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Documentation for the TIMES Model

PART I

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General Introduction to the TIMES Documentation

This documentation is composed of four Parts.

Part I provides a general description of the TIMES paradigm, with emphasis on the model's general structure and its economic significance. Part I also includes a simplified mathematical formulation of TIMES, a chapter comparing it to the MARKAL model, pointing to similarities and differences, and chapters describing new model options.

Part II constitutes a comprehensive reference manual intended for the technically minded modeler or programmer looking for an in-depth understanding of the complete model details, in particular the relationship between the input data and the model mathematics, or contemplating making changes to the model's equations. Part II includes a full description of the sets, attributes, variables, and equations of the TIMES model.

Part III describes the organization of the TIMES modeling environment and the GAMS control statements required to run the TIMES model. GAMS is a modeling language that translates a TIMES database into the Linear Programming matrix, and then submits this LP to an optimizer and generates the result files. Part III describes how the routines comprising the TIMES source code guide the model through compilation, execution, solve, and reporting; the files produced by the run process and their use; and the various switches that control the execution of the TIMES code according to the model instance, formulation options, and run options selected by the user. It also includes a section on identifying and resolving errors that may occur during the run process.

Part IV provides a step-by-step introduction to building a TIMES model in the VEDA2.0 user interface for model management and results analysis. It first offers an orientation to the basic features of VEDA2.0, including software layout, data files and tables, and model management features, both for handling the input and examining the results. It then describes in detail twelve Demo models (available for download from the ETSAP website) that progressively introduce VEDA-TIMES principles and modeling techniques.

PART I: TIMES CONCEPTS AND THEORY

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Organization of PART I

Part I comprises five divisions, each containing a number of chapters:

- Chapters 1 and 2 provide a general overview of the representation in TIMES of the Reference Energy System (RES) of a typical region or country, focusing on its basic elements, namely technologies and commodities.

- Chapters 3 to 7 describe the core TIMES model generator, i.e. the dynamic partial equilibrium version with perfect foresight: Chapter 3 discusses the economic rationale of the model, and Chapter 4 describes in more detail than chapter 3 the elastic demand feature and other economic and mathematical properties of the TIMES equilibrium. Chapter 5 presents a streamlined representation of the Linear Program used by TIMES to compute the equilibrium. Chapter 6 describes a new TIMES feature for conducting systematic sensitivity analyses. Chapter 7 describes the Climate Module of TIMES.

- Chapters 8 to 11 contain descriptions of 4 extensions or variants that, if used, depart from the assumptions of the core model in a way that alters the nature of the equilibrium: Chapter 8 covers the stochastic programming variant, which no longer assumes perfect foresight, but rather imperfect foresight; Chapter 9 describes the myopic use of TIMES, which violates the perfect foresight property and replaces it with limited foresight; Chapter 10 describes the lumpy investment variant where some decisions are discrete rather than continuous, and thus violate the convexity property; Chapter 11 describes the endogenous technology learning extension, also involving non-convex elements.

- Chapter 12 is devoted to two extensions that make TIMES into a General Equilibrium model, namely ES-MACRO and TIMES-MERGE-MACRO.

- Chapters 13 and 14 constitute appendices that may be of interest to readers at any point in their use of the rest of the text. Chapter 13 provides a brief history and comparison of TIMES and MARKAL, the modeling framework that preceded TIMES. Chapter 14 provides a short review of the theoretical foundation of Linear Programming and the interpretation of the dual solution of a linear program.

1 Introduction to the TIMES model

1.1 *A brief summary*

TIMES (an acronym for The Integrated MARKAL-EFOM[1] System) is an economic model generator for local, national, multi-regional, or global energy systems, which provides a technology-rich basis for representing energy dynamics over a multi-period time horizon. It is usually applied to the analysis of the entire energy sector, but may also be applied to study single sectors such as the electricity and district heat sector. Estimates of end-use energy service demands (e.g., car road travel; residential lighting; steam heat requirements in the paper industry; etc.) are provided by the user for each region to drive the reference scenario. In addition, the user provides estimates of the existing stocks of energy related equipment in all sectors, and the characteristics of available future technologies, as well as present and future sources of primary energy supply and their potentials.

Using these as inputs, the TIMES model aims to supply energy services at minimum global cost (more accurately at minimum loss of total surplus) by simultaneously making decisions on equipment investment and operation; primary energy supply; and energy trade for each region. For example, if there is an increase in residential lighting energy service relative to the reference scenario (perhaps due to a decline in the cost of residential lighting, or due to a different assumption on GDP growth), either existing generation equipment must be used more intensively or new – possibly more efficient – equipment must be installed. The choice by the model of the generation equipment (type and fuel) is based on the analysis of the characteristics of alternative generation technologies, on the economics of the energy supply, and on environmental criteria. TIMES is thus a vertically integrated model of the entire extended energy system.

The scope of the model extends beyond purely energy-oriented issues, to the representation of environmental emissions, and perhaps materials, related to the energy system. In addition, the model is suited to the analysis of energy-environmental policies, which may be represented with accuracy thanks to the explicitness of the representation of technologies and fuels in all sectors.

In TIMES – like in its MARKAL forebear – the quantities and prices of the various commodities are in equilibrium, i.e. their prices and quantities in each time period are

[1]MARKAL (MARket ALlocation model, Fishbone et al, 1981, 1983, Berger et al. 1992) and EFOM (Van Voort et al, 1984) are two bottom-up energy models that inspired the structure of TIMES.

such that the suppliers produce exactly the quantities demanded by the consumers. This equilibrium has the property that the total economic surplus is maximized.

1.2 *Driving a TIMES model via scenarios*

The TIMES model is particularly suited to the *exploration* of possible energy futures based on contrasted *scenarios*. Given the long horizons that are usually simulated with TIMES, the scenario approach is really the only choice (whereas for the shorter term, econometric methods may provide useful projections). Scenarios, unlike forecasts, do not pre-suppose knowledge of the main drivers of the energy system. Instead, a scenario consists of a set of *coherent assumptions* about the future trajectories of these drivers, leading to a coherent organization of the system under study. A scenario builder must therefore carefully test the scenario assumptions for internal coherence, via a credible *storyline*.

In TIMES, a complete scenario consists of four types of inputs: energy service demand curves, primary resource supply curves, a policy setting, and the descriptions of a complete set of technologies. We now present a few comments on each of these four components.

1.2.1 The Demand component of a TIMES scenario

In the case of the TIMES model, demand drivers (population, GDP, households, etc.) are obtained externally, via other models or from accepted other sources. As one example, several global instances of TIMES (e.g. Loulou, 2007) use the GEM-E3[2] to generate a set of *coherent* (national and sectoral) output growth rates in the various regions. Note that GEM-E3 or GEMINI-E3 themselves use other drivers as inputs, in order to derive GDP trajectories. These drivers consist of measures of technological progress, population, degree of market competitiveness, and a few other (perhaps qualitative) assumptions. For population and household projections, TIMES instances use the same exogenous sources (IPCC, Nakicenovic 2000, Moomaw and Moreira, 2001). Other approaches may be used to derive TIMES drivers, whether via models or other means.

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[2]European Commission, *The GEM-E3 Model, General Equilibrium Model for Economy, Energy and Environment*, https://ec.europa.eu/jrc/en/gem-e3/model.

For the global versions of TIMES, the main drivers are: Population, GDP, GDP per capita, number of households, and sectoral outputs. For sectoral TIMES models, the demand drivers may be different depending on the system boundaries.

Once the drivers for a TIMES model are determined and quantified the construction of the reference demand scenario requires computing a set of energy service demands over the horizon. This is done by choosing elasticities of demands to their respective drivers, in each region, using the following general formula:

$$Demand = Driver^{Elasticity}$$

As mentioned above, the demands are user provided for the reference scenario only. When the model is run for alternate scenarios (for instance for an emission constrained case, or for a set of alternate technological assumptions), it is likely that the demands will be affected. TIMES has the capability of estimating the response of the demands to the changing conditions of an alternate scenario. To do this, the model requires still another set of inputs, namely the assumed elasticities of the demands to their own prices. TIMES is then able to endogenously adjust the demands to the alternate cases without exogenous intervention. In fact, the TIMES model is driven not by demands but by *demand curves.*

To summarize: the TIMES demand scenario components consist of a set of assumptions on the drivers (GDP, population, households, outputs) and on the elasticities of the demands to the drivers and to their own prices.

1.2.2 The Supply component of a TIMES scenario

The second constituent of a scenario is a set of *supply curves* for primary energy and material resources. Multi-stepped supply curves are easily modeled in TIMES, each step representing a certain potential of the resource available at a particular cost. In some cases, the potential may be expressed as a cumulative potential over the model horizon (e.g. reserves of gas, crude oil, etc.), as a cumulative potential over the resource base (e.g. available areas for wind converters differentiated by velocities, available farmland for biocrops, roof areas for PV installations) and in others as an annual potential (e.g. maximum extraction rates, or for renewable resources the available wind, biomass, or hydro potentials). Note that the supply component also includes the identification of trading possibilities, where the amounts and prices of the traded commodities are determined endogenously (optionally within user imposed limits).

1.2.3 The Policy component of a TIMES scenario

Insofar as some policies impact on the energy system, they become an integral part of the scenario definition. For instance, a reference scenario may perfectly ignore emissions of various pollutants, while alternate policy scenarios may enforce emission restrictions, or emission taxes, etc. The detailed technological nature of TIMES allows the simulation of a wide variety of both micro measures (e.g. technology portfolios, or targeted subsidies to groups of technologies), and broader policy targets (such as general carbon tax, or permit trading system on air contaminants). A simpler example might be a nuclear policy that limits the future capacity of nuclear plants. Another example might be the imposition of fuel taxes, or of targeted capital subsidies, etc.

1.2.4 The Techno-economic component of a TIMES scenario

The fourth and last constituent of a scenario is the set of technical and economic parameters assumed for the transformation of primary resources into energy services. In TIMES, these techno-economic parameters are described in the form of *technologies* (or processes) that transform some commodities into others (fuels, materials, energy services, emissions). In TIMES, some technologies may be user imposed and others may simply be available for the model to choose from. The quality of a TIMES model rests on a rich, well developed set of technologies, both current and future, for the model to choose from. The emphasis put on the technological database is one of the main distinguishing factors of the class of Bottom-up models, to which TIMES belongs. Other classes of models will tend to emphasize other aspects of the system (e.g. interactions with the rest of the economy) and treat the technical system in a more succinct manner via aggregate production functions.

