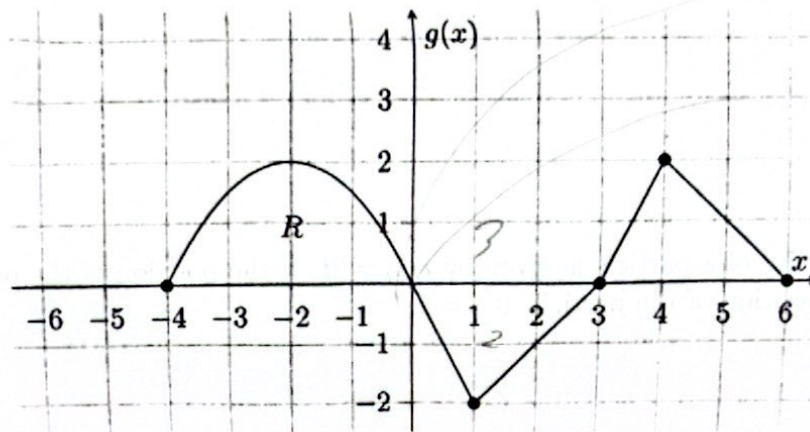
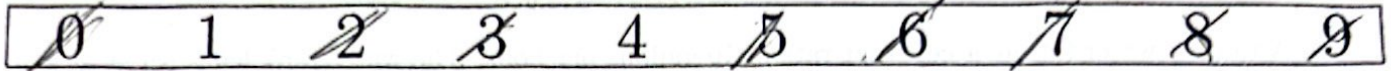


Big 10 Unit 8 Review

Instructions: In the box below are the numbers 0 – 9. Complete the following and cross off the number for each answer. If you complete all problems correctly, you will cross off each number exactly once!



Let g be the function whose graph is shown above. $g(x)$ is defined as $y = -\frac{1}{2}(x+2)^2 + 2$ on $-4 \leq x \leq 0$ and is piecewise linear from $x = 0$ to $x = 6$. Let R be the region enclosed by $g(x)$ and the x -axis from $x = -4$ to $x = 0$.

1. Find the area enclosed between the curves $y = 6\sqrt{x}$ and $y = 3x$.

$$\int_0^4 (6\sqrt{x} - 3x) dx = 6x^{3/2} - \frac{3}{2}x^2 \Big|_0^4 = 6(8) - \frac{3}{2}(16) = 48 - 24 = 24$$

2. The region R is the base of a solid. For this solid, the cross sections perpendicular to the x -axis are rectangles with height $\frac{1}{4}$. The volume of the solid can be expressed as the simplified fraction $\frac{a}{b}$. What is $a + b$?

$$V = \int_{-4}^0 \left(\frac{1}{4} \right) \left(-\frac{1}{2}(x+2)^2 + 2 \right) dx = \frac{1}{4} \int_{-4}^0 \left(-\frac{1}{2}(x+2)^2 + 2 \right) dx$$

$$= \frac{1}{4} \left[-\frac{1}{6}(x+2)^3 + 2x \right]_{-4}^0 = \frac{1}{4} \left(-\frac{1}{6}(8) + 0 - \left(-\frac{1}{6}(-8) + (-8) \right) \right) = \frac{1}{4} \left(-\frac{4}{3} + 8 \right) = \frac{1}{4} \left(\frac{20}{3} \right) = \frac{5}{3}$$

3. Suppose the velocity of a particle can be modeled with $g(t) = v(t)$ where $t \geq 0$. What is the total distance traveled of the particle from $t = 0$ to $t = 6$?

$$3 + 3 = 6$$

4. The region R is the base of a solid. For this solid, the cross sections perpendicular to the x -axis are isosceles right triangles with a leg on the region R . The volume of the solid can be expressed as the simplified fraction $\frac{a}{b}$. What is $a - 4b$?

