Lancaster University

MPhys Project Report

Muon Tomography

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Abstract

This report is about the investigation of the multiple Coulomb scattering of muons through four materials: Water, iron, lead and uranium using two simulations. The first simulation generates a histogram of the scattering strength, λ, for each material whilst the second simulates the movement of muons through a set up of detectors, material and magnetic field. Increasing the muon momentum to 5GeV from a base 3GeV decreased the mean scattering strength and decreasing the muon momentum to 1GeV increased the mean scattering strength and made it easier to tell the materials apart. Decreasing the range of angles at which the muons entered the material made it easier to tell the materials apart but decreased the mean λ whilst the opposite happened for increasing the range of angles. Over small length scales (1m) the detector resolution required to tell apart muons which scattered from water or uranium would need to be on the magnitude of 10^{-7}m whilst increasing rapidly over a larger length scale to 10^{-4}m (10m) to the point at which muons scattering off either uranium or water could easily be told apart just by inspection.

Acknowledgements

Thank you to Harald and Rob for your guidance throughout this project.

Contents

1 Introduction

Tomography is the process through which several images of an object are taken, where each image is a different cross section of the object. These cross sections are then reconstructed into a final image through the use of a reconstruction algorithm. This is done by the use of any penetrating wave, or high energy particle. Muons are used in this investigation. Information of the muon such as position and mometum can be measured before and after passing through a material and as such information can be obtained about the material, such as it's density, atomic make up and approximate size [1]

Muons have been used to probe matter for several decades now, the first instance of this was by E.P George in 1955. He was investigating the depth of the overburden of a tunnel in Australia, to do this he measured the muon flux outside the tunnel and then inside the tunnel and was able to approximate the depth by looking at the ratio [2]. Since then muons have been used for a wide variety of applications. Some notable examples are L. Alvarez in 1970, who wanted to search for hidden chambers in one of the pyramids of Giza through a similar process to E.P George, by measuring the change in muon flux [3]. Other large scale objects such as volcanoes have also been probed. H.K.M Tanaka and his team wished to investigate the internal structure of the Earth's crust via the density distribuiton of Mt. Asama, to do this they used a similar process to the previous two examples through the attenuated flux [4]. Not only can muon tomography be used to probe large objects on the scale of 100m, the can also be used on smaller objects on the length scale of 10m. A notable example is the Fukishima reactor after the earthquake hit in 2011. The power plant was to unsafe too send someone in to check the damages due to the radiation, muon tomograhpy was used to model the reactor and locate where the spent fuel rods were located, results are still waiting to be obtained [5].

This report describes the investigation into finding the optimal conditions for measuring the properties of muons before and after penetrating the object. Specifically, what sort of detector resolution is needed in order to tell apart different elements that muons are penetrating. Whether using cosmic ray muons that are produced naturally or a muon beam where the muon flux is a lot higher is better. What sort of muon momentum gives the biggest distinction between the different elements and the feasibility of muon tomography at varying length scales, from 1m to 10m.

2 Theory

2.1 Muons

Muons are one of the fundamental particles of the universe and are part of the lepton group. They are the second most massive of the group with a mass of 106MeV, this is 200 times the mass of their cousin, the electron and around 15 times smaller the mass of the tau, the most massive particle of the lepton group. Muons are negativley charged and have a life time of 2.20μs, which is quite long compared to other unstable particles[6]. Due to these properties muons are able to probe more deeply into matter than particles with less mass. Their large mass means they radiate less energy while inside matter, meaning it takes longer for muons to lose all their energy.

2.1.1 Muon Production

Almost all naturally occuring muons come from cosmic rays. Cosmic rays are high energy protons and α particles with their main source being the sun and they also come from deep space, usually produced in supernovae. These high energy particles collide with particles in the upper atmosphere of Earth causing a hadronic shower. This shower creates a range of particles, with π^+, π^- and π^0 being among the most common. The π^0 decays into two gamma rays which lose their energy relativley quickly due to various electromagnetic processes. The charged pions, π^+ and π^-, decay into muons and neutrinos via the weak interaction: $\pi^+ \rightarrow \mu^+ + \nu_\mu$ and $\pi^- \rightarrow \mu^- + \bar{\nu_\mu}$[9]. As mentioned before muons have a relatively long life time, this means that at the surface of the Earth the majority of particles seen are cosmic ray muons. Muons hit the Earth at a rate of 1 muon per square centimetre per minute.

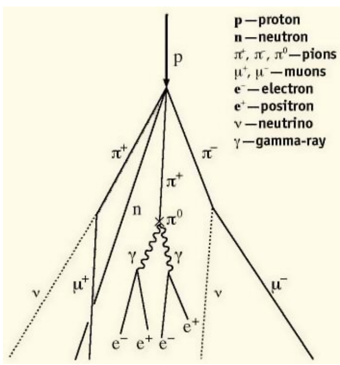

Figure 1: A diagram showing the production of μ^+ and μ^- via a cosmic ray shower.

Muons are incident on the surface of the Earth over a large distribution of angles and a wide variety of energies. The angular distribution of muons is proportional to $\cos^2(\theta)$ with the highest muon flux at $\theta = 0$, where θ is the angle between the path of the muon and a line at right angles to a detector if it were to be lay flat. The mean energy of muons at sea level is around 4GeV, at larger θ muons will have a higher energy as muons with low energies will decay before they reach the surface of the Earth.

For the purpose of the investigation, it makes more sense to use a muon beam, this is due to the fact that there is a much higher density of muons from a beam and they all move in near enough the same direction. The process of aquiring a muon beam is relativley simple: Start by having a high intensity beam of protons and firing that into a target. The resulting collision will form protons, neutrons and other particles such as π^+, π^-, π^0 K^+, K^- and K^0. A magnetic horn is then used to focus the charged particles, negativley charged particles will bend in one direction whilst positivley charged particles will move in the opposite direction, this also filters out the particles of neutral charge as they are unaffected by the magnetic field[10]. Within fifty meters the pions will decay: $\pi^+ \rightarrow \mu^+ + \nu_\mu$ and $\pi^- \rightarrow \mu^- + \bar{\nu_\mu}$. This will form a beam of μ^- of around 200MeV[11]. At this point, the beam is too low energy, around 1 to 3 GeV is required. The spread of the muons is also still quite large as they are moving in different directions, which results in a diverging beam.

To increase the energy of the beam, an oscillating electric field can be used along side a series of cells. The oscillation is timed in such a way that the muons are acellerated through the cells, increasing their energy. This also makes it so that the muons move with roughly the same momentum, instead of some being faster than others. The spread of the beam can be reduced through a process known as muon cooling[11].

2.1.2 Muon Physics

When muons travel through a material they are affected in a number of ways, this can be useful for muon tomography.

Firstly, muons can collide with electrons in the atoms of the material, this will cause the electron to ionize, freeing it from the atom. The muon loses energy equal to that of the ionization energy of the atom. Muons also lose energy due to photo nuclear interactions and Bremsstrahlung radiation, these radiated photons can produce an e^+ and e^- pair. The average rate of energy loss of the muon is given by the equation:

$$-\frac{dE}{dX} = a + bE, \tag{1}$$

where a is the energy loss due to ionization and b is the cumulitive energy loss due to Bremsstrahlung, photonuclear interactions and pair production[12]. One can measure this energy loss and be able to infer about the material, however measuring muon energy accurately is challenging and costly. A way around this would be to measure the change in muon flux, which is simply the rate at which muons pass through a given area.

Muons have a high mass, around 207 times the mass of an electron, resulting in muons not decelerating/accelerating as much as other particles when in the presence of an electromagnetic field. This means that the rate at which muons lose energy is quite low, allowing them to penertrate matter deeply[13]. For example a muon produced by cosmic rays at sea level is around 4 GeV, this is enough energy to penerate a few meters into a dense material such as uranium and even farther in something such as rock [8]. So while this is useful for inferring about large scale objects such as volcanoes or pyramids (see my literature review for more details [14]), this method can't be applied to smaller objects as the change in muon flux would be too little.

The second way in which muons are affected when travelling through a material is the multiple coulomb scattering. This occurs from muons interacting with atoms in the material, deviating them by a given angle. The deviation is due to the muon interacting with the electrons in the atom, this is known as the coulomb interaction. These scattered angles are usually small in size and the resultant scattering and displacement distributions are Gaussian in shape, with a mean of 0 and a variance given by:

$$\theta_0 = \frac{13.6\mathrm{MeV}}{\beta c p} z \sqrt{\frac{x}{X_0}} \Big[1 + 0.038 \ln(x/X_0) \Big], \tag{2}$$

where p is the scattered particles momentum, βc is the velocity of the particle, z is the charge number of the particle and x/X_0 is the thickness of the material the particle is passing through in radiation lengths. For the case of the muon, $\beta = 1$ and $z = 1$.

One can approximate the mean scattering angle if the logarithmic term is removed in equation 2:

$$\sigma_\theta \cong \frac{13.6\mathrm{MeV}}{p} \sqrt{\frac{x}{X_0}}. \tag{3}$$

A nominal muon momentum (most cases throughout this investigation use 1GeV) is then set and the mean square scattering of nominal muons per unit depth of a material is given to be:

$$\lambda_{mat} = \left(\frac{13.6}{p_0}\right)^2 \frac{1}{X_{0,\mathrm{mat}}} \cong \sigma^2_{\theta_0,\mathrm{mat}} \tag{4}$$

λ is known as the scattering strength. λ varies a lot with the atomic number, Z of the material. The higher the Z the greater λ is, this is because there is a dependence of $1/X_0$ and higher Z objects have smaller radiation lengths, therefore resulting in a larger mean square scattering, λ. While measuring the energy loss of a muon is not feasible, measuring the scattering is possible through the use of detectors and can allow us to infer about the material through the use of λ.