Remark: Two scenarios may differ in some or all of their components. For instance, the same demand scenario may very well lead to multiple scenarios by varying the primary resource potentials and/or technologies and/or policies, insofar as the alternative scenario assumptions do not alter the basic demand inputs (drivers and elasticities). The scenario builder must always be careful about the overall coherence of the various assumptions made on the four components of a scenario.

1.3 *Selected scenario types*

The purpose of this section is to show how certain policies may be simulated in a TIMES model. The enormous flexibility of TIMES, especially at the technology level, allows the representation of almost any policy, be it at the national, sector, or subsector level.

Policy 1: Carbon tax

A tax is levied on emissions of CO2 at point of source.

This policy is easily represented in TIMES a) making sure that all technologies that emit CO2 have an emission coefficient, and then defining a tax on these emissions (see 2.6.1.2). The policy may indicate that the tax be levied upstream for some end-use sectors (e.g. automobiles), in which case the emission coefficient is defined at the oil refinery level rather than at the level of individual car types.

Policy 2: Cap-and-trade on CO2

An upper limit on CO2 emissions is imposed at the national level (alternatively, separate upper limits are imposed at the sector level). If the model is multi-country, trade of emission permits is allowed between countries (and/or between sectors). The trade may also be upper bounded by a maximum percentage of the actual emissions, thus representing a form of the subsidiarity principle.

This type of policy is simulated by defining upper bounds on emissions, a straightforward feature in TIMES (sections2.6.1.3 and 2.6.2.3). By defining total sector emissions as a new commodity, the sector-restricted cap is just as easily implemented. The trade of national emissions makes use of the standard trade variables of TIMES (section 5.2).

Policy 3: Portfolio standard

A sector is submitted to a lower limit on its efficiency. For instance, the electricity subsector using fossil fuels must have an overall efficiency of 50%[3]. A similar example is an overall lower limit on the efficiency of light road vehicles.

[3]This standard may also be imposed on the entire electricity generation sector, in which case renewable electricity plants are assumed to have zero energy input.

This type of policy requires the definition of a new constraint that expresses that the ratio of electricity produced (via fossil fueled plants) over the amount of fuel used be more than 0.5. TIMES allows the modeller to define such new constraints via the user constraints (section 5.4.9).

Policy 4: Subsidies for some classes of technologies

The representation of this policy requires defining a capital subsidy for every new capacity of a class of technologies. This is quite straightforward in TIMES using the subsidy parameters (section 2.6.1.2.)

A more elaborate form of the subsidy might be to first levy an emission tax, and then use the proceeds of the tax to subsidize low-emitting and non-emitting technologies. Such a compound policy requires several sequential runs of TIMES, the first run establishing the proceeds of the carbon tax, followed by subsequent runs that distribute the proceeds among the targeted technologies. Several passes of these two runs may well be required in order to balance exactly the proceeds of the tax and the use of them as subsidies.

Assessing the robustness of policies

An important aspect of any policy is whether it will stay effective under various conditions. Examples of such conditions are oil prices, climate parameters, availability of certain resources, key technology costs or efficiency, etc. A policy that remains effective under a range of values for such conditions, is said to be ***robust***. In TIMES, robustness may be assessed using a variety of features, ranging from sensitivity analysis (chapter 6) to Stochastic Programming (chapter 8).

2 The basic structure of the core TIMES model

2.1 *The TIMES economy*

The TIMES energy economy is made up of producers and consumers of *commodities* such as energy carriers, materials, energy services, and emissions. By default, TIMES assumes competitive markets for all commodities, unless the modeler voluntarily imposes regulatory or other constraints on some parts of the energy system, in which case the equilibrium is (partially) regulated. The result is a supply-demand equilibrium that maximizes the *net total surplus* (i.e. the sum of producers' and consumers' surpluses) as fully discussed in chapters 3 and 4. TIMES may however depart from perfectly competitive market assumptions by the introduction of user-defined explicit constraints, such as limits to technological penetration, constraints on emissions, exogenous oil price, etc. Market imperfections can also be introduced in the form of taxes, subsidies and hurdle rates.

While computing the equilibrium, a TIMES run configures the *energy system* of a *set of regions*, over a certain *time horizon,* in such a way as to *minimize the net total cost* (or equivalently *maximize the net total surplus*) of the system, while satisfying a number of *constraints*. TIMES is run in a dynamic manner, which is to say that all investment decisions are made in each period with full knowledge of future events. The model is said to have *perfect foresight*[4] (or to be *clairvoyant*). The next subsection describes in detail the time dimension of the model.

2.2 *Time horizon*

The time horizon is divided into a user-chosen number of time-periods, each period containing a (possibly different) number of years.

In the standard version of TIMES each year in a given period is considered identical, except for the cost objective function which differentiates between payments in each year of a period. For all other quantities (capacities, commodity flows, operating levels, etc.) any model input or output related to period ***t*** applies to each of the years in that period,

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[4] However, there are TIMES variants – discussed in chapters 8 to 12, that depart significantly from these assumptions

with the exception of investment variables, which are usually made only once in a period[5].

Another version of TIMES is available, in which the TIMES variables (capacities and flows) are defined at some year in the midst of each period (called milestone year), and are assumed to evolve linearly between the successive milestone years. This option emulates that of the EFOM model and is discussed in section 5.5.

The initial period is usually considered a past period, over which the model has no freedom, and for which the quantities of interest are all fixed by the user at their historical values. It is often advisable to choose an initial period consisting of a single year, in order to facilitate calibration to standard energy statistics. Calibration to the initial period is one of the more important tasks required when setting up a TIMES model. The main variables to be calibrated are: the capacities and operating levels of all technologies, as well as the extracted, exported, imported, produced, and consumed quantities for all energy carriers, and the emissions if modeled.

In TIMES, years preceding the first period also play a role. Although no explicit variables are defined for these years, data may be provided by the modeler on past investments. Note carefully that the specification of past investments influences not only the initial period's calibration, but also partially determines the model's behavior over several future periods, since the past investments provide residual capacity in several years within the modeling horizon proper.

In addition to time-periods (which may be of variable length), there are time divisions within a year, also called *time-slices,* which may be defined at will by the user (see Figure 2.1). For instance, the user may want to define seasons, portions of the day/night, and/or weekdays/weekends. Time-slices are especially important whenever the mode and cost of production of an energy carrier at different times of the year are significantly different. This is the case for instance when the some energy commodity is expensive to store so that the matching of production and consumption of that commodity is itself an issue to be resolved by the model. The production technologies for the commodity may themselves have different characteristics depending on the time of year (e.g. wind turbines or run-of-the-river hydro plants). In such cases, the matching of supply and demand requires that the activities of the technologies producing and consuming the

[5] There are exceptional cases when an investment must be repeated more than once in a period, namely when the period is so long that it exceeds the technical life of the investment. These cases are described in detail in section 6.2.2 of PART II.

commodity be tracked for each time slice. Examples of commodities requiring time-slicing may include electricity, district heat, natural gas, industrial steam, and hydrogen. An additional reason for defining sub yearly time slices is the requirement of an expensive infrastructure whose capacity should be sufficient to allow the peak demand for the commodity to be satisfied. Technologies that store a commodity in one time slice, at a cost, for discharge in another time slice, may also be defined and modeled.
The net result of these conditions is that the deployment in time of the various production technologies may be very different in different time slices, and furthermore that specific investment decisions will be taken to insure adequate reserve capacity at peak.

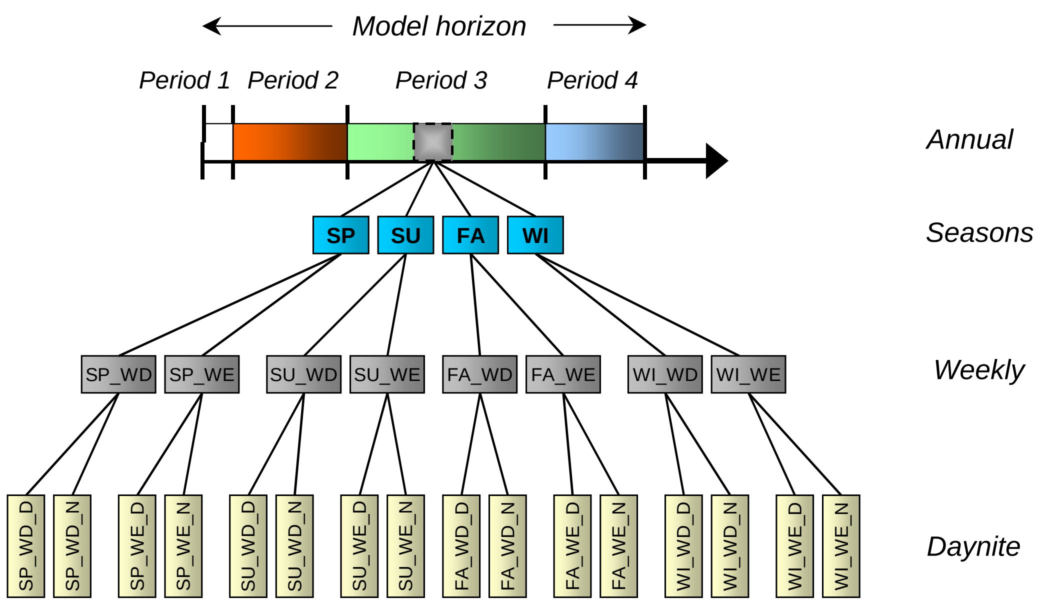

Figure 2.1: Example of a time-slice tree

2.3 ***Decoupling of data and model horizon***

In TIMES, special efforts have been made to decouple the specification of data from the definition of the time periods for which a model is run. Two TIMES features facilitate this decoupling.

First, the fact that investments made in past years are recognized by TIMES makes it much easier to modify the choice of the initial and subsequent periods without major revisions of the database.

Second, the specification of process and demand input data in TIMES is made by specifying the *calendar years* when the data apply, irrespective of how the model time periods have been defined. The model then takes care of interpolating and extrapolating the data for the *periods* chosen by the modeler for a particular model run. TIMES offers a particularly rich range of interpolation/extrapolation modes adapted to each type of data and freely overridden by the user. Section 3.1.1of Part II discusses this feature.

These two features combine to make a change in the definition of periods quite easy and error-free. For instance, if a modeler decides to change the initial year from 2010 to 2015, and perhaps change the number and durations of all other periods as well, only one type of data change is needed, namely to define the investments made from 2011 to 2015 as past investments. All other data specifications need not be altered[6]. This feature represents a great simplification of the modeler's work. In particular, it enables the user to define time periods that have varying lengths, without changing the input data.

