Lecture

Foundations of Artificial Intelligence

Part 4 – Uninformed Search

Dr. Mohsen Mesgar

Universität Duisburg-Essen

Organization

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

- **Prüfung:**
  - Die globale **Anmeldephase** läuft vom **02.05.2022** bis **13.05.2022**

Recall…

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

- Clearly define your task/goal?
- The PEAS description
- Performance measures
- Environment variables
- Agents
  - Simple reflex agent
  - Model-based agent
  - Goal-based agent
  - Utility-based agent
  - Learning-based agent

Any other open questions?

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

Goal: At the end of this lecture, you learn about

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

Problem-solving agents

Tree-search algorithms

- **Breadth-first search**
- **Depth-first search**
- **Iterative deepening**

Performance of a search algorithm

Problem-solving agents

Problem-solving agents

- special case of **goal-based agents**
- find sequences of actions that lead to desirable states

Goal-based agents

- can consider future actions and the desirability of their outcomes

Problem-solving agents

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

Uninformed problem-solving agents

- do **not have any information** except the problem definition

Informed problem-solving agents

- **have knowledge** where to look for solutions

Road trip through Romania

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

Task: find route from Arad to Bucharest

Assumption: agent has a map of Romania

1. **Formulate goal**
   - be in Bucharest

2. **Formulate environment (problem)**
   - states: various cities
   - actions: drive between cities

3. **Agent's program: Search**
   - Return a sequence of cities, e.g. [Arad, Sibiu, Rimnicu, Vilcea, Pitesti, Bucharest]

Environment

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

- environment is **fully observable**, **deterministic, sequential, static, discrete**, and **single-agent**
- **easiest** possible environment
- once we have a plan, we can execute it “with eyes closed”
- the solution to any problem is a fixed sequence of actions

This is an Abstraction

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

- Real-world conditions are too complex.
- Quality of the trip depends on
  - harmony with travel companions
  - weather
  - scenery
  - radio program
  - condition of roads
  - food supply

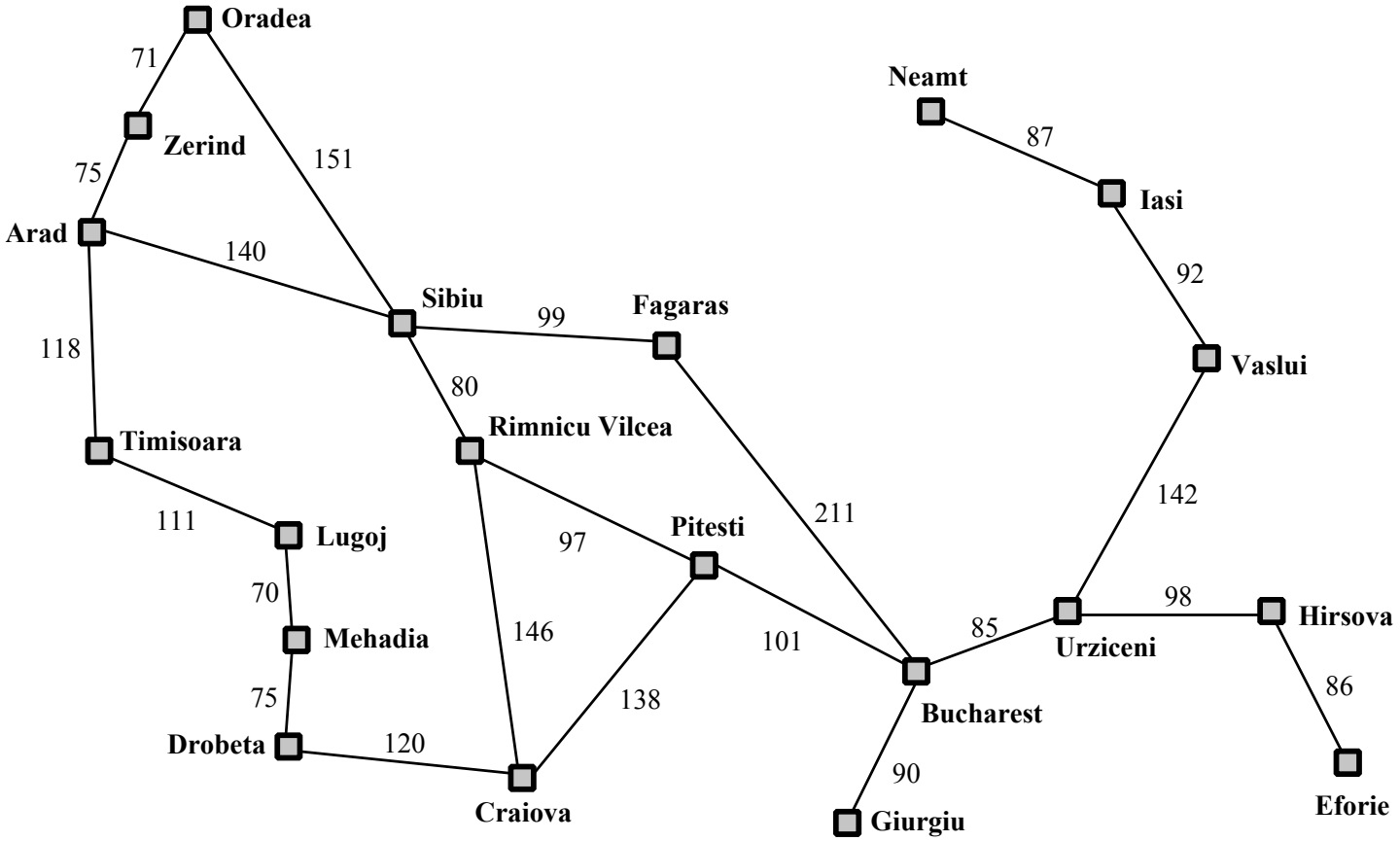
→ Leads to infeasibly large search space.

- **For finding a rational solution, we need to operate on an abstraction of the problem.**

Abstracted formulation

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken



Problem formulation

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

States

- in(x), x corresponds to all possible cities

Initial state

- start in a specific city: e.g. in(Arad)

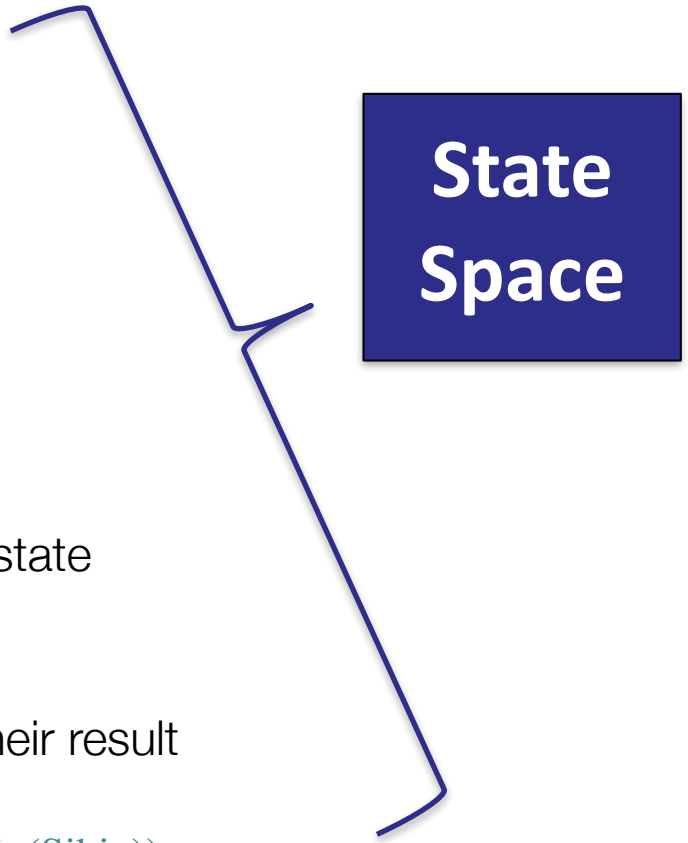
Actions

- go(x)
- only a limited set of actions is applicable in each state

Successor function

- models the applicable actions for each state and their result

Successors(in(Arad)) = {(go(Zerind), in(Zerind)), (go(Sibiu), in(Sibiu)), (go(Timisoara), in(Timisoara))}