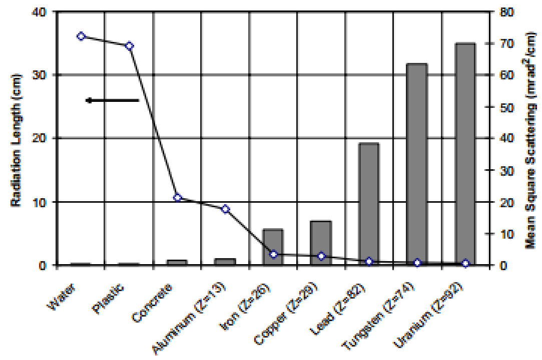

Figure 2: A graph showing various elements with their respective scattering strength and radiation length

2.1.3 Measuring Muon Momentum

In particle physics experiments, to measure the momentum of a particle, a magnetic field can be introduced and the radius of curvature can be measured, allowing you to calculate the particles momentum. When a charged particle is present in a magnetic field, the particle experiences a force:

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}, \tag{5}$$

where q is the charge of the particle in coulombs, $\mathbf{B}$ is the magnetic field in Tesla and $\mathbf{v}$ is the velocity of the particle in ms^{-1}. This causes the particle to move in a circular motion. The momentum of the particle $p_\perp$, can be calculated by measuring the sagitta of the curve (the component of momentum parallel to the magnetic field, $p_{\parallel} = 0$). The sagitta can be calculated using the Pythagorean theorem and some simple rearrangement:

$$s = \frac{L^2}{8r}, \tag{6}$$

where s is the sagitta, L is the measured length of track segment and r is the radius [15].

3 Simulating Muons

Configuring an experiment to investigate the multiple Coulomb scattering of muons through a device under test wouldn't be feasible, this is due to a number of reasons: Firstly would be the cost, obtaining multiple detectors, different blocks of elements of varying sizes, a muon beam etc. would be too expensive. Another would be health and safety, for my investigation I would need to use uranium which is a highly radioactive element and I do not have the apparatus required to contain that. If I were to perform this experiment in real life the muon detectors would not only detect the muons from the beam, they would also pick up cosmic ray muons. It would be impossible to tell the two different types of muons apart and as a result the data obtained wouldn't provide any significant results. I also wouldn't have the time or the space to perform such an experiment. A solution to all these problems would be to model the experiment through the use of a computer simulation.

3.1 λ Simulation

The first simulation was created by Daniel Parry and is simple in nature. The simulation outputs two 1D histograms of the scattering strength, λ for three different elements, iron, lead and uranium. The scattered angle is generated via a Gaussian with variance given by equation 4. The first histogram calculates λ from a generated value of the scattered angle and the material length. The second histogram takes into account the muon flux when calculating the scattered angle, to do this an incidence angle via a Gaussian with a mean of zero and a variance of 0.56 (which is a similar distribution to $\cos^2\theta$) is generated and then added on to the scattered angle. λ is calculated using a slightly different equation to equation 4, it takes into account direction in the x, y plane:

$$\hat{\lambda} = \left(\frac{p^2}{p_0^2}\frac{\theta_{xi}^2 + \theta_{yi}^2}{2L_i}\right), \tag{7}$$

where p is the muon momentum, p_0 is the nominal muon momentum. The value used for θ_{xi}, θ_{yi} and L_i changes depending on whether or not muon flux is taken into account for the calculations. If muon flux is not taken into account then θ_{xi} and θ_{yi} is simply the scattered angle in the x and y direction respectively and L_i is the length of the material($L_i = L_{mat}$), the muons are scattering through. If muon flux is taken into account then θ_{xi} and θ_{yi} is the sum of the generated incidence angle plus the scattered angle in the x and y direction respectively and L_i is the path length of the muon: $L_i = \sqrt{L_x^2 + L_y^2}$, where $L_{x,y} = \frac{L_{\text{mat}}}{\cos^2(\theta_{xi,yi})}$.

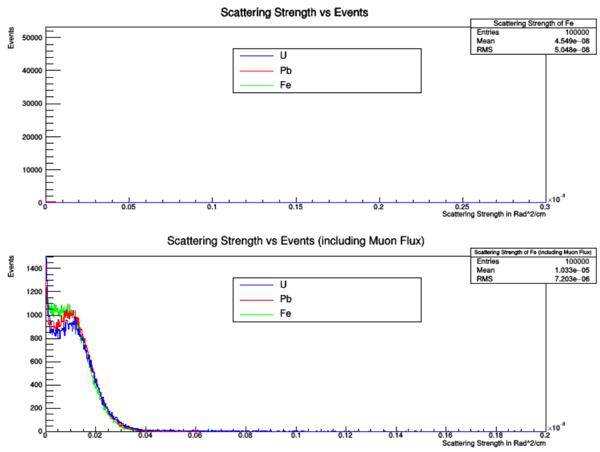

Figure 3: The output of the base code of simulation 1, with λ being calculated with and without muon flux (bottom and top histograms respectively)

To get familiar with the code, I included water as a fourth material and made the y-axis a logarithmic scale so it's easier to interpret.

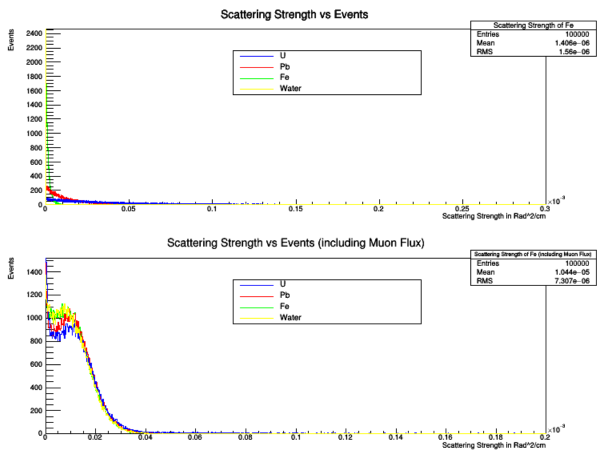

Figure 4: The output of simulation 1, with the inclusion of water

The uncertainty in muon momentum is: $\frac{\Delta p}{p} = E_p$, where Δp is the uncertainty in momentum. If this uncertainty is taken into account when calculating λ, then equation 7 is multiplied by a factor of $\frac{1}{(1+E_p^2)}$. The simulation was changed so a third 1D histogram of λ was output, this one using the uncertainty. It is worth noting that the muon flux was not taken into account with the inclusion of uncertainty.

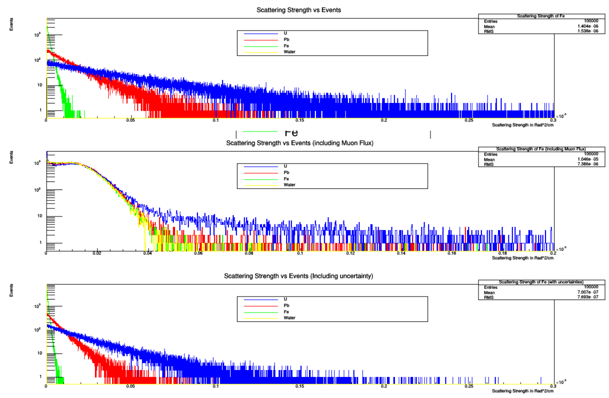

Figure 5: The output of λ simulation, with the inclusion of third histogram that takes the uncertainty of the momentum into account (bottom histogram)

An advantage of this simulation is you can generate a value of λ for a large number of muons (> 100000). As the motion of the muons is not being simulated, it's just a calculation, this results in better statistics and takes a lot less time to run than if you were to simulate the motion of that many muons. This simulation is fairly simple and cant really be built upon in any way other than adding more materials.

3.2 MCS Model Simulation

3.2.1 Simulation Goals

While this simulation is useful for seeing how λ varies for different elements it doesn't produce any other results of significance. An entirely new simulation was needed which could model the movement of the muons through a set-up of detectors and a device under test. This simulation was created by myself using ROOT and C++. Instead of generating values of λ instantaneously by simply inputting parameters, this simulation would model the multiple Coulomb scattering of muons through a material using a time step.

3.2.2 Simulation Set-up and Features

The overall set-up of the simulation is the same throughout: A muon beam which is placed at the start of the simulation, followed by two detectors, a magnetic field in order to measure the momentum of the muons, the device under test sandwiched between two detectors and finally two more detectors. The simulation is centred at 0.

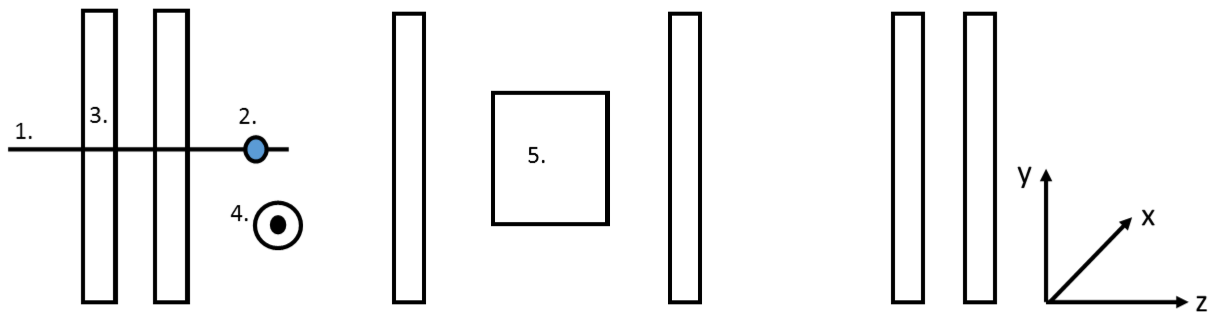

Figure 6: A simple diagram showing the set-up of the simulation. Not to scale.