2.4 *The components of a Reference Energy System (RES): processes, commodities, flows*

The TIMES energy economy consists of three types of entities:

- *Technologies* (also called *processes*) are representations of physical plants, vehicles, or other devices that transform some commodities into other commodities. They may be primary sources of commodities (e.g. mining processes, import processes), or transformation activities such as conversion plants that produce electricity, energy-processing plants such as refineries, or end-use demand devices such as cars and heating systems, that transform energy into a demand service;
- *Commodities* consisting of energy carriers, energy services, materials, monetary flows, and emissions. A commodity is produced by one or more processes and/or consumed by other processes; and
- *Commodity flows* are the links between processes and commodities. A flow is of the same nature as a commodity but is attached to a particular process, and represents one input or one output of that process. For instance, electricity produced by wind turbine type A at period *p*, time-slice *s*, in region *r*, is a commodity flow.

[6] However, if the horizon has been lengthened beyond the years already covered by the data, additional data for the new years at the end of the horizon must of course be provided.

2.4.1 The RES

It is helpful to picture the relationships among these various entities using a network diagram, referred to as a *Reference Energy System* (RES). In a TIMES RES, processes are represented as boxes and commodities as vertical lines. Commodity flows are represented as links between process boxes and commodity lines.

Figure 2.2 depicts a small portion of a hypothetical RES containing a single energy service demand, namely residential space heating. There are three end-use space heating technologies using the gas, electricity, and heating oil energy carriers (commodities), respectively. These energy carriers in turn are produced by other technologies, represented in the diagram by one gas plant, three electricity-generating plants (gas fired, coal fired, oil fired), and one oil refinery.

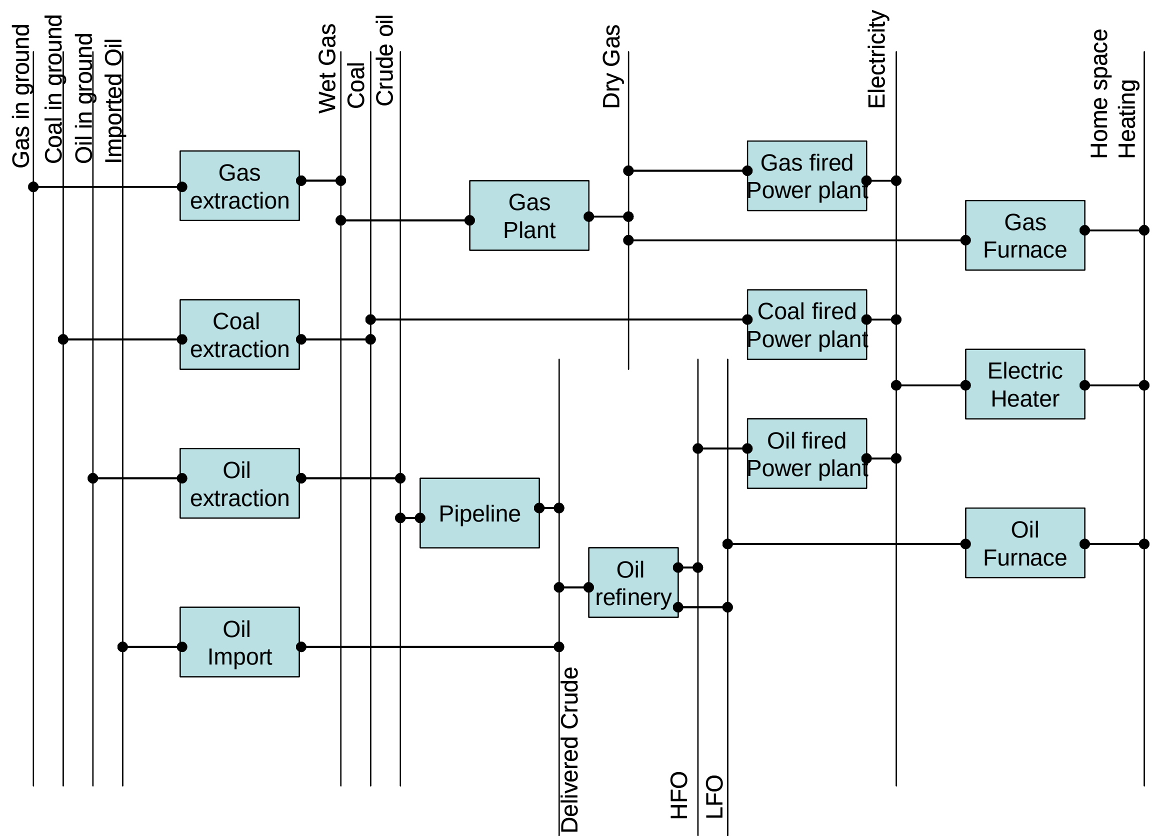

Figure 2.2. Partial view of a Reference Energy System (links are oriented left to right)

To complete the production chain on the primary energy side, the diagram also represents an extraction source for natural gas, an extraction source for coal, and two sources of crude oil (one extracted domestically and then transported by pipeline, and the other one imported). This simple RES has a total of 13 commodities and 13 processes. Note that in the RES every time a commodity enters/leaves a process (via a particular flow) its name is changed (e.g., wet gas becomes dry gas, crude becomes pipeline crude). This simple rule enables the inter-connections between the processes to be properly maintained throughout the network.

To organize the RES, and inform the modeling system of the nature of its components, the various technologies, commodities, and flows may be classified into *sets*. Each set regroups components of a similar nature. The entities belonging to a set are referred to as *members, items* or *elements* of that set. The same item may appear in multiple technology or commodity sets. While the topology of the RES can be represented by a multi-dimensional network, which maps the flow of the commodities to and from the various technologies, the set membership conveys the nature of the individual components and is often most relevant to post-processing (reporting) rather than influencing the model structure itself. However, the TIMES commodities are still classified into several *Major Groups*. There are five such groups: energy carriers, materials, energy services, emissions, and monetary flows. The use of these groups is essential in the definition of some TIMES constraints, as discussed in chapter 5.

2.4.2 Three classes of processes

We now give a brief overview of three classes of processes that need to be distinguished: Processes are *general processes, storage processe*s, and *inter-regional trading processes* (also called *inter-regional exchange processes*). The latter two classes need to be distinguished from general processes due to their special function requiring special rules and sometimes a different set of indices.

2.4.2.1 General processes

In TIMES most processes are endowed with essentially the same attributes (with the exceptions of storage and inter-regional exchange processes, see below), and unless the user decides otherwise (e.g. by providing values for some attributes and ignoring others), they have the same variables attached to them, and must obey similar constraints. Therefore, the differentiation between the various species of processes (or commodities) is made through data specification only, thus eliminating the need to define specialized membership sets, unless desired for processing results. Most of the TIMES features (e.g. sub-annual time-slice resolution, vintaging) are available for all processes and the modeler chooses the features being assigned to a particular process by specifying a corresponding indicator set (e.g. PRC_TSL, PRC_VINT).

A general process receives one or more commodity inputs (inflows) and produces one or more commodity outputs (outflows) in the same time-slice, period, and region.
As already mentioned, two classes of process do not follow these rules and deserve separate descriptions, namely *storage processes* and i*nter-regional exchange processes*.

2.4.2.2 Storage processes (class STG)

This advanced feature of TIMES allows the modeller to represent very intricate storage activities from real life energy systems. Storage processes are used to store a commodity either between periods or between time-slices in the same period. A process ***p*** is specified to be an ***inter-period storage (IPS) process*** for commodity ***c,*** or as ***general time-slice storage (TSS)***. A special case of time-slice storage is a so-called ***night-storage device (NST)*** where the commodity for charging and the one for discharging the storage are different.

An example of a night storage device is an electric heating technology that is charged during the night using electricity and produces heat during the day. Several time-slices may be specified as charging time-slices, the non-specified time-slices are assumed to be discharging time-slices. However, when the process is an end-use process that satisfies a service demand, the discharging occurs according to the load curve of the corresponding demand, and the charging is freely optimized by TIMES across time-slices. Such an exception for demand processes only exists if the demand is at the ANNUAL level. But if the demand is not ANNUAL, discharging can only occur in the non-charging time-slices.

An example of general time-slice storage is a pumped storage reservoir, where electricity is consumed during the night to store water in a reservoir, water which is then used to activate a turbine and produce electricity at a different time-slice.

An example of an inter-period storage process is a plant that accumulates organic refuse in order to produce methane some years later.

Besides the commodity being stored, other (auxiliary) commodity flows are also permitted and may be defined in relation to the stored commodity using the FLO_FUNC and/or the ACT_FLO parameters. The activity of a storage process is interpreted as the amount of the commodity being stored in the storage process. Accordingly the capacity of a storage process describes the maximum commodity amount that can be kept in storage.

2.4.2.3 Inter-regional exchange processes (class IRE)

Inter-regional exchange (IRE) processes are used for trading commodities between regions. Note that the name of the traded commodity is allowed to be different in both regions, depending on the chosen commodity names in both regions. There are two types of trade in TIMES, bi-lateral or multi-lateral.

Bi-lateral trade is the most detailed way to specify trade between regions. It takes place between specific pairs of regions. A pair of regions together with an exchange process and the direction of the commodity flow is first identified, and the model ensures that trade through the exchange process is balanced between these two regions (the amount is exported from region A to region B must be imported by region B from region A, possibly adjusted for transportation losses). If trade should occur only in one direction then only that direction is provided by the proper ordinal attribute. The process capacity and the process related costs (e.g. activity costs, investment costs) of the exchange process may be described individually for both regions by specifying the corresponding parameters in each region. This allows for instance the investment cost of a trade process to be shared between regions in user chosen proportions.

There are cases when it is not important to fully specify the pair of trading regions. An example is the trading of greenhouse gas (GHG) emission permits in a global market. In such cases, the *multi-lateral trade* option decreases the size of the model. Multi-lateral trade is based on the idea that a common marketplace exists for a traded commodity with several selling and several buying regions for the commodity (e.g. GHG emission permits). To model a marketplace the user must first identify (or create) one region that participates both in the production and consumption of the traded commodity. Then a single exchange process is used to link all regions with the marketplace region. Note however that some flexibility is lost when using multilateral trade. For instance, it is not possible to express transportation costs in a fully accurate manner, if such cost depends upon the precise pair of trading regions in a specific way.

2.5 ***Data-driven model structure***

It is useful to distinguish between a model's *structure* and a particular *instance* of its implementation. A model's structure exemplifies its fundamental approach for representing a problem—it does not change from one implementation to the next. All TIMES models exploit an identical underlying structure. However, because TIMES is *data*[7] *driven*, the *effective structure* of a particular instance of a model will vary according to the data inputs. This means that some of the TIMES features will not be activated if the corresponding data is not specified. For example, in a multi-region model one region may, as a matter of user data input, have undiscovered domestic oil reserves. Accordingly, TIMES automatically generates technologies and processes that account for the cost of discovery and field development. If, alternatively, user supplied data indicate that a region does not have undiscovered oil reserves no such technologies and processes would be included in the representation of that region's Reference Energy System (RES, see section 2.4). Due to this property TIMES may also be called a *model generator* that, based on the input information provided by the modeler, generates an instance of a model. In the following, if not stated otherwise, the word 'model' is used with two meanings indifferently: the instance of a TIMES model or more generally the model generator TIMES.

