Lecture

Foundations of Artificial Intelligence

Part 5 – Informed Search

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Organization

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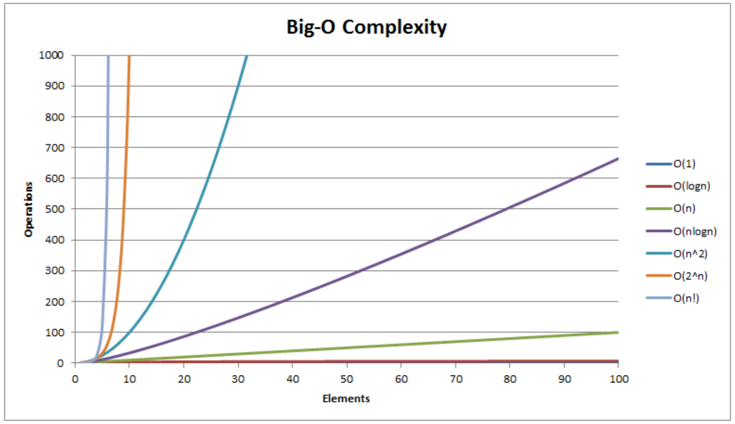
- **Exam**
  - **Questions:** English **AND** German.
  - **Answers:** English **OR** German **(you can choose only one language).**
  - **Dictionary:** is allowed (only hard copy).
  - **Anmeldephase** läuft vom **02.05.2022** bis **13.05.2022.**

- **Homework:**
  - There are German and English versions of homework.
  - You can **do your homework** in **German.**

Recall …

Search strategies

- **Breadth**-First Search (BFS),
- **Depth**-First Search (DFS),
- **Iterative** Deepening Search (IDS)



Parameters:

- **b**: maximum **branching factor** of the search tree
- **d**: **depth** of the **optimal solution**
- **m**: **maximum depth** of the tree (may be ∞)

O Notation

- **Introduction**

Evaluation of search strategies

- **completeness**
- **optimality**
- time complexity
- space complexity

Evaluation of search strategies

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Search strategies are compared along the following dimensions

- completeness: ✅
- optimality: ✅
- ⌛ **time complexity:** How long does the search take?
- 💽 **space complexity:** How much memory does the search need?

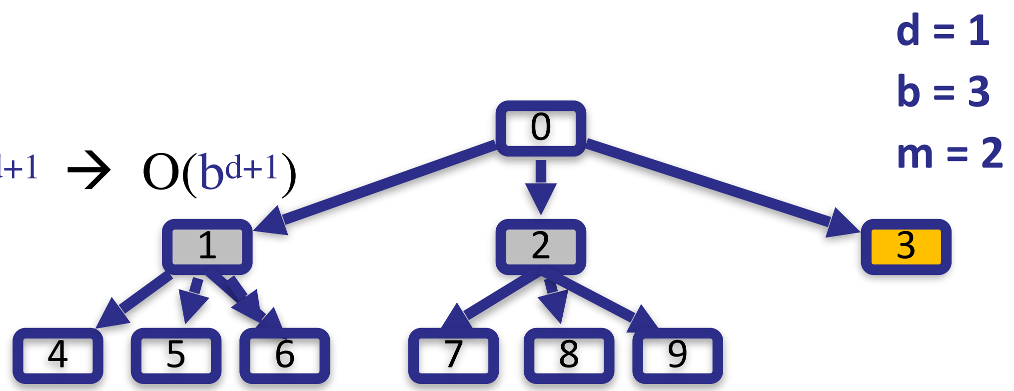
Time complexity of BFS

Time complexity: number of nodes that need to be generated

- **b**: maximum branching factor of the search tree
- **d**: **depth of the optimal solution**
- **m**: maximum depth of the state space (may be ∞)

- depth 0 → 1 node
- depth 1 → max b nodes
- depth 2 → max b * b = b^2 nodes
- depth d → max b^d nodes (goal state is here)
- depth d+1 → max b^{d+1} nodes

- Total: $1 + b + b^2 + b^3 \dots + b^d + b^{d+1}$ → $O(b^{d+1})$



Time complexity of DFS

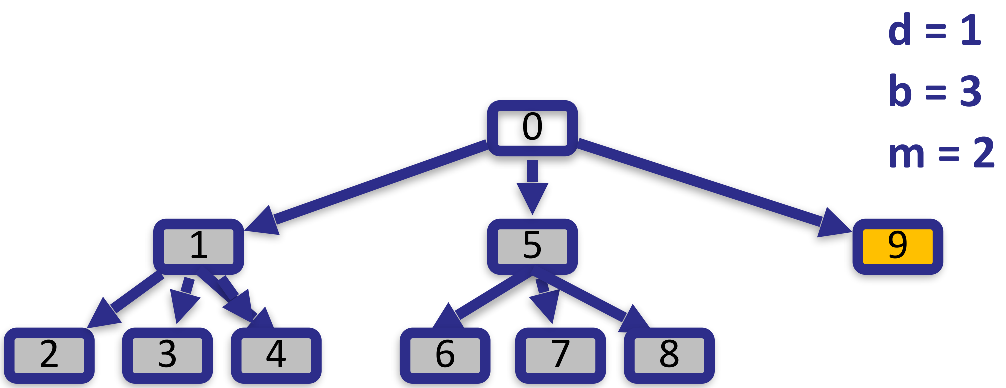
Time complexity: number of nodes that need to be generated

- follows each path until maximum depth m

  → $O(b^m)$

- terrible if m much larger than d

- **b**: maximum branching factor of the search tree
- **d**: depth of the optimal solution
- **m**: **maximum depth of the state space (may be ∞)**



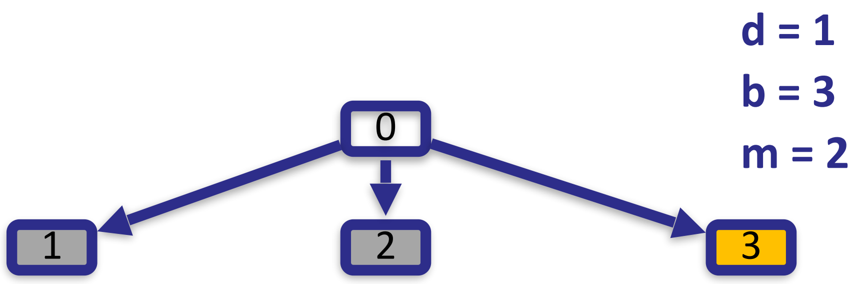
Time complexity of IDS

Time complexity: number of nodes that need to be generated

- m never goes beyond d

→ $O(b^d)$

- **b**: maximum branching factor of the search tree
- **d**: depth of the optimal solution
- **m**: **maximum depth of the state space (may be ∞)**



Space complexity of BFS

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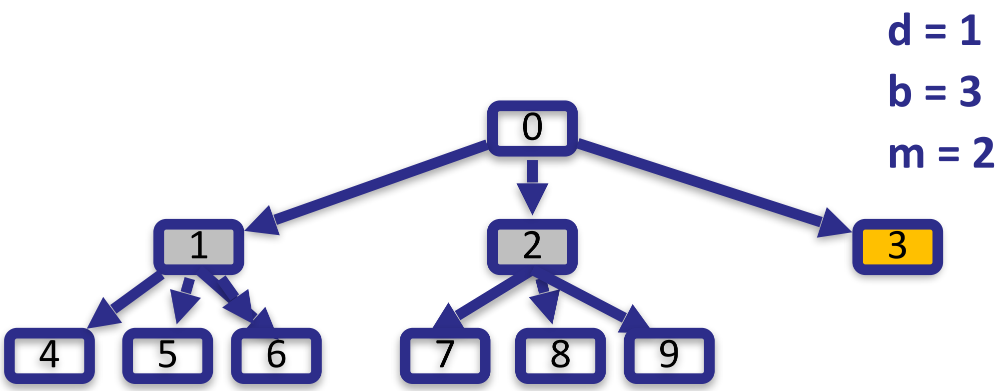
- **Space complexity:** maximum number of nodes in memory
  - all nodes in a level should remain in memory

  → $O(b^{d+1})$



Space complexity of DFS

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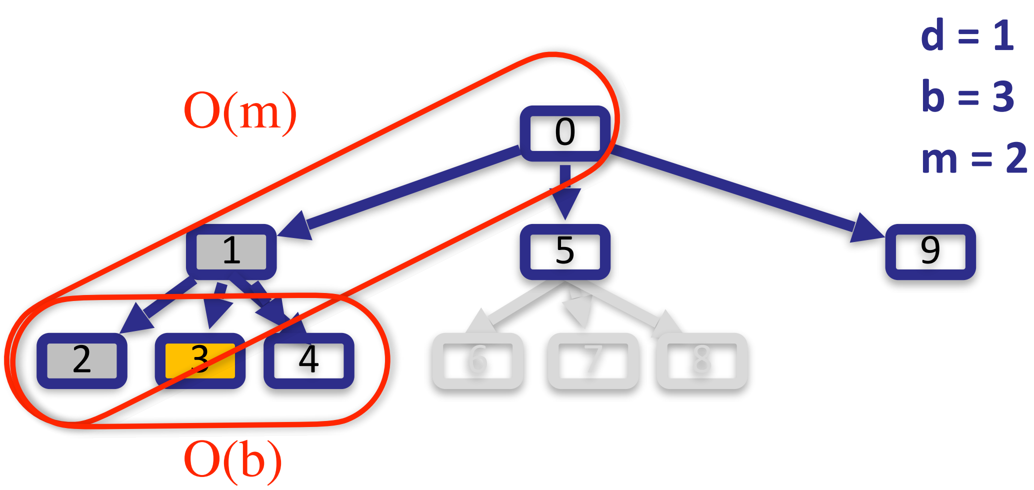
- **Space complexity:** maximum number of nodes in memory
- only nodes in current path and their unexpanded siblings need to be stored

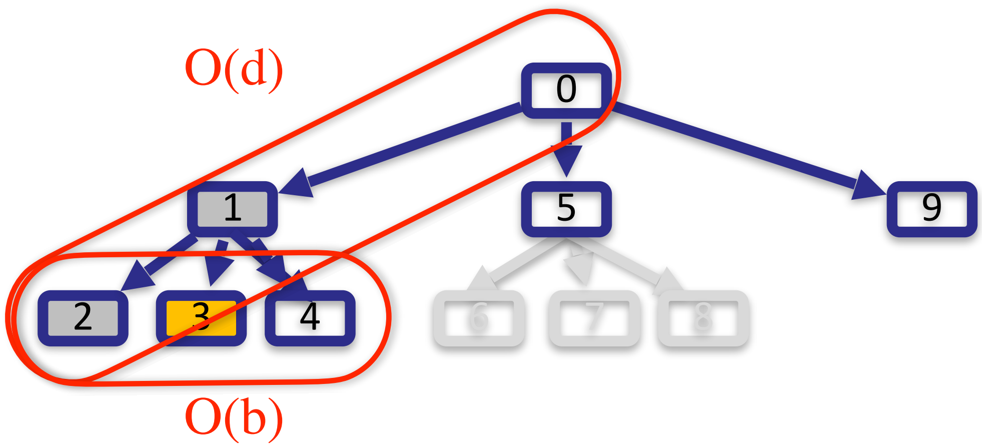
  → O(bm)



Space complexity of IDS

- **Space complexity:** maximum number of nodes in memory
- only nodes in **current path** and **their unexpanded siblings** need to be stored

→ O(bd)



Performance review summary

- Iterative deepening search uses only **linear space** and not much more time than other **uninformed algorithms**

	BFS	DFS	IDS
Complete?	Yes	No	Yes
Optimal?	Yes	No	Yes
Time complexity	b^{d+1}	b^m	b^d
Space complexity	b^{d+1}	bm	bd

Any open questions?

Where are we?

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- Intelligence ✅
- Agents ✅
- Search
  - Uninformed ✅
  - **Informed**
  - Local

In this lecture, you learn …

- **Informed search algorithms using heuristics**
  - Greedy **best-first** search
  - A* search
- **Heuristics**
  - Admissible
  - Consistent
  - How do we define and choose a good heuristic?

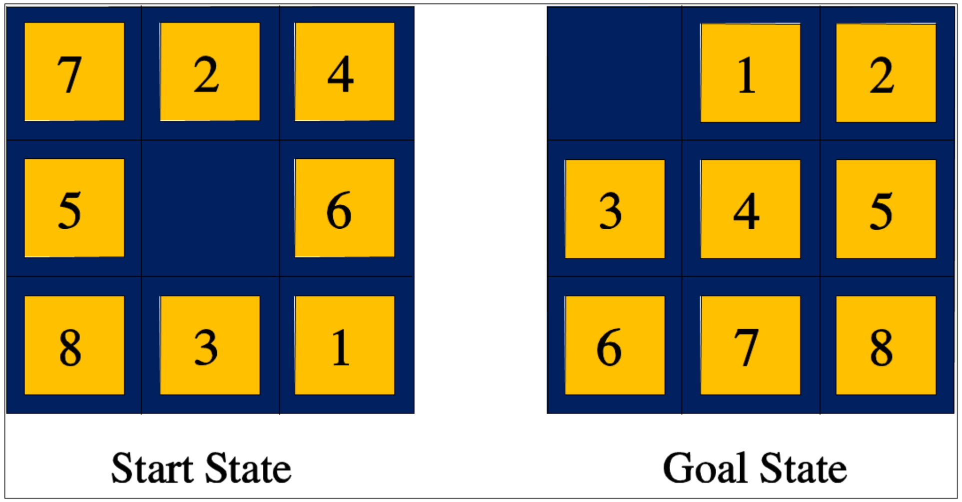
Today

Informed Search

Motivation

Uninformed search algorithms are **inefficient**

- they expand far **too many unpromising paths**



- average solution depth = 22
- BFS expands about 3.1 x 10^{10} nodes until depth 22
- Can we make it more efficient?

Idea …

- Instead of **randomly expanding nodes in a level,** …
- expand the **most promising node first**
- How do we determine which node is the most promising one?

