Genetic Algorithms

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1 Introduction

Evolutionary algorithms (EAs) are the most influential metaheuristics for optimization. Genetic Algorithm (GA) is the most popular form of EA. In this article, we first give an introduction to evolutionary computation. A state-of-the-art description of GA is then presented.

1.1 Introduction to Evolutionary Computation

Evolutionary algorithms (EAs) are a class of general-purpose stochastic global optimization algorithms under the universally accepted neo-Darwinian paradigm for simulating the natural evolution of biological systems. The neo-Darwinian paradigm is a combination of classical Darwinian evolutionary theory, the electionism of Weismann, and the genetics of Mendel. Evolution itself can be accelerated by integrating learning, either in the form of the Lamarckian strategy or based on the Baldwin effect. EAs are currently a major approach to adaptation and optimization. EAs and similar population-based methods are simple, parallel, general-purpose, global optimization methods. They are useful for any optimization problem, particularly when conventional calculus-based optimization techniques are difficult to implement or is inapplicable. EAs can reliably solve hard problems fast that are large, complex, noncontinuous, nondifferentiable, and multimodal. The approach is easy to hybridize, and can be directly interfaced to existing simulations and models. EAs can always reach the nearoptimum or the global maximum. EAs possess inherent parallelism by evaluating multipoints simultaneously. A typical EA may consist of a population generator and selector, a fitness estimator, and three basic genetic operators, namely, crossover(also called recombination), mutation, and selection. Individuals in a population compete and exchange information with one another. Biologically, both crossover and mutation are considered the driving forces of evolution. Crossover occurs when two parent chromosomes, normally two mologous instances of the same chromosome, break and then reconnect but to the different end pieces. Mutations can be caused by copying errors in the genetic material during cell division and by external environment factors. Although the overwhelming majority of mutations have no real effect, some can cause disease in organisms due to partially or fully nonfunctional proteins arising from the errors in the protein sequence. The procedure of a typical EA (in the form of GA) is given by Algorithm 1. The initial population is usually generated randomly, while the population of other generations are generated from some selection/reproduction procedure. The search process of an EA will terminate when a termination criterion is met. Otherwise a new generation will be produced and the search process continues. The termination criterion can be selected as a maximum number of generations, or the convergence of the genotypes of the individuals. Convergence of the genotypes occurs when all the values in the same positions of all the strings are identical, and crossover has no effect for further processes. Phenotypic onvergence without genotypic convergence is also possible. For a given system, the objective values are required to be mapped into fitness values so that the domain of the fitness function is ways greater than zero.

Algorithm 1 (EA)

- 1. Set t = 0.
- 2. Randomize initial population p(0).
- 3. Repeat:
 - (a) Evaluate fitness of each individual of P(t).
 - (b) Select individuals as parents from P(t) based on fitness.
 - (c) Apply search operators (crossover and mutation) to parents, and generate P(t+1).
 - (d) Set t = t + 1.

until termination criterion is satisfied

EAs are directed stochastic global search. They employ a structured, yet randomized, parallel multipoint search strategy that is biased toward reinforcing search points of high fitness. The evaluation function must be calculated for all the individuals of the population, thus resulting in a high computation load. The high computational cost can be reduced by introducing learning into EAs, depending on the prior knowledge of a given optimization problem. EAs can be broadly divided into genetic algorithms (GAs) , evolution strategies (ESs), evolutionary programming, genetic programming (GP) differential evolution (DE), and estimation of distribution algorithms (EDAs).

1.2 Terminologies of Evolutionary Computation

Some terminologies that are used in the evolutionary computation literature are listed below. These terminologies are an analogy to their biological counterparts.

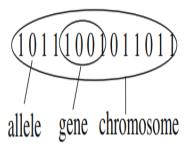
Definition 1.1 (Population). A set of individuals in a generation is called a population, $P(t) = x1, x2, \ldots, xNP$, where xi is the ith individual and NP is the size of the population.

Definition 1.2 (Chromosome). Each individual xi in a population is a single chromosome. A chromosome, sometimes called a genome, is a set of parameters that define a solution to the problem.

Definition 1.3 (Gene).In EAs, each chromosome x comprises of a string of elements xi, called genes, i.e., x = (x1, x2, ..., xn), where n is the number of genes in the chromosome. Each gene encodes a parameter of the problem into the chromosome. A gene is usually encoded as a binary string or areal number.