1. The muon beam is simply a line of muons, separated by a time step, t_s. Each time step, if the number of muons in the simulation is not the same as the total number of muons desired in the simulation, then a muon is created at the origin. This muon will have momentum $\mathbf{P}$, where P_z is equal to the nominal muon momentum, P_x and P_y are generated via a Gaussian with a mean of 0, this allows there to be a spread in the beam.

2. The position of the muon, $\boldsymbol{S_\mu}$ is updated every time step via the Euler-Cromer algorithm. The muon is treated as a point and for most cases has an acceleration, $\mathbf{a}$ of 0.

3. The detector is used to record the position of the muon. Every time step, the position of the muon is checked against the position of the detector, $\boldsymbol{S_d}$, if the muon is in the bounds of the detector, (x_d, y_d, z_d), then that position is saved. To prevent multiple positions of the same muon being saved, a boolean is used to mark whether or not the muon has already been detected. As well as detecting the muon, the detector will also scatter it.

4. The magnetic field is used to measure the muons momentum (explained in the theory section). Every time step the position of the muon is checked against the position of the magnetic field, $\boldsymbol{S_b}$ if the muon is in the bounds of the magnetic field, (x_b, y_b, z_b), then the acceleration of the muon is set to the acceleration experienced by a charged particle according to the Lorentz force law (non-relativistic).

5. The device under test scatters the muon. Every time step, the position of the muon is checked against the position of the device, $\boldsymbol{S_m}$ if the muon is in the bounds of the device, (x_m, y_m, z_m), then the muon is scattered. The

scattering angle is generated via a Gaussian with a mean of zero and variance calculated via equation4. Similarly to the detector, a boolean is used to make sure the muon is only scattered once once inside the material.

The physical length of the simulation, z_{sim} (make a line in your diagram to help show) can be adjusted so the simulation can be ran over a range of different lengths. $x_{d,b,m}, y_{d,b,m}$ and $z_{d,b,m}$ can also be adjusted to change the size of the detector/magnetic field/device under test. At the start of the simulation values for $\boldsymbol{S_d}$, $\boldsymbol{S_b}$ and $\boldsymbol{S_m}$ are set. This is done by setting the z value of these positions to a percentage of z_{sim}. The radiation lengths of both the detector and device under test, X_{0d} and X_{0m} respectively can be adjusted, this is useful as depending on the scale of the simulation different detectors made of different materials are needed, i.e silicon for small length scales (1m) and plastic scintillator for large length scales (100m). Having an adjustable radiation length for the device under test means the muon can scatter through any material, for example, $X_{0m} = 0.3166$cm corresponds to uranium. The value of the variance used in the Gaussian to generate P_x and P_y is adjustable resulting in a looser or a tighter beam. The total number of muons in the simulation and the nominal momentum of the muons are also adjustable.

3.2.3 Sample output

The simulation ends when the value of the time variable equals a set value. Upon reaching this set value, several graphs are output:

The first set of graphs are x, y scatter plots of each detector in the simulation. Each point represents a muon which hit the detector at that position in the x, y plane. The x and y range on the graph correspond to x_d and y_d.

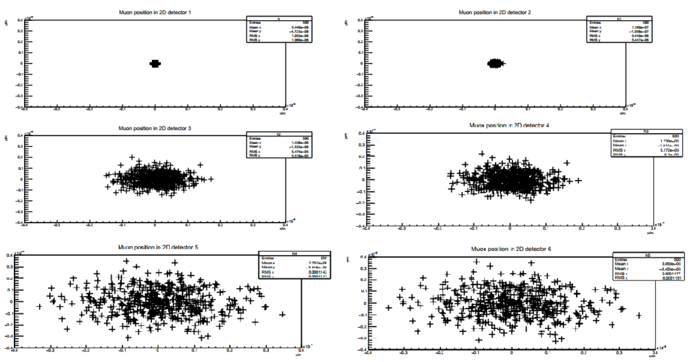

Figure 7: The first output of MCS model simulation simulation, an x, y scatter plot of each detector in the simulation

The second set of graphs produced are 1D scattered angle histograms. Specifically, the angle between $\boldsymbol{S_\mu}$ at detector 2 and detector 3 which corresponds to the deflection of the muon due to the magnetic field, and the angle between $\boldsymbol{S_\mu}$ at detector 3 and detector 4 which corresponds to the scattered angle from the material collision.

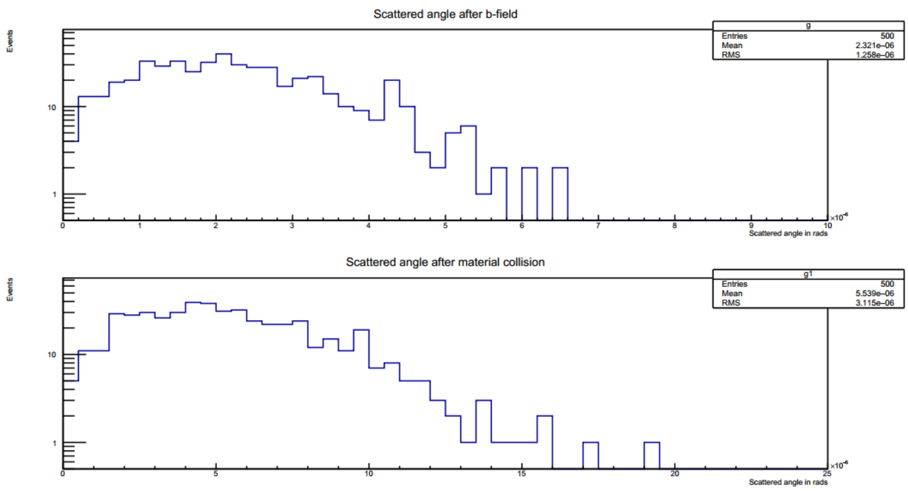

Figure 8: The second output of simulation, two 1D histograms of the angle between the muon position before and after deflecting due to the magnetic field and scattering due to the material respectively.

The third graph produced is an R against z plot, where $R = \sqrt{x^2 + y^2}$. x, y and z correspond to the x, y and z components of $\boldsymbol{S_\mu}$. The range of z is equal to that of z_{sim}.

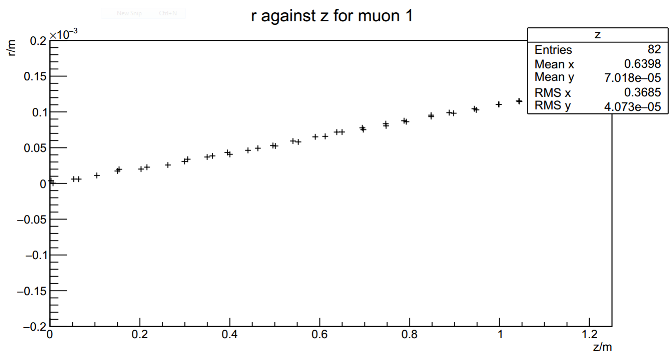

Figure 9: The third output of MCS model simulation simulation, R against z for the first muon in the simulation

The final graph produced is a 1D histogram of R at detector n where $n = 1, 2, 3, 4, 5, 6$.

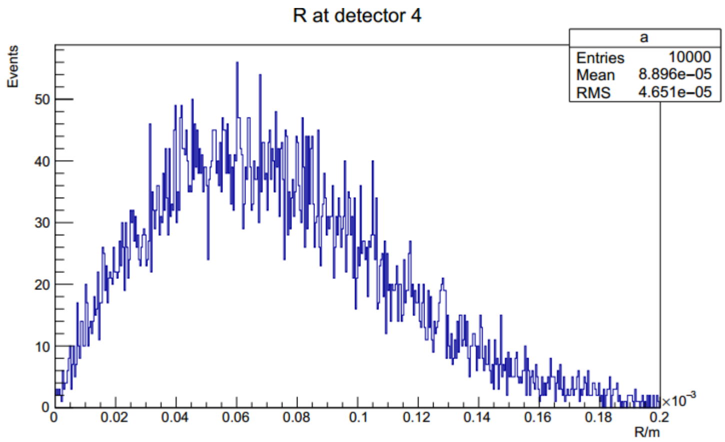

Figure 10: The fourth output of MCS model simulation simulation, a 1D histogram of R at detector n, where n = 4 in this case.

3.2.4 Limitations

MCS model simulation simulation uses the Euler-Cromer algorithm to update the position of the muon every time step, while this is a legitimate method there are other algorithms out there that produce a result that is more accurate, that is, a result that is closer to the actual value of the position. An example of a better algorithm would be Runge-Kutta 4.

The simulation ends and produces graphs when the value of the time reaches a set number, there are some flaws with ending the simulation this way. When running the simulation with a new set of values, the time at which the last muon reaches the last detector is not known resulting in a trial and error process to get the correct time. Depending on the values set in the simulation e.g the total number of muons, this could prove to be a lengthy process.

When the graphs are produced the range on the x-axis has to be manually input which similarly to finding the right time to run the simulation, is a trial and error process which can take multiple runs. Luckily the total number of muons can be set to a small number so you can find the value fairly quickly.

The detectors work by checking to see if the position of the muon is within the bounds of the detector every time step, if the muon is then the position is recorded.

This could be a problem though as if the muon is moving quick enough, the first instance of the $\boldsymbol{S_\mu}$ being recorded at the detector could be at a z greater than that of z of $\boldsymbol{S_d}$. If the muon is not moving in a straight line this would giving slightly different values of x and y of $\boldsymbol{S_\mu}$ also. A way around this is to make t_s small, $\approx$ 0.0001s for a nominal momentum of 1GeV.