→ **heuristics**

Heuristics

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Heuristics are **experience-based approximations** for solving a problem

- **Rule of thumb**
- **Heuristics may go wrong!**

https://upload.wikimedia.org/wikipedia/commons/2/2f/Eureka_arkimedi.jpg

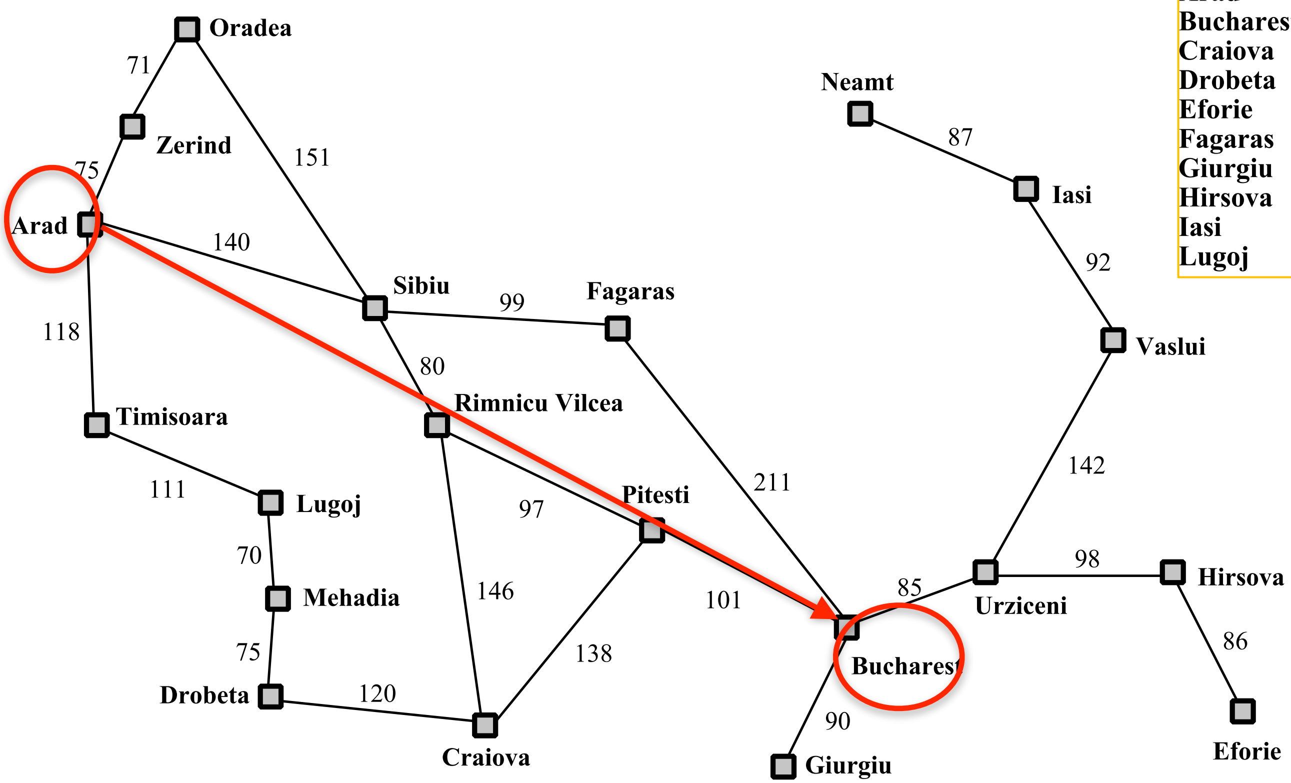
Heuristics

In tree-search algorithms, **a heuristic is a function** $\mathbf{f(n)}$ **for each node** $\mathbf{n}$

- it estimates **the ‘desirability’ of the node's state**

Getting informed

Straight line distances between cities



Straight line distances (SLD) to the goal state

Arad	366	**Mehadia**	241
Bucharest	0	**Neamt**	234
Craiova	160	**Oradea**	380
Drobeta	242	**Pitesti**	100
Eforie	161	**Rimnicu Vilcea**	193
Fagaras	176	**Sibiu**	253
Giurgiu	77	**Timisoara**	329
Hirsova	151	**Urziceni**	80
Iasi	226	**Vaslui**	199
Lugoj	244	**Zerind**	374

Let's refer to city names by their first letters

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Greedy Best-First Search

Greedy Best-first search

Heuristic function $f(n) = h(n)$

- $h(n)$: **goal-oriented heuristic that estimates the cost from node $\mathbf{n}$ to the goal**

Greedy Best-first search expands the node that **appears to be nearest to the goal —> It has the minimum cost**

Example: $h_{SLD}(n)$ = **S**traight **L**ine **D**istance from $\mathbf{n}$ to Bucharest

Greedy best-first search

Example: $h_{SLD}(n)$ = **S**traight **L**ine **D**istance from $\mathbf{n}$ to Bucharest

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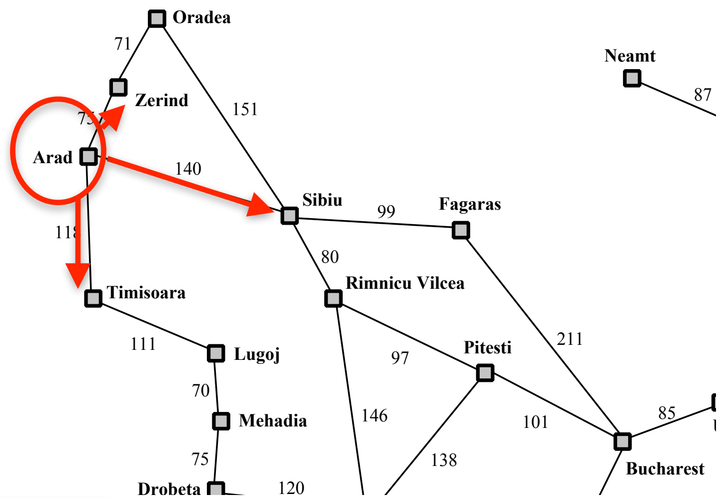
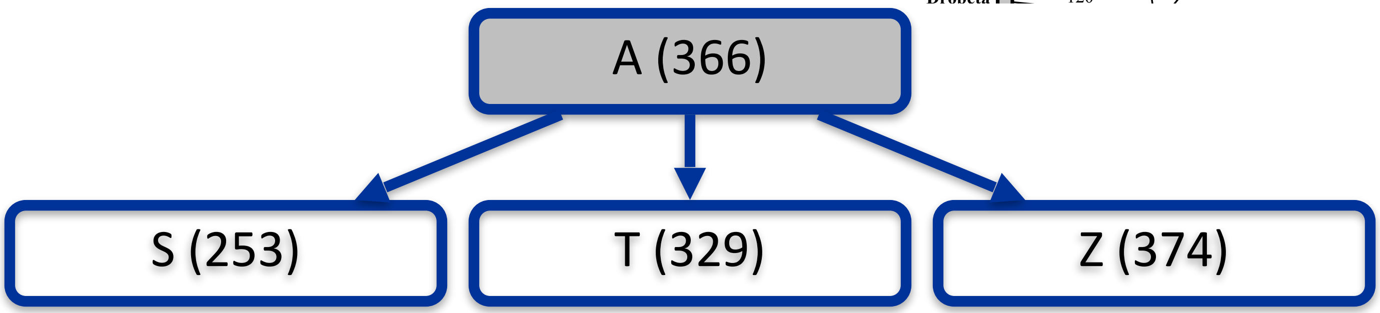
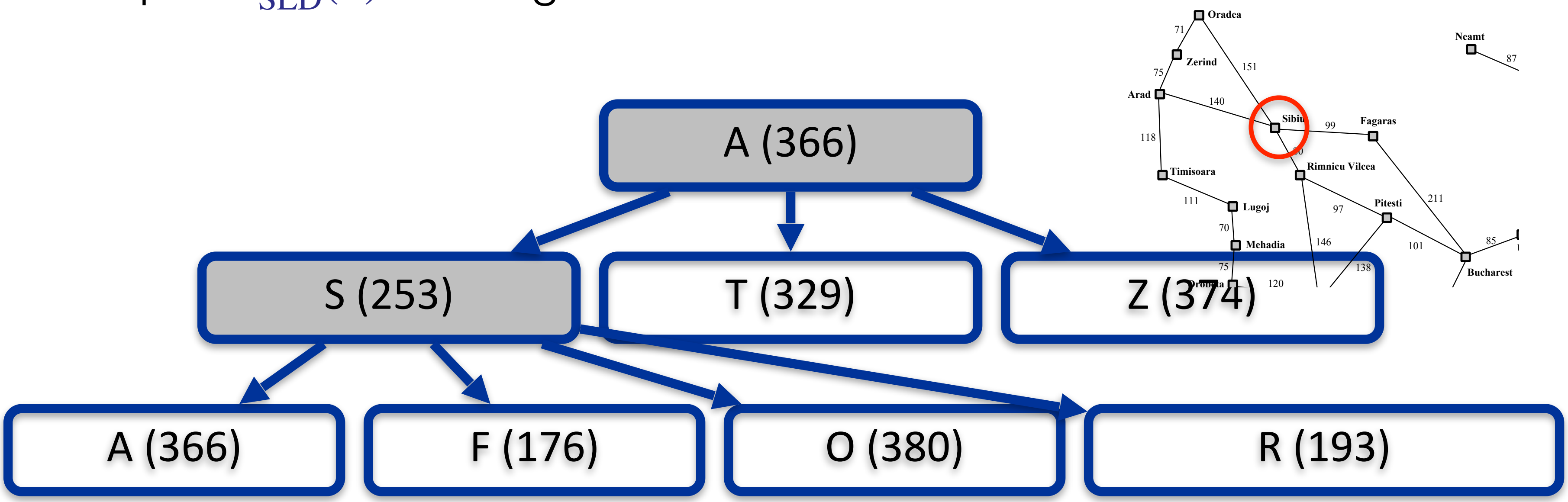




Example: Greedy best-first search

Example: $h_{SLD}(n)$ = straight-line distance from n to Bucharest



Example: Greedy best-first search

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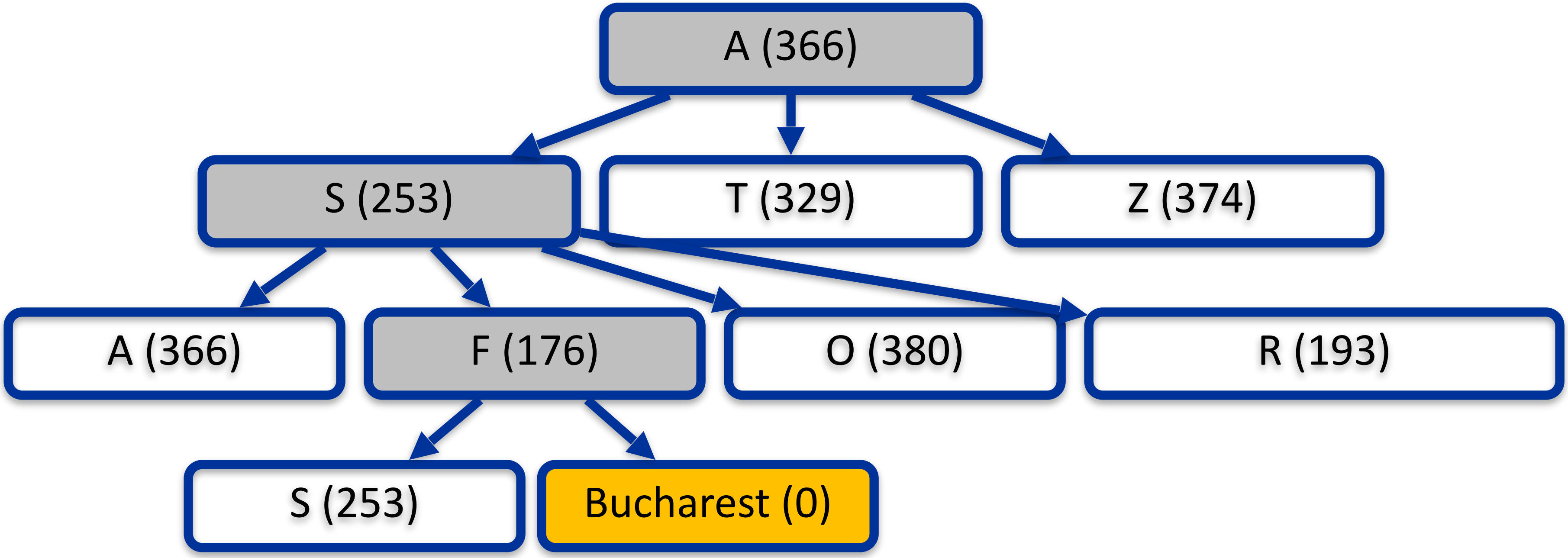

Example: Greedy best-first search

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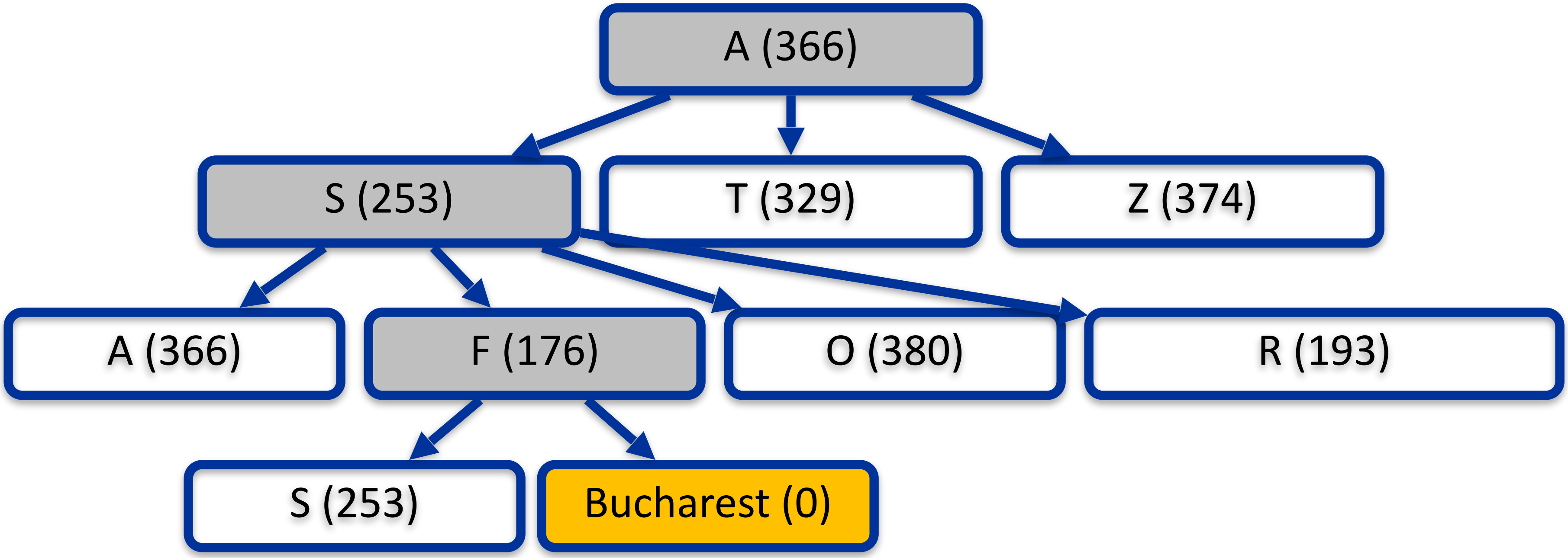
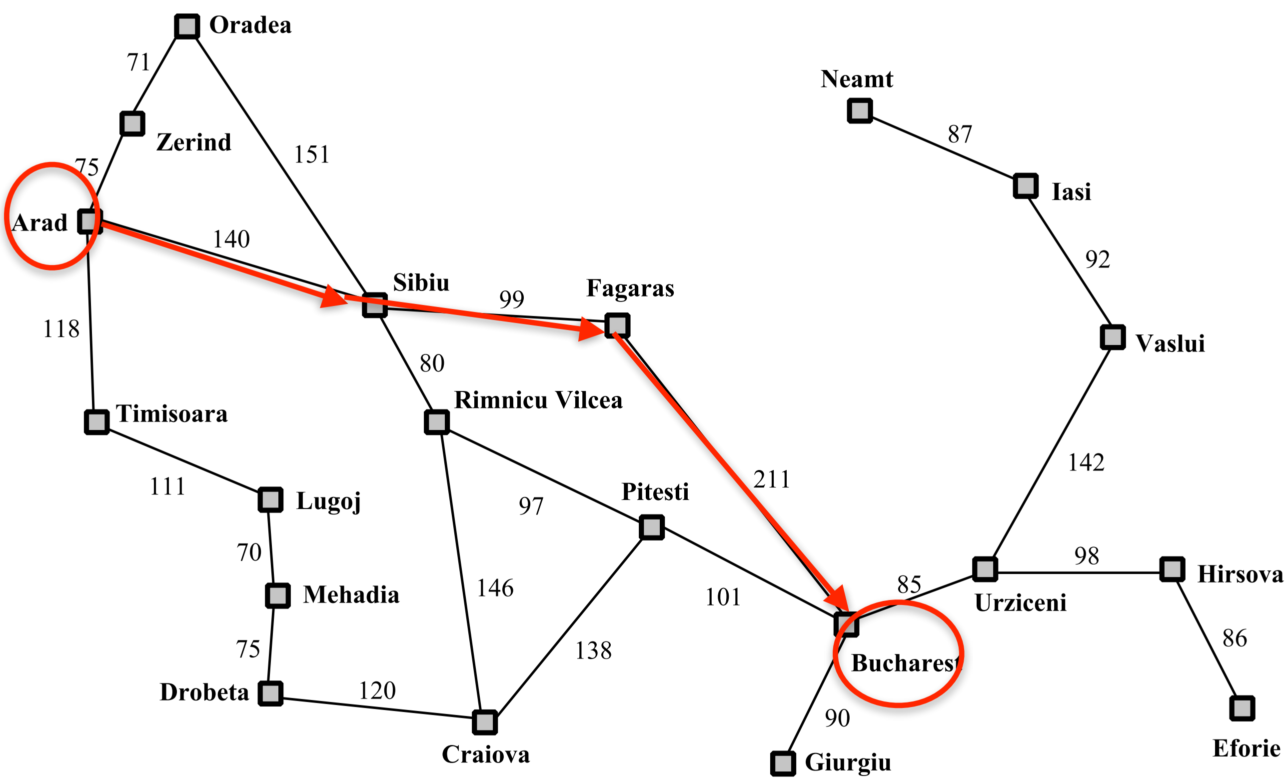
Example: $h_{SLD}(n)$ = straight-line distance from n to Bucharest