Thus, the structure of a TIMES model is ultimately defined by variables and equations created from the union of the underlying TIMES equations and the data input provided by the user. This information collectively defines each TIMES regional model database, and therefore the resulting mathematical representation of the RES for each region. The database itself contains both qualitative and quantitative data.

The *qualitative data* includes, for example, the list of commodities, and the list of those technologies that the modeler feels are applicable (to each region) over a specified time horizon. This information may be further classified into subgroups, for example commodities may include energy carriers (themselves split by type --e.g., fossil, nuclear, renewable, etc.), materials, emissions, energy services.

Quantitative data, in contrast, contains the technological and economic parameter assumptions specific to each technology, region, and time period. When constructing multi-region models it is often the case that a given technology is available for use in two or more regions; however, cost and performance assumptions may be quite different. The word ***attribute*** designates both qualitative and quantitative elements of the TIMES modeling system.

[7] Data in this context refers to parameter assumptions, technology characteristics, projections of energy service demands, etc. It does not refer to historical data series.

2.6 ***A brief overview of the TIMES attributes***

Due to the data driven nature of TIMES (see section 2.5), all TIMES constraints are activated and defined by specifying some attributes. Attributes are attached to processes, to commodities, to flows, or to special variables that have been created to define new TIMES features. Indeed, TIMES has many new attributes that were not available in earlier versions, corresponding to powerful new features that confer additional modeling flexibility. The complete list of attributes is fully described in section 3 of PART II, and we provide below only succinct comments on the types of attribute attached to each entity of the RES or to the RES as a whole. Additional attribute definitions may also be included in the chapters describing new features or variants of the TIMES generator.

Attributes may be *cardinal* (numbers) or *ordinal* (lists, sets). For example, some ordinal attributes are defined for processes to describe subsets of flows that are then used to construct specific flow constraints as described in section 5.4. PART II, section 2 shows the complete list of TIMES sets.

The cardinal attributes are usually called *parameters*. We give below a brief idea of the main types of parameters available in the TIMES model generator.

2.6.1 Parameters attached to processes

TIMES process-oriented parameters fall into several general categories.

2.6.1.1 Technical parameters

Technical parameters include process efficiency, availability factor(s)[8], commodity consumptions per unit of activity, shares of fuels per unit activity, technical life of the process, construction lead time, dismantling lead-time and duration, amounts of the commodities consumed (respectively released) by the construction (respectively dismantling) of one unit of the process, and contribution to the peak equations. The efficiency, availability factors, and commodity inputs and outputs of a process may be defined in several flexible ways depending on the desired process flexibility, on the time-slice resolution chosen for the process and on the time-slice resolution of the

[8] There are a variety of availability factors: annual or seasonal. Each may be specified as a maximum factor (the most frequent case), an exact factor, or even a minimum factor (in order to force some minimum utilization of the capacity of some equipment, as in a backup gas turbine for instance).

commodities involved. Certain parameters are only relevant to special processes, such as storage processes or processes that implement trade between regions.

2.6.1.2 Economic and policy parameters

A second class of process parameters comprises *economic and policy parameters* that include a variety of costs attached to the investment, dismantling, maintenance, and operation of a process. The investment cost of the technology is incurred once at the time of acquisition; the fixed annual cost is incurred each year per unit of the capacity of the technology, as long as the technology is kept alive (even if it is not actively functioning); the annual variable cost is incurred per unit of the activity of the technology. In addition to costs, taxes and subsidies (on investment and/or on activity) may be defined in a very flexible manner. Other economic parameters are: the economic life of a process (the time during which the investment cost of a process is amortized, which may differ from the operational lifetime) and the process specific discount rate, also called *hurdle rate*. Both these parameters serve to calculate the annualized payments on the process investment cost, which enters the expression for the total cost of the run (section 5.2).

2.6.1.3 Bounds

Another class of parameter is used to define the right-hand-side of some constraint. Such a parameter represents a ***bound*** and its specification triggers the constraint on the quantity concerned. Most frequently used bounds are those imposed on period investment, capacity, or activity of a process. Newly defined bounds allow the user to impose limits on the annual or annualized payments at some period or set of consecutive years.

A special type of bounding consists in imposing upper or lower limits on the ***growth rate*** of technologies. The most frequently quantities thus bounded are investment, capacity and activity of a process, for which a simplified formulation has been devised.

The growth constraints belong to the class of ***dynamic bounds*** that involve multiple periods. Many other dynamic bounds may be defined by the user. ***Bounds on cumulative quantities*** are also very useful. The accumulation may be over the entire horizon or over some user defined set of consecutive years. The variables on which such bounds apply may quite varied, such as: process capacity, process investment, process activity, annual or annuity payments, etc.

All bounds may be of four types: lower (LO), upper (UP), equality (FX), or neutral (N). The latter case does not introduce any restriction on the optimization, and is used only to generate a new reporting quantity.

2.6.1.4 Other parameters

Features that were added to TIMES over the years require new parameters. For instance, the Climate Module of TIMES (chapter 7), the Lumpy Investment feature (chapter 10), and several others. These will be alluded to in the corresponding chapters of this Part I, and more completely described in section 2 and Appendices of Part II.

An advanced feature allows the user to define certain process parameters as *vintaged* (i.e. dependent upon the date of installation of new capacity). For instance, the investment cost and fuel efficiency of a specific type of automobile will depend on the model year[9].

Finally, another advanced TIMES feature renders some parameters dependent *also on the age* of the technology. For instance, the annual maintenance cost of an automobile could be defined to remain constant for say 3 years and then increase in a specified manner each year after the third year.

2.6.2 Parameters attached to commodities

This subsection concerns parameters attached to each commodity, irrespective of how the commodity is produced or consumed. The next subsection concerns commodity flows. Commodity-oriented parameters fall into the same categories as those attached to processes.

2.6.2.1 Technical parameters

Technical parameters associated with commodities include overall efficiency (for instance the overall electric grid efficiency), and the time-slices over which that commodity is to be tracked. For demand commodities, in addition, the annual projected demand and load curves (if the commodity has a sub-annual time-slice resolution) can be specified.

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[9] Vintaging could also be introduced by defining a new technology for each vintage year, but this approach would be wasteful, as many parameters remain the same across all vintages.

2.6.2.2 Economic and policy parameters

Economic parameters include additional costs, taxes, and subsidies on the overall or net production of a commodity. These cost elements are then added to all other (implicit) costs of that commodity. In the case of a demand service, additional parameters define the demand curve (i.e. the relationship between the quantity of demand and its price). These parameters are: the demand's own-price elasticity, the total allowed range of variation of the demand value, and the number of steps to use for the discrete approximation of the curve.

Policy based parameters include bounds (at each period or cumulative over user defined years) on the gross or net production of a commodity, or on the imports or exports of a commodity by a region.

2.6.2.3 Bounds

In TIMES the net or the total production of each commodity may be explicitly represented by a variable, if needed for imposing a bound or a tax. A similar variety of bounding parameters exists for commodities as for processes.

2.6.3 Parameters attached to commodity flows

A *commodity flow* (more simply, a *flow*) is an amount of a given commodity produced or consumed by a given process. Some processes have several flows entering or leaving them, perhaps of different types (fuels, materials, demands, or emissions). In TIMES, each flow has a variable attached to it, as well as several attributes (parameters or sets). Flow related parameters confer enormous flexibility for modeling a large spectrum of conditions.

2.6.3.1 Technical parameters

Technical parameters, along with some set attributes, permit full control over the maximum and/or minimum share a given input or output flow may take within the same commodity group. For instance, a flexible turbine may accept oil and/or gas as input, and the modeler may use a parameter to limit the share of oil to, say, at most 40% of the total fuel input. Other parameters and sets define the amount of certain outflows in relation to certain inflows (e.g., efficiency, emission rate by fuel). For instance, in an oil refinery a parameter may be used to set the total amount of refined products equal to 92% of the total amount of crude oils (s) entering the refinery, or to calculate certain emissions as a fixed proportion of the amount of oil consumed. If a flow has a sub-annual time-slice resolution, a load curve can be specified for the flow. It is possible to define not only load curves for a flow, but also bounds on the share of a flow in a specific time-slice relative to the annual flow, e.g. the flow in the time-slice “Winter-Day” has to be at least 10 % of the total annual flow. Refer to section 5.4 describing TIMES constraints for details. Cumulative bounds on a process flow are also allowed.

2.6.3.2 Economic and policy parameters

Economic or policy parameters include delivery and other variable costs, taxes and subsidies attached to an individual process flow.

2.6.3.3 Bounds

Bounds may be defined for flows in similar variety that exists for commodities.

2.6.4 Parameters attached to the entire RES

These parameters include currency conversion factors (in a multi-regional model), region-specific time-slice definitions, a region-specific general discount rate, and reference year for calculating the discounted total cost (objective function). In addition, certain switches are needed to control the activation of the data interpolation procedure as well as special model features to be used. The complete set of switches is described in Part III.

2.7 ***Process and commodity classification***

Although TIMES does not explicitly differentiate processes or commodities that belong to different portions of the RES (with the notable exceptions of storage and trading processes), there are three ways in which some differentiation does occur.

First, TIMES requires the definition of Primary Commodity Groups (*pcg*), i.e. subsets of commodities *of the same nature* entering or leaving a process. TIMES utilizes the pcg to define the activity of the process, and also its capacity. For instance, the *pcg* of an oil refinery is defined as the set of energy forms produced by the plant; and the activity of the refinery is thus simply the sum of all its energy outputs (excluding any outputs that are non energy).

Besides establishing the process activity and capacity, these groups are convenient aids for defining certain complex quantities related to process flows, as discussed in chapter 5 and in PART II, section 2.1.

Even though TIMES *does not require* that the user provide many set memberships, the TIMES reporting step does pass some set declarations to the VEDA-BE result-processing system[10] to facilitate construction of results analysis tables. These include process subsets to distinguish demand devices, energy processes, material processes (by weight or volume), refineries, electric production plants, coupled heat and power plants, heating plants, storage technologies and distribution (link) technologies; and commodity subsets for energy, useful energy demands (split into six aggregate sub-sectors), environmental indicators, currencies, and materials.

