SEIR Model for Measles Disease

This is a MatLab code for SEIR model for modelling measles disease. Runge-Kutta Fourth-Order method is used in solving the Model numerically.

The SEIR equations model are given below:

$$\frac{dS}{dt} = b - \beta SI - \mu S, \tag{1}$$

$$\frac{dE}{dt} = \beta SI - (\mu + \alpha + \sigma)E \tag{2}$$

$$\frac{dI}{dt} = \alpha E - (\mu + \gamma)I \tag{3}$$

$$\frac{dR}{dt} = \gamma I + \sigma E - \mu R \tag{4}$$

These can be written:

$$f_1 = b - \beta SI - \mu S, \tag{5}$$

$$f_2 = \beta SI - (\mu + \alpha + \sigma)E \tag{6}$$

$$f_3 = \alpha E - (\mu + \gamma)I \tag{7}$$

$$f_4 = \gamma I + \sigma E - \mu R. \tag{8}$$

The Description of the Explicit Fourth-Order Runge-Kutta Method

Suppose we have a system of ordinary differential equations of the form:

$$\begin{aligned}
\dot{y}_1 &= f_1(t, y_1, y_2, ..., y_n), \\
\dot{y}_2 &= f_2(t, y_1, y_2, ..., y_n), \\
\dot{y}_3 &= f_3(t, y_1, y_2, ..., y_n), \\
&\vdots \\
\dot{y}_n &= f_n(t, y_1, y_2, ..., y_n).
\end{aligned}$$

with a given initial conditions:

$$y_i(0) = y_0^i, \;\; i = 0, 1, 2, ..., n.$$

We can put them together in a vector form to obtained:

$$\dot{Y}_n = F_n(t, y), \;\; Y(t_0) = y_0$$

Where:

$$Y(t) = (\ y\ )_1\, y_2 \vdots y_n; \;\; F(t, y) = (\ f\ )_1\, f_2 \vdots f_n.$$

The standard formula for explicit fourth-order Runge-Kutta method is given as:

$$y_{i+1} = y_i + \frac{h}{6}(K_1 + 2K_2 + 2K_3 + K_4).$$

Where:

$$K_1 = f(t_i, y_i),$$
$$K_2 = f(t_i + \frac{h}{2}, y_i + h\frac{K_1}{2}),$$
$$K_3 = f(t_i + \frac{h}{2}, y_i + h\frac{K_2}{2}),$$

and:

$$K_4 = f(t_i + \frac{h}{2}, y_i + hK_3).$$

Now, for our SEIR model equations (5) to (8); the explicit fourth-order Runge-Kutta method will become:

$$S_{i+1} = S_i + \frac{h}{6}(K_{1S} + 2K_{2S} + 2K_{3S} + K_{4S}),$$
$$E_{i+1} = E_i + \frac{h}{6}(K_{1E} + 2K_{2E} + 2K_{3E} + K_{4E}),$$
$$I_{i+1} = I_i + \frac{h}{6}(K_{1I} + 2K_{2I} + 2K_{3I} + K_{4I}),$$

and:

$$R_{i+1} = R_i + \frac{h}{6}(K_{1R} + 2K_{2R} + 2K_{3R} + K_{4R}).$$

Where:

$$K_{1S} = f_1\bigg(t_i, S_i, E_i, I_i, R_i\bigg),$$
$$K_{1E} = f_2\bigg(t_i, S_i, E_i, I_i, R_i\bigg),$$
$$K_{1I} = f_3\bigg(t_i, S_i, E_i, I_i, R_i\bigg),$$
$$K_{1R} = f_4\bigg(t_i, S_i, E_i, I_i, R_i\bigg),$$

$$K_{2S} = f_1\bigg(t_i + \frac{h}{2}, S_i + \frac{h}{2}K_{1S}, E_i + \frac{h}{2}K_{1E}, I_i + \frac{h}{2}K_{1I}, R_i + \frac{h}{2}K_{1R}\bigg),$$
$$K_{2E} = f_2\bigg(t_i + \frac{h}{2}, S_i + \frac{h}{2}K_{1S}, E_i + \frac{h}{2}K_{1E}, I_i + \frac{h}{2}K_{1I}, R_i + \frac{h}{2}K_{1R}\bigg),$$
$$K_{2I} = f_3\bigg(t_i + \frac{h}{2}, S_i + \frac{h}{2}K_{1S}, E_i + \frac{h}{2}K_{1E}, I_i + \frac{h}{2}K_{1I}, R_i + \frac{h}{2}K_{1R}\bigg),$$

$$K_{2R} = f_4\left(t_i + \frac{h}{2}, S_i + \frac{h}{2}K_{1S}, E_i + \frac{h}{2}K_{1E}, I_i + \frac{h}{2}K_{1I}, R_i + \frac{h}{2}K_{1R}\right),$$

$$K_{3S} = f_1\left(t_i + \frac{h}{2}, S_i + \frac{h}{2}K_{2S}, E_i + \frac{h}{2}K_{2E}, I_i\frac{h}{2}K_{2I}, R_i + \frac{h}{2}K_{2R}\right),$$

$$K_{3E} = f_2\left(t_i + \frac{h}{2}, S_i + \frac{h}{2}K_{2S}, E_i + \frac{h}{2}K_{2E}, I_i\frac{h}{2}K_{2I}, R_i + \frac{h}{2}K_{2R}\right),$$

$$K_{3I} = f_3\left(t_i + \frac{h}{2}, S_i + \frac{h}{2}K_{2S}, E_i + \frac{h}{2}K_{2E}, I_i + \frac{h}{2}K_{2I}, R_i + \frac{h}{2}K_{2R}\right),$$

$$K_{3R} = f_4\left(t_i + \frac{h}{2}, S_i + \frac{h}{2}K_{2S}, E_i + \frac{h}{2}K_{2E}, I_i + \frac{h}{2}K_{2I}, R_i + \frac{h}{2}K_{2R}\right),$$

$$K_{4S} = f_1\left(t_i + h, S_i + hK_{3S}, E_i + hK_{3E}, I_i + hK_{3I}, R_i + hK_{3R}\right),$$

$$K_{4E} = f_2\left(t_i + h, S_i + hK_{3S}, E_i + hK_{3E}, I_i + hK_{3I}, R_i + hK_{3R}\right),$$

$$K_{4I} = f_3\left(t_i + h, S_i + hK_{3S}, E_i + hK_{3E}, I_i + hK_{3I}, R_i + hK_{3R}\right),$$

$$K_{4R} = f_4\left(t_i + h, S_i + hK_{3S}, E_i + hK_{3E}, I_i + hK_{3I}, R_i + hK_{3R}\right).$$