4 λ Simulation Results

The results obtained for λ simulation came from experimenting with the simulation controls. Specifically, the nominal momentum, the variance of the nominal momentum and the variance of the incoming angle. The reasoning behind this is because one can change similar variables in MCS model simulation simulation and use results in λ simulation to cross compare. As the two simulations work in different ways it is good to compare the two. Another reason for doing this is to make sure the simulation is working correctly and the physics is right. Other than this, all other simulation controls were kept the same throughout: 10000 muon events and a material length of 10000cm. All the histograms shown are when λ is calculated with the inclusion of muon flux.

4.1 Changing Momentum Values

This section covers the results obtained changing the nominal muon momentum and its variance.

4.1.1 Nominal Momentum = 3GeV, Variance = 0.5GeV

This is the default setting of the simulation. All future histograms for λ simulation will be compared to this one:

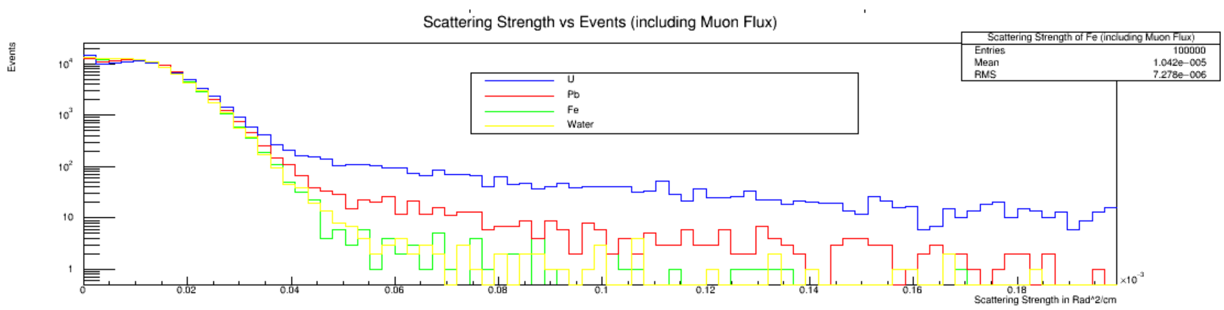

Figure 11: A 1D histogram of λ for water, uranium, iron and lead. With a nominal momentum of 3GeV and a variance of 0.5GeV

By looking at the graph we can see that the simulation is working as intended. Uranium has the most events at the drop off for λ (past 0.04×10^{-6} Rad^2/cm), followed by lead, iron then water. The mean λ given on the graph is that of iron and will be for every histogram covered in this section. The mean λ is 1.040×10^{-5} Rad^2/cm.

4.1.2 Nominal Momentum = 1GeV, Variance = 0.5GeV

The value of nominal momentum was changed from 3GeV to 1GeV. A reduction in the nominal momentum should result in a higher mean scattering strength as the muons will be scattered via larger angles:

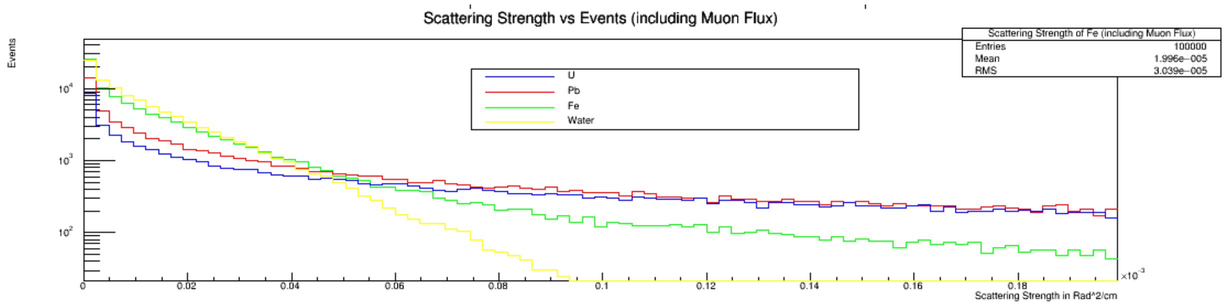

Figure 12: A 1D histogram of λ for water, uranium, iron and lead. With a nominal momentum of 1GeV and a variance of 0.5GeV

Looking at the graph we can see this is the case both by inspection and by the mean value of λ. Compared to figure 11, it is clear that the drop in events is more gradual and the distinction between iron and water is clearer. If the x-axis range were longer we could see the drop off for lead. The mean value of λ given is 1.996×10^{-5} Rad^2/cm, larger than that given by figure 11 which is expected.

4.1.3 Nominal Momentum = 5GeV, Variance = 0.5GeV

The value of nominal momentum has been increased from 3GeV to 5GeV. An increase in nominal momentum should result in a lower mean scattering strength as the muons are travelling faster they will not deviate at as high an angle, this is because the interaction time with the atoms in the material is less:

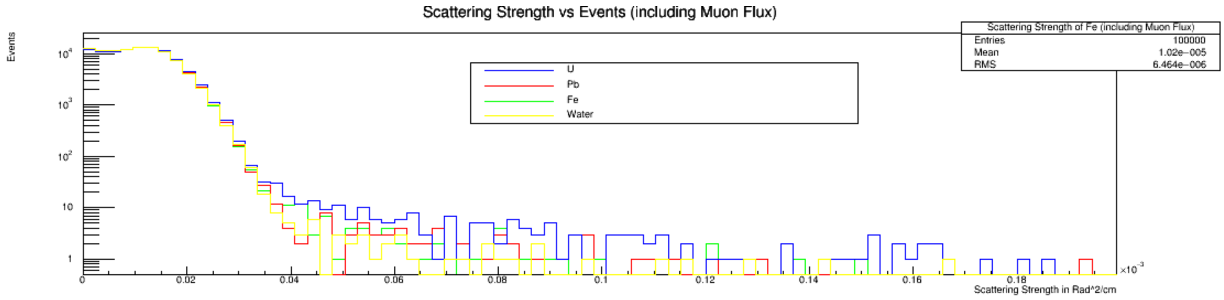

Figure 13: A 1D histogram of λ for water, uranium, iron and lead. With a nominal momentum of 5GeV and a variance of 0.5GeV

This is clearly seen, the drop in events occurs at a lower value of λ around 0.03×10^{-3} Rad^2/cm. It is also hard to distinguish between the 4 materials, this is because the muons have such a high momentum. The mean value of λ given is 1.020×10^{-5} Rad^2/cm, a lower mean than figure 11, which is to be expected.

4.1.4 Nominal Momentum = 3GeV, Variance = 1GeV

The value of nominal momentum is kept the same at 3GeV but the value of variance is doubled to 1GeV. This will result in a wider distribution of momenta and as a result the drop off for λ will be larger and more gradual:

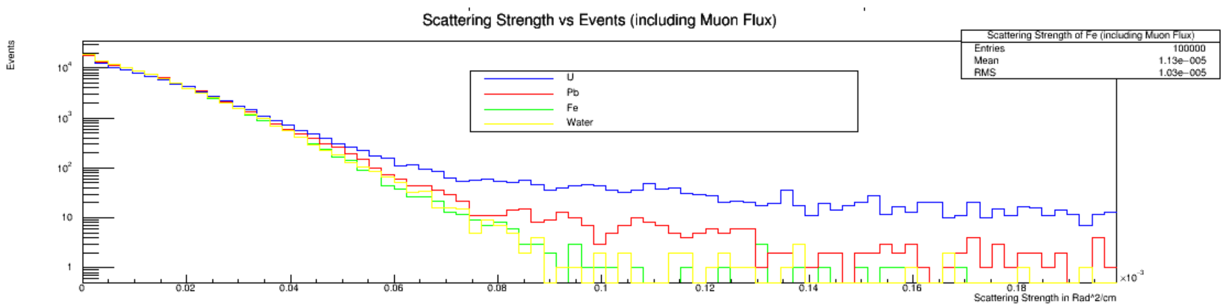

Figure 14: A 1D histogram of λ for water, uranium, iron and lead. With a nominal momentum of 3GeV and a variance of 1GeV

This appears to be the case, the drop in λ is gradual and occurs at around 0.06×10^{-3} Rad^2/cm. The mean value of λ given is 1.130×10^{-5} Rad^2/cm, only slightly bigger than that given by figure 11. This is to be expected, as the nominal muon momentum is the same in both cases but the maximum momentum that can be obtained is higher in this case as the variance is doubled.

4.1.5 Nominal Momentum = 1GeV, Variance = 1GeV

The value of nominal momentum is reduced down to 1GeV and the variance is doubled to 1GeV. This will result in a histogram similar to that of a nominal momentum of 1GeV and a variance of 0.5GeV except the drop off for λ should be even higher:

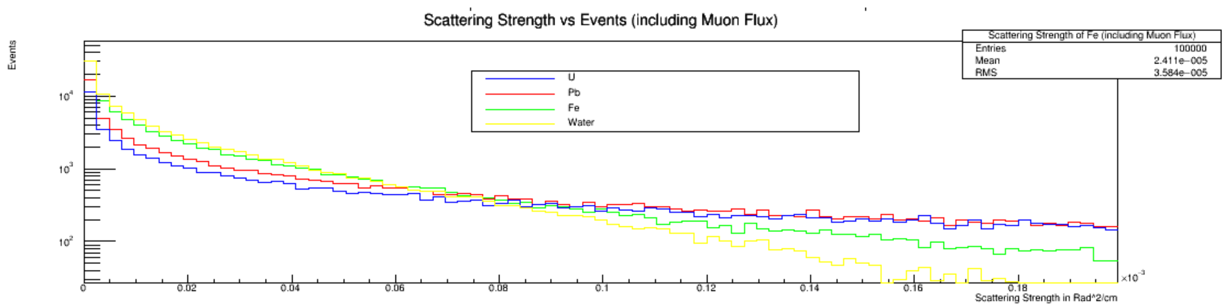

Figure 15: A 1D histogram of λ for water, uranium, iron and lead. With a nominal momentum of 1GeV and a variance of 1GeV

This can clearly be seen, this is because the range of momenta the muons can have is a lot larger. The muons can have the lowest possible momentum and as a result experience the most scattering meaning λ will be at its maximum value. The drop off here for λ is around 0.16×10^{-3} Rad^2/cm, although it is hard to tell as the drop off is so gradual. The mean value of λ given is 2.411×10^{-5} Rad^2/cm, a lot larger than that given by figure 11.