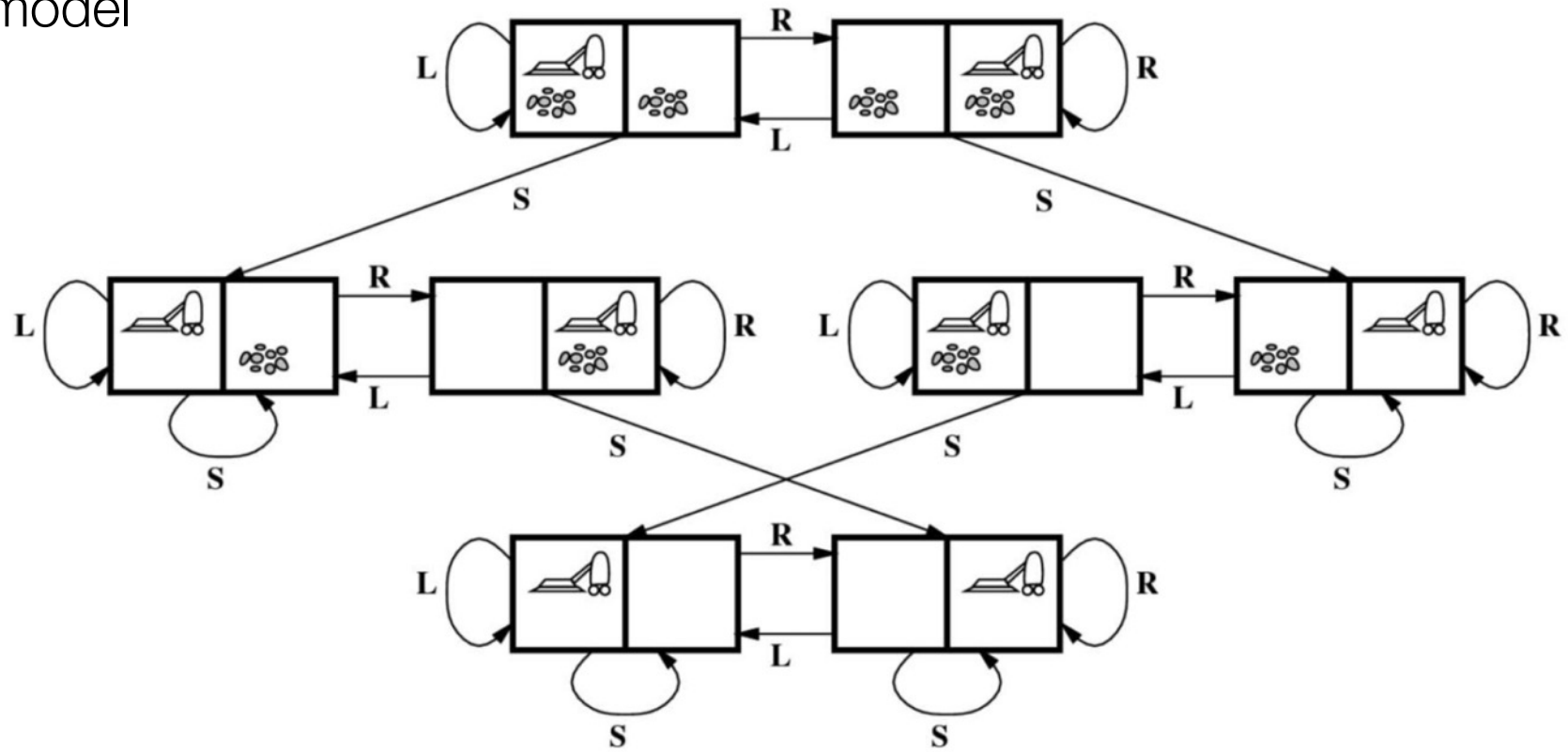


State space

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

- The **set of all states reachable from the initial state**
- Implicitly defined by the **initial state**, the actions, and the transition model



Search parameters

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

Goal test

- explicit goal state: in(Bucharest)
- abstract property: checkmate

Path

- a sequence of states connected by a sequence of actions

Path cost

- cumulated cost of actions connecting a sequence of states
- e.g.: sum of distances, driving time
- cost depends on performance measure

Solution

- a path that leads from the initial state to a goal state

Optimal solution

- solution with the minimum path cost

Recall!

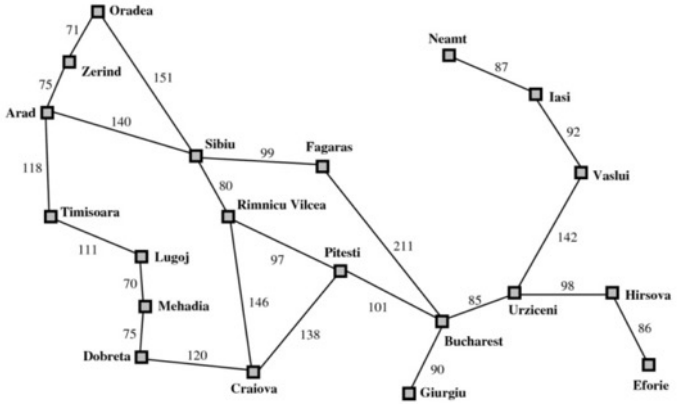
UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

Abstract actions correspond to a complex combination of real **actions**

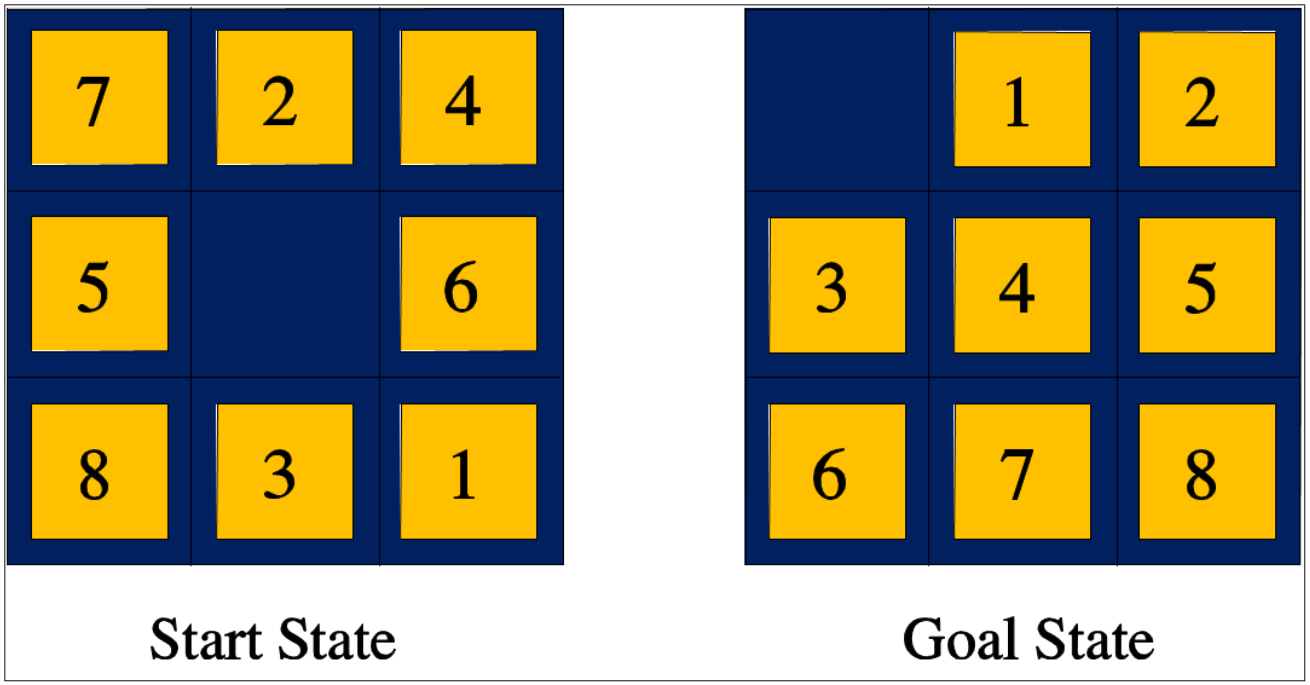
- e.g. go(Bucharest) represents a complex set of possible routes, detours, rest stops, etc.
- **abstract actions** are a simplification of the original actions

An **abstract solution** corresponds to a set of **real paths** that are **solutions in the real world.**



Toy Problems

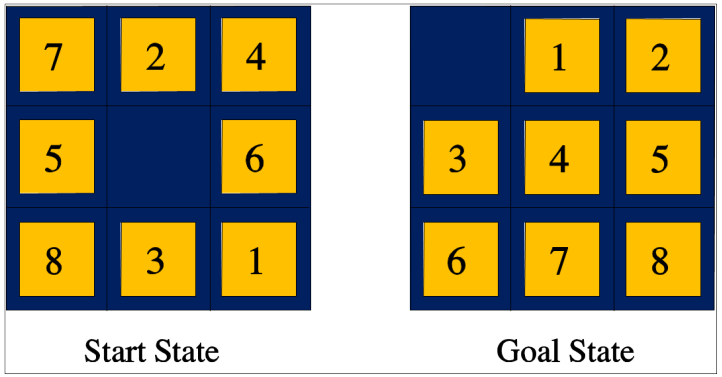
Example: The 8-puzzle



Search configuration of the 8-puzzle

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken



States

- location of tiles

Initial state

- any configuration of tiles

Successor function

- **actions:** move blank tile left, right, up, down
- efficient, but counterintuitive: move blank left ~> move 5 right
- **result state:** new configuration of tiles

Goal test

- configuration equals goal state

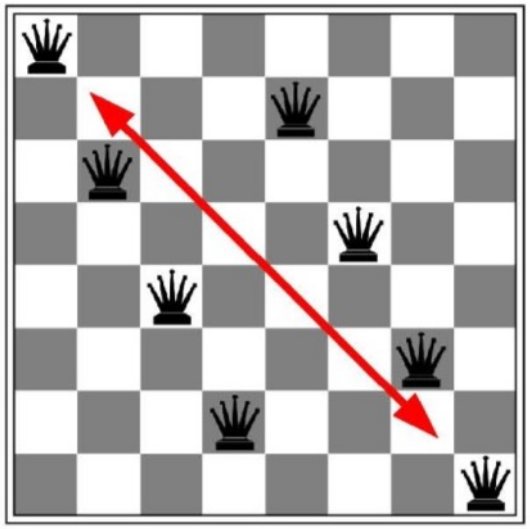
Path cost

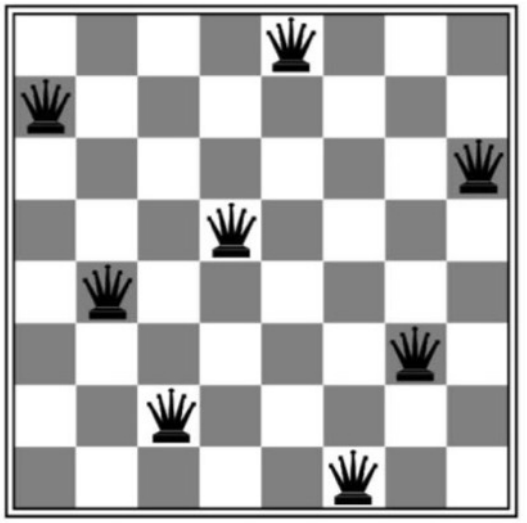
- number of steps in path

Example: The 8-Queens Problem

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken





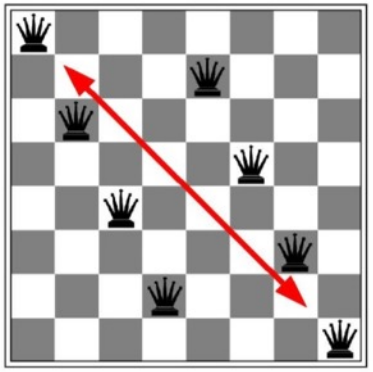
Naïve incremental approach

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

States

- 0-8 queens are positioned on the board

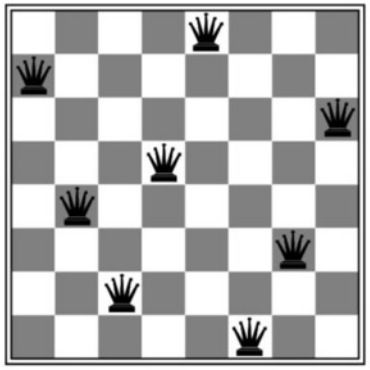


Initial state

- no queens on board

Successor function

- **action:** add queen to any empty square
- **result state:** new configuration of queens