Definition 1.4 (Allele). The biological definition for an allele is any one of a number of alternative forms of the same gene occupying a given position called a locus on a chromosome. The gene's position in the chromosome is called locus.

Definition 1.5 (Genotype). A genotype is biologically referred to the underlying genetic coding of a living organism, usually in the form of DNA. In EAs, a genotype represents a coded solution, that is, an individual's chromosome.



Definition 1.6 (Phenotype).Biologically, the phenotype of an organism is either its total physical appearance and constitution or a specific manifestation of a trait. There is a phenotype associated with each individual. The phenotype of an individual is the set of all its traits (including its fitness and its genotype).

Definition 1.7 (Fitness). Fitness in biology refers to the ability of an individual of certain genotype to reproduce. The set of all possible genotypes and their respective fitness values is called a fitness landscape.

Definition 1.8 (Natural Selection). Natural selection alters biological populations over timeby propagating heritable traits affecting individual organisms to survive and reproduce. It adapts a species to its environment. Natural selection does not distinguish between its two forms, namely, ecological selection and sexual selection, but it is concerned with those traits that help individuals to survive the environment and to reproduce. Natural selection causes traits to become more prevalent when they contribute to fitness.

Definition 1.9 (Genetic Drift). As opposed to natural selection, genetic drift is a stochastic process that arises from random sampling in the reproduction. Genetic drift is the tendency of the selection mechanism to converge over time toward auniform distribution of mutants of the fittest individual. It changes allele frequencies (gene variations) in a population over many generations and affects traits that are more neutral.

2 Methodologies

2.1 Encoding/Decoding

GA uses binary coding. A chromosome x is a potential solution, denoted by a concatenation of the parameters $x = (x1, x2, \dots, xn)$, where each xi is a gene, and the value of xi is an allele. x is encoded in the form

$$\underbrace{01\ldots00}_{x_1}\underbrace{10\ldots10}_{x_2}\ldots\underbrace{10\ldots11}_{x_n}.$$

If the chromosome is l-bit long, it has 2^l possible values. If the variable xi is in the range [xi, xi+] with a coding sli...s2s1, where li is its bit-length in the chromosome and si0, 1, then the decoding function is given by

$$X_i = X_i^- + (X_i^+ - X_i^-) \frac{1}{2^{l_i} - 1} (\sum_{j=0}^{l_i - 1} s_j 2^j)$$

In binary coding, there is the so-called Hamming cliffs phenomenon, where large Hamming distances between the binary codes of adjacent integers occur. Gray coding is another approach to encoding the parameters into bits. The decimal value of a Gray-encoded integer variable increases or decreases by 1 if only one bit is changed. However, the Hamming distance does not monotonically increase with the difference in integer values. For a long period, Gray encoding was believed to outperform binary encoding in GA. However, based on a Markov chain analysis of GA, there is little difference between the performance of binary and Gray codings for all possible functions. Also, Gray coding does not necessarily improve the performance for functions that have fewer local minima in the Gray representation than in the binary representation. This reiterates the no free lunch theorem, namely, no representation is superior for all classes of problems.

The performance of GA depends on the choice of the encoding techniques. GA usually uses fixed-length binary coding, which results in limited accuracy and slow convergence when approaching the optimum. This drawback can also be eliminated by introducing adaptation into GA. Greater accuracy in the final solution is obtained and convergence speed is increased by dynamically controlling the coding of the search space. Examples of adaptive coding include delta coding, dynamic parameter encoding, and fuzzy coding. There are also variable-length encoding methods. In messy GA, both the value and the position of each bit are encoded in the chromosome. The addition of fuzzy rules to control coding changes provides a more uniform performance in GA search. Examples of fuzzy encoding techniques for GA are the fuzzy GA parameter coding and the fuzzy coding for chromosomes. Compared with other coding methods, each parameter in the fuzzy coding always falls within the desired range, thus

removing the additional overheads on the genetic operators. Prior knowledge from the problem domain can be integrated easily.