A —> S —> F —>B

Here is the solution

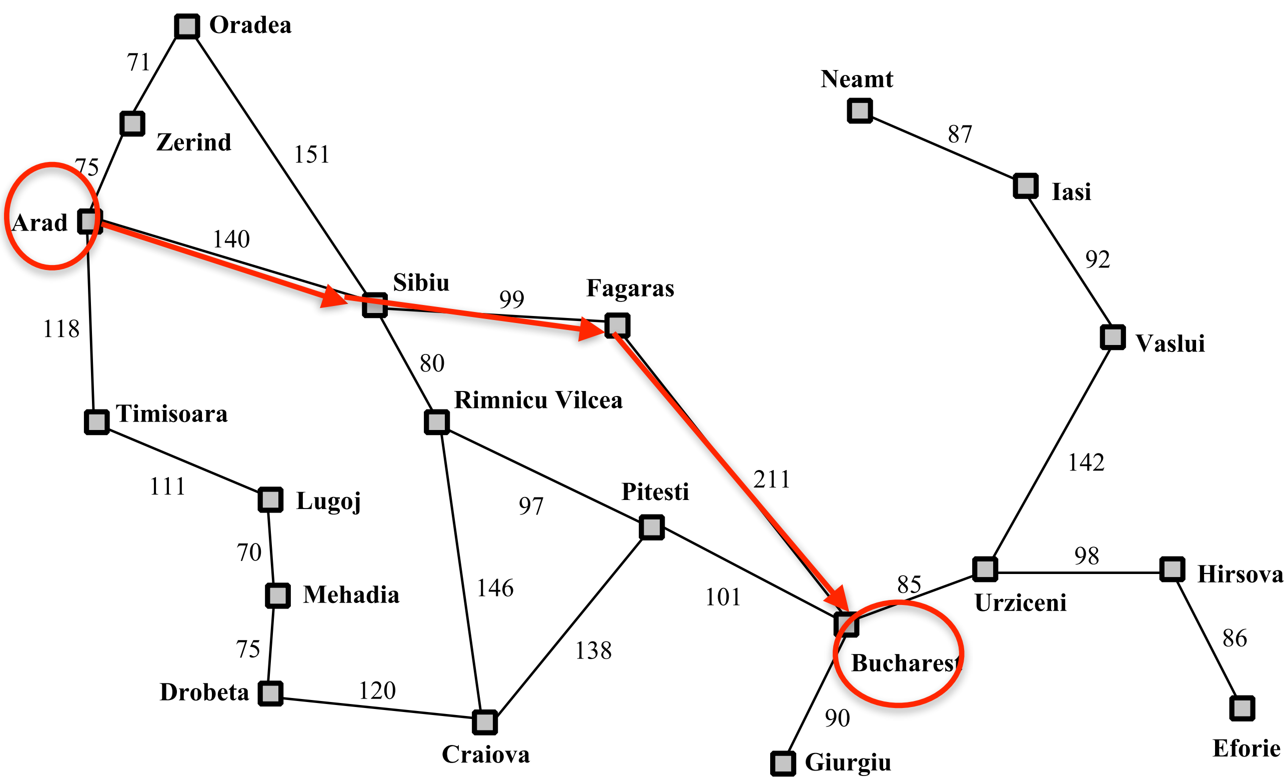
A —> S —> F —>B



Here is the solution

A —> S —> F —>B

What is the cost of this path (solution)?

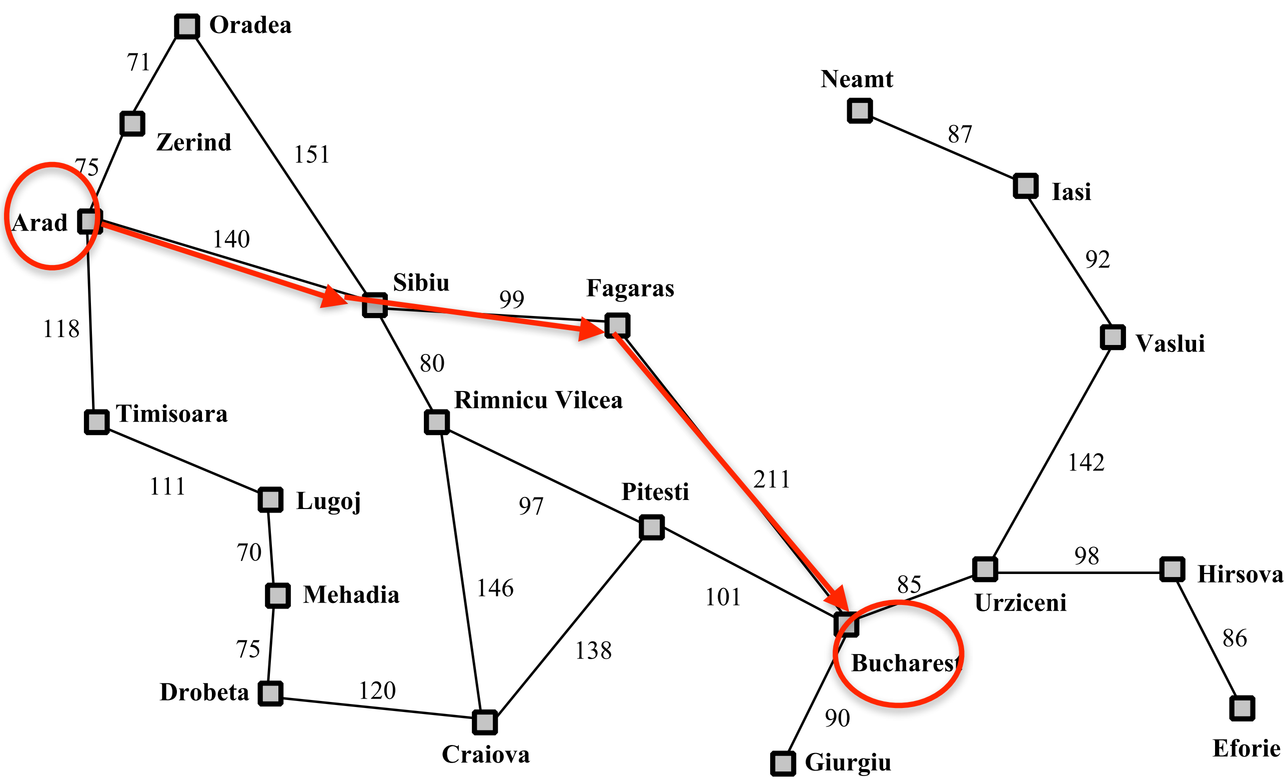


Here is the solution

A —> S —> F —>B

What is the cost of this path (solution)?

140 + 99 + 211 = 450



Here is the solution

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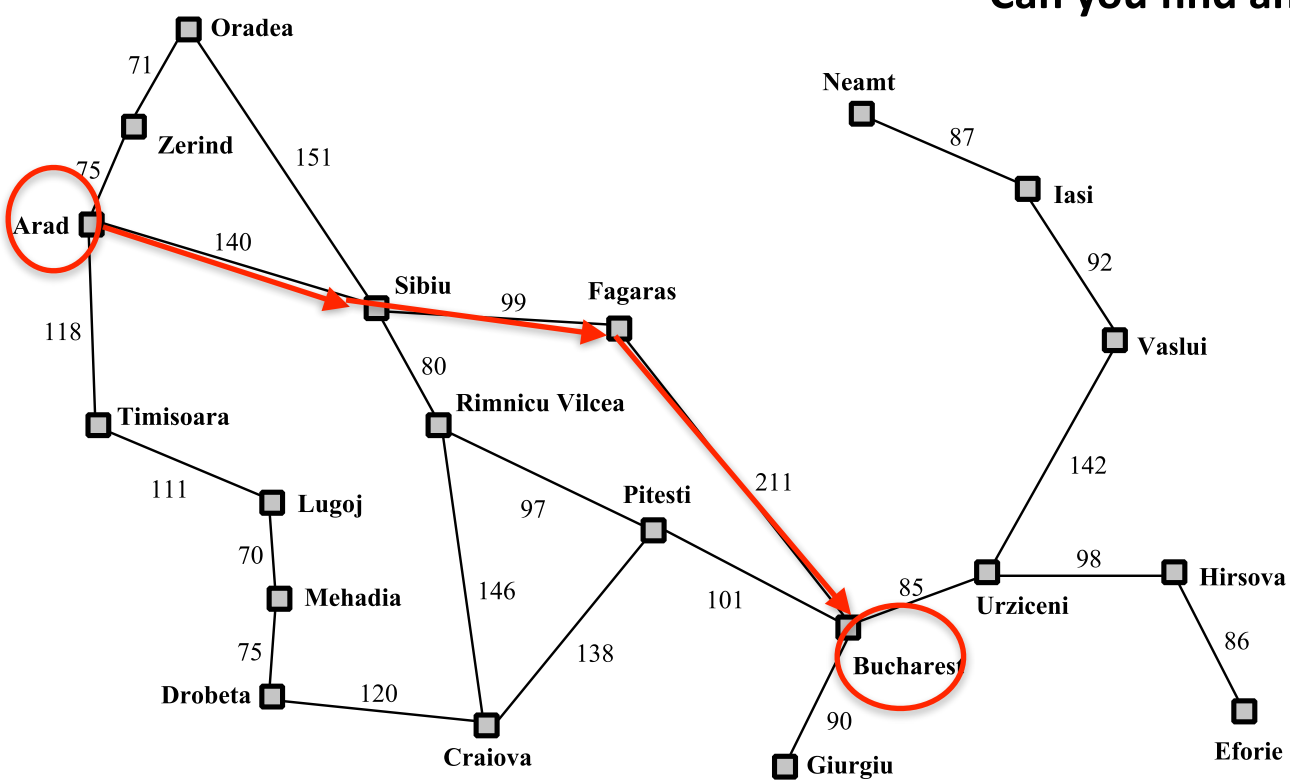
A —> S —> F —>B

What is the cost of this path (solution)?

140 + 99 + 211 = 450

Can you find another path with a lower cost?



Here is the solution

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A —> S —> F —>B

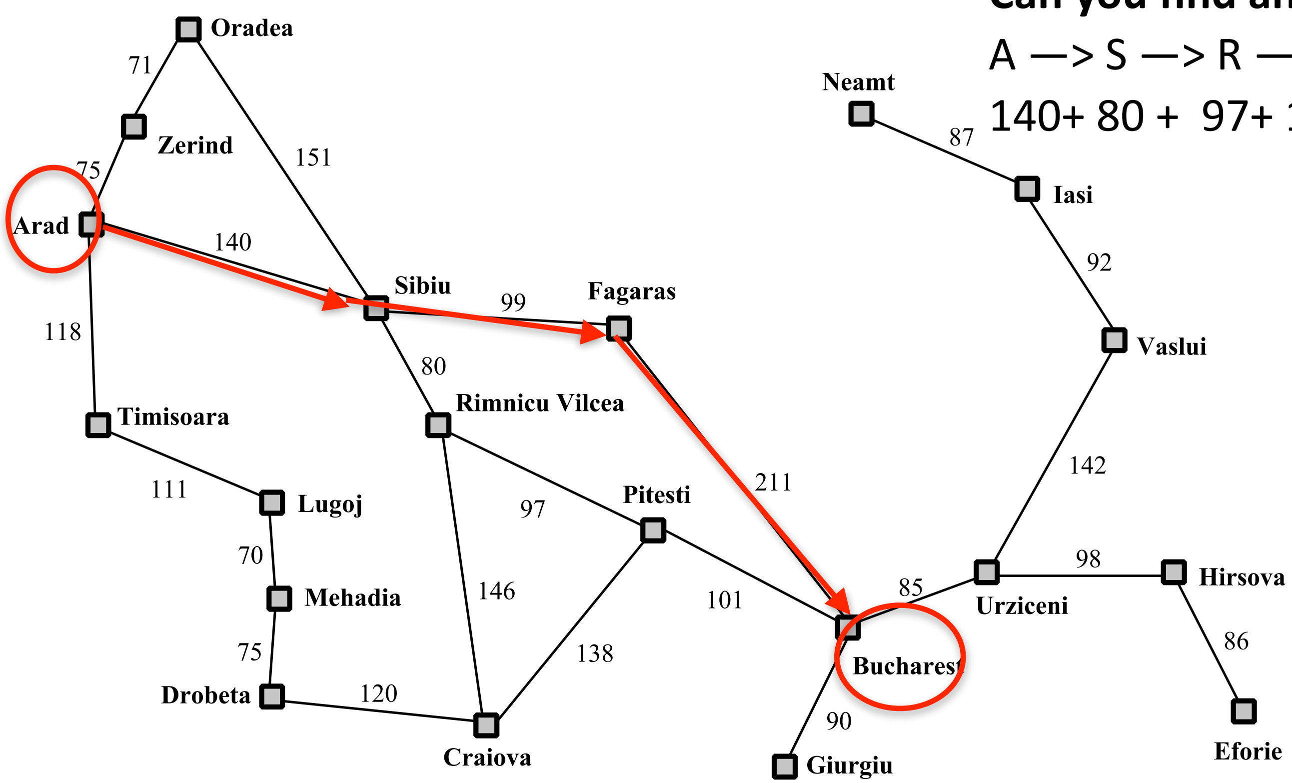
What is the cost of this path (solution)?

140 + 99 + 211 = 450

Can you find another path with a lower cost?