Goal test

- 8 queens on board, none attacked

→ inefficient: $64 \cdot 63 \cdot \ldots \cdot 57 \approx 1.8 * 10^{14}$ states

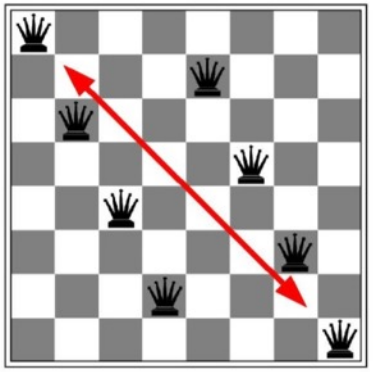
Less naïve approach

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

States

- any configuration of 0 to 8 queens in the leftmost columns, no queen attacking each other

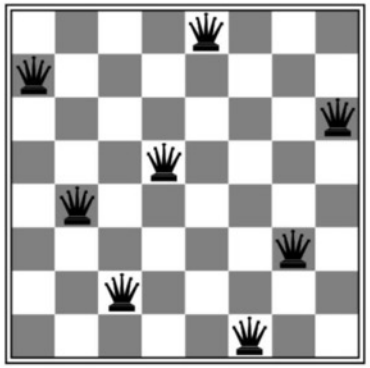


Initial state

- no queens on board

Successor function

- **action: add queen to leftmost empty column such that it is not attacked**
- result state: new configuration of queens



Goal test

- 8 queens on board, none attacked

→ reduction to 2057 possible states

Tree Search Algorithms

Search Tree

UNIVERSITÄT DUISBURG ESSEN

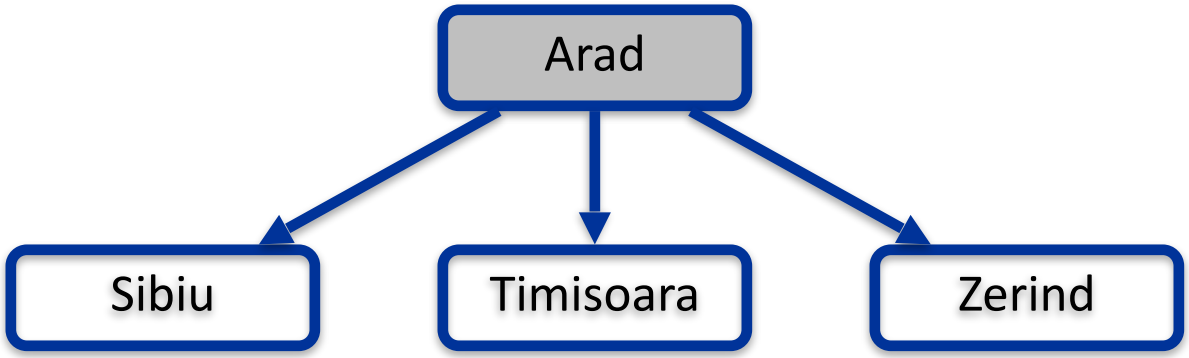
Offen im Denken

Treat the state-space graph as a tree

- **root:** initial state
- **successor nodes:** result states after applying possible actions

Search

- expand the successor nodes
- **search strategy:** decide which nodes to expand next