2.2 Selection/Reproduction

Selection embodies the principle of survival of the fittest, which provides a driving force in GA. Selection is based on the fitness of the individuals. From a population P(t), those individuals with strong fitness will be selected for reproduction so as to generate a population of the next generation, P(t+1). Chromosomes with larger fitness are selected and are assigned a higher probability of reproduction. Sampling chromosomes from the sample space can be in a stochastic manner, a deterministic manner, or their mixed mode. The roulette-wheel selection is a stochastic selection method, while the ranking selection and the tournament selection are mixed mode selection methods. Other approaches that incorporate mating preferences into evolutionary systems are correlative tournament selection and seduction.

2.2.1 Roulette-Wheel Selection

The roulette-wheel or proportional selection is a simple and popular selection scheme. Segments of the roulette wheel are allocated to individuals of the population in proportion to the individuals' relative fitness scores. Selection of parents is carried out by successive spins of the roulette wheel, and an individual's possibility of being selected is based on its fitness:

$$p_i = \frac{f(x_i)}{\sum_{i=1}^{N_p} f(x_i)}, \qquad i = 1, 2, ..., N_p$$

Consequently, a chromosome with larger fitness has a possibility of getting more offspring.

Only two chromosomes will be selected to undergo genetic operations. Typically, the population size NP is relatively small, and this proportional selection may select a disproportionately large number of unfit chromosomes. This easily induces premature convergence when all the individuals in the population become very similar after a few generations. GA thus degenerates into a Monte Carlo-type search method.

2.2.2 Ranking Selection

Ranking selection can eliminate some of the problems inherent in proportional selection. It can maintain a more constant selective pressure. Individuals are sorted according to their fitness values. The best individual is assigned the maximum rank NP and the worst individual the lowest rank 1. The selection probability is linearly assigned according to their ranks

$$P_i = \frac{1}{N_p}(\beta - 2(\beta - 1)\frac{i - 1}{N_p - 1}),$$
 $i = 1, 2, ..., N_p$

where β is selected in [0, 2].

2.2.3 Tournament Selection

Tournament selection involves h individuals at a time. The h chromosomes are compared and a copy of the best performing individual becomes part of the mating pool. The tournament will be performed repeatedly NP times until the mating pool is filled. Typically, the tournament size h, which controls the selective pressure, is selected as 2. Tournament selection only uses local information. It is very easy to implement in parallel and its time complexity is small. However, tournament selection suffers from selection bias, and the best one will not be selected. Unbiased tournament selection is suggested to diminish the selective error. Boltzmann tournament selection introduces probability into tournament selection. In binary Boltzmann tournament selection, two individuals i and j are picked up randomly with replacement. The probability of i winning the tournament is given by $p_i = \frac{1}{1 + exp(\frac{f_j - f_i}{T})}$ where T is a temperature decreasing as an annealing process, and f_i and f_j are fitness values of individual i and j, respectively.

2.2.4 Elitism Strategy

The elitism strategy for selecting the individual with best fitness can improve the convergence of GA [78]. The elitism strategy always copies the best individual of a generation to the next generation. Although elitism may increase the possibility of premature convergence, it improves the performance of GA in most cases and thus, is integrated in most GA implementations. Truncation selection is also an elitism strategy. It ranks all the individuals in the current population according to their fitness and selects the best ones as parents. Truncation selection is used as the basic selection scheme in ES and is also used in breeder GA. Breeder GA was designed according to the methods used in livestock breeding, and is based on artificial selection. Stud GA uses the fittest individual (the stud) in the population as one of the parents in all recombination operations. Only one parent is selected stochastically.

2.2.5 Fitness-Uniform Selection/Deletion

Fitness-uniform selection and fitness-uniform deletion chieve a population which is uniformly distributed across fitness values, thus diversity is always preserved in the population. Fitness-uniform selection generates selection pressure toward sparsely populated fitness regions, not necessarily toward higher fitness. Fitnessuniform deletion always deletes those individuals with very commonly occurring fitness values. As fitness-uniform deletion is only a deletion scheme, EA still requires a selection scheme. However, within a given fitness level genetic drift can occur, although the presence of many individuals in other fitness levels to breed with will reduce this effect.