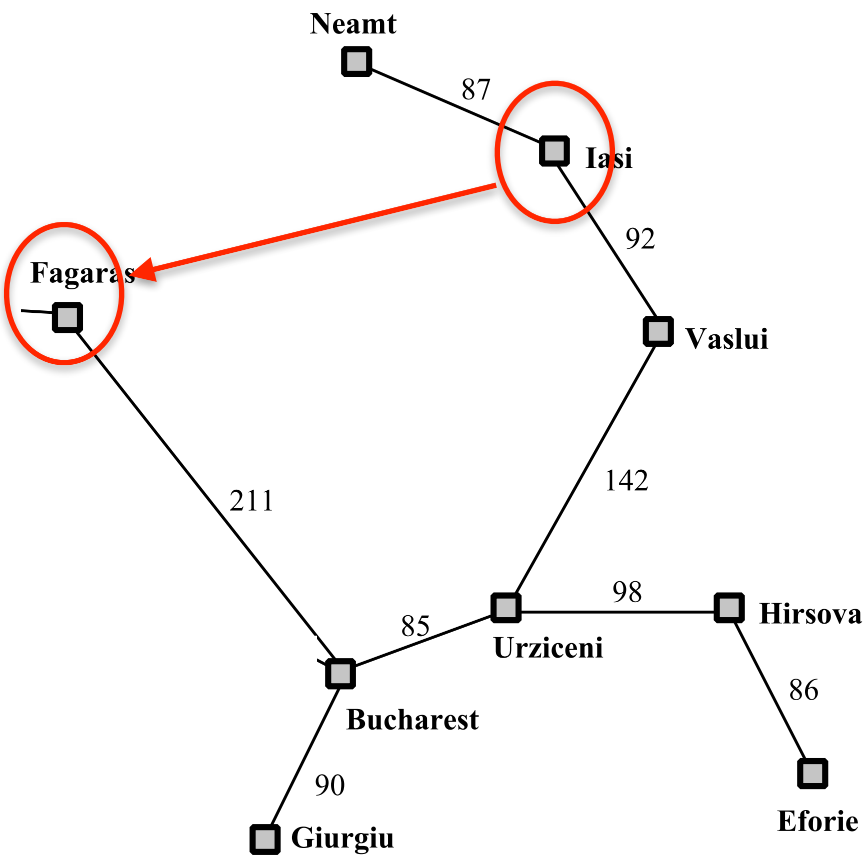
A —> S —> R —> P —> B

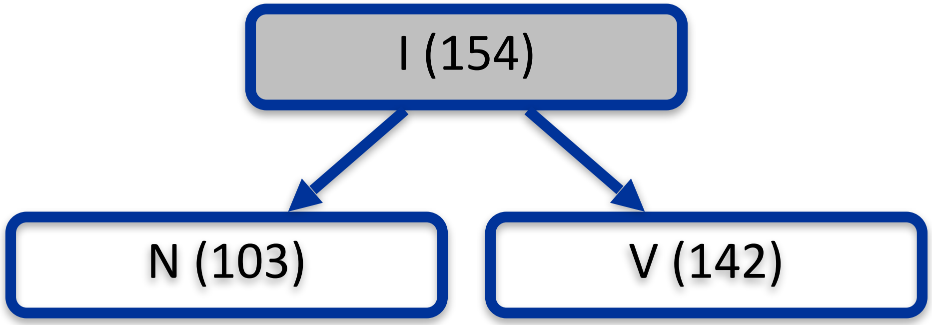
140+ 80 + 97+ 101 = 418



Another example

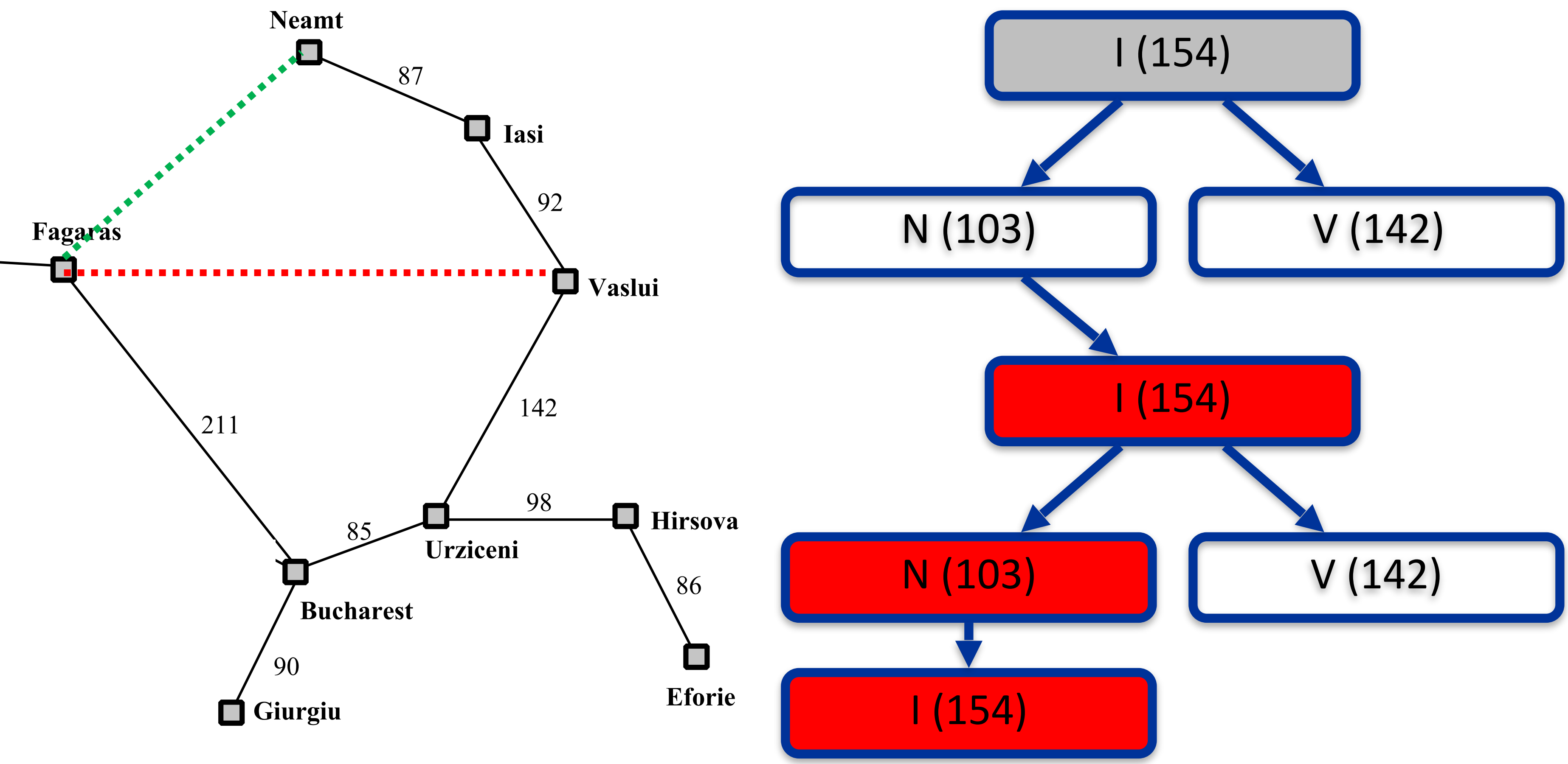
Find a path from **Iasi (I)** to **Fagaras (F)** (with greedy best-first search)





Loops in greedy best-first search

Find a path from Iasi to Fagaras (with greedy best-first search)



Properties of greedy best-first search

Completeness: Do we always **find a solution** if one exists?

- **No** –> can get stuck in loops

→ complete in finite state space **if we check for repeated states**

Optimality: Do we always find **an optimal solution**?

- **No**: the path Arad -> Sibiu -> Fagaras -> Bucharest is not optimal

Time complexity:

- **O(b^m)**, **like DFS**
- optimal case: best choice in each step → only *d* steps
- a good heuristic improves chances for encountering optimal case

Space Complexity

- has to keep all nodes in memory → same complexity as best-first search

A* Search

A* search

Idea

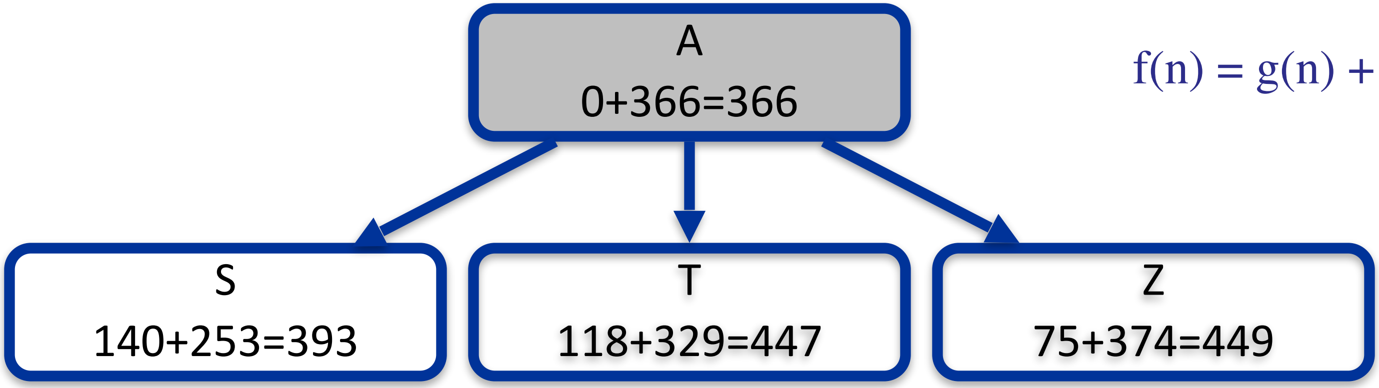
- **avoid expanding paths** that are **already expensive**

→ **evaluate complete path cost (not only the cost to reach goal)**

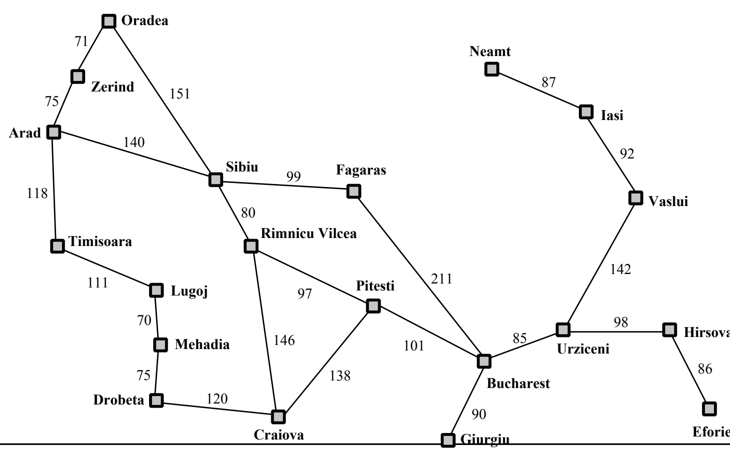
Heuristic function $f(n) = g(n) + h(n)$

- $g(n)$ = **real cost** to reach node n from source
- $h(n)$ = estimated cost **to get from n to goal G**
- $f(n)$ = **estimated cost of path to goal via n**

A* search



$$f(n) = g(n) + h(n)$$



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H	151	U	80
I	226	V	199
L	244	Z	374

A* search

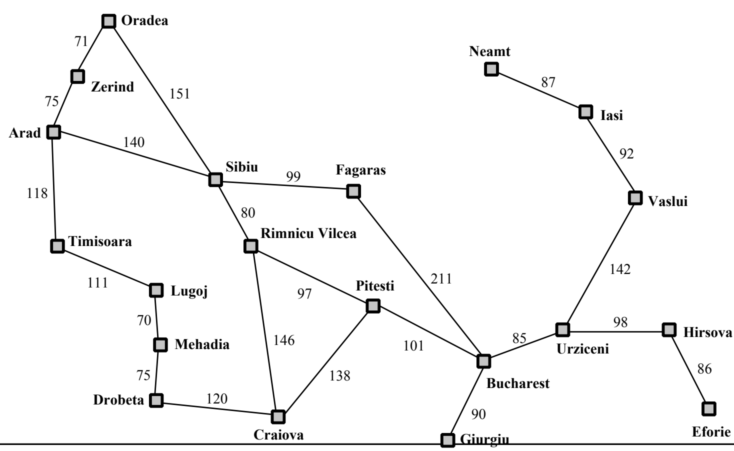
$$f(n) = g(n) + h(n)$$

A
0+366=366

S
140+253=393

T
118+329=447

Z
75+374=449



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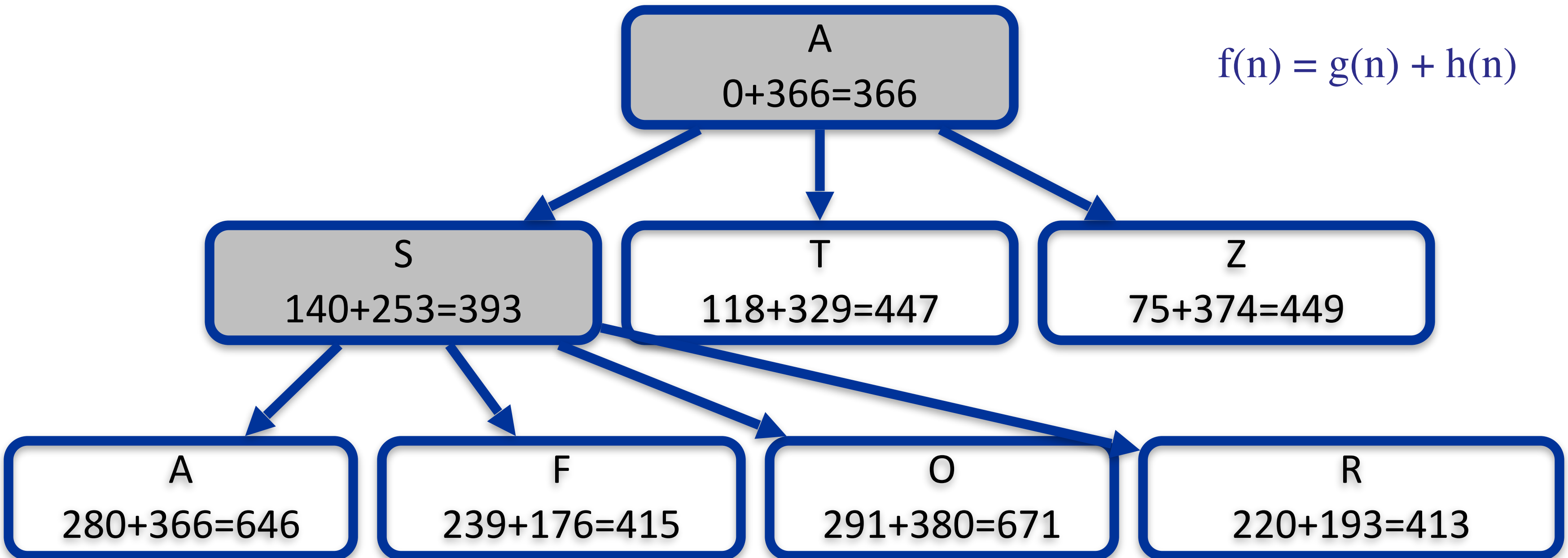
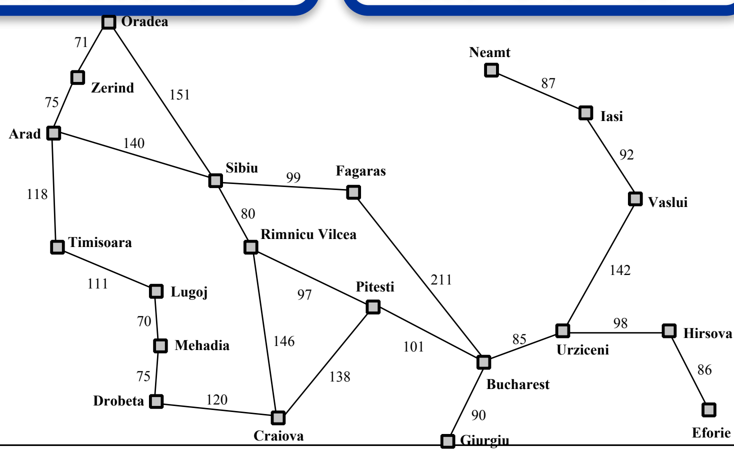
A* search