$$V = \int_{-4}^0 \frac{1}{2} \left(-\frac{1}{2}(x+2)^2 + 2 \right)^2 dx = \frac{1}{2} \int_{-4}^0 \left(\frac{1}{4}(x+2)^4 - (x+2)^2 + 4 \right) dx$$

$$= \frac{1}{2} \left[\frac{1}{20}(x+2)^5 - \frac{1}{3}(x+2)^3 + 4x \right]_{-4}^0 = \frac{1}{2} \left(\frac{1}{20}(32) - \frac{1}{3}(8) + 0 - \left(\frac{1}{20}(-8) - \frac{1}{3}(-8) + (-16) \right) \right)$$

$$= \frac{1}{2} \left(\frac{8}{5} - \frac{8}{3} + \frac{8}{5} + \frac{8}{3} - 16 \right) = \frac{1}{2} (-16) = -8$$

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5. What is the average value of $f(x) = x^3 - 1$ on the interval $[1, 3]$?

$$\frac{1}{2} \int_1^3 (x^3 - 1) dx = \frac{1}{2} \left[\frac{x^4}{4} - x \right]_1^3 = \frac{1}{2} \left(\frac{81}{4} - 3 - \left(\frac{1}{4} - 1 \right) \right) = \frac{1}{2} \left(\frac{80}{4} - 2 \right) = \frac{1}{2} (20 - 2) = 9$$

6. A tank of water fills at a constant rate of 16 gallons per hour. The water tank leaks water at a rate $D(t) = t^2 + 7$ gallons per hour. At what time is the amount of water in the tank a maximum on the interval $[0, 10]$?

$$16 - (t^2 + 7) = 9 - t^2 = 0 \Rightarrow t = 3$$

7. Suppose the velocity of a particle is given by $v(t) = 2t$. If the position of the particle $x(t)$ is -3 at $t = 1$, what is the position of the particle at $t = 2$?

$$\int_1^2 2t dt = t^2 \Big|_1^2 = 4 - 1 = 3 \Rightarrow -3 + 3 = 0$$

8. The rate at which lava erupts from a volcano and lands on the ground is given by $L(t) = t^2 + \frac{1}{6}$ where $L(t)$ is measured in cubic kilometers per second and t is measured in seconds. If there is 2.5 cubic kilometers of lava on the ground at $t = 1$ second, how much lava is on the ground $t = 2$ seconds?

$$2.5 + \int_1^2 (t^2 + \frac{1}{6}) dt = 2.5 + \left[\frac{t^3}{3} + \frac{1}{6}t \right]_1^2 = 2.5 + \left(\frac{8}{3} + \frac{2}{6} - \left(\frac{1}{3} + \frac{1}{6} \right) \right) = 2.5 + \left(\frac{8}{3} + \frac{1}{3} - \frac{1}{2} \right) = 2.5 + \frac{17}{6} = \frac{5}{2} + \frac{17}{6} = \frac{15}{6} + \frac{17}{6} = \frac{32}{6} = \frac{16}{3}$$

9. If $m(x) = 9x$ and $n(x) = 3x^2 + 3x$. What is the average vertical distance between $m(x)$ and $n(x)$ on the interval $[0, 2]$?

$$\frac{1}{2} \int_0^2 (3x^2 + 3x - 9x) dx = \frac{1}{2} \int_0^2 (3x^2 - 6x) dx = \frac{1}{2} \left[x^3 - 3x^2 \right]_0^2 = \frac{1}{2} (8 - 12) = -2$$

10. Let S be the region enclosed by the curves $x = 1$ and $x = y^2$. The volume of the solid generated when S is rotated about the vertical line $x = 1$ can be expressed as the simplified fraction $\frac{a\pi}{b}$. What is $a - b$?

$$\pi \int_0^1 (1 - y^2)^2 dy = \pi \int_0^1 (1 - 2y^2 + y^4) dy = \pi \left[y - \frac{2}{3}y^3 + \frac{1}{5}y^5 \right]_0^1 = \pi \left(1 - \frac{2}{3} + \frac{1}{5} \right) = \pi \left(\frac{15}{15} - \frac{10}{15} + \frac{3}{15} \right) = \pi \left(\frac{8}{15} \right) = \frac{8\pi}{15}$$

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