' public debt. Here we derive public debt where interest rate change effect on AD from change in Ig and change in DS equals zero, that is $dI_g(r) + dDS(r) = dAD(r) = 0$.

1. $I_g = \alpha r^{\bar{\varepsilon}}$,
2. $let \ldots \alpha = 1$,
3. $dI_g = \bar{\varepsilon} * r^{\bar{\varepsilon}-1} dr$,
4. $DS = mpc * Dp * (r - rAdjust)$,
5. $dDS = mpc * Dp * dr$,
6. $let \ldots dI_g + dDS = dAD = 0$,
7. $-[\bar{\varepsilon} * r^{\bar{\varepsilon}-1}] = mpc * Dp$,
8. $\therefore Dp = \dfrac{-[\bar{\varepsilon} * r^{\bar{\varepsilon}-1}]}{mpc}$.

Results for 'equilibrium' public debt (Dp) for mpc = 0.9, ε = –0.25, and investment interest rates of 3 per cent to 20 per cent are shown in Table 3.5 and Figure 3.11. With an interest rate of 10 per cent, if the public debt is $4.94 trillion (or about 50 per cent of GDP), interest rate hikes can stimulate aggregate demand.

Table 3.5 'Equilibrium' Public Debt

Investment Interest Rate r	Change in Investment –dlg = –ε*r^(ε–1)	'Equilibrium' Public Debt Dp = –dlg/mpc
0.03	20.0234	22.2483
0.04	13.9754	15.5282
0.05	10.5737	11.7486
0.06	8.4188	9.3542
0.07	6.9433	7.7148
0.08	5.8759	6.5288
0.09	5.0715	5.6350
0.10	4.4457	4.9397
0.11	3.9464	4.3849
0.12	3.5397	3.9330
0.13	3.2027	3.5585
0.14	2.9193	3.2437
0.15	2.6781	2.9757
0.16	2.4705	2.7450
0.17	2.2902	2.5447
0.18	2.1323	2.3692
0.19	1.9930	2.2144
0.20	1.8692	2.0769

Admittedly, this is a simple model that includes only one negative effect: the interest rate elasticity of investment. If other private spending is also interest rate-sensitive, and if the usual assumption that net debtors have higher spending propensities holds, then higher debt ratios will be required to obtain perverse results. However, note that Italy had a government debt ratio above 100 per cent and interest rates on government debt above 10 per cent in the late 1980s. Turkey flirts with interest rates of 28 per cent and government deficits above 25 per cent of GDP, perhaps sufficiently high for the lowering of rates actually to cool the economy. Finally, Japan has the highest deficits and debt ratio in the developed world, with high net private saving ratios and substantial private sector financial

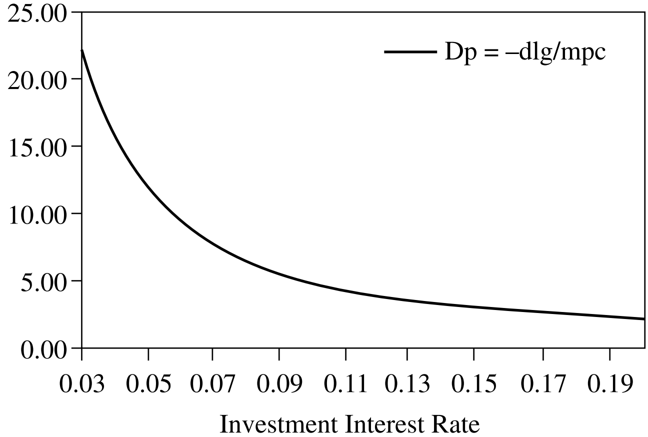

Figure 3.11 'Equilibrium' public debt by interest rate

wealth, all in the context of zero interest rates. It is entirely possible that raising rates in Japan would actually stimulate the economy by increasing private sector interest income.

CONCLUSIONS

This exercise has demonstrated that under not-too-implausible conditions, raising interest rates could actually stimulate aggregate demand through debt service payments made by the government on its outstanding debt. This is more likely if private sector indebtedness is small, if private spending is not interest rate-elastic, if interest rates are high, and if government debt is large (above 50 per cent) relative to GDP. In addition, the reset period of the government's debt affects the rapidity with which interest rate changes are transmitted to spending. Our analysis used fairly simple models and hence represents a first attempt at modelling these impacts.

In future work we need to take account of the distribution of ownership of the public debt – domestically and internationally. Foreign holdings of sovereign debt presumably would diminish the debt service effects we have explored; further, institutional ownership of the debt probably reduces the spending propensities of received government interest payments. Most government debt is not directly held by 'widows and orphans', but rather is held indirectly through financial institutions, pension funds, and so on. The 'pass through' effects of increased government interest payments on household income and thus on spending are not immediately clear in the case of institutionalized holdings.

Still, it is plausible that part of the reason for empirical evidence not supporting the conventional wisdom about monetary policy can be attributed to the debt service effects analysed here.

NOTES

1. The term 'System Dynamics' pertains to the practice of Industrial Dynamics applied to social systems modelling.
2. It should be noted that the application of feedback theory to problems of understanding organizational behaviour had been previously explored by Simon (1957) and to cybernetic models of human behaviour by Wiener (1948). See Richardson (1991) for additional examples.
3. To avoid negative DS rates, in the model the DS rate floor is set at 1 per cent. In this study, since rIg never goes below 9 per cent, rDS never goes below 6.5077 per cent.
4. The data points generated from the simulation are variably spaced in time (see figure and Table 3.3 variable 'Qtrs'). Each data point represents the 'equilibrium' level of AD/GDP immediately before the next change in rIg. As can be seen this occurs at irregular intervals becoming smaller from Quarter #0 (start of simulated time) to Quarter #55, and longer after Quarter #58. This has the effect of distorting the slope of the variables relative to the changes in interest rate. This should be kept in mind when interpreting the results temporally.

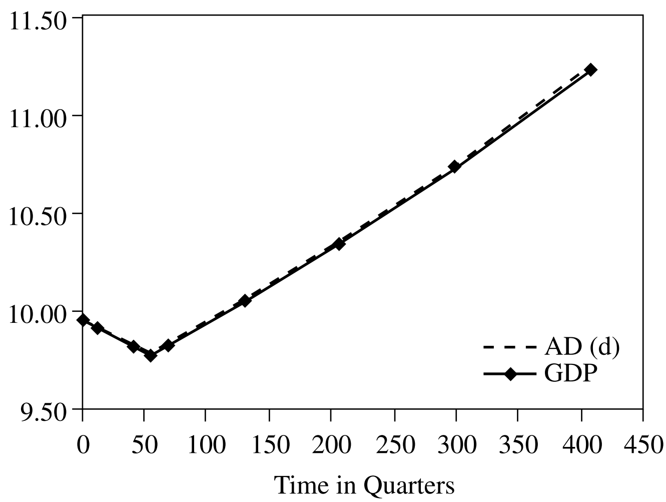

Figure 3.12 Dynamic aggregate demand/GDP by time in quarters

5. The total change on ΔDS as Dp and rI_g vary is the sum of the partial changes in both Dp and rDS, that is $dDS = rDS*\partial DP + \partial rDS*Dp$.
6. In this model $I_g(r)$ is independent of GDP and AD. A refinement might be to make I_g dependent on expected or perceived AD, shifting $I_g(r)$ as a function of dGDP.

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