$$f(n) = g(n) + h(n)$$





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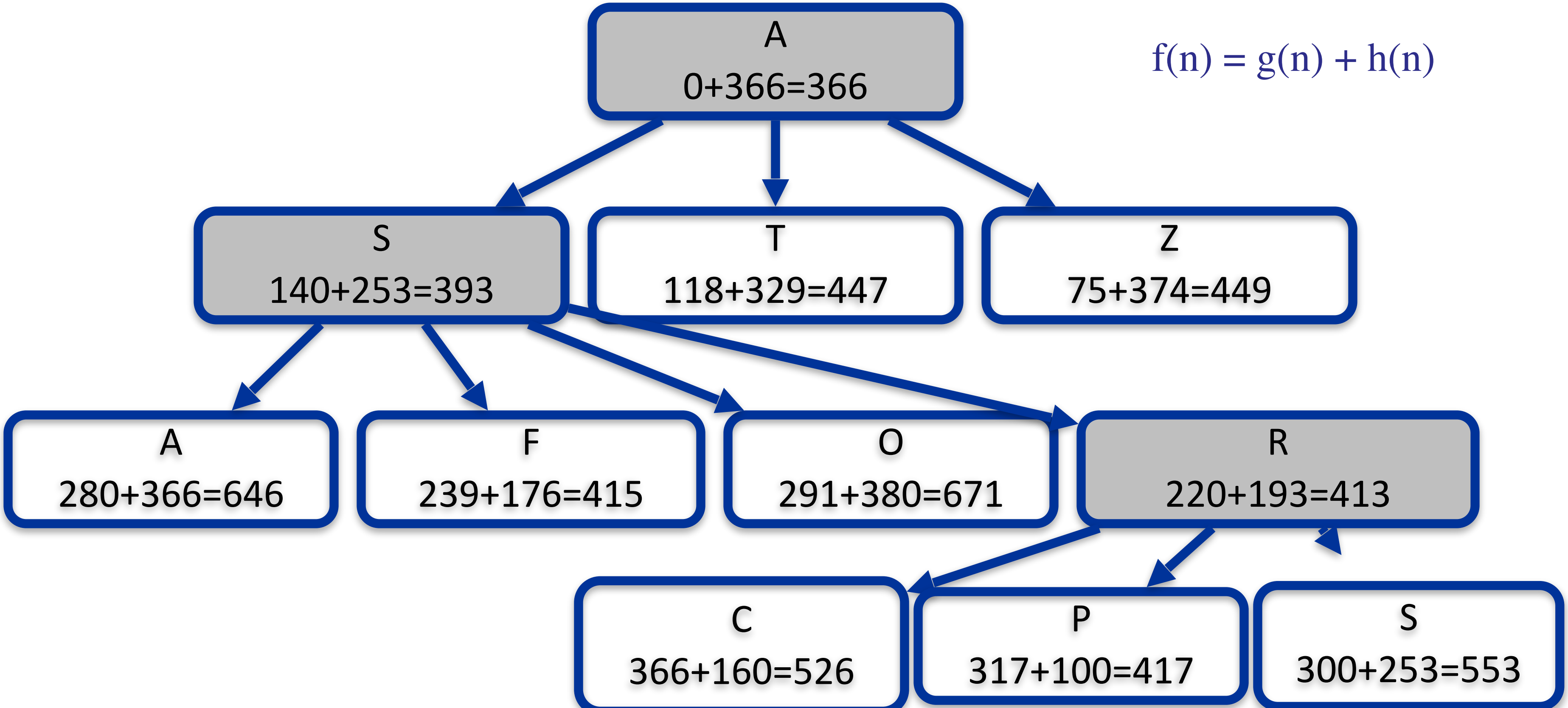
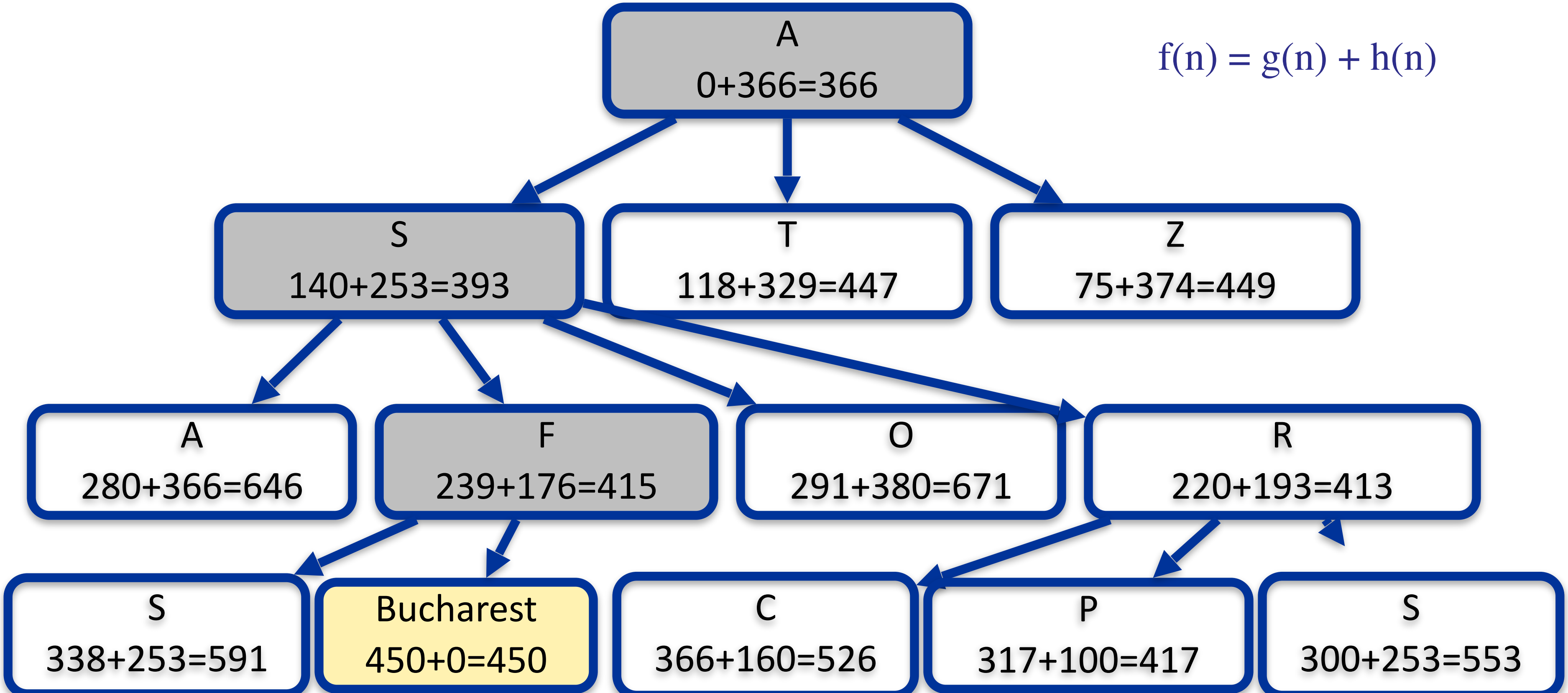
A* search



$f(n) = g(n) + h(n)$

R (Rimnicu Vilcea) will be expanded because it has the lowest value for $f(n)$.

A* search

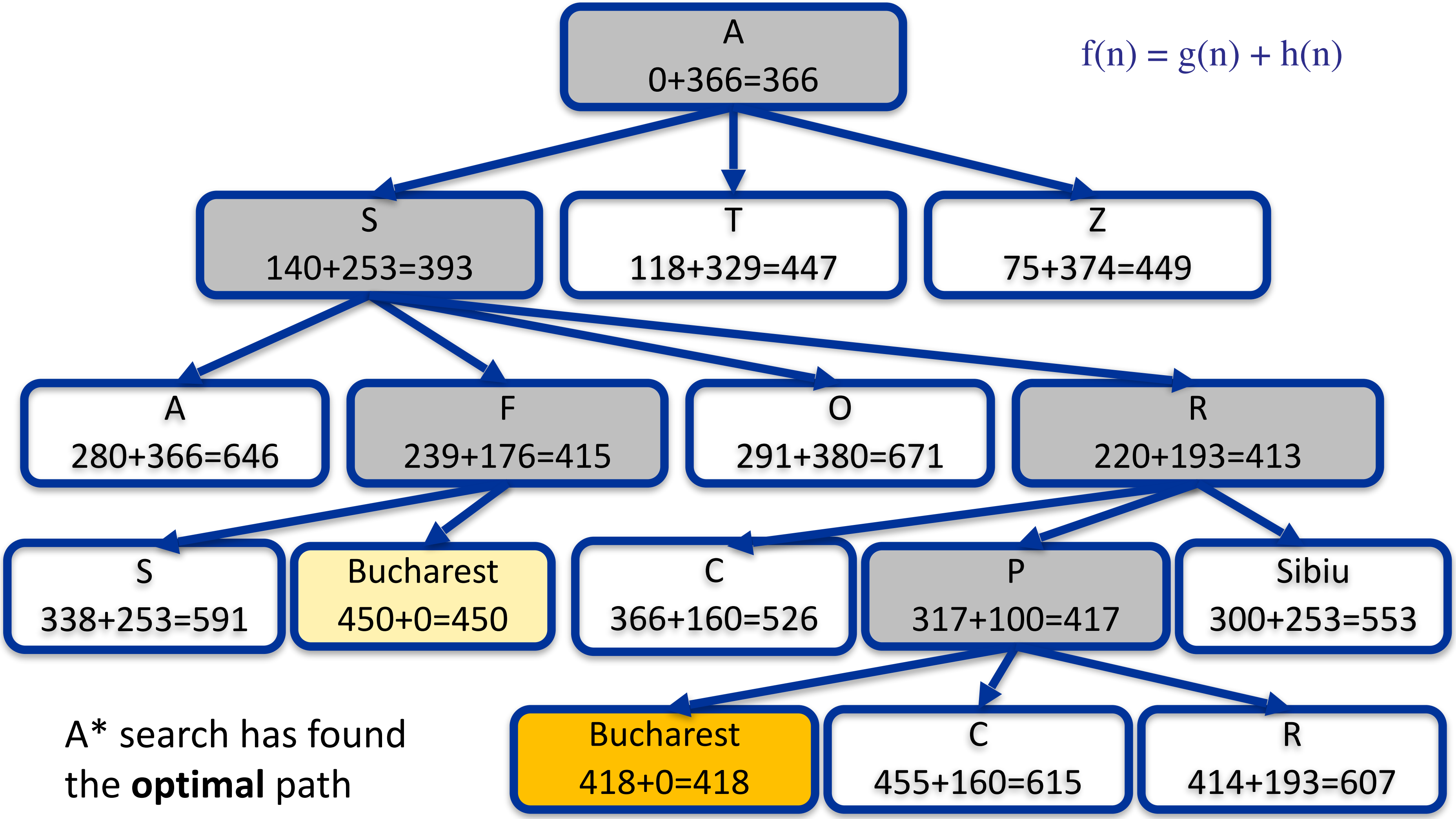


$f(n) = g(n) + h(n)$

A* search has found one possible path …

But: A* expands P next
Greedy search would **not** do that.

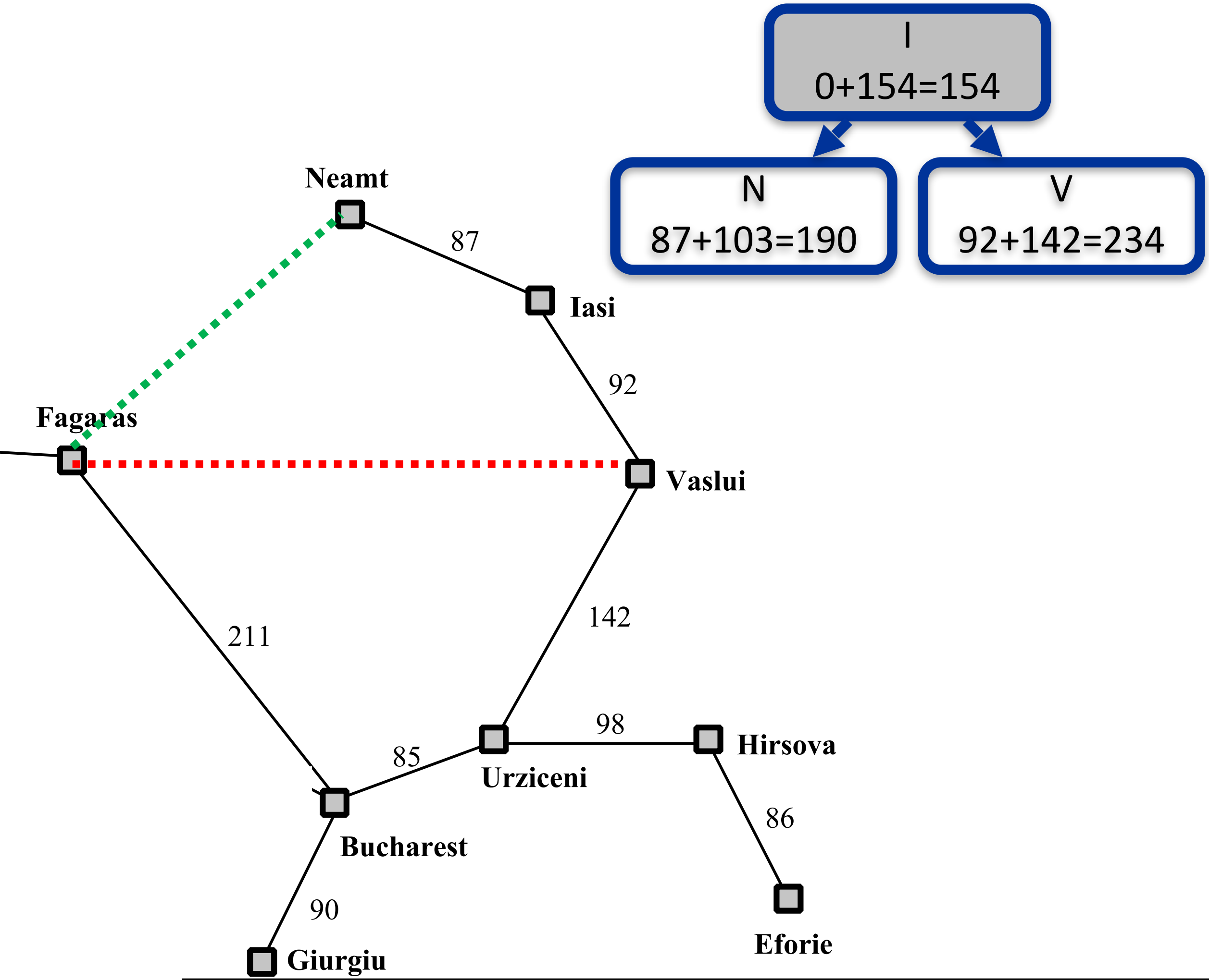
A* search



$f(n) = g(n) + h(n)$

A* search has found the **optimal** path

Avoiding loops in A* search: Find a path from Iasi to Fagaras



City 1	City 2	SLD
I	F	154
N	F	103
V	F	142
U	F	205
Bucharest	F	176

City 1	City 2	Cost
I	N	87
I	V	92
V	U	142
U	Bucharest	85
Bucharest	F	211

Avoiding loops in A* search: Find a path from Iasi to Fagaras

City 1	City 2	SLD
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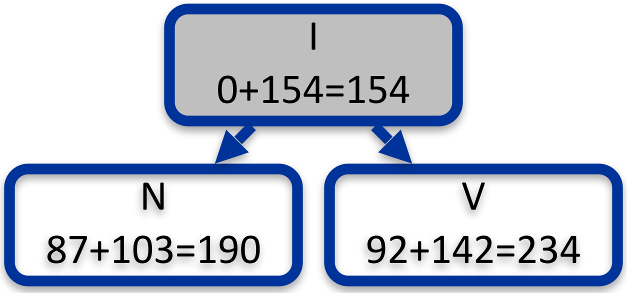

City 1	City 2	Cost
I	N	87
I	V	92
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Always expand the node with the lowest value of the h function

Avoiding loops in A* search: Find a path from Iasi to Fagaras

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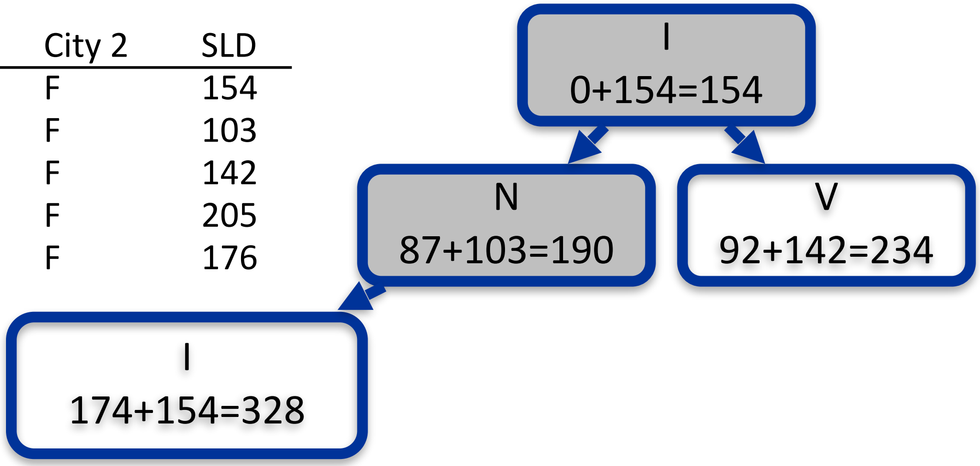

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Avoiding loops with A* search: Find a path from Iasi to Fagaras

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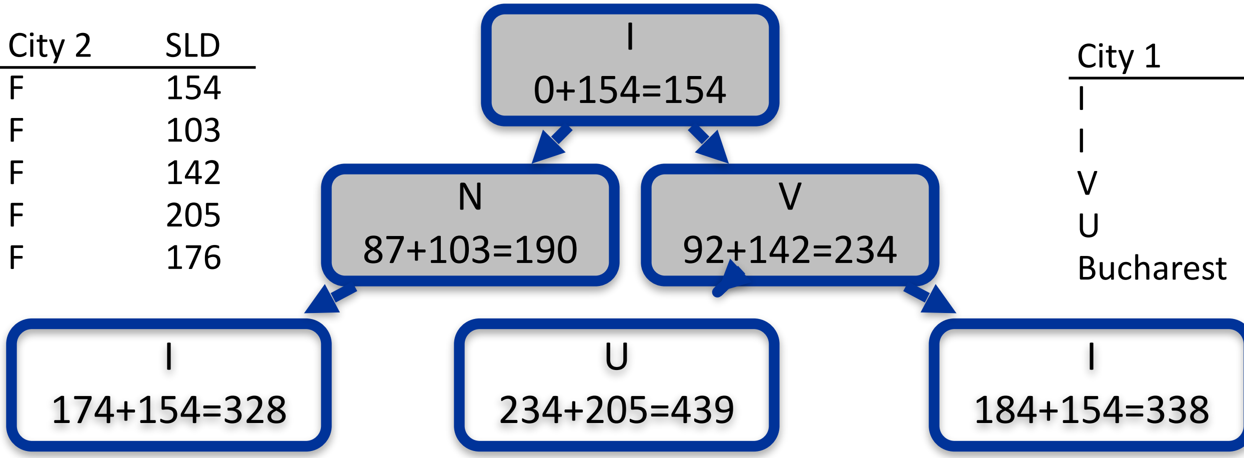