Besides the definition of *pcg*'s and that of VEDA reporting sets, there is a third instance of commodity or process differentiation which is not embedded in TIMES, but rests entirely on the modeler. A modeler may well want to choose process and commodity names in a judicious manner so as to more easily identify them when browsing through the input database or when examining results. As an example, the TIAM-World multi-regional TIMES model (Loulou, 2007) adopts a naming convention whereby the first three characters of a commodity’s name denote the sector and the next three the fuel (e.g., light fuel oil used in the residential sector is denoted RESLFO). Similarly, process names are chosen so as to identify the sub-sector or end-use (first three characters), the main fuel used (next three), and the specific technology (last four). For instance, a standard (0001) residential water heater (RWH) using electricity (ELC) is named RWHELC0001. Naming conventions may thus play a critical role in allowing the easy identification of an element’s position in the RES and thus facilitate the analysis and reporting of results.

Similarly, energy services may be labeled so that they are more easily recognized. For instance, the first letter may indicate the broad sector (e.g. ‘T’ for transport) and the second letter designate any homogenous sub-sectors (e.g. ‘R’ for road transport), the third character being free.

In the same fashion, fuels, materials, and emissions may be identified so as to immediately designate the sector and sub-sector where they are produced or consumed. To achieve this, some fuels have to change names when they change sectors. This is accomplished via processes whose primary role is to change the name of a fuel. In

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[10] See Appendix A for the VEDA-FE, VEDA-BE, and ANSWER modeling and analysis systems, used to maintain and manage TIMES databases, conduct model runs, and organize results.

addition, such a process may serve as a bearer of sector wide parameters such as distribution cost, price markup, tax, that are specific to that sector and fuel. For instance, a tax may be levied on industrial distillate use but not on agricultural distillate use, even though the two commodities are physically identical.

3 Economic rationale of the TIMES modeling approach

This chapter provides a detailed economic interpretation of TIMES and other partial equilibrium models based on maximizing total surplus. Partial equilibrium models have one common feature – they simultaneously configure the production and consumption of commodities (i.e. fuels, materials, and energy services) and their prices. The price of producing a commodity affects the demand for that commodity, while at the same time the demand affects the commodity's price. A market is said to have reached an equilibrium at prices p^* and quantities q^* when no consumer wishes to purchase less than q^* and no producer wishes to produce more than q^* at price p^*. Both p^* and q^* are vectors whose dimension is equal to the number of different commodities being modeled. As will be explained below, when all markets are in equilibrium the total economic surplus is maximized.

The concept of total surplus maximization extends the direct cost minimization approach upon which earlier bottom-up energy system models were based. These simpler models had fixed energy service demands, and thus were limited to minimizing the cost of supplying these demands. In contrast, the TIMES demands for energy services are themselves elastic to their own prices, thus allowing the model to compute a *bona fide* supply-demand equilibrium. This feature is a fundamental step toward capturing the main feedback from the economy to the energy system.

Section 3.1 provides a brief review of different types of energy models. Section 3.2 discusses the economic rationale of the TIMES model with emphasis on the features that distinguish TIMES from other bottom-up models (such as the early incarnations of MARKAL, see Fishbone and Abilock, 1981 and Berger et al., 1992, though MARKAL has since been extended beyond these early versions). Section 3.3 describes the details of how price elastic demands are modeled in TIMES, and section 3.4 provides additional discussion of the economic properties of the model.

3.1. A brief classification of energy models

Many energy models are in current use around the world, each designed to emphasize a particular facet of interest. Differences include: economic rationale, level of disaggregation of the variables, time horizon over which decisions are made (which is closely related to the type of decisions, i.e., only operational planning or also investment decisions), and geographic scope. One of the most significant differentiating features among energy models is the degree of detail with which commodities and technologies are represented, which will guide our classification of models into two major classes, as explained in the following very streamlined classification.

3.1.1 'Top-down' models

At one end of the spectrum are aggregated *General Equilibrium* (GE) models. In these each sector is represented by a production function designed to simulate the potential substitutions between the main factors of production (also highly aggregated into a few variables such as: energy, capital, and labor) in the production of each sector's output. In this model category are found a number of models of national or global energy systems. These models are usually called "top-down", because they represent an entire economy via a relatively small number of aggregate variables and equations. In these models, production function parameters are calculated for each sector such that inputs and outputs reproduce a single base historical year.[11] In policy runs, the mix of inputs[12] required to produce one unit of a sector's output is allowed to vary according to user-selected elasticities of substitution. Sectoral production functions most typically have the following general form:

$$X_s = A_0\left(B_K \cdot K_s^{\rho} + B_L \cdot L_S^{\rho} + B_E \cdot E_S^{\rho}\right)^{1/\rho} \qquad (3\text{–}1)$$

where $\boldsymbol{X_S}$ is the output of sector *S*,

$\boldsymbol{K_S}$, $\boldsymbol{L_S}$, *and* $\boldsymbol{E_S}$ are the inputs of capital, labor and energy needed to produce one unit of output in sector *S*,

$\boldsymbol{\rho}$ is the elasticity of substitution parameter,

$\boldsymbol{A_0}$ and the $\boldsymbol{B}$'s are scaling coefficients.

The choice of ρ determines the ease or difficulty with which one production factor may be substituted for another: the smaller ρ is (but still greater than or equal to 1), the easier it is to substitute the factors to produce the same amount of output from sector *S*. Also note that the degree of factor substitutability does not vary among the factors of production — the ease with which capital can be substituted for labor is equal to the ease

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[11] These models assume that the relationships (as defined by the form of the production functions as well as the calculated parameters) between sector level inputs and outputs are in equilibrium in the base year.

[12] Most models use inputs such as labor, energy, and capital, but other input factors may conceivably be added, such as arable land, water, or even technical know-how. Similarly, labor may be further subdivided into several categories.

with which capital can be substituted for energy, while maintaining the same level of output. GE models may also use alternate forms of production function (3-1), but retain the basic idea of an explicit substitutability of production factors.

3.1.2 'Bottom-up' models

At the other end of the spectrum are the very detailed, *technology explicit* models that focus primarily on the energy sector of an economy. In these models, each important energy-using technology is identified by a detailed description of its inputs, outputs, unit costs, and several other technical and economic characteristics. In these so-called 'bottom-up' models, a sector is constituted by a (usually large) number of logically arranged technologies, linked together by their inputs and outputs (*commodities*, which may be energy forms or carriers, materials, emissions and/or demand services). Some bottom-up models compute a partial equilibrium via maximization of the total net (consumer and producer) surplus, while others simulate other types of behavior by economic agents, as will be discussed below. In bottom-up models, one unit of sectoral output (e.g., a billion vehicle kilometers, one billion tonnes transported by heavy trucks or one petajoule of residential cooling service) is produced using a mix of individual technologies' outputs. Thus the production function of a sector is *implicitly* constructed, rather than explicitly specified as in more aggregated models. Such implicit production functions may be quite complex, depending on the complexity of the reference energy system of each sector (sub-RES).

3.1.3 Hybrid approaches

While the above dichotomy applied fairly well to earlier models, these distinctions now tend to be somewhat blurred by advances in both categories of model. In the case of aggregate top-down models, several general equilibrium models now include a fair amount of fuel and technology disaggregation in the key energy producing sectors (for instance: electricity production, oil and gas supply). This is the case with MERGE[13] and SGM[14], among others.

In the other direction, the more advanced bottom-up models are 'reaching up' to capture some of the effects of the entire economy on the energy system. The TIMES model has

[13] Model for Evaluating Regional and Global Effects (Manne et al., 1995)
[14] Second Generation Model (Edmonds et al., 1991)

end-use demands (including demands for industrial output) that are sensitive to their own prices, and thus captures the impact of rising energy prices on economic output and *vice versa*. Recent incarnations of technology-rich models (including TIMES) are multi-regional, and thus are able to consider the impacts of energy-related decisions on trade. It is worth noting that while the multi-regional top-down models have always represented trade, they have done so with a very limited set of traded commodities – typically one or two, whereas there may be quite a number of traded energy forms and materials in multi-regional bottom-up models.

MARKAL-MACRO (Manne and Wene, 1992) and TIMES-MACRO (Kypreos and Lehtila, 2013) are hybrid models combining the technological detail of MARKAL with a succinct representation of the macro-economy consisting of a single producing sector in a single region. Because of its succinct single-sector production function, MARKAL-MACRO is able to compute a general equilibrium in a single optimization step. More recently, TIMES_MACRO-MSA (section 12.2) is based on the computation of a multi-regional global equilibrium, but requires an iterative process to do so. MESSAGE (Messner and Strubegger, 1995) links a bottom-up model based on the EFOM paradigm with a macro module, and computes a global, multi-regional equilibrium iteratively. The NEMS (US EIA, 2000) model is another example of a full linkage between several technology rich modules of the various energy subsectors and a set of macro-economic equations, and requires iterative resolution methods.

In spite of these advances in both classes of models, there remain important differences. Specifically:

- Top-down models encompass macroeconomic variables beyond the energy sector proper, such as wages, consumption, and interest rates, and
- Bottom-up models have a rich representation of the variety of technologies (existing and/or future) available to meet energy needs, and, they often have the capability to track a much wider variety of traded commodities. They are also more adapted to the representation of micro policies targeting specific technologies or commodities.

The top-down vs. bottom-up approach is not the only relevant difference among energy models. Among top-down models, the so-called Computable General Equilibrium models (CGE) described above differ markedly from the *macro econometric models*. The latter do not compute equilibrium solutions, but rather simulate the flows of capital and other monetized quantities between sectors (see, e.g., Meade, 1996 on the LIFT model). They use econometrically derived input-output coefficients to compute the impacts of these flows on the main sectoral indicators, including economic output (GDP) and other variables (labor, investments). The sector variables are then aggregated into national indicators of consumption, interest rate, GDP, labor, and wages.

Among technology explicit models also, two main classes are usually distinguished: the first class is that of the partial equilibrium models such as MARKAL, MESSAGE, and TIMES, that use optimization techniques to compute a least cost (or maximum surplus) path for the energy system. The second class is that of *simulation* models, where the emphasis is on representing a system not governed purely by financial costs and profits. In these simulation models (e.g., CIMS, Jaccard et al. 2003), investment decisions taken by a representative agent (firm or consumer) are only partially based on profit maximization, and technologies may capture a share of the market even though their life-cycle cost may be higher than that of other technologies. Simulation models use market-sharing formulas that preclude the easy computation of equilibrium – at least not in a single pass. The SAGE (US EIA, 2002) incarnation of the MARKAL model possesses a market sharing mechanism that allows it to reproduce certain behavioral characteristics of observed markets.