4.1.6 Nominal Momentum = 5GeV, Variance = 1GeV

The value of nominal momentum is increased to 5GeV and the variance is doubled to 1GeV. This will result in a histogram similar to that of a nominal momentum of 5GeV and a variance of 0.5GeV expect the drop off for λ should be larger:

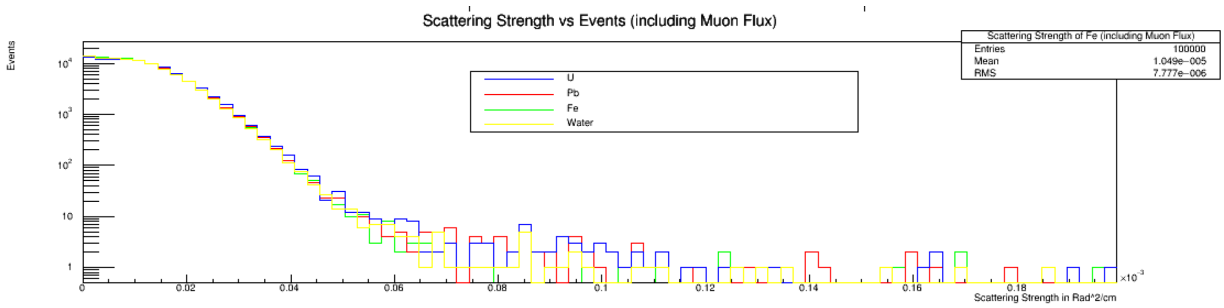

Figure 16: A 1D histogram of λ for water, uranium, iron and lead. With a nominal momentum of 5GeV and a variance of 1GeV

This is as predicted, although the muons have a high momentum, there is a chance that the momentum of one muon could be lower that what's possible in the graph previously mentioned due to a higher variance. The drop off is around 0.04×10^{-5} Rad^2/cm and the mean value for λ is given to be 1.049×10^{-5} Rad^2/cm.

4.1.7 Summary of Results

To summarise, it was found that increasing the nominal muon momentum from 3GeV to 5GeV decreased the mean scattering strength, λ, from 1.040×10^{-5} rad^2/cm to 1.020×10^{-5} Rad^2/cm and made it harder to tell the materials apart from inspection. Decreasing the nominal muon momentum down to 1GeV increased the mean scattering strength to 1.996×10^{-5} rad^2/cm and made it easier to tell the materials apart. Increasing the variance from 0.5GeV to 1GeV meant that the range of momenta for the muons was larger, resulting in larger λ for 1GeV, 3GeV but not 5GeV than if the variance were set to 0.5GeV.

4.2 Changing Incident Angle Values

This section covers the results obtained from changing the variance of the incident angle. The incident angle is generated via a Gaussian with a mean of 0 and a variance of 0.56. The variance is set to 0.56 as this is similar to a $\cos^2\theta$ distribution which is the angular distribution of cosmic ray muons. Changing the mean value is not necessary as all it would do is off centre the muons, changing the variance however provides interesting results.

4.2.1 Variance = 0.1

Dropping the variance means that the distribution of angles is not as wide as cosmic ray muons. A low variance such as 0.1 would mean that there was very little spread in the muons incident on the material. This would mimic a muon beam and give an almost identical histogram to that if λ were calculated without taking muon flux into account:

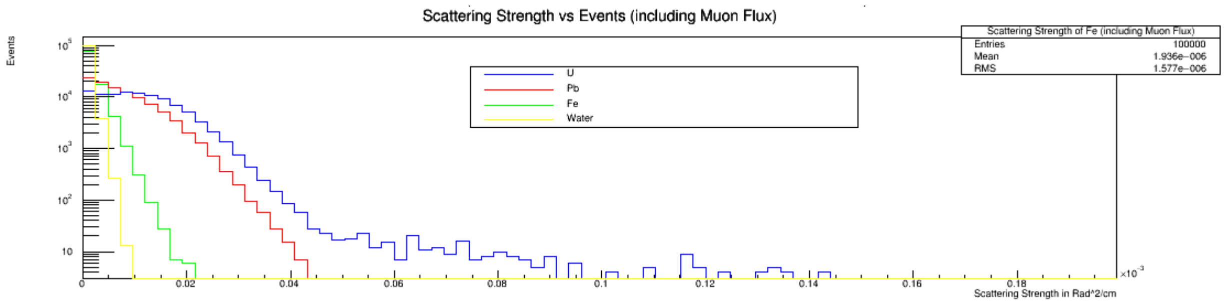

Figure 17: A 1D histogram of λ for water, uranium, iron and lead. With an incident angle variance of 0.1

As you can see there is a much bigger distinction between the materials. This result alone shows why using a muon beam would be better that cosmic ray muons. The mean λ is given to be 1.936×10^{-6} Rad^2/cm, much smaller than that given by figure 11. All that has been done is limit the angles at which the muon enters the material.

4.2.2 Variance = 0.9

Increasing the variance means the muons enter the material at a much wider distribution of angles. This would result in the opposite to the previous histogram, it will difficult to tell each material apart:

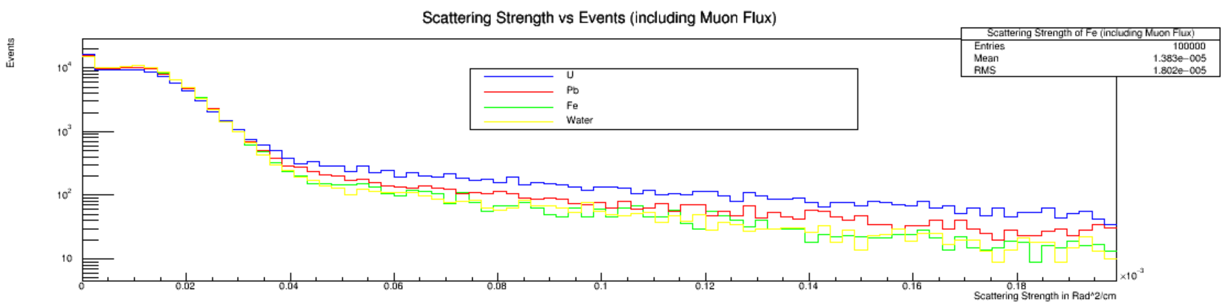

Figure 18: A 1D histogram of λ for water, uranium, iron and lead. With an incident angle variance of 0.9

As you can see this is the case, even at large values of λ (0.09×10^{-3} Rad^2/cm) it's hard to tell them apart. The mean λ is given to be 1.383×10^{-5} Rad^2/cm.

4.2.3 Summary of Results

To summarise it was found that decreasing the variance of the incident angle from 0.56 to 0.1 made it so the muons were more like a muon beam than cosmic ray muons. This is because the range of incident angles the muons can have is a lot less and as a results will enter the material in near enough the same direction. Using a variance of 0.1 decreased the mean scattering strength significantly, λ, from 1.040×10^{-5} rad^2/cm to 1.936×10^{-6} and made it so the 4 materials were incredibly easy to distinguish. Increasing the variance to 0.9 increases the mean scattering strength to 1.383×10^{-5} Rad^2/cm and makes it harder to distinguish between the 4 materials.

4.3 Momentum Uncertainty Histograms

As mentioned before, when taking the uncertainty in muon momentum into account the final calculate of λ is multiplied by a factor of $\frac{1}{(1+E_p^2)}$. Where E_p is the uncertainty in muon momentum. This section briefly looks at results obtained by changing the value of the momentum uncertainty. This is the histogram when the uncertainty in muon momentum is zero percent:

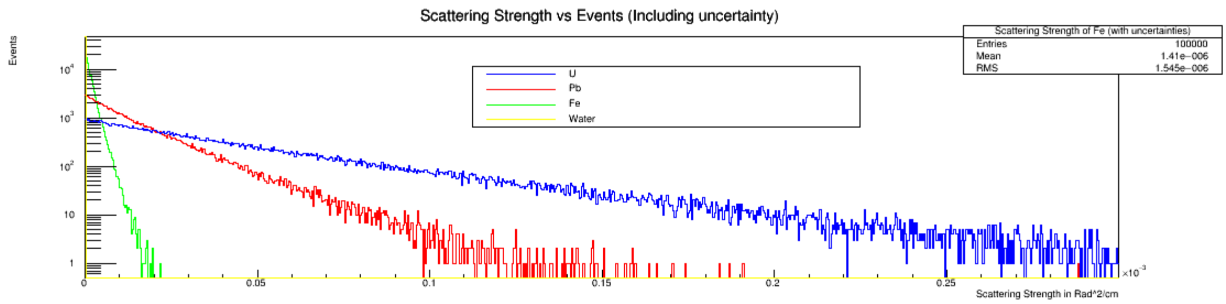

Figure 19: A 1D histogram of λ for water, uranium, iron and lead. Where λ is calculated with a momentum uncertainty of 0%

4.3.1 10% Uncertainty

The uncertainty of muon momentum was set to 10%. This means the value of scattering strength will be multiplied by a factor of 0.90

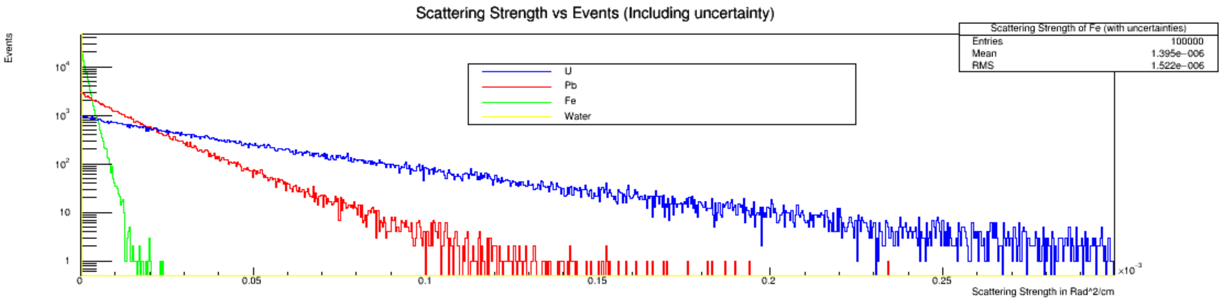

Figure 20: A 1D histogram of λ for water, uranium, iron and lead. Where λ is calculated with a momentum uncertainty of 1%

Looking at the mean we can see this is the case. The mean λ given is 1.395×10^{-6} Rad^2/cm, this is approximately 0.90 times the mean λ given by figure 19.