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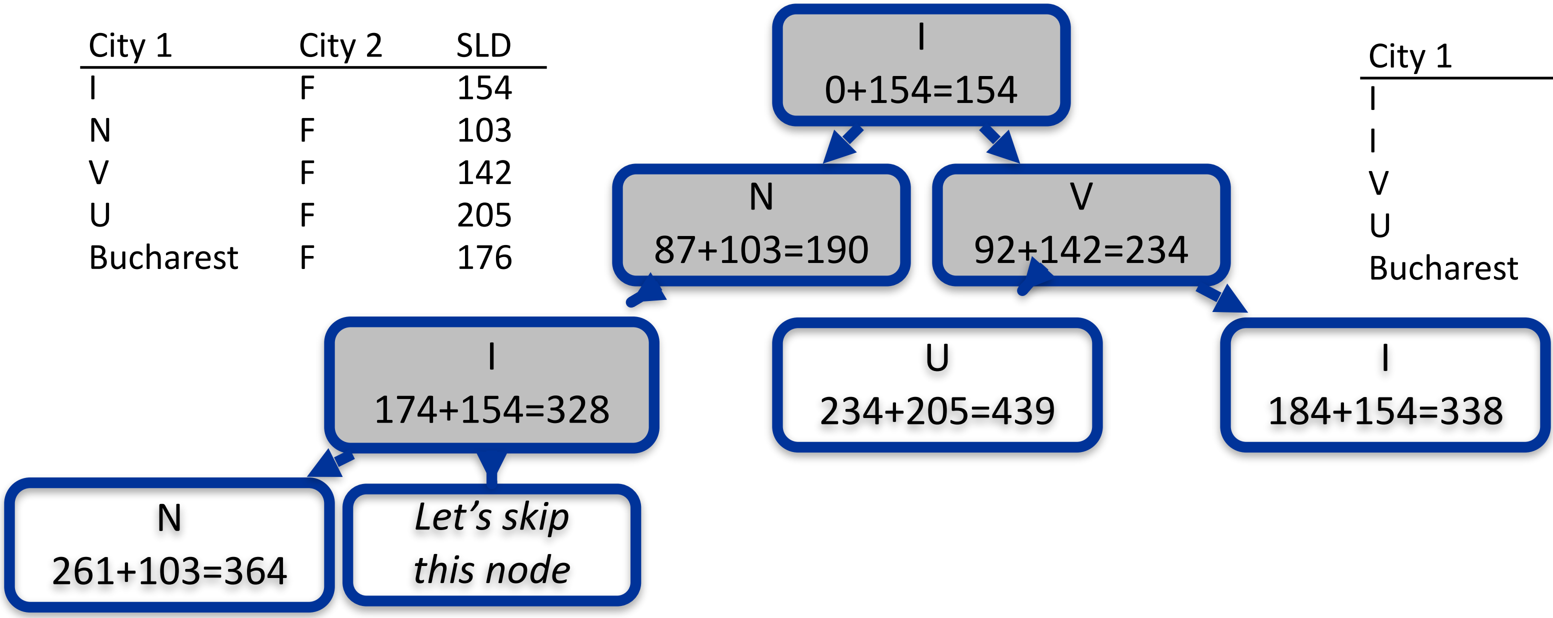

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We skip irrelevant nodes to keep the visualization readable.

Avoiding loops in A* search: Find a path from Iasi to Fagaras

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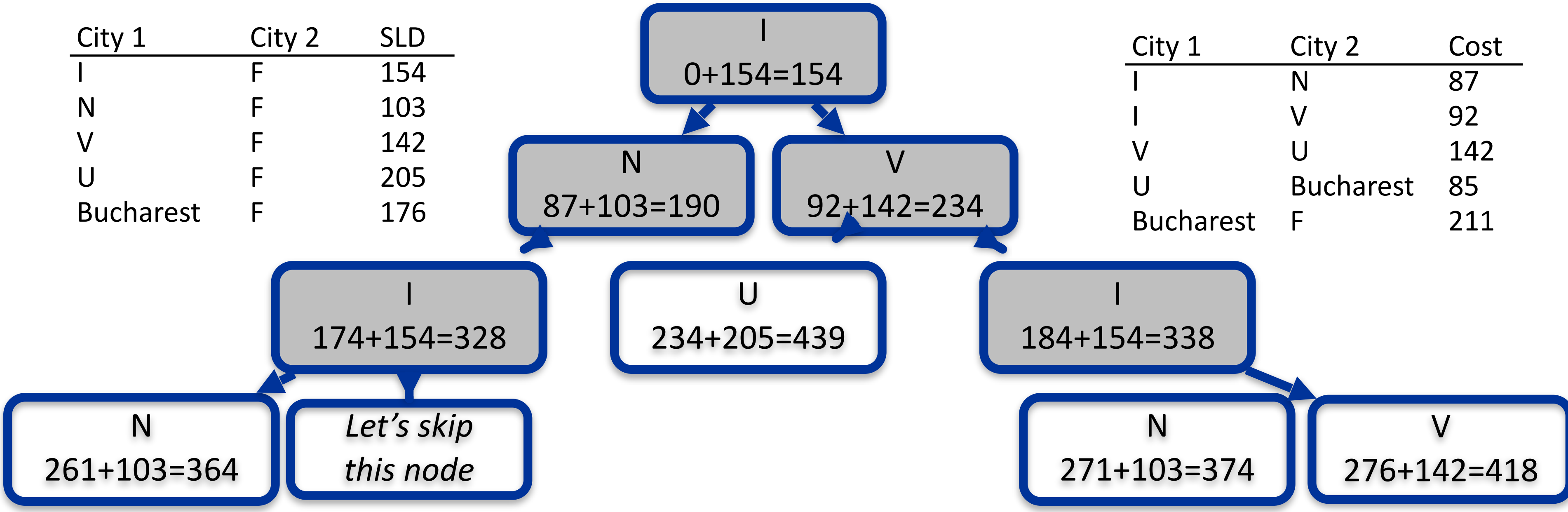

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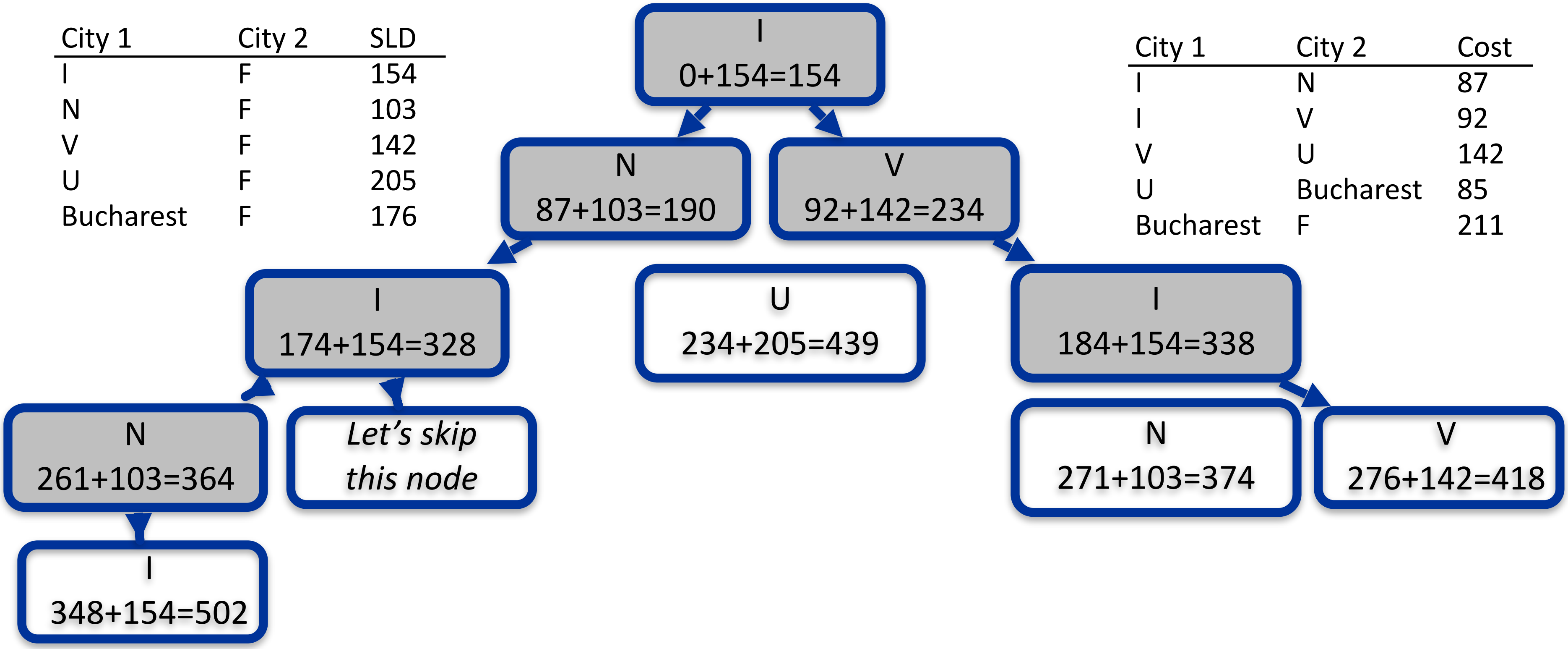
U	F	205
Bucharest	F	176



City 1	City 2	Cost
I	N	87
I	V	92
V	U	142
U	Bucharest	85
Bucharest	F	211

Avoiding loops in A* search: Find a path from Iasi to Fagaras

City 1	City 2	SLD
I	F	154
N	F	103
V	F	142
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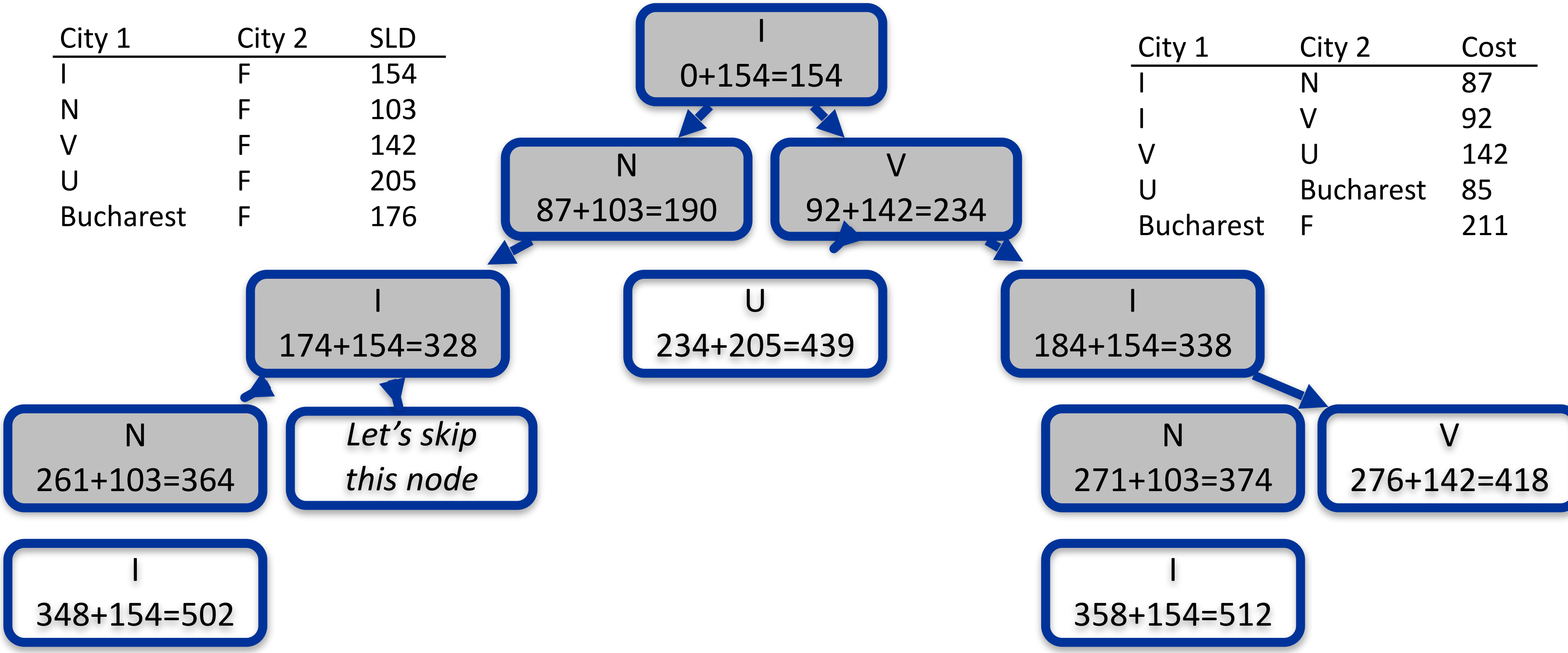

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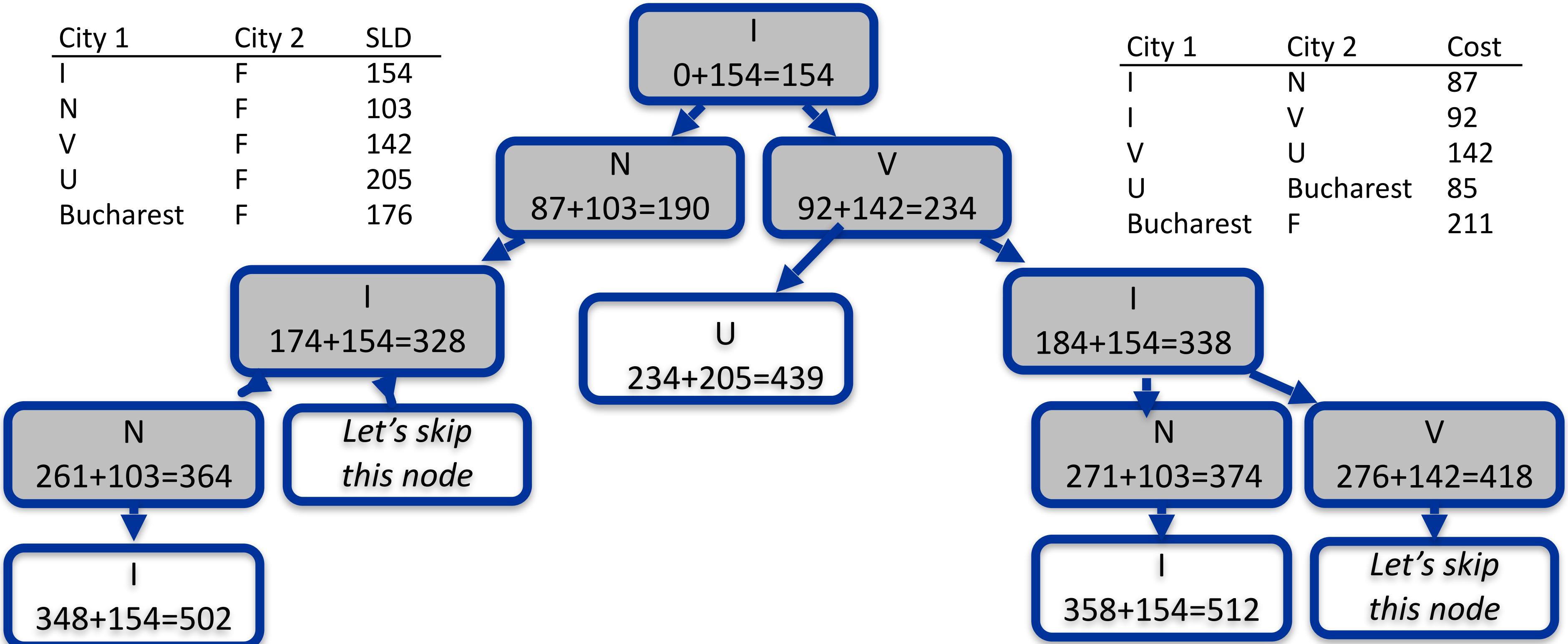