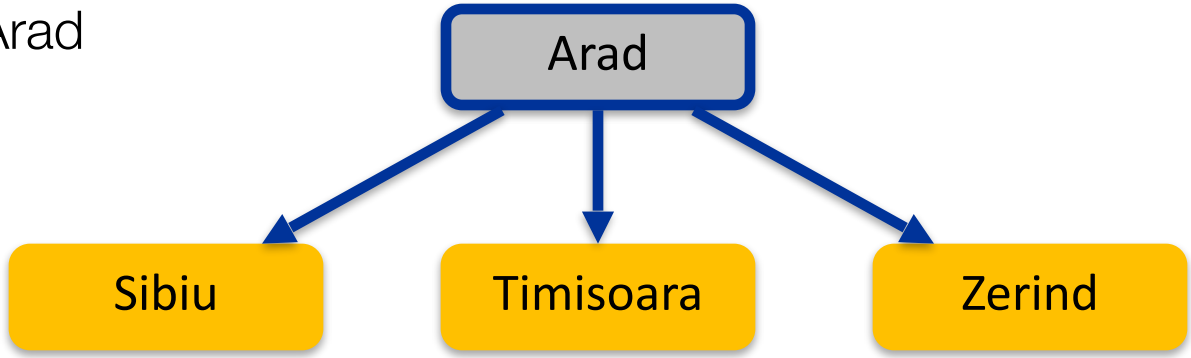


Tree search example

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

- Initial State: In(Arad)
- Expand Arad

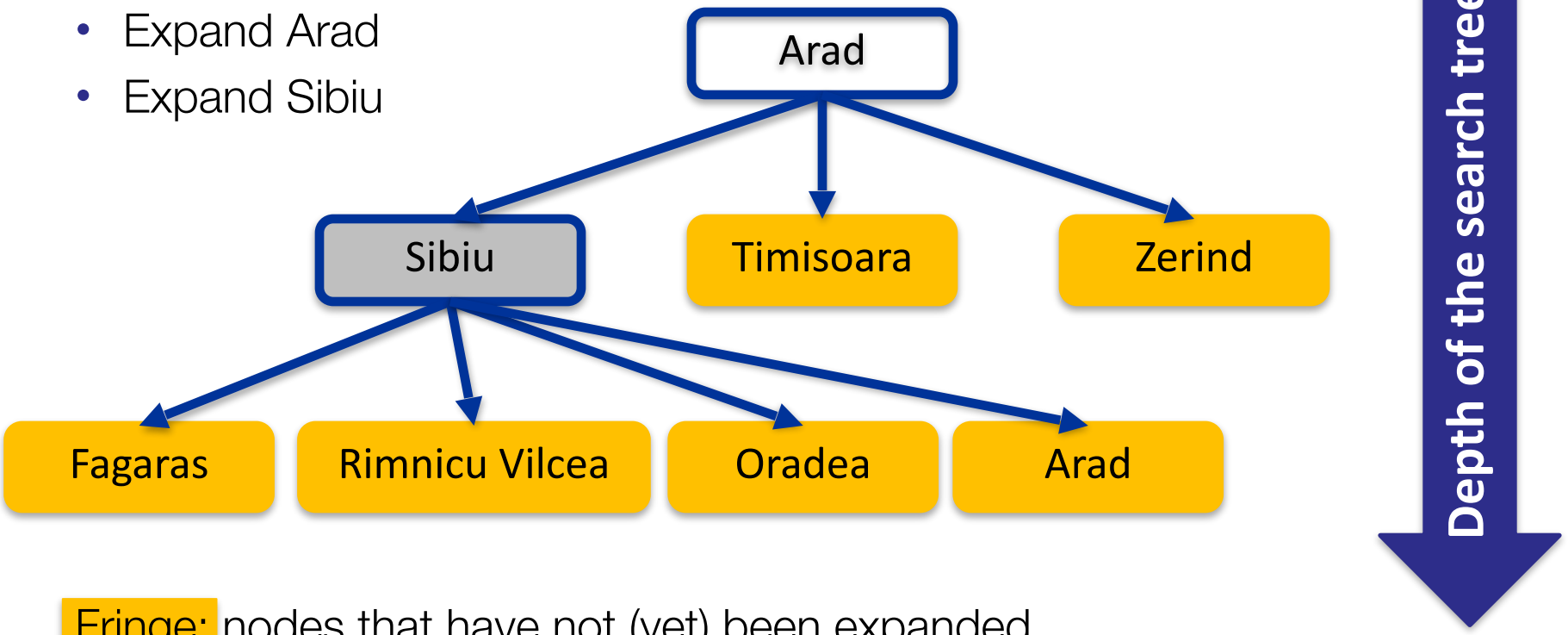


Tree search example

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

- Initial State: In(Arad)
- Expand Arad
- Expand Sibiu



Fringe: nodes that have not (yet) been expanded

→ leave nodes of the current tree

States vs. nodes

UNIVERSITÄT DUISBURG ESSEN

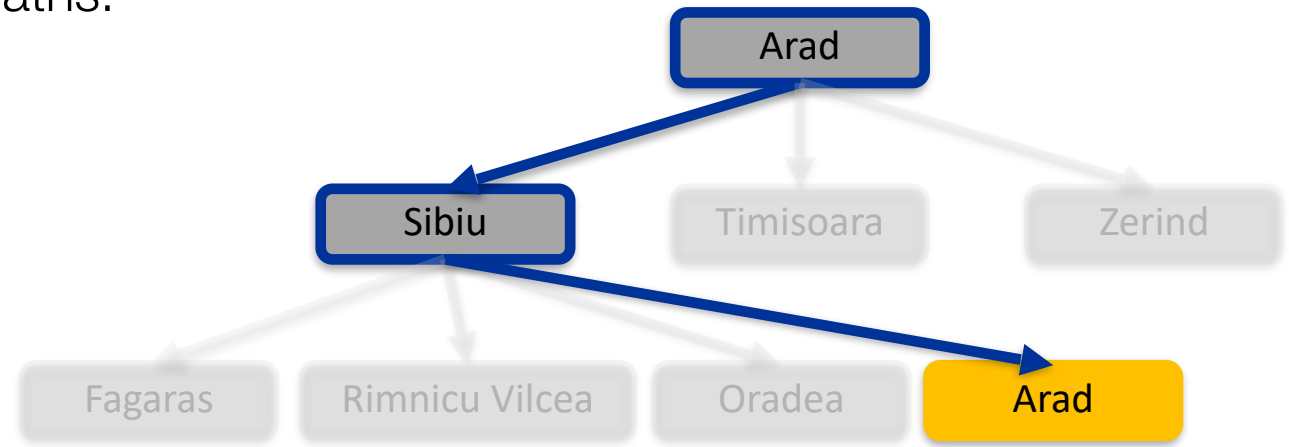
Offen im Denken

State

- (representation of) a problem configuration

Node

- data structure constituting a part of a search tree
- can have parent and successor nodes
- two different nodes can point to the same state which has been reached by different paths:



Search strategies

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

Uninformed search strategies use only the information available in the problem definition → **blind search**

- Breadth-first search (BFS)
- Depth-first search (DFS)
- Iterative deepening search
- Bidirectional search

Informed search strategies have knowledge that allows to guide the search to promising regions → **heuristic search**

- Greedy search
- A* best-first search

Outline

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

- Problem-solving agents
- **Tree-search algorithms**
  - **Breadth-first search**
  - Depth-first search
  - Iterative deepening
- Performance of a search algorithm

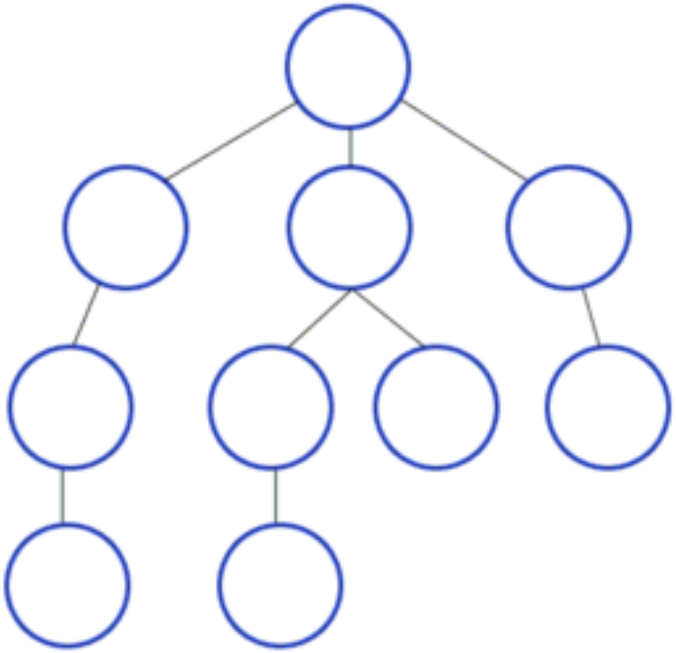
Breadth-first strategy

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

Expand all neighbors of a node before any of its successors is expanded

- breadth before depth
- expand the shallowest unexpanded node



Check animation on: https://de.wikipedia.org/wiki/Breitensuche

Breadth-first implementation

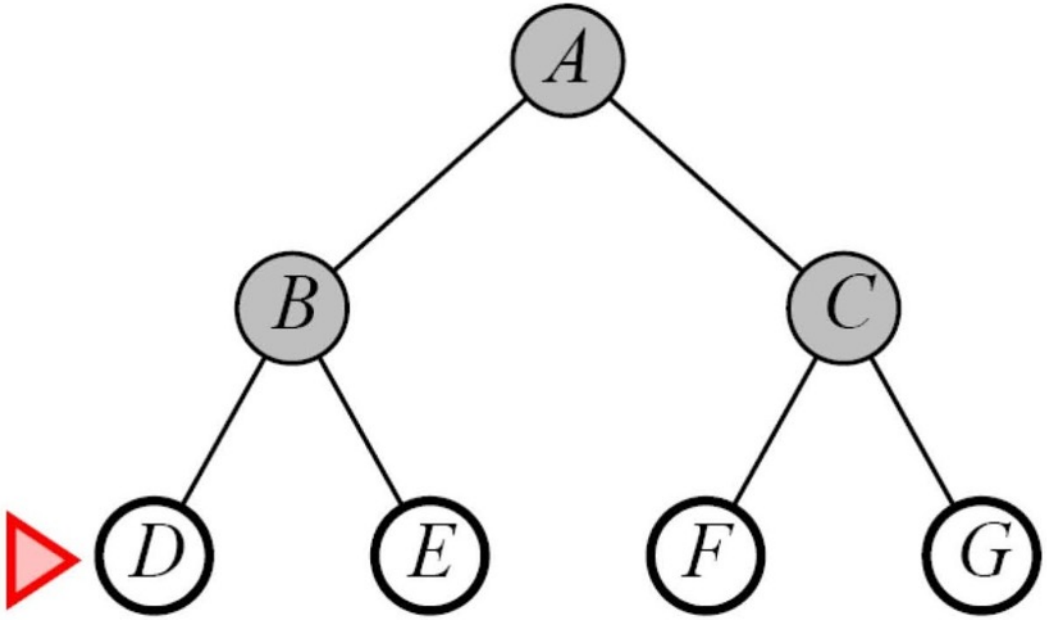
UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

Breadth-first search is implemented as a FIFO queue:

- FIFO = First in, first out

- Expand start node A → Queue: B, C
- Expand node B → Queue: C, D, E
- Expand node C → Queue: D, E, F, G
- Expand node D ....



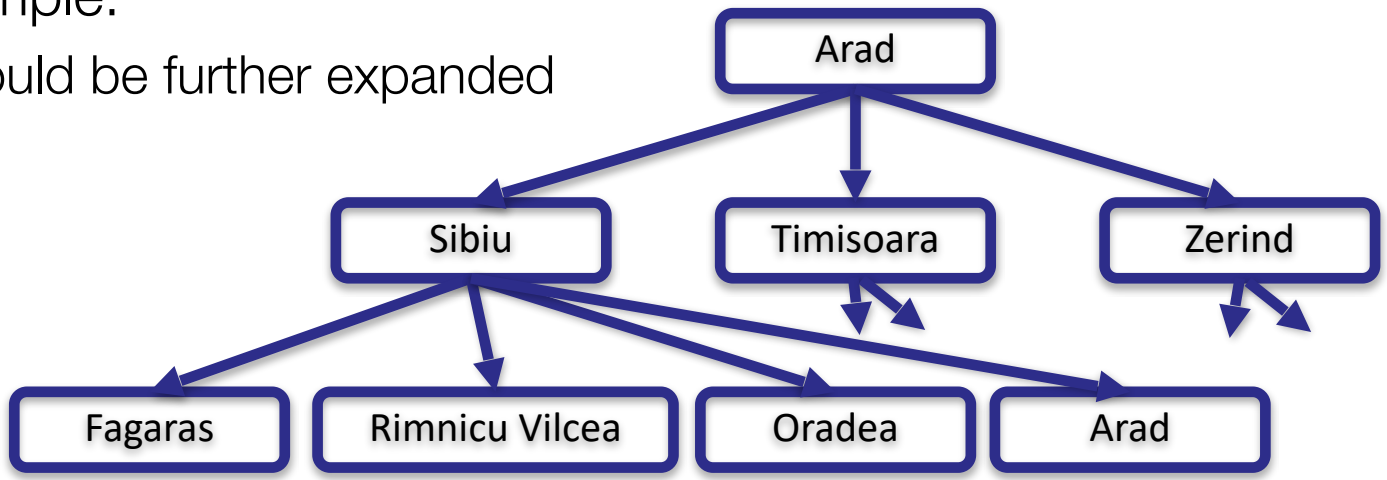
Tree Shapes

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

- d = depth (starting at 0)
- b = maximum branching factor = maximum number of successor nodes
- b_i = maximum branching factor at depth i

- Romania-Example:
  - partial tree, could be further expanded
  - d = 2
  - b = 4
  - b_0 = 3

