Probability distributions
Lectures, questions and solutions

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1 LECTURE 1

An insurance company receives on a certain day two claims $X, Y \geq 0$. We will find the PMF of the loss $Z = X + Y$ under different assumptions.

The CDF $F_{X,Y}$ and PMF $p_{X,Y}$ are assumed known.

Ex 1.1. Why is it not interesting to consider the case $i = j = 0$?

Ex 1.2. Find an expression for $F_Z(k)$.

Suppose $p_{X,Y}(i,j) = c \sum_{i,j} I_{i=j} I_{1 \leq i \leq 4}$.

Ex 1.3. What is c?

Ex 1.4. What is $F_x(i)$? What is $F_Y(j)$?

Ex 1.5. Are X and Y dependent? If so, why, because $1 = F_{X,Y}(4,4) = F_X(4)F_Y(4)$

Ex 1.6. What is $\mathsf{P}\{Z = k\}$?

Ex 1.7. What is $\mathsf{V}[Z]$?

Now take X, Y iid $\sim$ Unif$(\{1,2,3,4\})$.

Ex 1.8. What is $\mathsf{P}\{Z = 4\}$?

Remark 1.1. We can make lots of variations on this theme.

1. Let $X \in \{1,2,3\}$ and $Y \in \{1,2,3,4\}$.
2. Take $X \sim \text{Pois}(\lambda)$ and $Y \sim \text{Pois}(\mu)$. (Use the chicken-egg story)
3. We can make X and Y such that they are (both) continuous, i.e., have densities. The conceptual ideas[1] don't change much, except that the summations become integrals.
4. Why do people often/sometimes (?) model the claim sizes as iid $\sim \text{Norm}(\mu, \sigma^2)$? There is a slight problem with this model (can claim sizes be negative), but what is the way out?
5. The example is more versatile than you might think. Here is another interpretation.

   A supermarket has 5 packets of rice on the shelf. Two customers buy rice, with amounts X and Y. What is the probability of a lost sale, i.e., $\mathsf{P}\{X + Y > 5\}$? What is the expected amount lost, i.e., $\mathsf{E}[\max X + Y - 5, 0]$?

   Here is yet another. Two patients arrive in to the first aid of a hospital. They need X and Y amounts of service, and there is one doctor. When it is 2 pm, what is the probability that the doctor has work in overtime, i.e., $\mathsf{P}\{X + Y > 5pm - 2pm\}$?

[1] Unless you start digging deeper. Then things change drastically, but we skip this technical stuff.

2 LECTURE 2

See memoryless_excursions.pdf.