2.2.6 Multikulti Selection

The natural mate selection of preferring somewhat different individuals has been proved to increase the resistance to infection of the resulting offspring and thus fitness. Multikulti methods choose the individuals that are going to be sent to other nodes based on the principle of multiculturality in an island model. In general, multikulti policies outperform the usual migration policy of sending the best or a random individual; however, the size of this advantage tends to be greater as the number of nodes increases.

2.2.7 Replacement Strategies

The selection procedure needs to decide as to how many individuals in one population will be replaced by the newly generated individuals so as to produce the population for the new generation. Thus, the selection mechanism is split into two phases, namely, parental selection and replacement strategy. There are many replacement strategies such as the complete generational replacement, replace-random, replaceworst, replace-oldest, and deletion by kill tournament. In the crowding strategy, an offspring replaces one of the parents whom it most resembles using the similarity measure of the Hamming distance. These replacement strategies may result in a situation where the best individuals in a generation may fail to reproduce. Elitism strategy cures the problem by storing the best individuals obtained so far.

Statistically, the selective pressure for different replacement strategies are ranked as: replace worst ξ kill tournament ξ age-based replacement replace random. Elitism increases the selective pressure. Elitism can be combined with the kill tournament, the age-based replacement, and the replace random rule. One can define a probability for replacement so that the individual selected by the replacement rule will have a chance to survive. This technique decreases the selective pressure.

2.3 Crossover

In sexually reproducing animals, genetic recombination occurs during the fusion of sperm and egg cells (gametes); this process is called meiosis. Genetic recombination actually occurs in the initial stage of meiosis. During meiosis, chromosomes in a diploid cell resegregate, forming four haploid cells. DNA replication has already occurred prior to meiosis. Each of the chromosomes within the cell have already been doubled forming pairs of sister chromatids or dyads held together by the kinetochore.

The primary exploration operator in GA is crossover, which searches the range of possible solutions based on existing solutions. Crossover, as a binary operator, is to exchange information between two selected parent chromosomes at randomly selected positions and to produce two new offspring (individuals). Both the children will be different from either of their parents, yet retain some features of both.

The crossover method is highly dependent on the method of the genetic coding. Some of the commonly used crossover techniques are one-point crossover, two-point crossover, multipoint crossover, and uniform crossover. The crossover points are typically at the same, random positions for both parent chromosomes. These crossover operators are illustrated in **Figure 2.1**.

2.3.1 One-Point Crossover

One-point crossover requires one crossover point on the parent chromosomes, and all the data beyond that point are swapped between the two parent chromosomes. It is easy to model analytically. The operator generates bias toward bits at the ends of the strings.

2.3.2 Two-Point Crossover

Two-point crossover selects two points on the parent chromosomes, and everything between the two points is swapped. The operator causes a smaller schema disruption than one-point crossover. It eliminates the disadvantage of one-point crossover, but generates bias at a different level.

Two-point crossover does not sample all regions of the string equally, and the ends of the string are rarely sampled. This problem can be solved by wrapping around the string, such that the substring outside the region from the first cut point to the second is crossed.

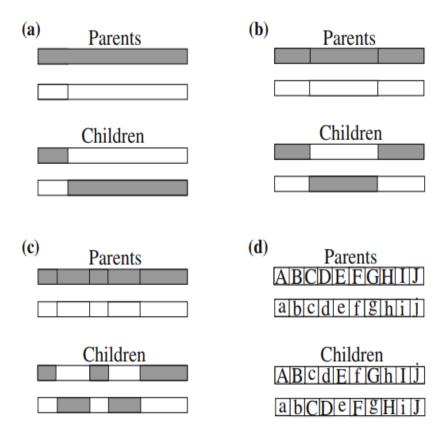


Figure 2.1 Illustration of crossover operators. **a** One-point crossover. **b** Two-point crossover. **c** Multipoint crossover. **d** Uniform crossover. For multipoint crossover and uniform crossover, the exchange between crossover points takes place at a fixed probability.

2.3.3 Multipoint Crossover

Multipoint crossover treats each string as a ring of bits divided by m crossover points into m segments, and each segment is exchanged at a fixed probability.

2.3.4 Uniform Crossover

Uniform crossover exchanges bits of a string rather than segments. Individual bits in the parent chromosomes are compared, and each of the nonmatching bits is probabilistically swapped with a fixed probability, typically 0.5. The operator is unbiased with respect to defining length. In half-uniform crossover, exactly half of the nonmatching bits are swapped.