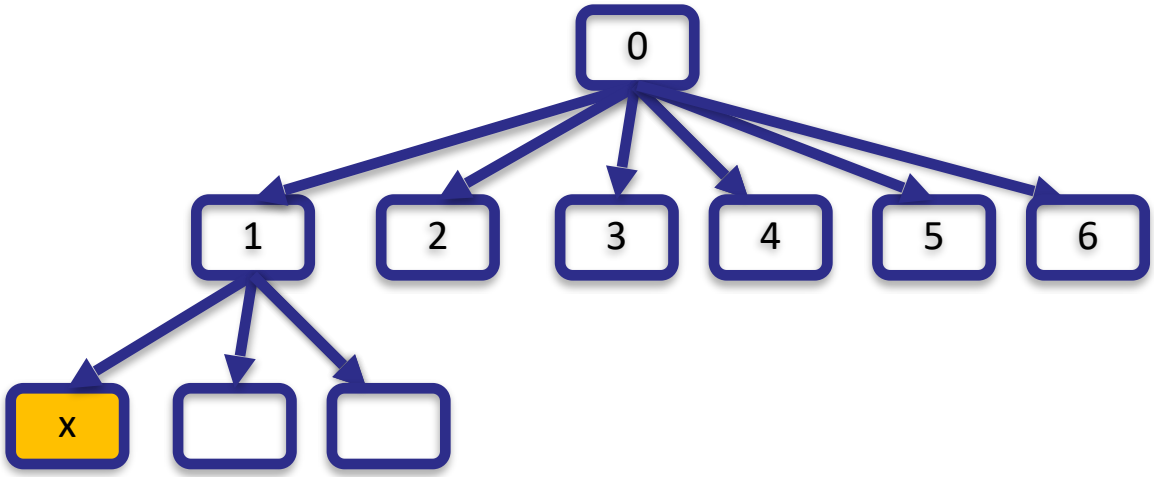


Tree examples: Breadth-first

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

- d = 4
- b = 1
- Breadth-first search steps to x = 4
- Path cost = 4



◊ d = 2

◊ b = 6

◊ Breadth-first search steps to x = 7

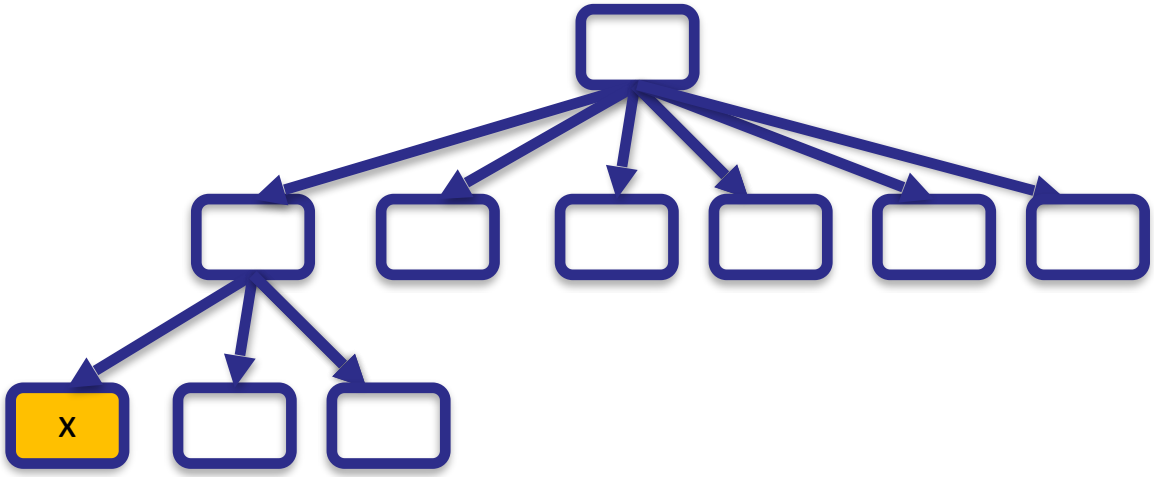
◊ Path cost = 2

Shortcomings of BFS

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

- Needs to keep track of all discovered nodes for later expansion
- → large memory consumption



Outline

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

- Problem-solving agents
- **Tree-search algorithms**
  - Breadth-first search
  - **Depth-first search**
  - Iterative deepening
- Performance of a search algorithm

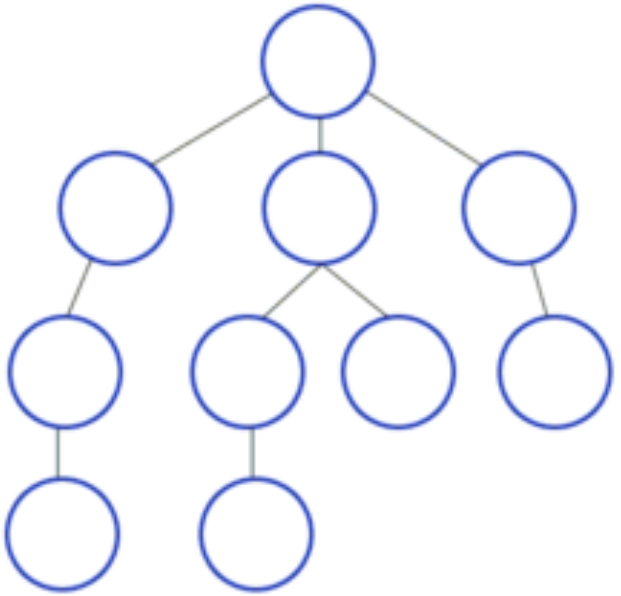
Depth-first Search (DFS)

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

Expand all successors of a node before any of its neighbors is expanded

- depth before breadth
- expand the deepest unexpanded node



Check animation on: https://de.wikipedia.org/wiki/Tiefensuche

Depth-first implementation

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

Depth-first search is implemented as a LIFO stack

LIFO: Last in ↓, first out ↑

Stack

FIFO: → first in, → first out

Queue

Comparing Implementations

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

```
def bfs(graph, root):
    seen = set([root])
    queue = collections.deque([root])
    while queue:
        print("Queue: ", queue)
        vertex = queue.popleft()
        print("Expanding: ", vertex)
        for node in graph[vertex]:
            if node not in seen:
                seen.add(node)
                queue.append(node)
```

```
def dfs(graph, root):
    seen = set([root])
    stack = [root]
    while stack:
        print("Stack: ", stack)
        vertex = stack.pop()
        print("Expanding: ", vertex)
        for node in graph[vertex]:
            if node not in seen:
                seen.add(node)
                stack.append(node)
```

DFS implementation

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

```
graph = {
    "A": ["B","C"],
    "B": ["D","E"],
    "C": ["F","G"],
    "D": ["H","I"],
    "E": ["J","K"],
    "F": [], "G": [], "H": [], "I": [], "J": [], "K": []
}
```

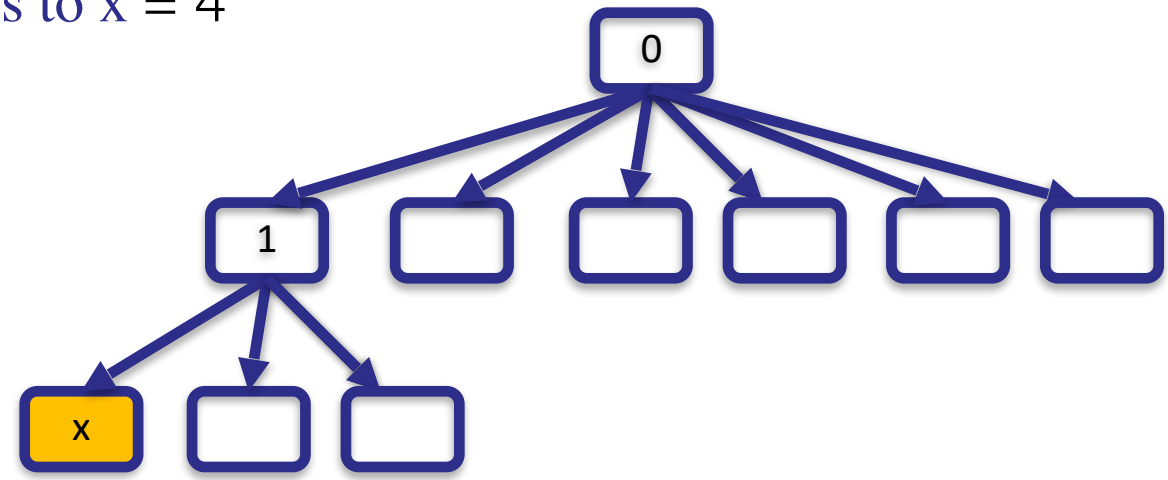
```
DFS
Stack:  ['A']
Expanding:  A
Stack:  ['B', 'C']
Expanding:  C
Stack:  ['B', 'F', 'G']
Expanding:  G
Stack:  ['B', 'F']
Expanding:  F
Stack:  ['B']
Expanding:  B
Stack:  ['D', 'E']
Expanding:  E
Stack:  ['D', 'J', 'K']
Expanding:  K
Stack:  ['D', 'J']
Expanding:  J
Stack:  ['D']
Expanding:  D
Stack:  ['H', 'I']
Expanding:  I
Stack:  ['H']
Expanding:  H
```

Tree Shapes

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

- d = 4, b = 1
- Breadth-first search steps to x = 4
- Depth-first search steps to x = 4
- Path cost = 4



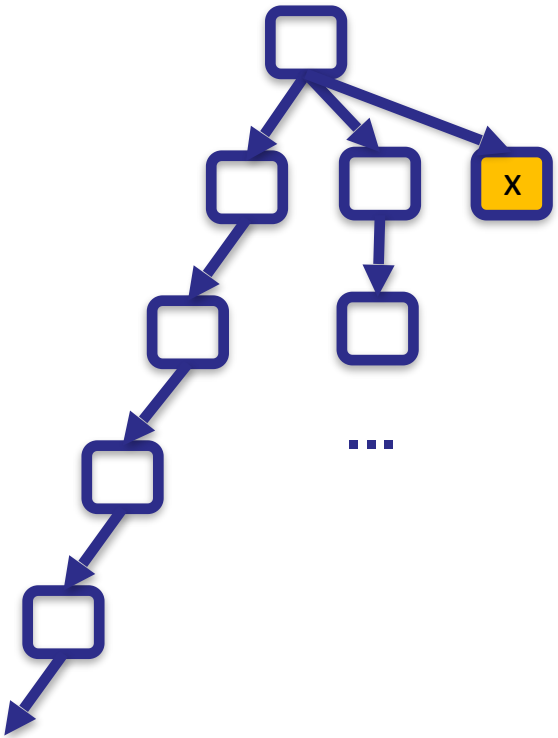
◊ d = 2, b = 6

◊ Breadth-first search steps to x = 7

◊ Depth-first search steps to x = 2

◊ Path cost = 2

Shortcomings of DFS



- Might explore very deep path, before finding solution higher in tree.
- Could even get stuck in sub-tree with infinite depth.