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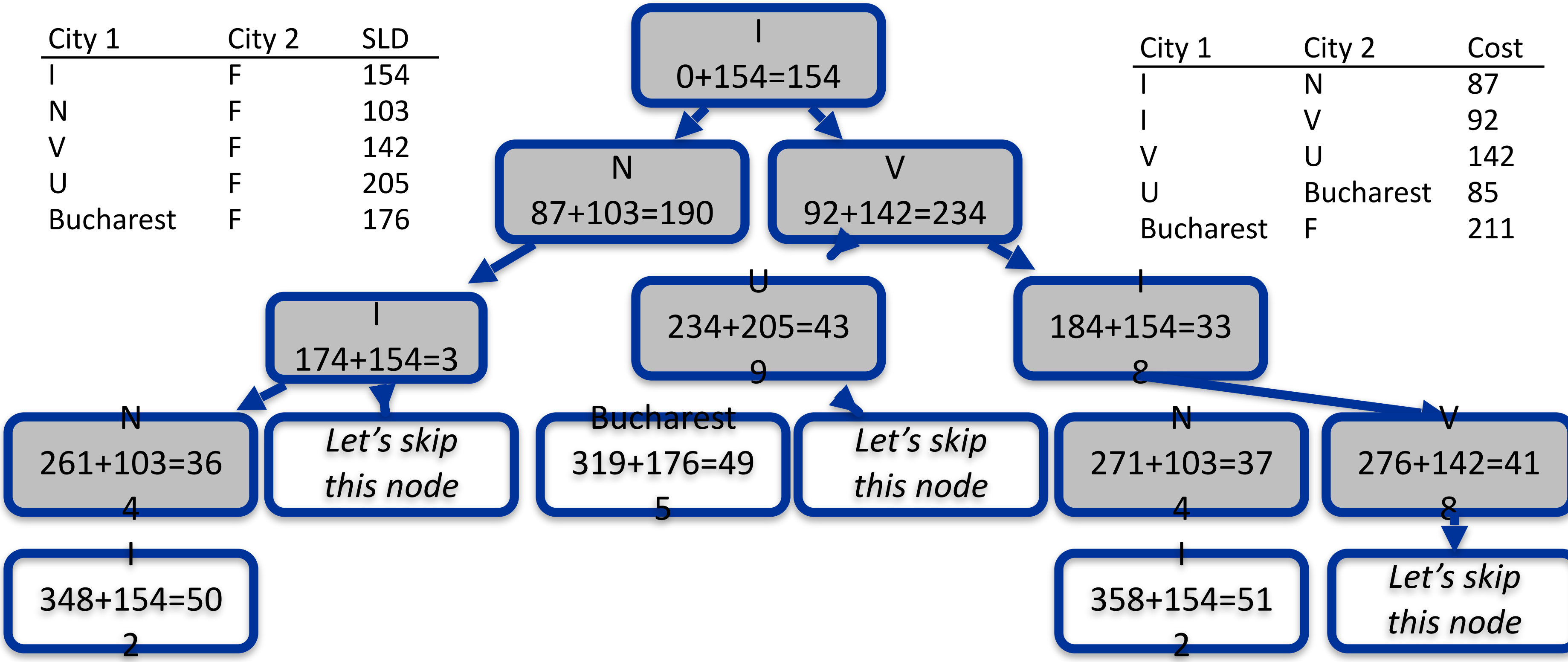

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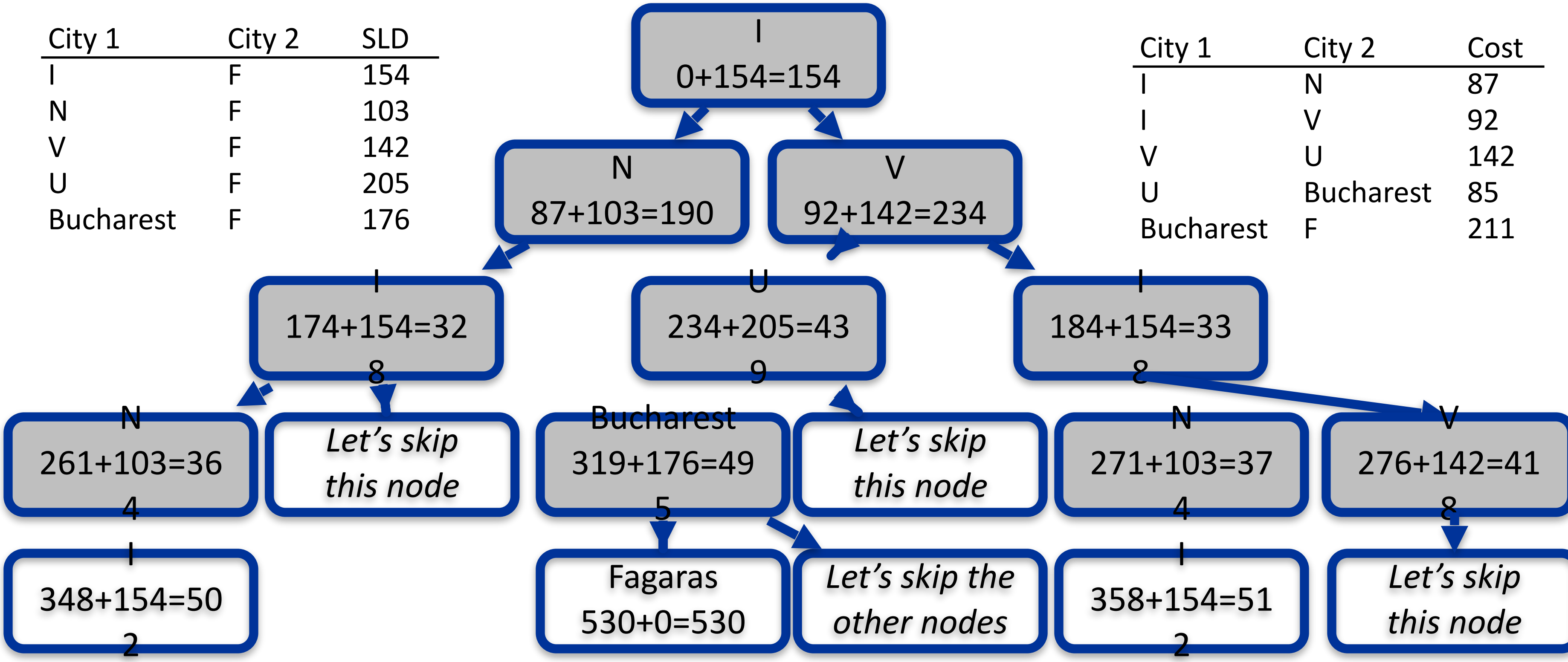

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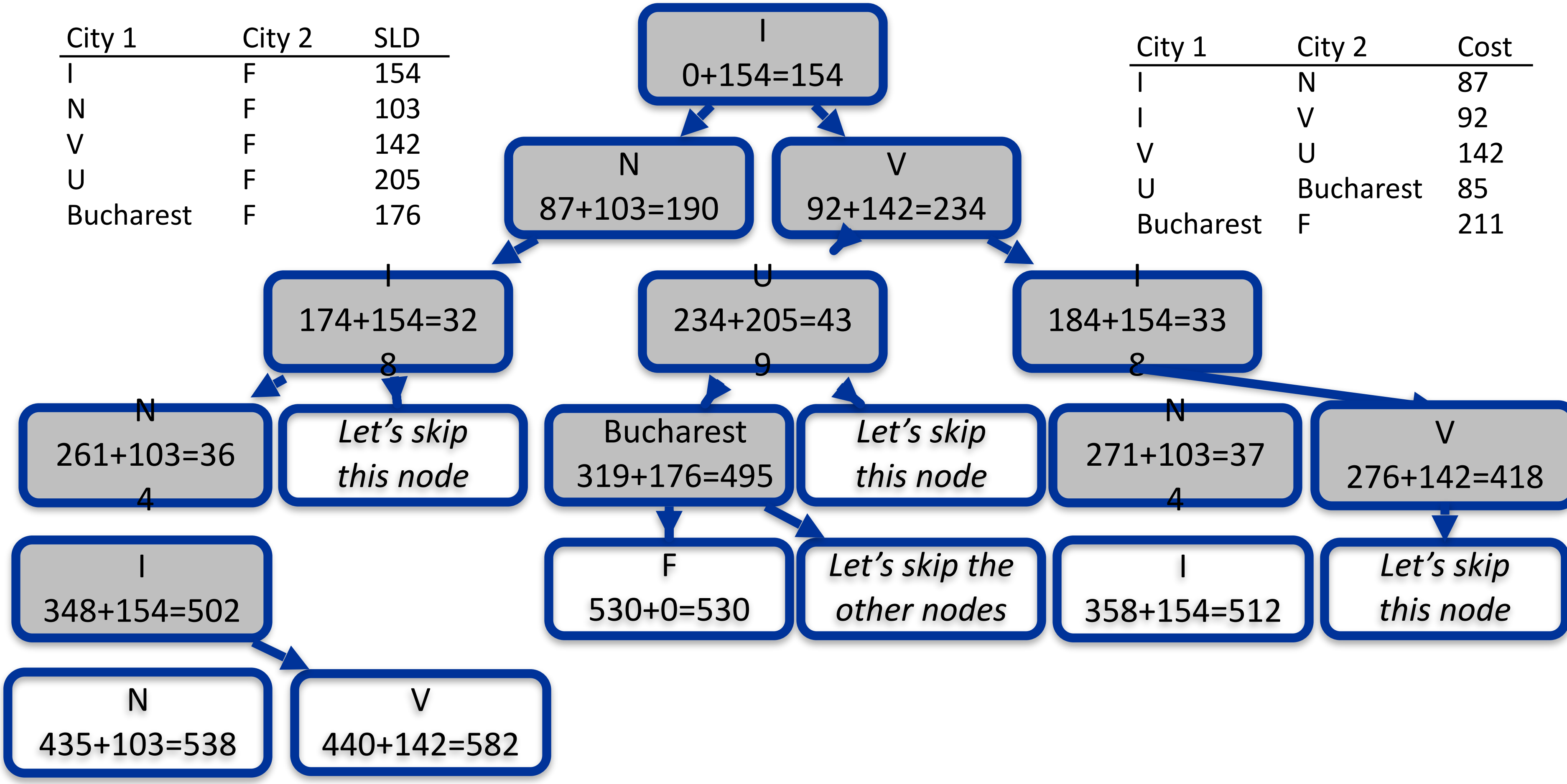

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Find a path from Iasi to Fagaras

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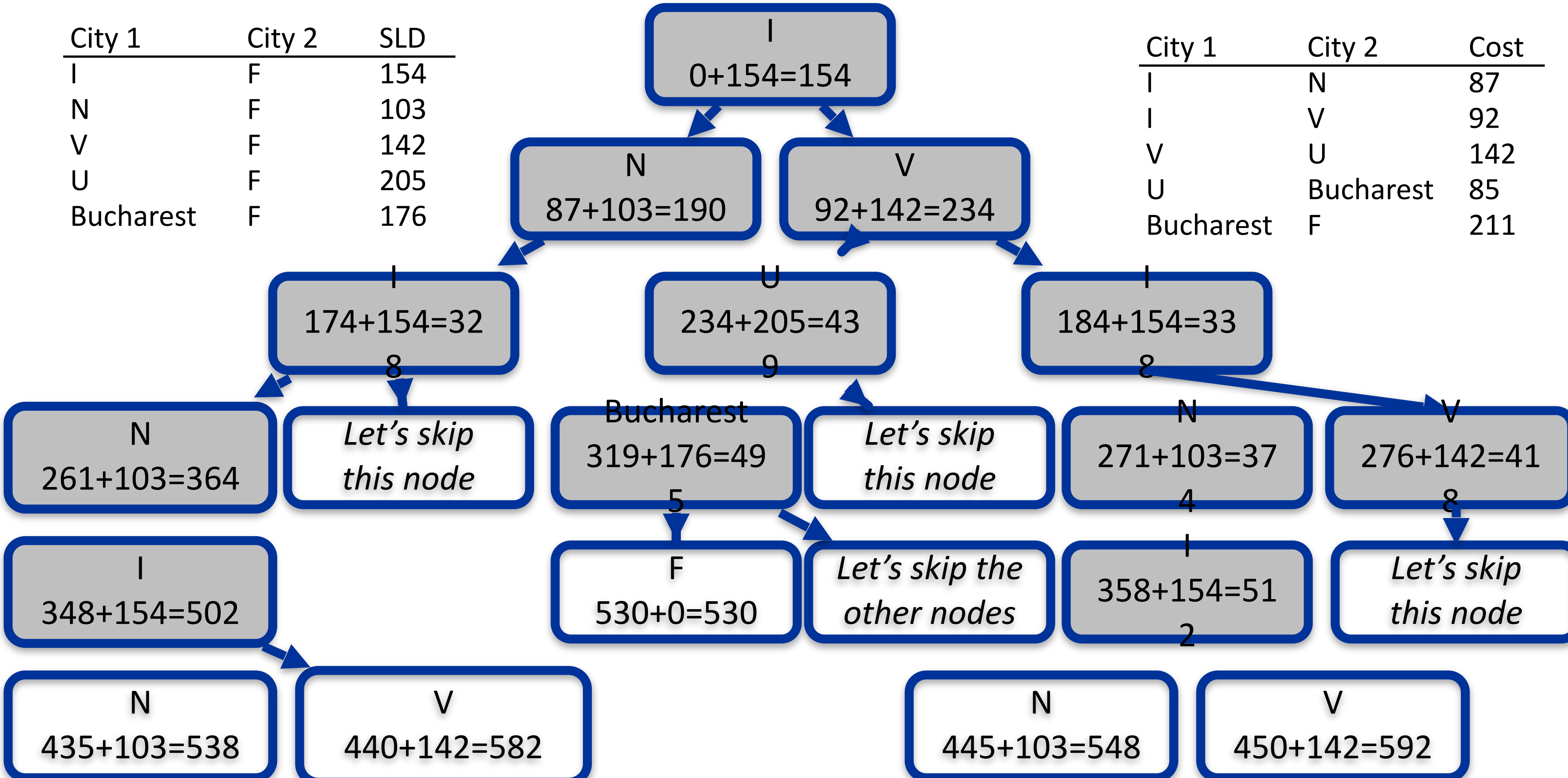

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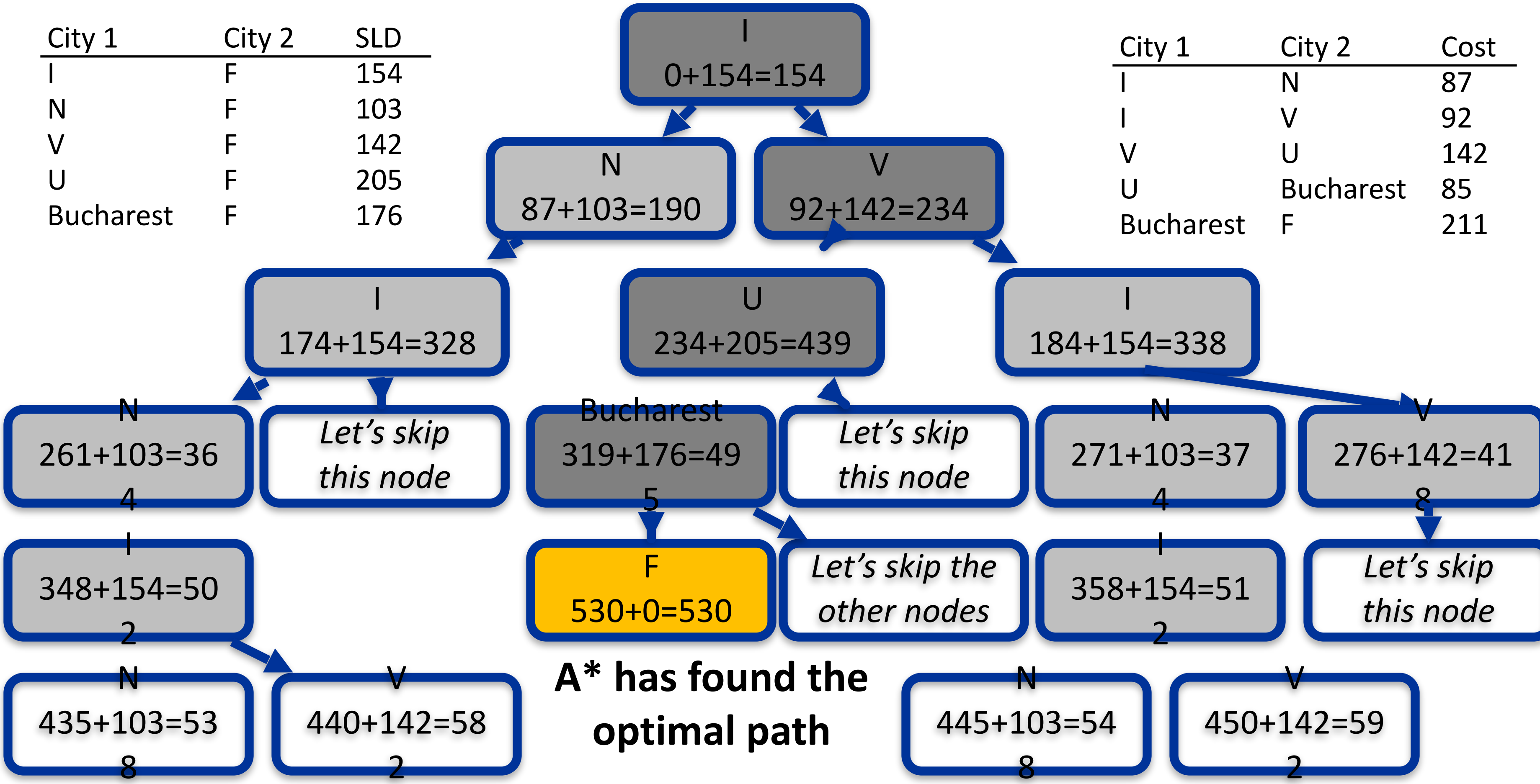