4.3.2 100% Uncertainty

The uncertainty of muon momentum was set to 100%. This means that the value of scattering strength will be reduced by a factor of 2:

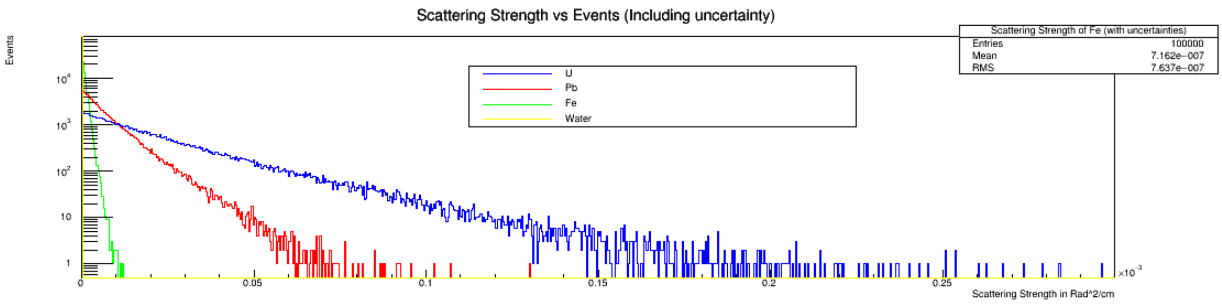

Figure 21: A 1D histogram of λ for water, uranium, iron and lead. Where λ is calculated with a momentum uncertainty of 100%

Looking at the mean we can see this is the case. The mean λ given is 7.162×10^{-7} Rad^2/cm, this is approximately half of the mean given by figure 19.

5 MCS Model Simulation Results

The results from MCS model simulation simulation came from running the simulation over two different length scales, 1.2m and 10.2m. This is an overview of the results obtained. The nominal muon momentum was set to 1GeV as from the results of λ simulation, it's easy to tell apart the different materials that the muons scattered through.

5.1 Set-up Length = 1.2m

Initially, the simulation was set up so that the multiple coulomb scattering of muons could be investigated over a small distance. The simulation was set up with the following controls:

Material Controls			
MaterialZPosition	x_m	y_m	z_m
0.6 m	0.05 m	0.1 m	0.05 m

Detector Controls								
1ZPos	2ZPos	3ZPos	4ZPos	5ZPos	6ZPos	x_d	y_d	z_d
0.024 m	0.060 m	0.564 m	0.720 m	1.140 m	1.176 m	0.02 m	0.10 m	0.02 m

Table 1: Table of simulation controls where nZpos is the Z position of detector n (n = 1, 2,...6).

The radiation length of the detector was set to that of silicon, $X_0 = 9.370cm$ as that would be an appropriate detector to use in small length scales due to its micro meter resolution. The simulation was ran for a time of 0.06s, this was to ensure that all muons scattered through the device under test and were detected in the final detector.

5.1.1 Scattered Angle Plot

The simulation was ran for 1000 muons for 4 materials: water, iron, lead and uranium. Each scattered angle plot was then combined into one single graph:

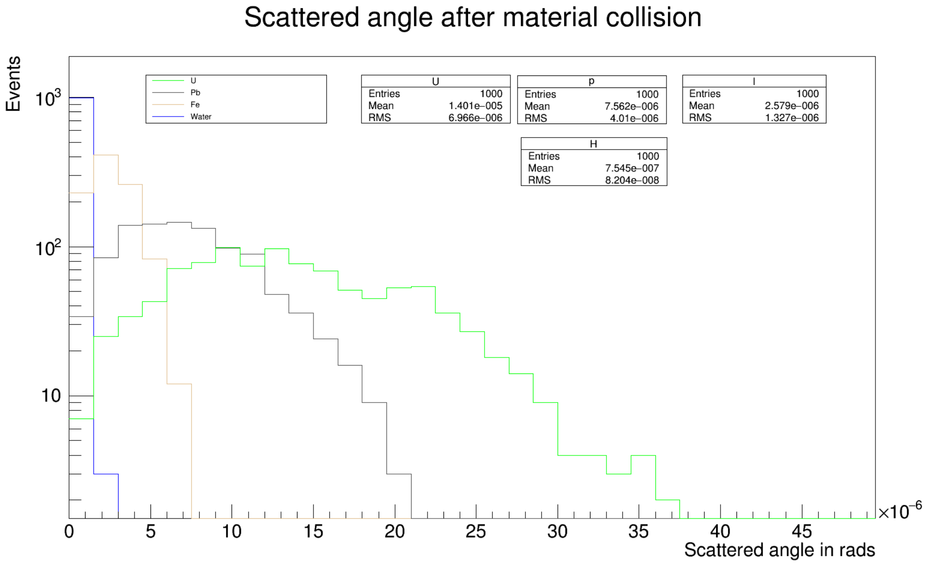

Figure 22: A graph of the scattered angle for four materials: uranium, lead, iron and water.

The mean scattered angle for uranium, lead, iron and water are given as: 1.401×10^{-5} rad, 7.562×10^{-6} rad, 2.579×10^{-6} rad and 5.605×10^{-7} rad respectively.

5.1.2 R at detector 4

The value of R was at detector 4 was recorded for each muon for 2 different materials, uranium and water. The reasoning behind using these two materials is because in the previous section, uranium gave the highest mean scattered angle and water gave the lowest. This means that the difference in R between these two materials should be the maximum possible difference between all 4 materials. The simulation was ran a total of 20 times, 10 for each material, 10000 muons in total:

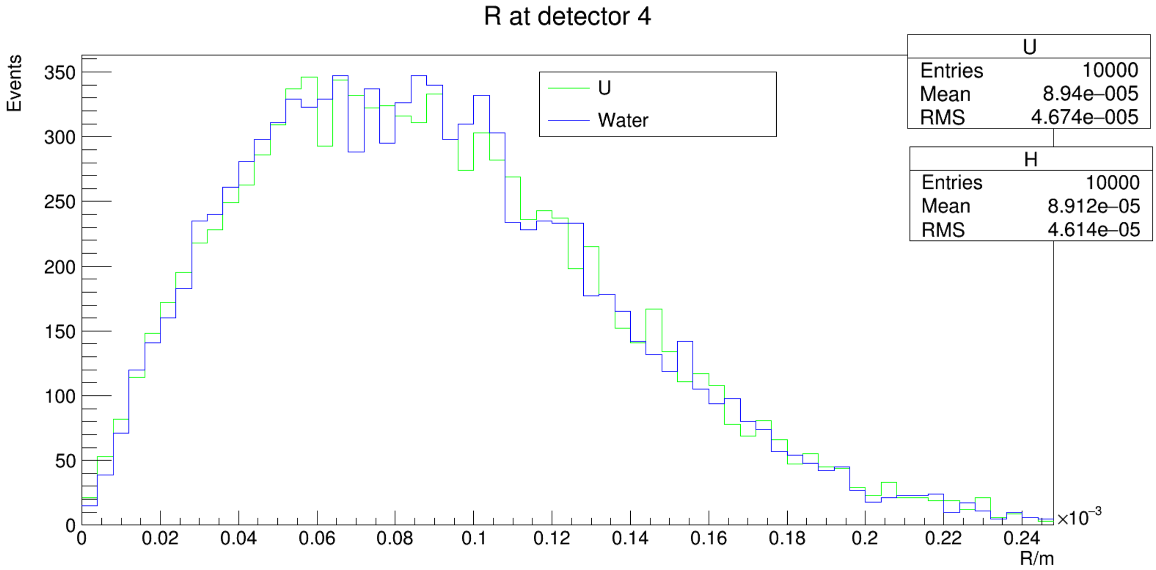

Figure 23: A histogram of R at detector 4 for both water (blue) and uranium (green) for simulation length = 1.2m

It is very hard to distinguish the two materials. The difference in mean R for water and uranium is very small, 3.3×10^{-7}m where uranium has a mean R of 8.945×10^{-5}m and water has a mean R of 8.912×10^{-5}m. This was unexpected as the difference in mean scattering angle between the two is so large. I feel this is due to two reasons: Firstly, the distance from the material to the fourth detector is very small, around 0.12m apart. It could be that the muons haven't had enough of a distance to deviate from their original position due to the scattering, a way around this would be to check R at detector 5 and see if the difference in mean R is any larger. Another reason why the difference between the mean R for water and uranium is small could be due to the fact that the scattered angle is too small. Even though the difference between the two mean scattered angles is great, they could both be very small values such that the muons hardly deviate from their original position. A way around this would be to increase the length of the material as the magnitude of the scattered angle depends on this, this will be done later.