Outline

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

- Problem-solving agents
- **Tree-search algorithms**
  - Breadth-first search
  - Depth-first search
  - **Iterative deepening**
- Performance of a search algorithm

Limit the depth

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

Overcome shortcomings of DFS by setting a maximum depth L

- How to set L?
- If L is too small, the solution is not found.
- If L is too big, the search takes too long

Iterative deepening

- Try all possible L = 0, 1, 2, 3, ...
- The costs are dominated by the last iteration

→ overhead of iterative process can be neglected

Iterative Deepening

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

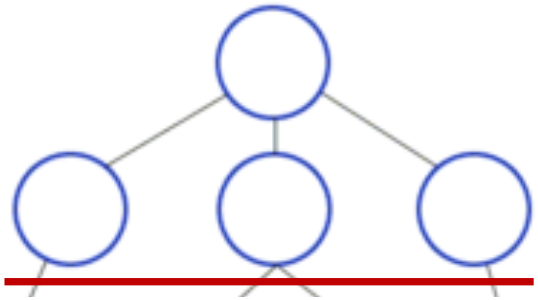
- Limit = 0

Iterative Deepening

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

- Limit = 1

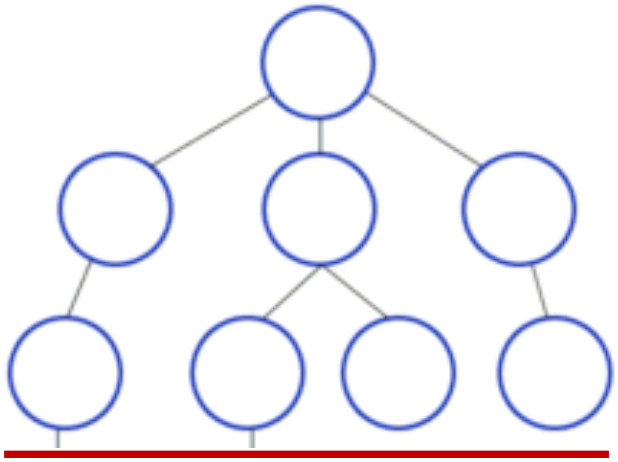


Iterative Deepening

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

- Limit = 2

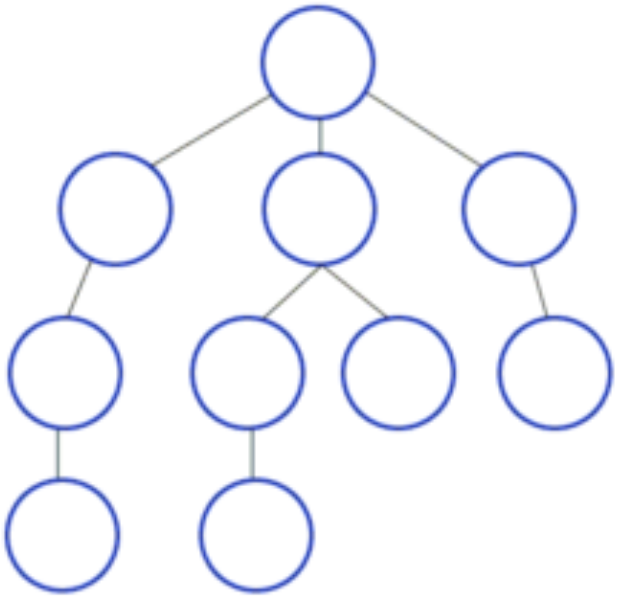


Iterative Deepening

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

- Limit = 3



- In the last iteration, we have standard depth-first search again:

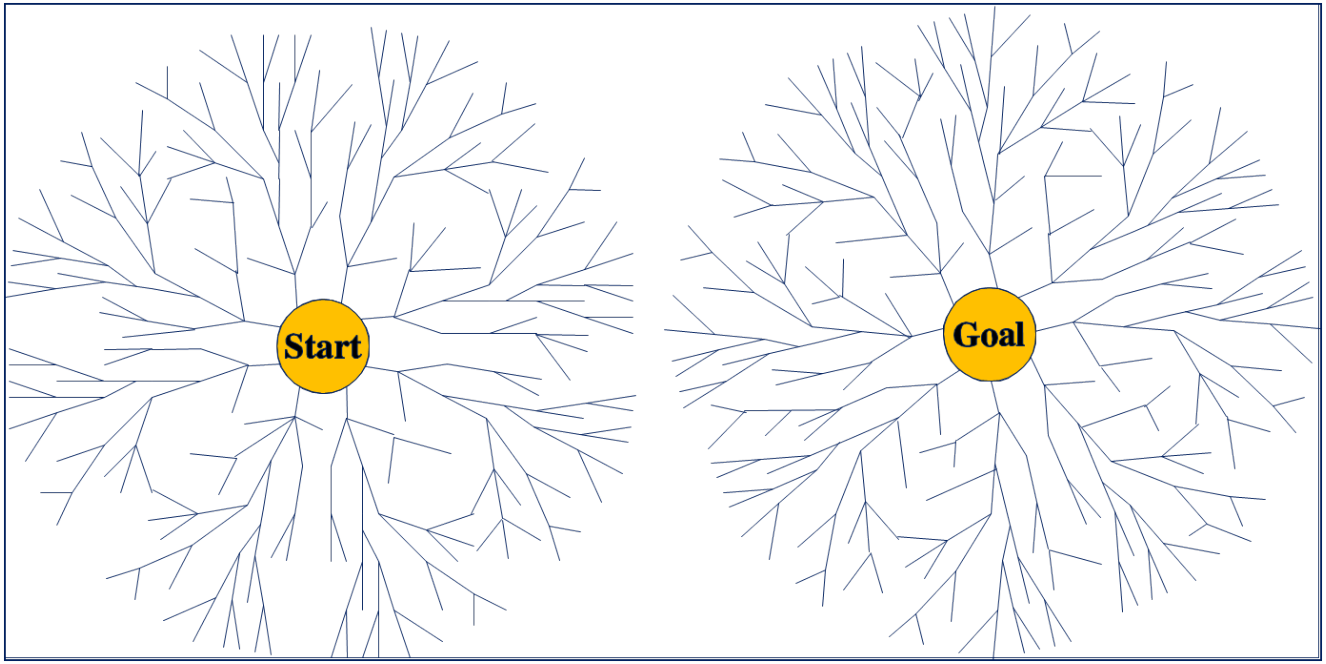
Bidirectional search

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

Perform two searches simultaneously

- forward: starting with initial state
- backward: starting with goal state



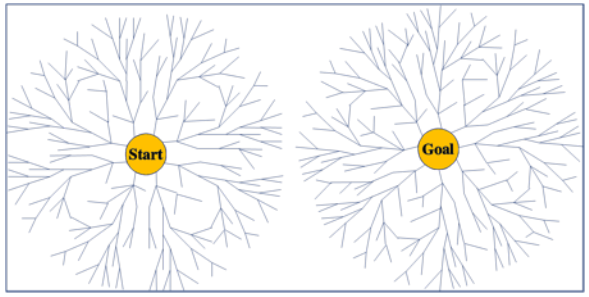
Bidirectional search

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

Perform two searches simultaneously

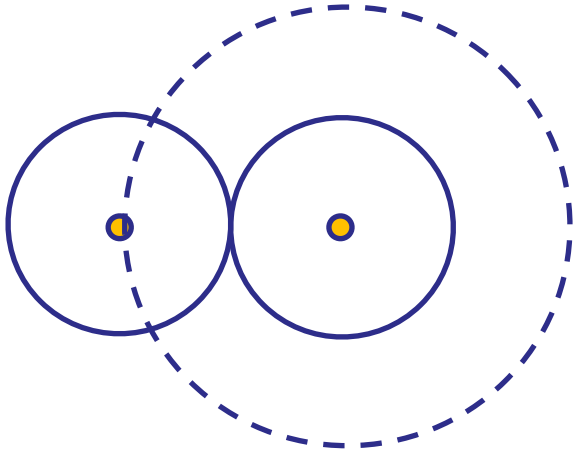
- ◊ forward starting with initial state
- ◊ backward starting with goal state



Check whether generated node is in fringe of the other search

Properties

- ◊ reduction in complexity
- ◊ only possible if actions can be reversed
- ◊ search paths may not meet for DFS ⇒ use BFS



Outline

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

- Problem-solving agents
- Tree-search algorithms
  - Breadth-first search
  - Depth-first search
  - Iterative deepening
- **Performance of a search algorithm**

Search strategies

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

Different search strategies have different strength and weaknesses.

Search strategies are compared along the following dimensions:

- **Completeness**: Does it always find a solution if one exists?
- **Optimality**: Does it always find a least-cost solution?

Completeness & Optimality

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

Completeness:

Do we always find a solution if one exists?

- **Breadth-first:** yes
- **Depth-first:** no, fails in infinite-depth search spaces and spaces with loops
- **Iterative deepening:** yes

Optimality:

Do we always find a least-cost solution?

- **Breadth-first:** yes, for uniform costs
- **Depth-first:** no, solutions with longer paths may be found earlier than solutions with shorter path
- **Iterative deepening:** yes

Search strategies

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

Different search strategies have different strength and weaknesses.