One-point and two-point crossover operations preserve schemata due to low disruption rates. In contrast, uniform crossover swaps are more exploratory, but have a high disruptive nature. Uniform crossover is more suitable for small populations, while two-point crossover is better for large populations. Two-point crossover performs consistently better than one-point crossover.

When all the chromosomes are very similar or even the same in the population, it is difficult to generate a new structure by crossover only and premature convergence takes place. Mutation operation can introduce genetic diversity into the population. This prevents premature convergence from happening when all the individuals in the population become very similar.

2.4 Mutation

Mutation is a unary operator that requires only one parent to generate an offspring. A mutation operator typically selects a random position of a random chromosome and replaces the corresponding gene or bit by other information. Mutation helps to regain the lost alleles into the population.

Mutations can be classified into point mutations and large-scale mutations. Point mutations are changes to a single position, which can be substitutions, deletions, or insertions of a gene or a bit. Large-scale mutations can be similar to the point mutations, but operate in multiple positions simultaneously, or at one point with multiple genes or bits, or even on the chromosome scale. Functionally, mutation introduces the necessary amount of noise to perform hill-climbing.

Inversion and rearrangement operators are also large-scale mutation operators. Inversion operator picks up a portion between two randomly selected positions within a chromosome and then reverses it. Swap is the most primitive reordering operator, based on which many new unary operators including inversion can be derived. The rearrangement operator reshuffles a portion of a chromosome such that the juxtaposition of the genes or bits is changed. Some mutation operations are illustrated in **Figure 2.2**.

Uniform bit-flip mutation is a popular mutation for binary string representations. It independently changes each bit of a chromosome with a probability of p. Typically, p = 1/L for a string of L bits. This in expectation changes one bit in each chromosome. The probability distribution of fitness values after the operation can be exactly computed as a polynomial in p.

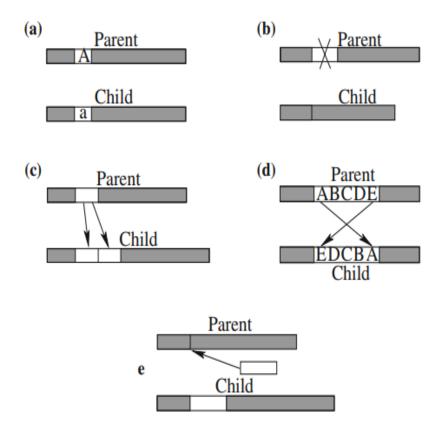


Figure 2.2 Illustration of some mutation operators. a Substitution. b Deletion.c Duplication. d Inversion. e Insertion.

A high mutation rate can lead genetic search to random search. It may change the value of an important bit, and thus slow down the fast convergence of a good solution or slow down the process of convergence of the final stage of the iterations. In simple GA, mutation is typically selected as a substitution operation that changes one random bit in the chromosome at a time. An empirically derived formula that can be used as the probability of mutation P_m at a starting point is $p_m = \frac{1}{T\sqrt{l}}$ for a total number of T generations and a string length of l.

The random nature of mutation and its low probability of occurrence leads to slow convergence of GA. The search process can be expedited by using the directed mutation technique [6] that deterministically introduces new points into the population by using gradient or extrapolation of the information acquired so far.

It is commonly agreed that crossover plays a more important role if the population size is large, and mutation is more important if the population size

is small.

In addition to traditional mutation operators, hill-climbing and bit climber are two well-known local search operators, which can be treated as mutation operators. Hill-climbing operators find an alternative similar individual that represents a local minimum close to the original individual in the solution space. The bit climber is a simple stochastic bit-flipping operator. The fitness is computed for an initial string. A bit of the string is randomly selected and flipped, and the fitness is computed at the new point. If the fitness is lower than its earlier value, the new string is updated as the current string. The operation repeats until no bit flip improves the fitness. The bit-based descent algorithm is several times faster than an efficient GA.

2.5 Exploitation Versus Exploration

For EAs, two fundamental processes that drive the evolution of a population are the exploration process and the exploitation process. *Exploitation* means taking advantage of the information already obtained, while *exploration* means searching different regions of the search space. Exploitation is achieved by the selection procedure, while exploration is achieved by genetic operators, which preserve genetic diversity in the population. The two objectives are conflicting: increasing the selective pressure leads to decreasing diversity, while keeping the diversity can result in delayed convergence.