5.1.3 R at detector 5

Measuring R at detector 5 for both water and uranium might give a better distinction between the two. The distance between the material and detector 5 is 0.54m instead of 0.12m, this could be enough time for the muons to spread a good enough distance from their original path such that the scattering from uranium and water can be distinguished: Surprisingly there seems to be no difference than figure 23.

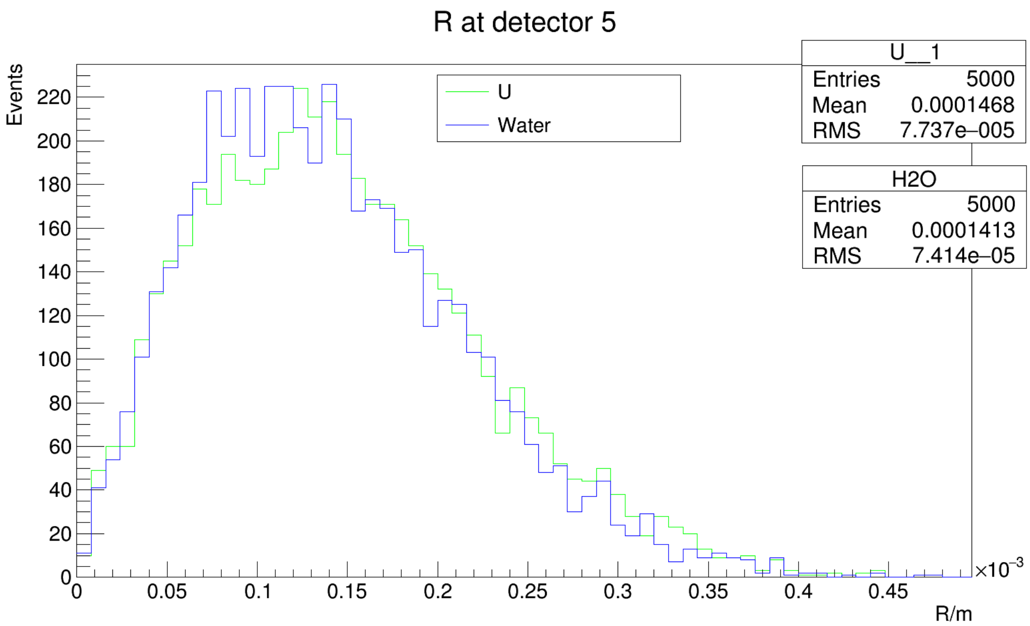

Figure 24: A histogram of R at detector 5 for both water (blue) and uranium (green) for simulation length = 1.2m

The two materials are still indistinguishable even though the distance from the material to the detector has increased four fold. The difference in mean R for water and uranium is 5.6×10^{-6}m where uranium has a mean R of 1.469×10^{-4}m and water has a mean R of 1.413×10^{-4}m. The difference in mean R has increased from figure 23, but not enough. This must be due to the fact that the material length is so small the muons aren't scattering at a high enough angle. Increasing the material length should do it.

5.2 Set-up Length = 10.2m

The simulation was then scaled up by a factor of 10 to investigate if increasing the material length could help distinguish muons scattering from uranium or water. The nominal muon momentum was kept at 1GeV. The simulation was set up with the following controls:

Material Controls			
MaterialZPosition	x_m	y_m	z_m
5.10 m	0.50 m	1.00 m	0.50 m

Detector Controls								
1ZPos	2ZPos	3ZPos	4ZPos	5ZPos	6ZPos	x_d	y_d	z_d
0.204 m	0.510 m	4.794 m	6.120 m	9.690 m	9.996 m	1.00 m	1.00 m	0.05 m

Table 2: Table of simulation controls where nZpos is the Z position of detector n (n = 1, 2,...6).

Again the radiation length of the detector was set to that of silicon and the run time of the simulation was set to 0.2s.

5.2.1 R at detector 4

The same two materials were used, water and uranium. However, the distance between detector 4 and the material increased up to 1.1m, approximately the entire length of the simulation used before. This larger distance should give the muons enough time to deviate from their original path and spread. The material length has also been increased by a factor of 10, so the scattered angle for both water and uranium should be a lot larger:

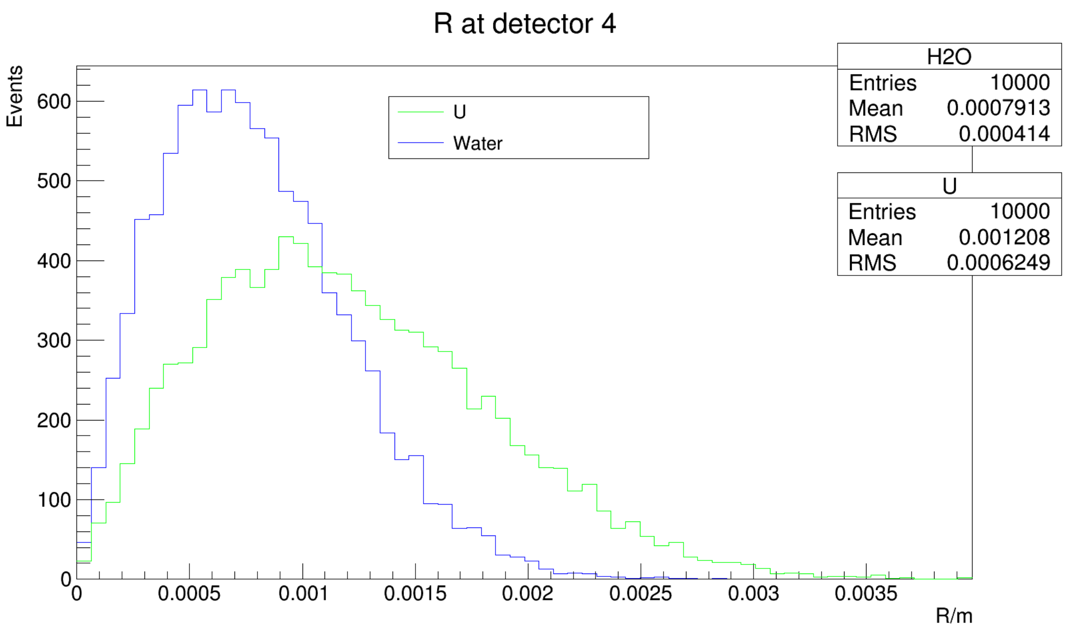

Figure 25: A histogram of R at detector 4 for both water (blue) and uranium (green) for simulation length = 10.2m

The two materials are clearly distinguishable. What is also worth noting is the shape of the two curves, they both appear to be Gaussian in shape. Uranium appears to have a larger variance than that of water. This is to be expected as the scattered angle is generated via a Gaussian with a variance calculated via equation 4. As uranium has a higher calculated variance, the range of scattered angles will be larger, resulting in a wider range of values of R at detector 5. The difference in variance for the two curves will only get larger the further the distance from the point of scattering you go. The difference in mean R for water and uranium is 4.164×10^{-4}m where uranium has a mean R of 1.208×10^{-3}m and water has a mean R of 7.916×10^{-4}m. While it seems obvious that the difference in R has increased as the entire simulation size has been scaled up, what is interesting is when you compare the ratio of the mean R of uranium to the mean R of water at detector 4 for both simulation set ups.

The ratio of R's is 1.0037 and 1.5260 for a set up length of 1.2m and 10.2m respectively. This shows that increasing the material length, increases the scattered angle and makes it easier to distinguish between different elements used for the device under test.

5.2.2 R at detector 5

Measuring R at detector 5 should make it so the distinction between the two materials is even larger. The distance from the material to detector 5 in this set up is 8.69m approximately 7 times the length of the first set up:

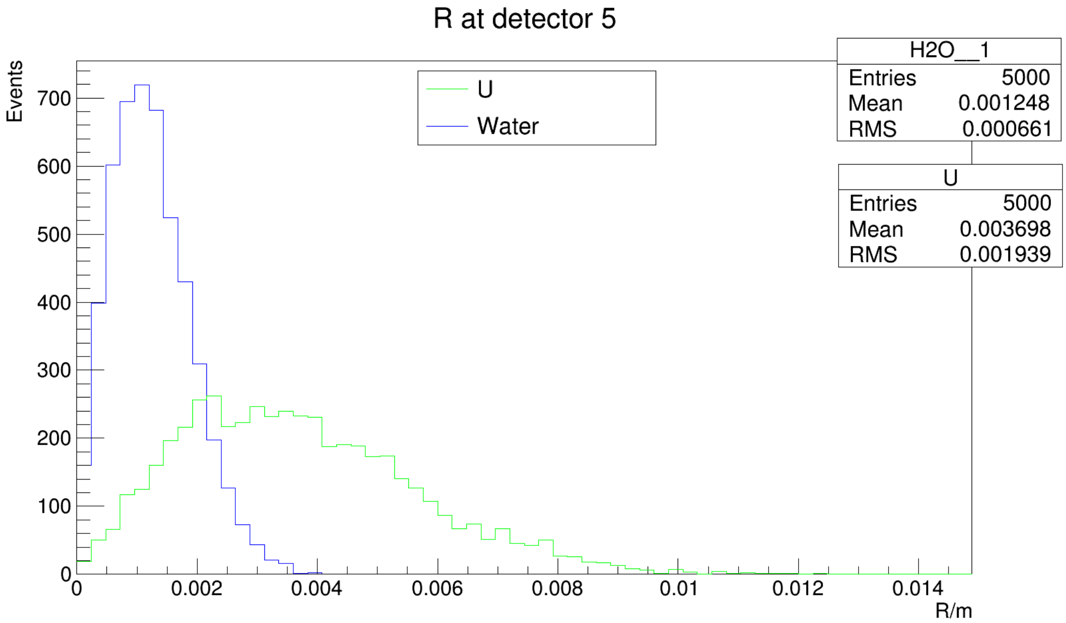

Figure 26: A histogram of R at detector 5 for both water (blue) and uranium (green) for simulation length = 10.2m

The reason the height of the uranium curve appears to be a lot smaller than in figure 25 is because the number of muons simulated was halved to 5000 due to the time constraints. What is interesting here is that the variance of the Gaussian for water has decreased significantly, this could be due to the fact that number of muons has halved but this seems unlikely. The difference in mean R for water and uranium is 3.698×10^{-3}m where uranium has a mean R of 3.698×10^{-3}m and water has a mean R of 1.249×10^{-3}m. Comparing the ratio of R's for uranium and water at detector 5 for both simulation set ups you get: 1.0396 and 2.9608 for a set up length of 1.2m and 10.2m respectively.