Search strategies are compared along the following dimensions

- **completeness:** Does it always find a solution if one exists?
- **optimality:** Does it always find a least-cost solution?
- **time complexity:** How long does the search take?
- **space complexity:** How much memory does the search need?

→ Time and space complexity are presented by the O-notation.

O-Notation

Linear time complexity

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

- Number of calculation steps that need to be performed
- Example: addition
  - 2 numbers, 1 step: $a + b$
  - 3 numbers, 2 steps: $(a + b) + c$
  - 4 numbers, 3 steps: $((a + b) + c) + d$
  - n numbers → $n - 1$ steps
  - 2n numbers → $2n - 1$ steps
- Number of calculation steps grows linearly with the number of input elements
- The constant -1 becomes less and less important for large numbers
- Order of magnitude → $O(n)$

Quadratic time complexity

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

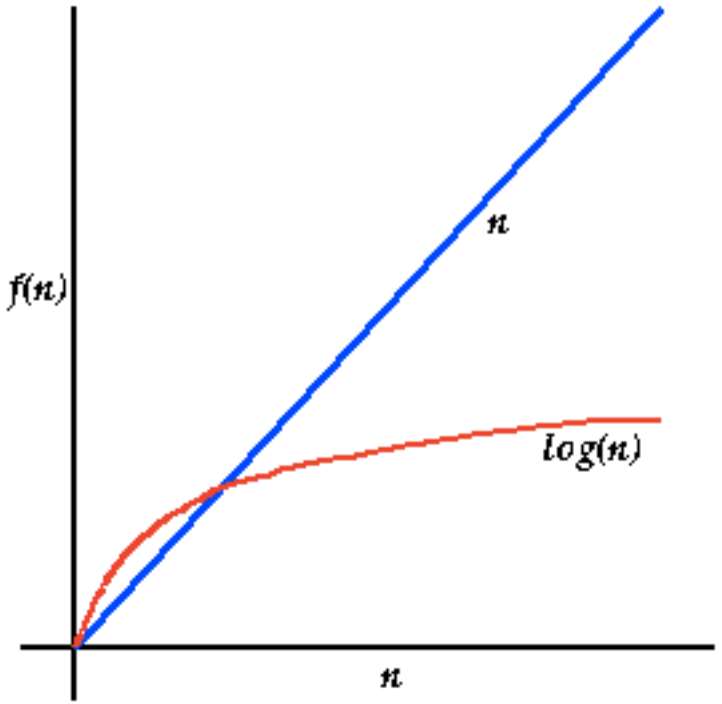
- Example: check for duplicates in a list
  - n = 2, 1 step
    input: [a, b], compare: a/b
  - n = 3, 3 steps
    input: [a, b, c], compare: a/b, a/c, b/c,
  - n = 4, 6 steps
    input: [a, b, c, d], compare: a/b, a/c, a/d, b/c, b/d, c/d
  - n = 5, 10 steps
    input: [a, b, c, d, e], compare: a/b, a/c, a/d, a/e, b/c, b/d, b/e, c/d, c/e, d/e
  - n inputs → $\frac{n * (n - 1)}{2}$ steps
  - 2n inputs → $\frac{2n * (2n - 1)}{2}$ steps
- Number of calculation steps grows quadratically with number of input elements
- Order of magnitude → $O(n^2)$

Logarithmic time complexity

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

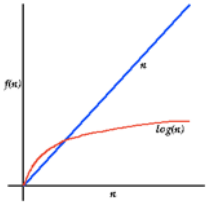
- Recall logarithms:
  - $2^x = n$
  - $\log_2 n = x$
  - n = input, x = time steps



Logarithmic time complexity

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken



- Searching a number in a phone book of n pages
- Start in the middle: $\frac{n}{2}$, check name
  - names on page are later in the alphabet → go to middle of left part: $\frac{n}{4}$
    - ....
  - names on page are earlier in the alphabet → go to middle of right part: $\frac{3}{4}n$
- If we double the number of pages to $2n$ pages, we only need to add one step:
  - Start in the middle: $\frac{2n}{2} = n,$ check name
    - names on page are later in alphabet → go to middle of left part: $\frac{2n}{4} = \frac{n}{2}$
    - ...
- Number of calculation steps grows logarithmically with number of input elements
- Order of magnitude → $O(\log(n))$

O-notation

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

The complexity is calculated for the worst-case scenario.

- If I search for an element in a list of size n, I might find it at position 1 (best case) or position n (worst case) or somewhere in between
- If I search for an element in a list of size $2n$, I might find it at position 1 (best case) or position $2n$ (worst case) or somewhere in between
- best-case scenario remains constant → $O(1)$
- worst-case scenario grows linearly → $O(n)$

Determining the O-notation

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

Constants are ignored.

- $100000n^2 = O(n^2)$

Order of magnitude is dominated by the highest-order term.

- $7n^3 + 2n^5 + 24n = O(n^5)$

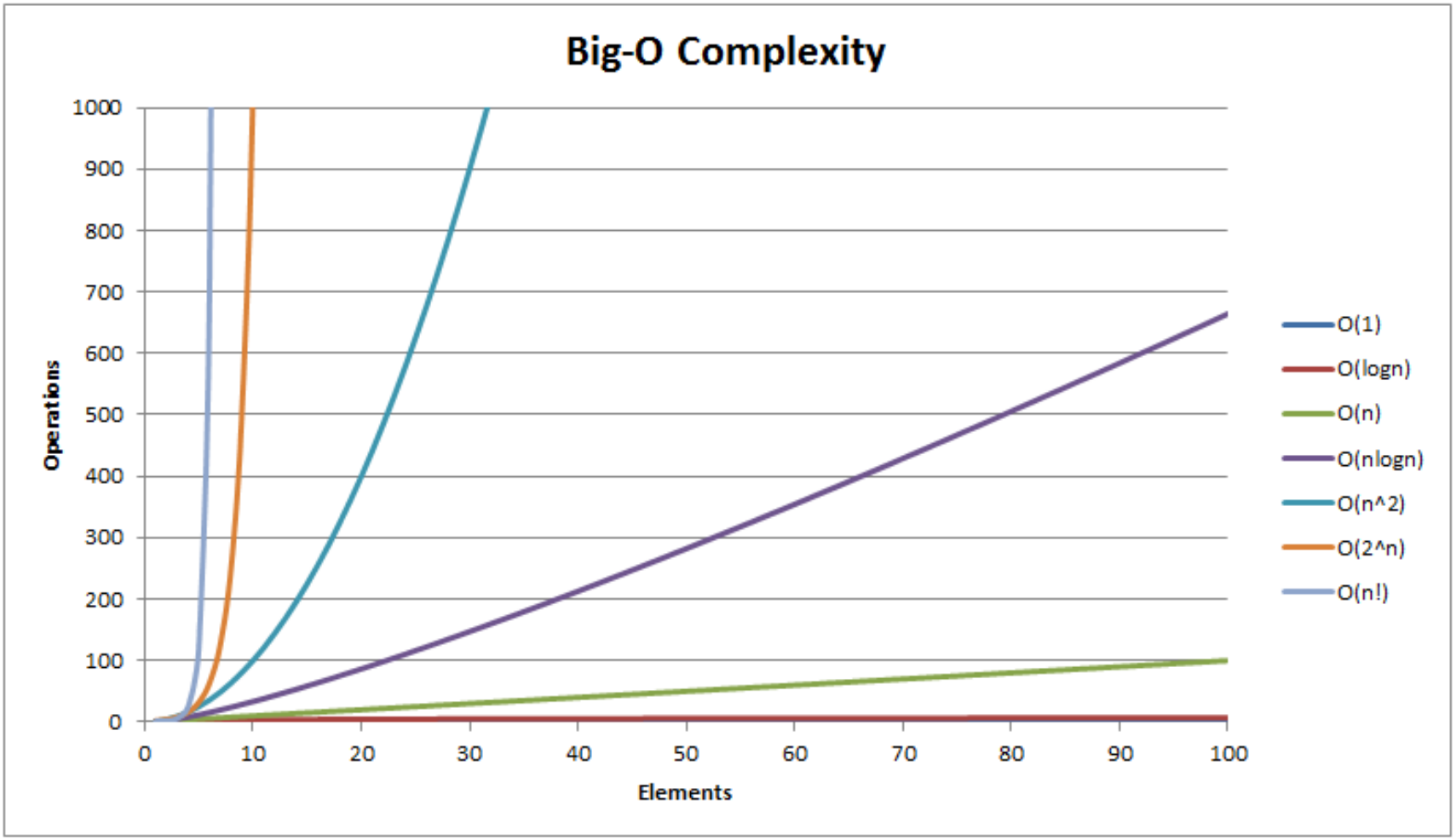
Complexity should be expressed by minimal O.

- $2n$ is contained in $O(n^2)$ and $O(n^3)$,
  but it is more accurate to say $2n = O(n)$

Function growth rates

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken



https://medium.com/@devontem/simple-guide-to-big-o-notation-time-complexity-space-complexity-9e906c60f7f9

Performance Review Continued

Search strategies

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

Different search strategies have different strength and weaknesses.

Search strategies are compared along the following dimensions

- completeness: Does it always find a solution if one exists?
- optimality: Does it always find a least-cost solution?
- **time complexity:** How long does the search take?
- **space complexity:** How much memory does the search need?