GA often converges rather prematurely before the optimal solution is found. To prevent premature convergence, an appropriate diversity in the population has to be maintained. Otherwise, the entire population tends to be very similar, and crossover will be useless and GA reduces to parallel mutation climbing. The trade-off between exploitation (convergence) and exploration (diversity) controls the performance of GA and is determined by the choice of the control parameters, namely, the probability of crossover P_c , the probability of mutation P_m , and the population size N_P . Some trade-offs are made for selecting the optimal control parameters:

- Increasing P_c results in fast exploration at the price of increasing the disruption of good strings.
- Increasing P_m tends to transform genetic search into a random search, while it helps reintroduce lost alleles into the population.
- Increasing N_P increases the genetic diversity in the population and reduces the probability of premature convergence, at the price of an increased time of convergence.

These control parameters depend on one another, and their choices depend on the nature of the problem. In GA practice, for small NP one can select relatively large P_m and P_c , while for large N_P smaller P_c and P_m are desirable. Empirical results show that GA with $N_P = 20 - 30$, $P_c = 0.75 - 0.95$, and $P_m = 0.005 - 0.01$ performs well. When crossover is not used, GA can start with large

 P_m , decreasing toward the end of the run. In, the optimal P_m is analytically derived as $P_m = 1/L$ for a string length L.

It is concluded from a systematic benchmark investigation on the seven parameters of GA in that crossovermost significantly influenced the success of GA, followed by mutation rate and population size and then by rerandomization point and elite strategy. Selection method and the representation precision for numerical values had least influence.

2.5.1 Adapting Control Parameters

Adaptation of control parameters is necessary for the best search process. At the beginning of a search process, GA should have more emphasis on exploration, while at a later stage more emphasis should be on exploitation.

Increasing Pm and Pc promotes exploration at the expense of exploitation. A simple method to adapt P_m is implemented by linearly decreasing P_m with the number of generations, t. P_m can also be modified by

$$P_m = \frac{\alpha_0 e^{-\frac{\gamma_0^t}{2}}}{N_p \sqrt{l}} \tag{2.5.1}$$

where the constants $\alpha_0 > 0, \gamma_0 0, and list helength of the chromosome. In, \alpha_0 = 1.76 and \gamma = 0.$

a fitness-based rule is used to assign mutation and recombination rates, with higher rates being assigned to those genotypes that are most different in fitness from the fittest individual in the population. This results in a reduced probability of crossover for the best solutions available in an attempt to protect them. When all the individuals in the population are very similar, the exploration drive will be lost. Rank GA is obtained by assigning the mutation rate through a ranking of the population by fitness. This protects only the current maximal fitness found, while the rest perform random walks with different step sizes. The worst individuals will undergo the most changes.

Dynamic control of GA parameters can be based on fuzzy logic techniques. In, the population sizes, and crossover and mutation rates are determined from average and maximum fitness values and differentials of the fitness value by fuzzy reasoning.

2.5.2 Controlling Diversity

The genetic diversity of the population can be easily improved so as to prevent premature convergence by adapting the size of the population [1,34] and using partial restart. Partial restart is a simple approach to maintain genetic diversity. It can be implemented by a fixed restart schedule at a fixed number of generations, or implemented when premature convergence occurs.

Periodic population reinitialization can increase the diversity of the population. One methodology combining the effects of the two strategies is saw-tooth GA, which follows a saw-tooth population scheme with a specific amplitude and period of variation.

Duplicate removal can enhance the diversity substantially. The uniqueness operator [63] allows a child to be inserted into the population only if its Hamming distance to all members of the population is greater than a threshold. Analysis of an EA with $N_P > 1$ using uniform bit mutation but no crossover shows that the duplicate removal method changes the time complexity of optimizing a plateau function from exponential to polynomial. Each child is required to compare with all the solutions in the current population.

Diversity-guided EA uses the distance-to-average-point measure to alternate between phases of exploration (mutation) and phases of exploitation (recombination and selection). The diversity-guided EA has shown remarkable results not only in terms of fitness, but also in terms of saving a substantial amount of fitness evaluations compared to simple EA.