As mentioned previously both these curves appear to be Gaussian in shape, fitting a Gaussian to these curves yields the following results:

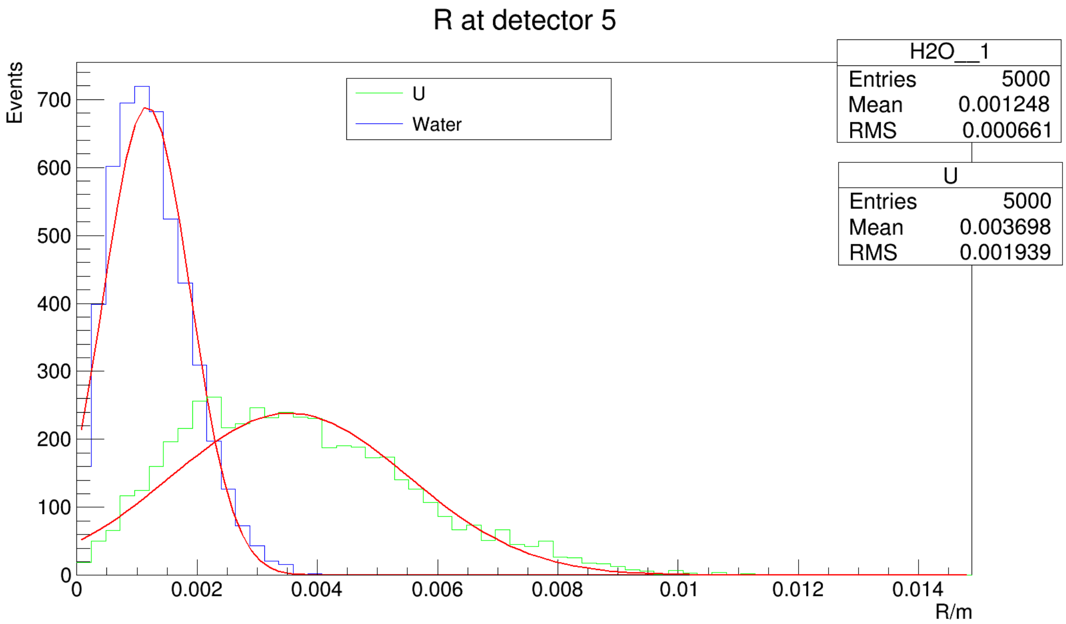

Figure 27: A histogram of R at detector 5 for both water (blue) and uranium (green) for simulation length = 10.2m where both curves are fitted by a Gaussian distribution.

Where the Gaussian distribution for uranium has a mean of 0.00349 and a variance, σ, of 0.00190 and the Gaussian distribution for water has a mean of 0.00121 and a σ of 0.000657.

5.2.3 Scattered Angle vs Momentum

As mentioned previously, decreasing the muon momentum will increase the scattered angle. This is a graph of the scattered angle vs nominal muon momentum for 100 muons:

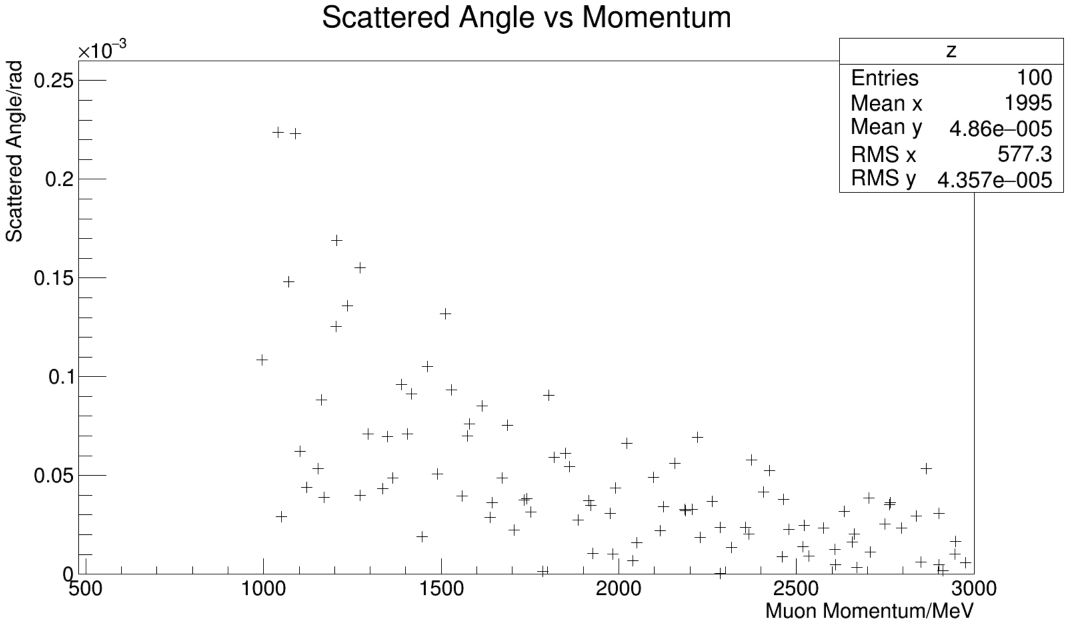

Figure 28: A graph of scattered angle against nominal muon momentum for a set-up length = 10.2m for 100 muons.

As you can see, this is the case. The reason there are low scattered angles for low momentum values is because the scattered angle is generated via a Gaussian, so there is a chance that the generated angle is small due to the variance.

6 Conclusions and Future Work

6.1 Conclusion

In conclusion it was found that changing the nominal momentum from the base momentum of 3GeV changed the scattering strength λ and the ability to tell apart different elements. Specifically, decreasing the nominal momentum from 3GeV to 1GeV, it was found that the mean λ increased from 1.040×10^{-5} rad^2/cm to 1.996×10^{-5} rad^2/cm and meant that the materials were easier to tell apart. Increasing the nominal momentum to 5GeV showed a decrease in the mean λ down to 1.020×10^{-5} Rad^2/cm and made it so the materials were mostly indistinguishable. This means it would be better to probe matter with muons with a lower momentum, but not so low that they can't penetrate all the way through. Changing the variance of the incident angle from 0.56 was found to also change the value of λ. Decreasing the variance to 0.1 results in a thinner Gaussian Distribution through which incident angle values are generated, as a result the incident muons mimic that of a muon beam, very little spread. Using a small variance was found to decrease the mean λ to 1.936×10^{-6} and give the ability to very easily tell apart the 4 different materials just by inspection. Increasing the variance to 0.9 increases the mean scattering strength to 1.383×10^{-5} Rad^2/cm and makes it harder to distinguish between the 4 materials. This means it would be better to use a muon beam than cosmic ray muons if you wish to distinguish between certain materials.

From MCS model simulation simulation it was found that the the difference in R between uranium and water for detector 4 and 5 was very small for a set up length of 1.2m, 3.3×10^{-7}m and 5.6×10^{-6} respectively. This means to tell apart muons that scattered from uranium or water would require a detector with a very good resolution on the order of 10^{-7}m. Comparing this to a set up length of 10.2m, the difference in R between the two was found to be 4.2×10^{-4}m and 3.7×10^{-3}m, just by looking at the curves of the two materials it's easy to tell the materials apart by inspection as this set up length. This means that when you increase the material length the scattered angle increases, and increasing the distance from the material to the detector gives time for the muons to deviate from their original path.

6.2 Future Work

There is still a lot more that I would have liked to have done with this project but couldn't due to time constraints.

The next step after this would be to find the minimum number of muons required to differentiate between the two materials for a set-up of this size. This would be done by fitting the two Gaussian distributions for uranium and water onto the R histogram for 50 muons and have an acceptance of around 3σ.

The simulation size could then be increased to 100m and one could investigate how R changes as you increase both the material length and the distance between the material and detectors 4 and 5.

A grid could have been laid onto each detector with squares of all the same area, then investigate what sort of area would be needed to tell apart muons that scatter through different materials, effectively finding a detector resolution.

In the results for λ simulation the nominal muon momentum was changed and the scattering strength was found to change dramatically, it would have been interesting to see how changing the nominal momentum for MCS model simulation simulation would effect what sort of detector resolution would be needed.

At the moment the simulation is set up so the muons only scatter through one material. While this does provide us with some information it can't really be applied to any real life scenarios. I would have liked to have added multiple materials and investigate what number of muons and what sort of detector resolution would be needed to tell them apart. For example being able to detect a submarine in the sea.

7 Appendix

Links to the code used for λ simulation and MCS model simulation simulation can be found in my log book as well as this website: http://www.hep.lancs.ac.uk/ jmurk-in/

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