3.2 *The core TIMES paradigm*

In the rest of this chapter, we present the properties of the **core TIMES** paradigm. As will be seen in chapters8 to 12, some of these properties are not applicable to several important TIMES variants. The reader should keep this caveat in mind when contemplating the use of some features that are described in these 5 chapters.

Since certain portions of this and the next sections require an understanding of the concepts and terminology of Linear Programming, the reader requiring a brush-up on this topic may first read Appendix B, and then, if needed, some standard textbook on LP, such as Hillier and Lieberman (2009), Chvàtal (1983), or Schrijver (1986). The application of Linear Programming to microeconomic theory is covered in two historically important references, Gale (1960 and 11th edition 1989), and in Dorfman, Samuelson, and Solow (1958, and 1987 reprint).

A brief description of the core TIMES model generator would express that it is:

- *Technologically explicit, integrated*;
- *Multi-regional*; and

- *Partial equilibrium* (with *price elastic* demands for energy services) in *competitive markets* with *perfect foresight*. It will be seen that such an equilibrium entails *marginal value pricing* of all commodities.

We now proceed to flesh out each of these properties.

3.2.1 A technologically explicit integrated model

As already presented in chapter 2 (and described in much more detail in Part II, section 3), each technology is described in TIMES by a number of technical and economic parameters. Thus each technology is explicitly identified (given a unique name) and distinguished from all others in the model. A mature TIMES model may include several thousand technologies in all sectors of the energy system (energy procurement, conversion, processing, transmission, and end-uses) in each region. Thus TIMES is not only technologically explicit, it is technology rich and it is integrated as well. Furthermore, the number of technologies and their relative topology may be changed at will, purely via data input specification, without the user ever having to modify the model's equations. The model is thus to a large extent *data driven*.

3.2.2 Multi-regional

Some existing TIMES models comprise several dozen regional modules, or more. The number of regions in a model is limited only by the difficulty of solving LP's of very large size. The individual regional modules are linked by energy and material trading variables, and by emission permit trading variables, if desired. The linking variables transform the set of regional modules into a *single* multi-regional (possibly global) energy model, where actions taken in one region may affect all other regions. This feature is essential when global as well as regional energy and emission policies are being simulated. Thus a multi-regional TIMES model is geographically integrated.

3.2.3 Partial equilibrium

The core version of TIMES computes a partial equilibrium on energy markets. This means that the model computes both the *flows* of energy forms and materials as well as their *prices*, in such a way that, at the prices computed by the model, the suppliers of energy produce exactly the amounts that the consumers are willing to buy. This equilibrium feature is present at every stage of the energy system: primary energy forms, secondary energy forms, and energy services[15]. A supply-demand equilibrium model has as its economic rationale the maximization of the total surplus, defined as the sum of all suppliers' and consumers' surpluses. The mathematical method used to maximize the surplus must be adapted to the particular mathematical properties of the model. In TIMES, these properties are as follows:

- Outputs of a technology are linear functions of its inputs (subsection 3.2.3.1)[16];
- Total economic surplus is maximized over the entire horizon (3.2.3.2); and
- Energy markets are competitive, with perfect foresight (3.2.3.3)[17].

As a result of these assumptions the following additional properties hold:

- The market price of each commodity is equal to its marginal value in the overall system (3.2.4); and
- Each economic agent maximizes its own profit or utility (3.2.5).

3.2.3.1 Linearity

A linear input-to-output relationship first means that each technology represented may be implemented at any capacity, from zero to some upper limit, without economies or dis-economies of scale. In a real economy, a given technology is usually available in discrete sizes, rather than on a continuum. In particular, for some real life technologies, there may be a minimum size below which the technology may not be implemented (or else at a prohibitive cost), as for instance a nuclear power plant, or a hydroelectric project. In such cases, because TIMES assumes that all technologies may be implemented in any size, it may happen that the model's solution shows some technology's capacity at an unrealistically small size. It should however be noted that in most applications, such a situation is relatively infrequent and often innocuous, since the scope of application is at the country or region's level, and thus large enough so that small capacities are unlikely to occur.

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[15]It has been argued, based on strong experimental evidence, that the change in demands for energy services indeed captures the main economic impact of energy system policies on the economy at large (Loulou and Kanudia, 2000)

[16] This property does not hold in three TIMES extensions presented in Chapters 10-12.

[17] These two properties do not hold in the time-stepped extension of TIMES (chapter 9) and in Stochastic TIMES (Chapter 8.)

On the other hand, there may be situations where plant size matters, for instance when the region being modeled is very small. In such cases, it is possible to enforce a rule by which certain capacities are allowed only in multiples of a given size (e.g., build or not a gas pipeline), by introducing *integer variables*. This option, referred to as lumpy investment (LI), is available in TIMES and is discussed in chapter 10. This approach should, however, be used sparingly because it may greatly increase solution time.
It is the linearity property that allows the TIMES equilibrium to be computed using Linear Programming techniques. In the case where economies of scale or some other non-convex relationship is important to the problem being investigated, the optimization program would no longer be linear or even convex. We shall examine such cases in chapters 9 to 12.

We must now mention a common misconception regarding linearity: the fact that TIMES equations are linear *does not mean that production functions behave in a linear fashion; far from it*. Indeed, the TIMES production functions are usually highly non-linear (although convex), consisting of a stepped sequence of linear functions. As a simple example, a supply of some resource is almost always represented as a sequence of segments, each with rising (but constant within an interval) unit cost. The modeler defines the 'width' of each interval so that the resulting supply curve may simulate any non-linear convex function. In brief, diseconomies of scale are easily represented in linear models.

3.2.3.2 Maximization of total surplus: Price equals marginal value

The *total surplus* of an economy is the sum of the suppliers' and the consumers' surpluses. The term *supplier* designates any economic agent that produces (and/or sells) one or more commodities i.e., in TIMES, an energy form, a material, an emission permit, and/or an energy service. A *consumer* is a buyer of one or more commodities. In TIMES, the suppliers of a commodity are technologies that procure a given commodity, and the consumers of a commodity are technologies or service segments that consume a given commodity. Some (indeed most) technologies are both suppliers and consumers. Therefore, for each commodity the RES defines a complex set of suppliers and consumers.

It is customary in microeconomics to represent the set of suppliers of a commodity by their *inverse production function*, that plots the marginal production cost of the commodity (vertical axis) as a function of the quantity supplied (horizontal axis). In TIMES, as in other linear optimization models, the supply curve of a commodity, with the exception of end-use demands, is entirely determined endogenously by the model. It is a standard result of Linear Programming theory that the inverse supply function is step-wise constant and increasing in each factor (see Figures 3.1 and 3.3 for the case of a single commodity[18]). Each horizontal step of the inverse supply function indicates that the commodity is produced by a certain technology or set of technologies in a strictly linear fashion. As the quantity produced increases, one or more resources in the mix (either a technological potential or some resource's availability) is exhausted, and therefore the system must start using a different (more expensive) technology or set of technologies in order to produce additional units of the commodity, albeit at higher unit cost. Thus, each change in production mix generates one step of the staircase production function with a value higher than the preceding step. The width of any particular step depends upon the technological potential and/or resource availability associated with the set of technologies represented by that step.

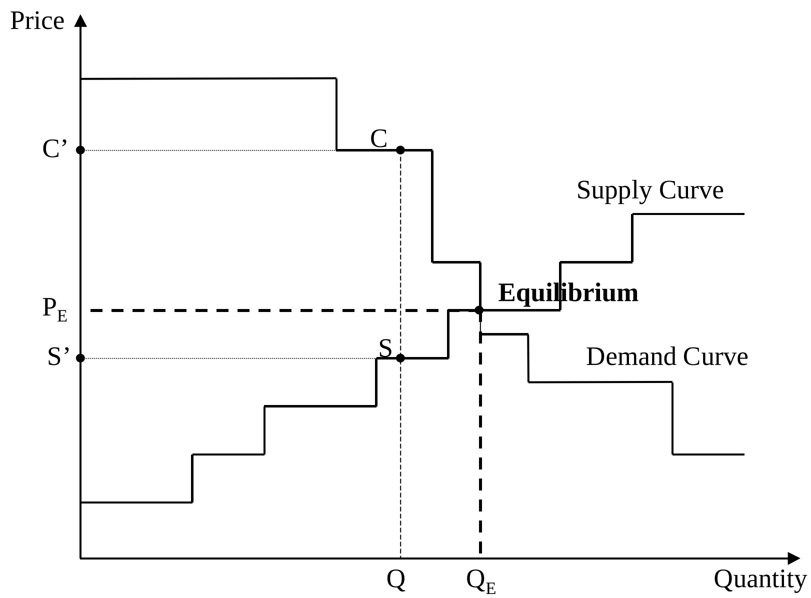

Figure 3.1. Equilibrium in the case of an energy form: the model implicitly constructs both the supply and the demand curves (note that the equilibrium is multiple in this configuration)

In a similar manner, each TIMES model instance defines a series of inverse demand functions. In the case of demands, two cases are distinguished. First, if the commodity in question is an energy carrier whose production and consumption are endogenous to the model, then its demand function is *implicitly* constructed within TIMES, and is a step-

[18] This is so because in Linear Programming the shadow price of a constraint remains constant over a certain interval, and then changes abruptly, giving rise to a stepwise constant functional shape.

wise constant, decreasing function of the quantity demanded, as illustrated in Figure 3.1 for a single commodity. If on the other hand the commodity is a demand for an energy service, then its demand curve is *defined by the user* via the specification of the own-price elasticity of that demand, and the curve is in this instance a smoothly decreasing curve as illustrated in Figure 3.2[19]. In both cases, the supply-demand equilibrium is at the intersection of the supply function and the demand function, and corresponds to an equilibrium quantity Q_E and an equilibrium price P_E[20]. At price P_E, suppliers are willing to supply the quantity Q_E and consumers are willing to buy exactly that same quantity $Q_{E.}$ Of course, the TIMES equilibrium concerns a large number of commodities simultaneously, and thus the equilibrium is a multi-dimensional analog of the above, where Q_E and P_E are now vectors rather than scalars.

As already mentioned, the demand curves of most TIMES commodities (i.e. energy carriers, materials, emission permits) are implicitly constructed endogenously as an integral part of the solution of the LP. For each commodity that is an energy service, the user *explicitly* defines the demand *function* by specifying its own price elasticity. In TIMES, each energy service demand is assumed to have a constant own price elasticity function of the form (see Figure 3.2):

$$DM/DM_0 = (P/P_0)^E \qquad (3\text{–}2)$$

Where $\{DM_0 ,P_0\}$ is a reference pair of demand and price values for that energy service over the forecast horizon, and E is the (negative) own price elasticity of that energy service demand, as specified by the user (note that although not obvious from the notation, this price elasticity may vary over time). The pair $\{DM_0, P_0\}$ is obtained by solving TIMES for a reference scenario. More precisely, $DM_0$is the demand projection estimated by the user in the reference scenario (usually based upon explicitly defined relationships to economic and demographic drivers), and P_0 is the shadow price of that energy service demand in the dual solution of the reference case scenario. The precise manner in which the demand functions are discretized and incorporated in the TIMES objective function is explained in chapter 4.