Since the selection operator has a tendency to reduce the population variance, population variance can be increased by the variation operator to maintain adequate diversity in the population. A variation operator is a combination of the recombination and the mutation operator. For a variation operator, population mean decision variable vector should remain the same before and after the variation operator.

2.5.3 Varying Population Size

Population sizing schemes for EAs may rely on the population sizing theory, or include the concepts of age, lifetime, and competition among species for limited resources. In, a thorough analysis of the role of the offspring population size in an EA is presented using a simplified, but still realistic EA. The result suggests a simple way to dynamically adapt this parameter when necessary.

MessyGA starts with a large initial population and halves it at regular intervals during the primordial stage. In the primordial stage only a selection operation is applied. This helps the population to get enriched with good building blocks. Fast messy GA is an improved version of messy GA.

GENITOR employs an elitist selection that is a deterministic, rank-based selection method so that the best N_P individuals found so far are preserved by using a crossgenerational competition. Crossover produces only one offspring that immediately enters the population. Offspring do not replace their parents, except for those least-fit individuals in the population. This selection strategy is similar to the $(\lambda + \mu)$ strategy of ES.

CHC algorithm stands for crossgenerational elitist selection, heterogeneous recombination, and cataclysmic mutation. Like GENITOR, it also borrows from the $(\lambda + \mu)$ strategy of ES. Incest prevention is introduced so that similar individuals are prevented from mating. Half-uniform crossover is applied, and mutation is not performed. Diversity is reintroduced by restarting partial population whenever convergence is detected. This is implemented by randomly flipping a fixed proportion of the best individual found so far as template, and introducing the better offspring into the population.

Parameterless population pyramid is an efficient, general, parameterless evolutionary approach without user-specified parameters. It replaces the genera-

tional model with a pyramid of multiple populations that are iteratively created and expanded. The approach scales to the difficulty of the problem when combined with local search, advanced crossover, and addition of diversity.

2.5.4 Aging

Aging provides a mechanism to make room for the development of the next generation. Aging is a general mechanism to increase genetic diversity. An optimal lifespan plays an important role in improving the effectiveness of evolution. For intelligent species which are able to learn from experience, aging avoids excessive experience accumulation of older individuals to avoid their being always the superior competitors.

Aging is often used by assigning age 0 to each new offspring. The age is increased by 1in each generation. In selection for replacement the age is taken into account: Search points exceeding a predefined maximal age are removed from the collection of search points.

In cohort GA, a string of high fitness produces offspring quickly, while a string of low fitness may have to wait a long time before reproducing. All strings can have the same number of offspring, say two, at the time they reproduce. To implement this delayed-reproduction idea, the population of cohort GA is divided into an ordered set of nonoverlapping subpopulations called cohorts. Reproduction is carried out by cycling through the cohorts in the given order.

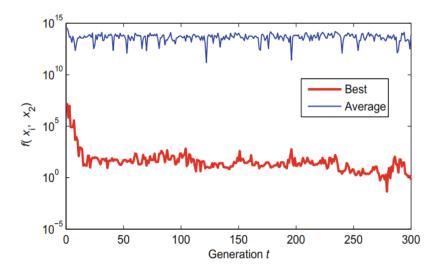


Figure 2.3 The evolution of a random run of simple GA: the maximum and average objectives.

3 Numerical Methods

Example 3.1: The conversion from binary coding to Gray coding is formulated as

$$g_i = \begin{cases} b_1, & i = 1 \\ b_i \oplus b_{i-1}, & i > 1 \end{cases}$$

where g_i and b_i are, respectively, the *i*th Gray code bit and the *i*th binary code bit, which are numbered from 1 to n starting on the left, and \oplus denotes addition mod 2, i.e., exclusive-or. Gray coding can be converted into binary coding by where g_i and bi are, respectively, the *i*th Gray code bit and the *i*th binary code bit, which are numbered from 1 to n starting on the left, and \oplus denotes addition mod 2, i.e., exclusive-or. Gray coding can be converted into binary coding by

$$b_i = \sum_{j=1}^i$$

where the summation denotes summation mod 2. As an example, we can check the equivalence between the binary code 1011011011 and the gray code 1110110110. From the two equations, the most significant i bits of the binary code determine the most significant i bits of the Gray code and vice versa.