Parameters:

- b: maximum branching factor of the search tree
- d: depth of the least-cost solution
- m: maximum depth of the state space (may be ∞)

Time complexity

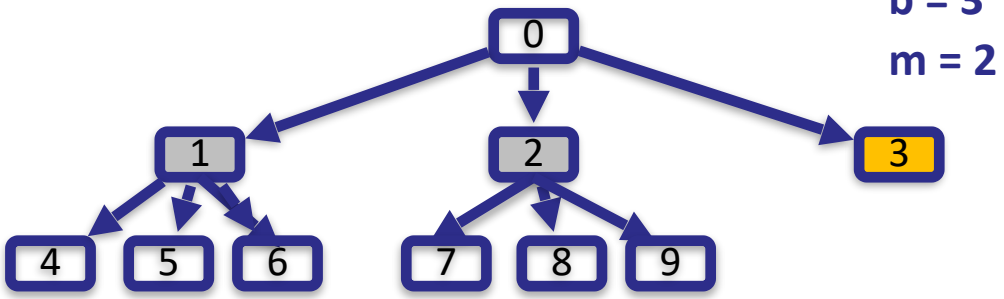
UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

Time complexity: number of nodes that need to be generated

Breadth-first:

- depth 0 has 1 node
- depth 1 has max. 1 * b nodes
- depth 2 has max. 1 *b * b nodes
- goal is in level d
- worst case: goal is last node in level d
- search steps: $b + b^2 + b^3 \ldots + b^d + (b^{d+1} - b)$
- → $O(b^{d+1})$



Time complexity

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

Time complexity: number of nodes that need to be generated

Breadth-first:

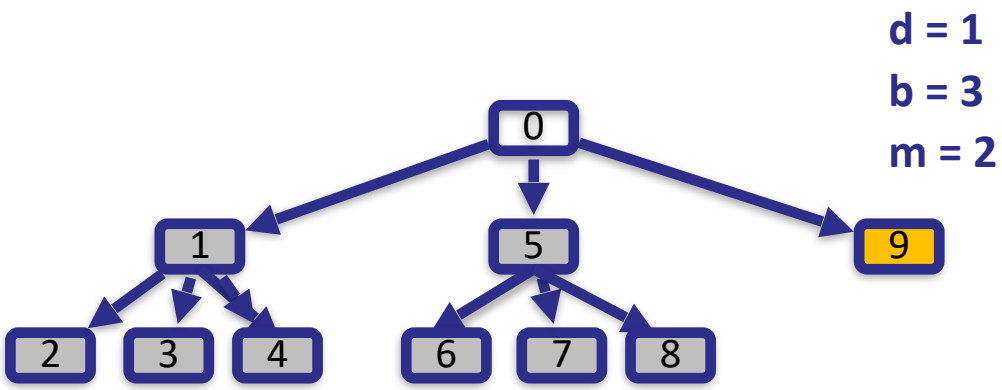
→ $O(b^{d+1})$

Depth-first:

- follows each path until maximum depth m

  → $O(b^m)$

- terrible if m much larger than d



Time complexity

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

Time complexity: number of nodes that need to be generated

Breadth-first:

→ $O(b^{d+1})$

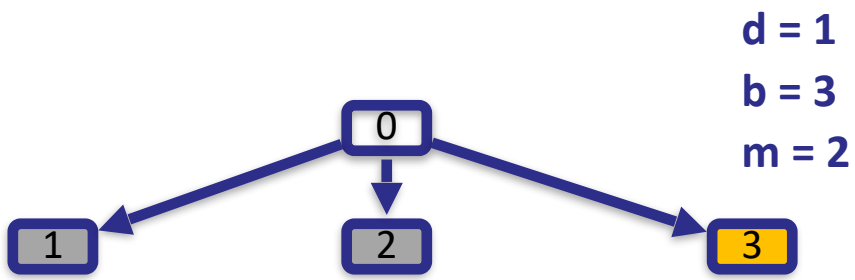
Depth-first:

→ $O(b^m)$

Iterative deepening:

- m never goes beyond d

→ $O(b^d)$



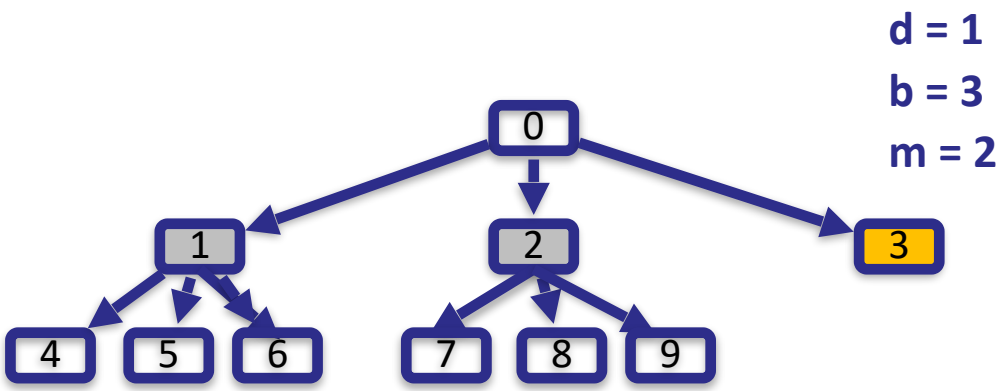
Space Complexity

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

- **Space complexity:** maximum number of nodes in memory
- **Breadth-first:**
  - Every node must remain in memory because it is either a fringe node or an ancestor of a fringe node
  - In the end, the goal will be in the fringe, and its ancestors will be needed for the solution path

  → $O(b^{d+1})$



Space Complexity

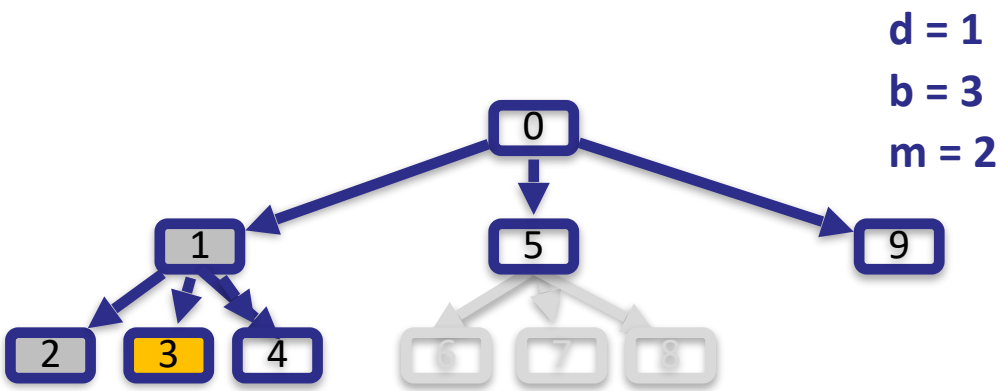
UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

- Space complexity: maximum number of nodes in memory
- Breadth-first:

  → $O(b^{d+1})$
- **Depth-first:**
  - only nodes in current path and their unexpanded siblings need to be stored

    → $O(bm)$



Space Complexity

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

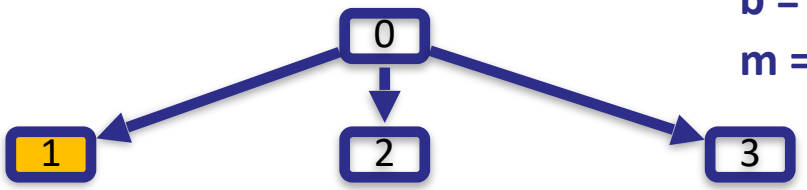
- Space complexity: maximum number of nodes in memory
- Breadth-first:

  → $O(b^{d+1})$
- Depth-first:

  → $O(bm)$
- Iterative deepening:
  - only nodes in current path and their unexpanded siblings need to be stored

    → $O(bd)$

d = 1
b = 3
m = 2



Performance review summary

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

- Iterative deepening search uses only linear space and not much more time than other uninformed algorithms

	Breadth-first	Depth-first	Iterative deepening
Complete?	Yes	No	Yes
Optimal?	Yes	No	Yes
Time complexity	b^{d+1}	b^m	b^d
Space complexity	b^{d+1}	bm	bd

Complexity in Practice

Issue: Size of constant factor

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

Which algorithm would you choose?

Algorithm 1

- 2,000 n = O(n)

Algorithm 2

- 2 n^2 = O(n^2)

Depends on problem size

- for problem size with n<1,000 Algorithm 2 is faster

Issue: Memory usage

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

Which algorithm would you choose?

Algorithm 1

- time-complexity O(n log n)
- memory usage O(2^n)

Algorithm 2

- time-complexity O(n^2)
- memory usage O(n)

Depends on how much memory you have

- but Algorithm 1 will only fit in memory for very small n

Issue: Programming time

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

Which algorithm would you choose?

Algorithm 1

- time-complexity O(n log n)
- programming time 1 month

Algorithm 2

- time-complexity $O(n^2)$
- programming time 1 hour

Depends on how much time you have ☺

- if you need to run it often or for large problems, might be worth the time
- but faster algorithms are usually also more complex and more error-prone

Issue: Quality of solution

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

Which algorithm would you choose?

Algorithm 1

- time-complexity $O(2^n)$
- finds the optimal answer

Algorithm 2

- time-complexity $O(n^2)$
- finds a good answer

Depends on whether you need the optimal answer

- often an **approximation** is good enough
  - can usually be found much, much faster than optimal solution

Summary

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

Tree search algorithms operate on an abstraction of the real problem.

Search strategies

- Breadth-First (BFS)
- Depth-First (DFS)
- Iterative Deepening (IDS)

Evaluation of search strategies

- completeness
- optimality
- time complexity
- space complexity

- O-Notation

Readings

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken

Mandatory

- Russell & Norvig, Section 3, *Solving Problems by Searching*, 3.1-3.4

Optional

- Russell & Norvig, Rest of Section 3
- Joachim Ziegler: Tutorial on O-notation
  http://www.leda-tutorial.org/de/offiziell/ch02s02s03.html

Next lecture: Find your way like Google

UNIVERSITÄT DUISBURG ESSEN

Offen im Denken