Using Figure 3.1 as an example, the definition of the suppliers' surplus corresponding to a certain point S on the inverse supply curve is the difference between the total revenue

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[19] This smooth curve will be discretized later for computational purposes, and thus become a staircase function, as described in section 4.2

[20] As may be seen in figure 3.1, the equilibrium is not necessarily unique. In the case shown, any point on the vertical segment containing the equilibrium is also an equilibrium, with the same quantity Q_E but a different price. In other situations, the multiple equilibria may have a single price but multiple quantities.

and the total cost of supplying a commodity, i.e. the gross profit. In Figure 3.1, the surplus is thus the area between the horizontal segment SS' and the inverse supply curve. Similarly, the consumers' surplus for a point C on the inverse demand curve, is defined as the area between line segment CC' and the inverse demand curve. This area is a consumer's analog to a producer's profit; more precisely it is the cumulative opportunity gain of all consumers who purchase the commodity at a price lower than the price they would have been willing to pay. Thus, for a given quantity Q, the total surplus (suppliers' plus consumers') is simply the area between the two inverse curves situated at the left of Q. It should be clear from Figure 3.1 that the total surplus is maximized when Q is exactly equal to the equilibrium quantity Q_E. Therefore, we may state (in the single commodity case) the following Equivalence Principle:

"The supply-demand equilibrium is reached when the total surplus is maximized."

This is a remarkably useful result, as it leads to a method for computing the equilibrium, as will be see in much detail in Chapter 4.

In the multi-dimensional case, the proof of the above statement is less obvious, and requires a certain qualifying property (called the integrability property) to hold (Samuelson, 1952, Takayama and Judge, 1972). One sufficient condition for the integrability property to be satisfied is realized when the cross-price elasticities of any two energy forms are equal, viz.

$$\partial P_j / \partial Q_i = \partial P_i / \partial Q_j \text{ for all } i, j$$

In the case of commodities that are end-use energy services, these conditions are trivially satisfied in TIMES because we have assumed zero cross price elasticities. In the case of an endogenous energy carrier, where the demand curve is implicitly derived, it is also easy to show that the integrability property is always satisfied[21]. Thus the equivalence principle is valid in all cases.

---

[21] This results from the fact that in TIMES each price $\boldsymbol{P_i}$ is the shadow price of a balance constraint (see section 5.4.4), and may thus be (loosely) expressed as the derivative of the objective function $\boldsymbol{F}$ with respect to the right-hand-side of a balance constraint, i.e. $\partial \boldsymbol{F} / \partial \boldsymbol{Q_i}$. When that price is further differentiated with respect to another quantity $\boldsymbol{Q_j}$, one gets $\partial^2 \boldsymbol{F} / \partial \boldsymbol{Q_i} \bullet \partial \boldsymbol{Q_j}$, which, under mild conditions is always equal to $\partial^2 \boldsymbol{F} / \partial \boldsymbol{Q_j} \bullet \partial \boldsymbol{Q_i}$, as desired.

In summary, the equivalence principle guarantees that the TIMES supply-demand equilibrium maximizes total surplus. The total surplus concept has long been a mainstay of social welfare economics because it takes into account both the surpluses of consumers and of producers.[22]

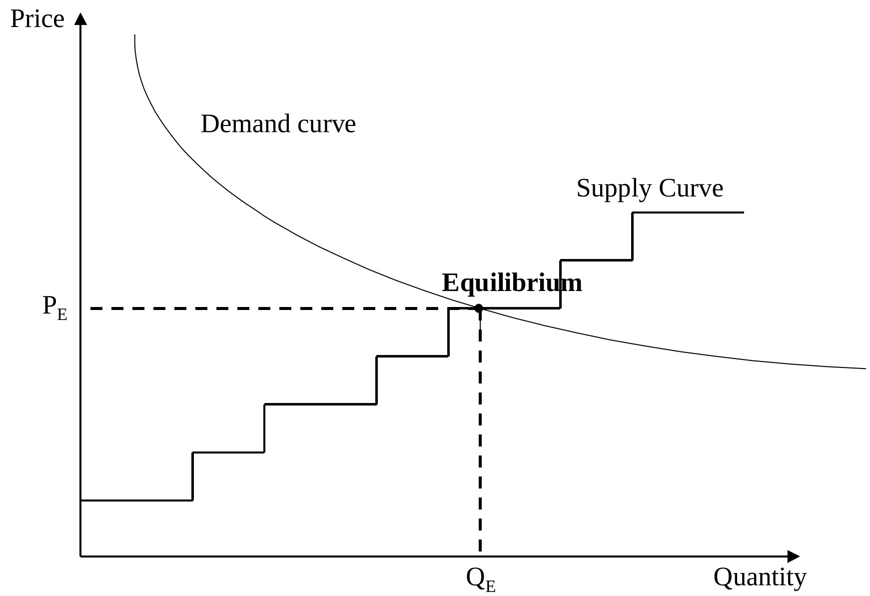

Figure 3.2. Equilibrium in the case of an energy service: the user explicitly provides the demand curve, usually using a simple functional form (see text for details)

Remark: In older versions of MARKAL, and in several other least-cost bottom-up models, energy service demands are exogenously specified by the modeler, and only the cost of supplying these energy services is minimized. Such a case is illustrated in Figure 3.3 where the "inverse demand curve" is a vertical line. The objective of such models was simply the minimization of the total cost of meeting exogenously specified levels of energy service.

[22] See e.g. Samuelson and Nordhaus (1977)

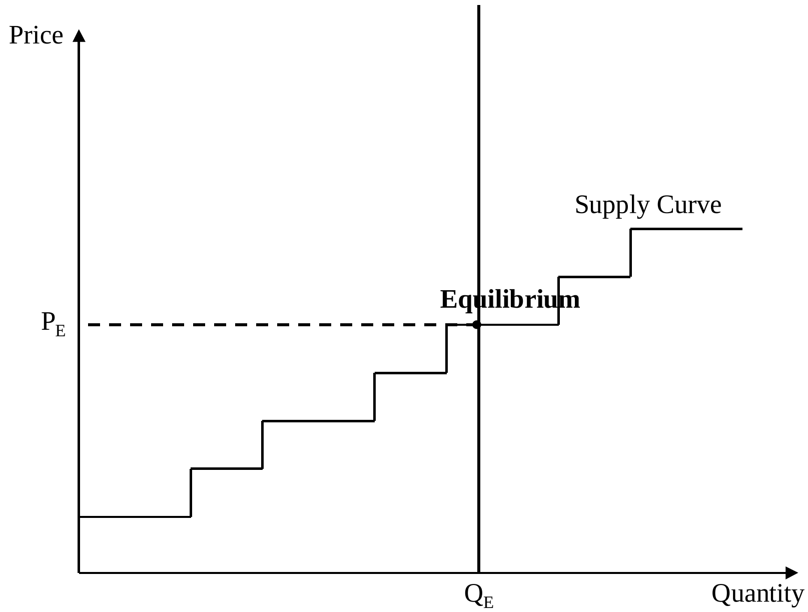

Figure 3.3. Equilibrium when an energy service demand is fixed

3.2.3.3 Competitive energy markets with perfect foresight

Competitive energy markets are characterized by perfect information and atomic economic agents, which together preclude any of them from exercising market power. That is, neither the level at which any individual producer supplies, nor the level any individual consumer acquires, affects the equilibrium market price (because there are many other buyers and sellers to replace them). It is a standard result of microeconomic theory that the assumption of competitive markets entails that the market price of a commodity is equal to its marginal value in the economy (Samuelson, 1952). This is of course also verified in the TIMES economy, as discussed in the next subsection.

Of course, real world energy markets are not always competitive. For instance, an electric utility company may be a (regulated) monopoly within an entire country, or a cartel of oil producing countries may have market power on oil markets. There are ways around these so-called “market imperfections”. For instance, concerning the monopolistic utility, a socially desirable approach would be to first use the assumption of marginal cost pricing, so as to determine a socially optimal plan for the monopoly, and then to have the regulatory agency enforce such a plan, including the principle of marginal cost pricing. The case of the oil producers’ cartel is less simple, since there is no global regulatory agency to ensure that oil producers act in a socially optimal fashion. There are however ways to use equilibrium models such as TIMES in order to faithfully represent the market power of certain economic agents, as exemplified in (Loulou et al., 2007).

In the core version of TIMES, the perfect information assumption extends to the entire planning horizon, so that each agent has perfect foresight, i.e. complete knowledge of the market's parameters, present and future. Hence, the equilibrium is computed by maximizing total surplus in one pass for the entire set of periods. Such a farsighted equilibrium is also called an *inter-temporal dynamic equilibrium*.

Note that there are at least two ways in which the perfect foresight assumption may be voided: in one variant, agents are assumed to have foresight over a limited portion of the horizon, say one or a few periods. Such an assumption of limited foresight is embodied in the TIMES feature discussed in chapter 9, as well as in the SAGE variant of MARKAL (US EIA, 2002). In another variant, foresight is assumed to be imperfect, meaning that agents may only *probabilistically* know certain key future events. This assumption is at the basis of the TIMES Stochastic Programming option, discussed in chapter 8.

3.2.4 Marginal value pricing

We have seen in the preceding subsections that the TIMES equilibrium occurs at the intersection of the inverse supply and inverse demand curves. It follows that the equilibrium prices are equal to the marginal system values of the various commodities. From a different angle, the duality theory of Linear Programming (chapter 14) indicates that for each constraint of the TIMES linear program there is a *dual variable*. This dual variable (when an optimal solution is reached) is also called the constraint's *shadow price*[23], and is equal to the marginal change of the objective function per unit increase of the constraint's right-hand-side. For instance, the shadow price of the balance constraint of a commodity (whether it be an energy form, material, a service demand, or an emission) represents the competitive market price of the commodity.