All other unexpanded nodes have a higher value

Properties of A*

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Completeness

- **Yes**, unless there are infinitely many nodes with $f(n) < f(Goal)$

Optimality

- depends **on the heuristic** $h(n)$

Time Complexity

- **the number of nodes grows exponentially** unless the error between the true cost $h^*(n)$ and the heuristic $h(n)$ grows logarithmically.

$$\left| h(n) - h^*(n) \right| \leq O(\log h^*(n))$$

Space Complexity

- **has to keep all nodes in memory, typically the main problem with A***

Addressing the space problem

- High memory consumption is the main problem with A* → **memory-bounded algorithms**

- **Iterative-deepening A***
  - like iterative deepening search
  - iteratively increase limit of the function ($f = g + h$)

- **Recursive BFS**
  - recursive algorithm that attempts to mimic standard BFS with linear space
  - keeps track of the cost of the best alternative path available from any ancestor of the current node

- **(Simple) memory-bounded A***
  - drop the worst leaf node when memory is full

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Heuristics

Admissible heuristics

A heuristic $h(n)$ is **admissible**

- if it **never overestimates** the true **true cost** $h^*(n)$

$$\forall\, n:\ h(n) \leq\ h^*(n)$$

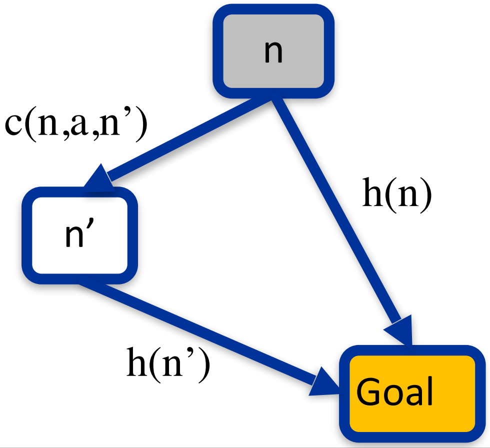
Consistent heuristics

A heuristic $h(n)$ is **consistent**

- if **the estimated cost from n to the goal** is always less than the sum of **the path cost to any successor n' of n** and t**he estimated cost from n' to the goal.**

$$\forall\, n:\ h(n) \leq\ c(n,\ a, n') + h(n')$$

→ **Triangle inequality**



Properties of heuristics for searching

- Every **consistent** heuristic is **admissible**.
- The reverse case is not always true (it rarely occurs).

Properties of A*

Completeness

- **Yes**, unless there are infinitely many nodes with $f(n) < f(Goal)$

Optimality

- if $h(n)$ is **consistent**

Time Complexity

- the number of nodes grows exponentially unless the error between the true cost $h^*(n)$ and the heuristic $h(n)$ grows only logarithmically.

$$\left| h(n) - h^*(n) \right| \leq O(\log h^*(n))$$

Space Complexity

- has to keep all nodes in memory, typically the main problem with A*

Today

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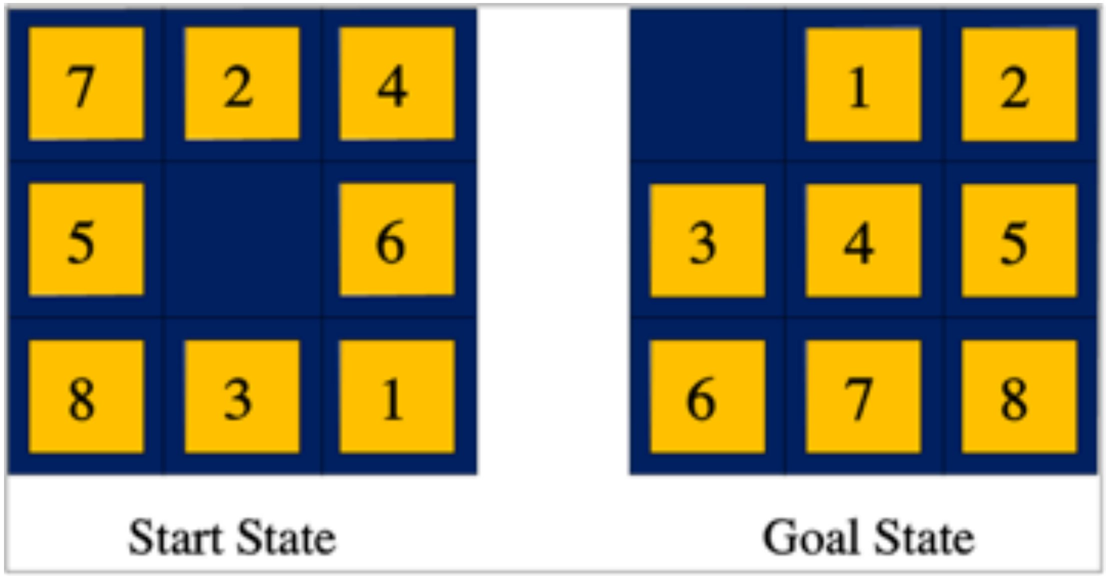
How do we define a good heuristic?

Try to relax the problem

- A problem with a **fewer restrictions** on the **environment** is called **a relaxed problem**.

- The **cost of an optimal solution to a relaxed problem** is an **admissible heuristic** for the **original problem**.

Admissible heuristic for the 8-puzzle?



Admissible heuristic for the 8-puzzle

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- **Relax** the rules of the **8-puzzle** so that **a tile can move anywhere**

→ $h_{MIS}(n)$ = **number of misplaced tiles**

- **Relax** the rules of the **8-puzzle** so that **a tile can move to any adjacent square (not only empty ones),**

→ $h_{MAN}(n)$ = **Manhattan distance of each tile to its final position (summed up)**

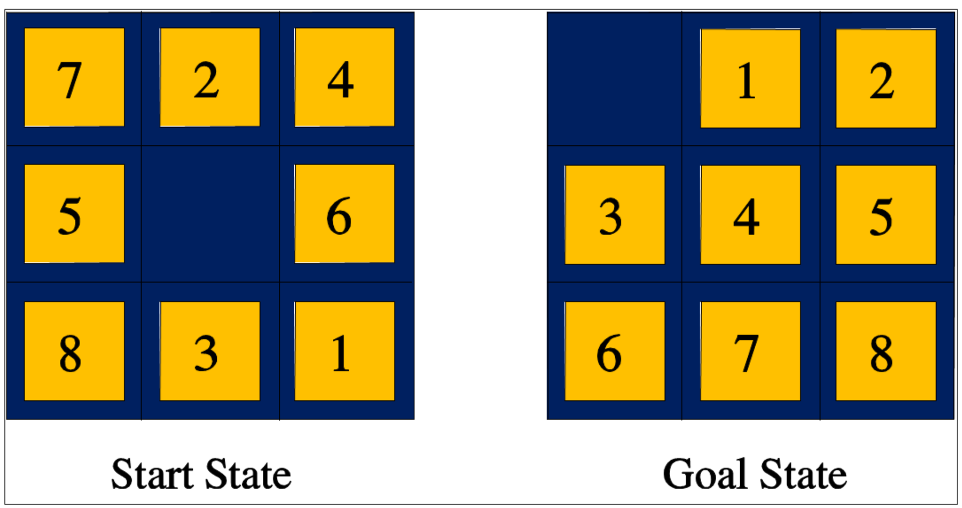
Admissible heuristics for 8-puzzle

$h_{MIS}(n)$ = number of **misplaced tiles**

- **admissible** because **each misplaced tile must be moved at least once**

$h_{MAN}(n)$ = **Manhattan distance to final position (summed up over all tiles)**

- **admissible** because this is **the minimum distance of each tile to its target square**



$h_{MIS}(start)$ = 8

$h_{MAN}(start)$ = 18

$h^*(start)$ = 26

Which admissible h to choose?

If two heuristics are **admissible, choose the one that dominates the other one.**

Consider two **admissible** heuristics h_1 and h_2 :

h_2 **dominates** h_1 if h_2 always estimates higher or equal costs than h_1

What if more than one admissible h exist

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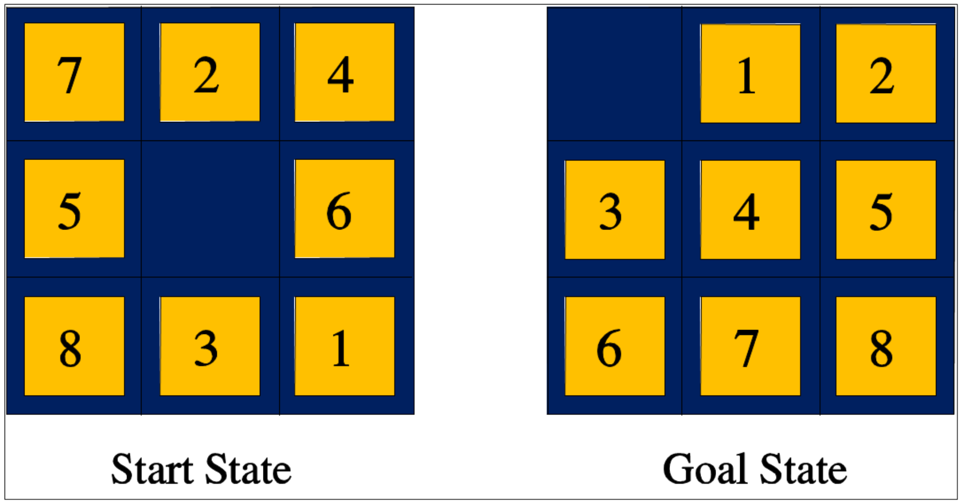


$h_{MIS}(start) = 8$

$h_{MAN}(start) = 18$

$h^*(start) = 26$

- h_{MAN} dominates h_{MIS} because both are admissible and each misplaced tile needs to be replaced at least once.

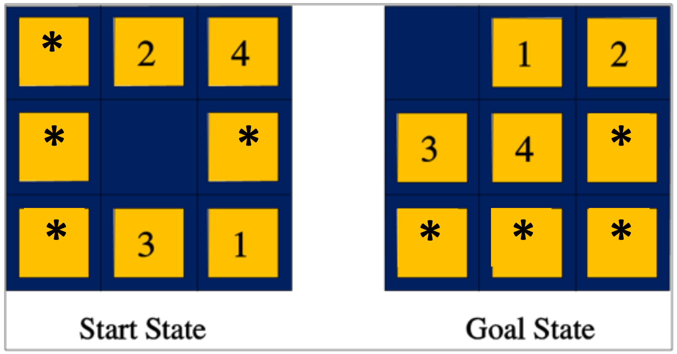
- The dominating heuristic is better because it will be closer to the true cost h^*.

Pattern databases

Admissible heuristics can also be derived from the solution cost of a sub-problem of a given problem

- this cost is a lower bound on the cost of the real problem

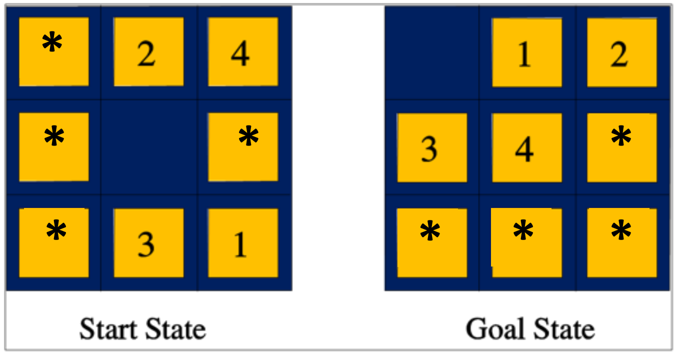
A pattern database (PDB) is a heuristic function implemented as a lookup table that stores the lengths of optimal solutions for subproblem instances.



Pattern databases

Constructed once for all by searching backwards from the goal and recording every possible pattern

Example: store exact solution costs for solving 4 tiles of the 8-puzzle



Pattern databases for chess

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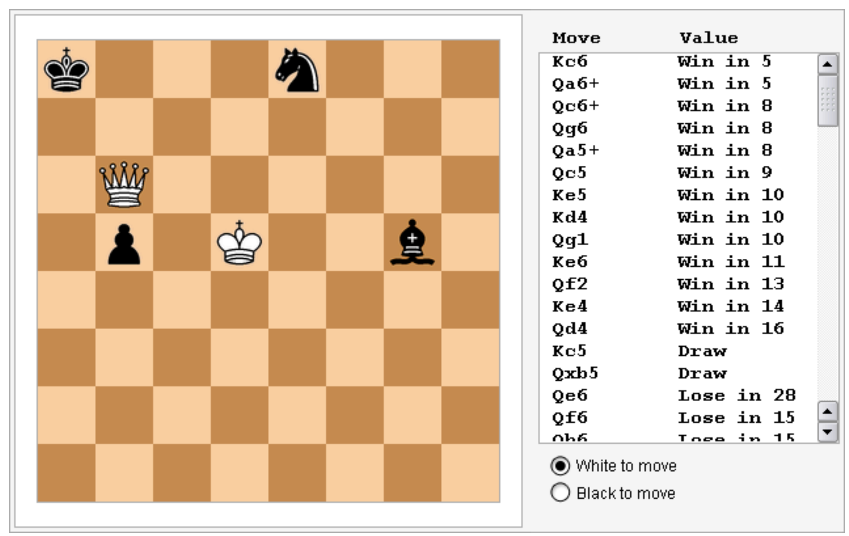
- Storing patterns from the start is infeasible.

Depth	Nodes
0	1
1	20
2	400
3	8,902
4	197,281
5	4,865,609
6	119,060,324
7	3,195,901,860
8	84,998,978,956
9	2,439,530,234,167
10	69,352,859,712,417
11	2,097,651,003,696,806
12	62,854,969,236,701,747
13	1,981,066,775,000,396,239
14	61,885,021,521,585,529,237
15	2,015,099,950,053,364,471,960

- ◊ Storing patterns by looking at the end?
- ◊ 2012: database complete for all moves for seven remaining chess pieces
- ◊ 140 terabyte



https://chessprogramming.wikispaces.com/Perft%20Results

https://de.m.wikipedia.org/wiki/Endspieldatenbank

Summary

- **Informed search algorithms using heuristics**
  - Greedy **best**-first search ✅
  - A* search ✅
- **Heuristics**
  - Admissible ✅
  - Consistent ✅
  - How do we define and choose a good heuristic? ✅

Readings

Mandatory

- Russell & Norvig:
  Informed (Heuristic) Search Strategies
  3.5.1 & 3.5.2, p. 92-102

Optional

- Rest of 3.5