Example 3.2:In this example, we apply simple GA to solve optimization of Rosenbrock function plotted in Example 1.1. The domain is x1, x2 [2048, 2048]. For simple GA without elite strategy, the size of population is 100, and the representation for each variable is 30-bit Gray coding. Single-point crossover with Pc = 0.98 is applied, and Pm = 0.02. The selection scheme is the roulette-wheel selection. In each generation, 90 % of the population is newly generated. The evolution for 300 generations for a typical random run is shown in Figure 3.4. At the end of the 300th generation, the best solution is x1 = 1 4171, x2 = 1.9413, and f = 0.6198. The best solution is present at the end of the 279th generation, x1 = 0.8060, x2 = 0.6409, and y2 = 0.6198. The global minimum is y3 = 0 at y3 = 0.

Example 3.3: In this example, we solve optimization of Rosenbrock function given in Example 1.1 by using real-coded GA. The domain is x1, x2 [2, 048, 2, 048]. The global optimum fitness is 0 at (1, 1). For most numerical optimization problems, real coding can usually generate a performance better than that of binary coding. Here we include the elitism strategy in the real-coded GA realization. Our numerical test shows that a high mutation rate can usually

yield good results. We select $N_P=100,\ P_c=0.9,\ P_m=0.9$. Roulettewheel selection scheme is adopted. The crossover operator generates, by averaging two parents, only one offspring. One point mutation is employed. The mutation operator rejects infeasible chromosomes that are beyond the domain. An annealing variance $\sigma=\sigma_0(1\frac{1}{T})+\sigma_1$ is selected for Gaussian mutation, where $\sigma_0=30$, and $\sigma_1=0.1$, and t corresponds to the number of generations. Except for the largest chromosome of the old generation, all chromosomes in the old generations are replaced by the new offspring.

The evolution for T = 300 generations for a typical random run is shown in Figure 3.1. At the end of the 300th generation, the solution of a typical run is $x_1 = 0.9338$, $x_2 = 0.8719$, and f = 0.0044. The adaptation is shown in Figure 3.1. Realcoded GA typically leads to a performance better than that of simple GA based on a random run.

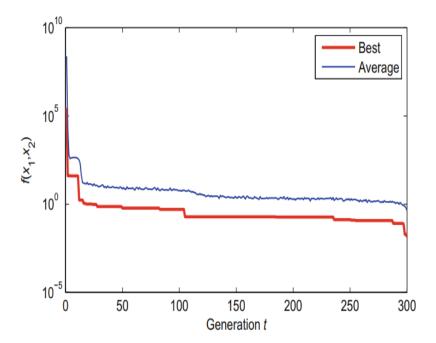


Figure 3.1 The evolution of a random run of the real-coded GA with the elitism strategy: the maximum and average objectives.

4 Conclusion

In genetics, a strong distinction is drawn between the genotype and the phenotype; the former contains genetic information, whereas the latter is the physical manifestation of this information. Both play a role in evolution as the biological processes of diversity generation act on the genotype, while the 'worth' or fitness of this genotype in the environment depends on the survival and reproductive success of its corresponding phenotype. Similarly, in the canonical GA a distinction is made between the encoding of a solution (the 'genotype'), to which simulated genetic operators are applied, and the phenotype associated with that encoding. These phenotypes can have many diverse forms depending on the application of interest. Unlike traditional optimisation techniques the GA maintains and iteratively improves a population of solution encodings. Evolutionary algorithms, including the GA, can be broadly characterised as

$$x[t+1] = r(v(s(x[t])))$$

where x[t] is the population of encodings at timestep t, v(.) is the random variation operator (crossover and mutation), s(.) is the selection for mating operator, and r(.) is the replacement selection operator.

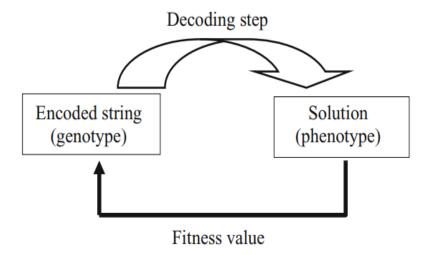


Fig. 4.1. Decoding of genotype into a solution in order to calculate fitness

5 References

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