[23] The term *shadow price* is often used in the mathematical economics literature, whenever the price is derived from the marginal value of a commodity. The qualifier 'shadow' is used to distinguish the competitive market price from the price observed in the real world, which may be different, as is the case in regulated industries or in sectors where either consumers or producers exercise market power, or again when other market imperfections exist. When the equilibrium is computed using LP optimization, as is the case for TIMES, the shadow price of each commodity is computed as the dual variable of that commodity's balance constraint, see chapter 14

The fact that the price of a commodity is equal to its marginal value is an important feature of competitive markets. Duality theory does not necessarily indicate that the marginal value of a commodity is equal to the marginal cost of *producing* that commodity. For instance, in the equilibrium shown in Figure 3.4 the price does not correspond to *any* marginal supply cost, since it is situated at a discontinuity of the inverse supply curve. In this case, the price is precisely determined by demand rather than by supply, and the term *marginal cost pricing* (so often used in the context of optimizing models) is *sensu stricto* incorrect. The term *marginal value pricing* is a more appropriate term to use.

It is important to reiterate that marginal value pricing *does not imply that suppliers have zero profit*. Profit is exactly equal to the suppliers' surplus, and is generally positive. Only the last few units produced may have zero profit, if, and when, their production cost equals the equilibrium price.

In TIMES the shadow prices of commodities play a very important diagnostic role. If some shadow price is clearly out of line (i.e. if it seems much too small or too large compared to the anticipated market prices), this indicates that the model's database may contain some errors. The examination of shadow prices is just as important as the analysis of the quantities produced and consumed of each commodity and of the technological investments.

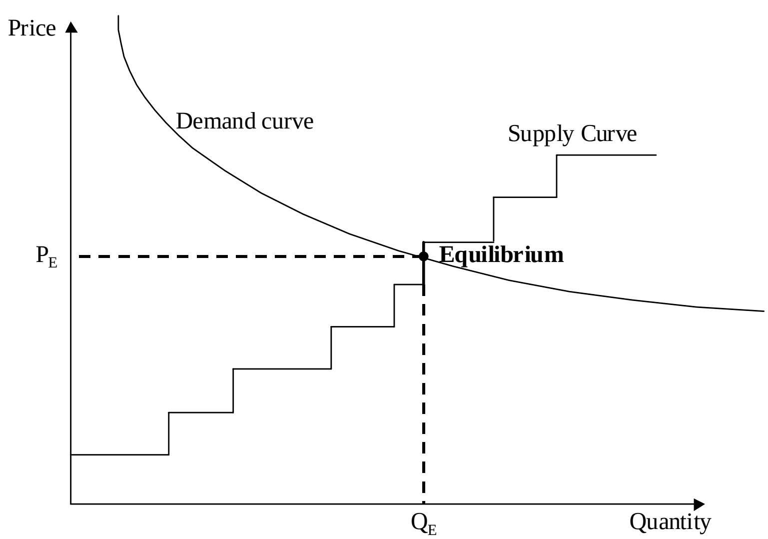

Figure 3.4. Case where the equilibrium price is not equal to any marginal supply cost.

3.2.5 Profit maximization: the Invisible Hand

An interesting property may be derived from the assumptions of competitiveness. While the avowed objective of the TIMES model is to maximize the overall surplus, it is also true that each economic agent in TIMES maximizes its own surplus. This property is akin to the famous ‘invisible hand’ property of competitive markets, and may be established rigorously by the following theorem that we state in an informal manner:

> *Theorem: Let (p*,q*) be a pair of equilibrium vectors that maximize total surplus. If we now replace the original TIMES linear program by one where all commodity prices are fixed at value p*, and we let each agent maximize its own surplus, the vector of optimal quantities produced or purchased by the agents also maximizes the total surplus*[24].

This property is important inasmuch as it provides an alternative justification for the class of equilibria based on the maximization of total surplus. It is now possible to shift the model’s rationale from a global, societal one (total surplus maximization), to a local, decentralized one (individual utility maximization). Of course, the equivalence suggested by the theorem is valid only insofar as the marginal value pricing mechanism is strictly enforced—that is, neither an individual producer nor an individual consumer may affect market prices—both are price takers. Clearly, some markets are not competitive in the sense the term has been used here. For example, the behavior of a few oil producers has a dramatic impact on world oil prices, which then depart from their marginal system value. Market power[25] may also exist in cases where a few consumers dominate a market.

[24] However, the resulting Linear Program has multiple optimal solutions. Therefore, although q^* is an optimal solution, it is not necessarily the one found when the modified LP is solved.

[25] An agent has market power if its decisions, all other things being equal, have an impact on the market price. Monopolies and oligopolies are example of markets where one or several agents have market power.

4 Core TIMES model: Mathematics of the computation of the supply-demand equilibrium

In the preceding chapter, we have seen that TIMES does more than minimize the cost of supplying energy services. Instead, it computes a supply-demand equilibrium where both the energy supplies and the energy service demands are endogenously determined by the model. The equilibrium is driven by the user-defined specification of demand functions, which determine how each energy service demand varies as a function of the current market price of that energy service. The TIMES code assumes that each demand has constant own-price elasticity in a given time period, and that cross price elasticities are zero. We have also seen that economic theory establishes that the equilibrium thus computed corresponds to the maximization of the net total surplus, defined as the sum of the suppliers' and consumers' surpluses. We have argued in section 3.2 that the total net surplus has often been considered a valid metric of societal welfare in microeconomic literature, and this fact confers strong validity to the equilibrium computed by TIMES. Thus although TIMES falls short of computing a general equilibrium, it does capture a major element of the feedback effects not previously accounted for in bottom-up energy models.

In this chapter we provide the details on how the equilibrium is transformed into an optimization problem and solved accordingly.

Historically, the approach was first used in the Project Independence Energy System (PIES, see Hogan, 1975), although in the context of demands for final energy rather than for energy services as in TIMES or MARKAL. It was then proposed for MARKAL model by Tosato (1980) and Altdorfer (1982), and later made available as a standard MARKAL option by Loulou and Lavigne (1995). The TIMES implementation is identical to the MARKAL one.

4.1 *Theoretical considerations: the Equivalence Theorem*

The computational method is based on the equivalence theorem presented in chapter 3, which we restate here:

"A supply/demand economic equilibrium is reached when the sum of the producers and the consumers surpluses is maximized"

Figure 3.2 of Chapter 3 provides a graphical illustration of this theorem in a case where only one commodity is considered.

4.2 ***Mathematics of the TIMES equilibrium***

4.2.1 Defining demand functions

From chapter 3, we have the following demand function for each demand category ***i***

$$DM_i/DM_i{}^0 = (p_i/p_i^0)^{E_i} \qquad (4-1)$$

Or its inverse:

$$p_i = p_i^0 \cdot (DM_i/DM_i{}^0)^{1/E_i}$$

where the superscript '0' indicates the reference case, and the elasticity $\boldsymbol{E_i}$ is negative. Note also that the elasticity may have two different values, one for upward changes in demand, the other for downward changes.

4.2.2 Formulating the TIMES equilibrium

With inelastic demands (i.e. pure cost minimization), the TIMES model may be written as the following Linear Program:

$$Min\ c \cdot X \qquad (4-2)$$

$$s.t\ \sum_k VAR_{ACT\,k,j}(t) \geq DM_i(t) \quad i = 1,2,\ldots,I;\ t = 1,\ldots,T \qquad (4-3)$$

$$and\ \ B \cdot X \geq b \qquad (4-4)$$

where ***X*** is the vector of all TIMES variables and ***I*** is the number of demand categories. In words:

- (4-2) expresses the total discounted cost to be minimized. See chapter 5 for details on the list of TIMES variables ***X***, and on the cost vector ***c.***
- (4-3) is the set of demand satisfaction constraints (where the ***VAR_ACT*** variables are the activity levels of end-use technologies, and the ***DM*** right-hand-sides are the exogenous demands to satisfy).
- (4-4) is the set of all other TIMES constraints, which need not be explicated here, and are presented in chapter 5.

When demand are elastic, TIMES must compute a supply/demand equilibrium of the optimization problem (4-2) through (4-4), where the demand side adjusts to changes in prices, and the prevailing demand prices are the marginal costs of the demand categories (i.e. p_i is the marginal cost of producing demand DM_i). *A priori* this seems to be a difficult task, because the demand prices are computed as part of the dual solution to that optimization problem. The Equivalence Theorem, however, states that the equilibrium is reached as the solution of the following mathematical program, where the objective is to maximize the net total surplus:

$$Max \sum_i \sum_t \left(p_i^0(t) \cdot \left[DM_i^0(t) \right]^{-1/E_i} \bullet \int_a^{DM_i(t)} q^{1/E_i} \cdot dq \right) - c \cdot X \qquad (4-5)$$

$$s.t. \quad \sum_k VAR\_ACT_{k,i}(t) - DM_i(t) \geq 0 \qquad i = 1,..,I; \; t = 1,..,T \qquad (4-6)$$

$$and \quad B \cdot X \geq b \qquad (4-7)$$

where $\boldsymbol{X}$ is the vector of all TIMES variables, (4-5) expresses the total net surplus, and ***DM(t)*** is now a vector of *variables* in (4-6), rather than fixed demands. The integral in (4-5) is easily computed, yielding the following maximization program:

$$Max \sum_i \sum_t \left(p_i^0(t) \cdot \left[DM_i^0(t) \right]^{-1/E_i} \bullet DM_i(t)^{1+1/E_i} / (1 + 1/E_i) \right) - c \cdot X \qquad (4-5)'$$

$$s.t. \quad \sum_k VAR\_ACT_{k,i}(t) \geq DM_i(t) \qquad i = 1,..,I; \; t = 1,..,T \qquad (4-6)'$$

$$B \cdot X \geq b \qquad (4-7)'$$

We are almost there, but not quite, since the $\boldsymbol{[DM_i(t)]^{-1/Ei}}$ are non linear expressions and thus not directly usable in an L.P.

4.2.3 Linearization of the Mathematical Program

The Mathematical Program embodied in (4-5)’, (4-6)’ and (4-7)’ has a non-linear objective function. Because the latter is separable (i.e. does not include cross terms) and concave in the DM_i variables, each of its terms is easily linearized by piece-wise linear functions which approximate the integrals in (4-5). This is the same as saying that the inverse demand curves are approximated by staircase functions, as illustrated in figure

4.1. By so doing, the resulting optimization problem becomes linear again. The linearization proceeds as follows.

a) For each demand category ***i***, and each time period ***t***, the user selects a range $\boldsymbol{R(t)_i}$, i.e. the distance between some values $\boldsymbol{DM_i(t)_{min}}$ and $\boldsymbol{DM_i(t)_{max}}$. The user estimates that the demand value $\boldsymbol{DM_i(t)}$ will always remain within such a range, even after adjustment for price effects (for instance the range could be equal to the reference demand $\boldsymbol{DM^o_i(t)}$ plus or minus 50%).

b) Select a grid that divides each range into a number *n* of equal width intervals. Let $ß_i(t)$ be the resulting common width of the grid, $\boldsymbol{ß_i(t)= R_i(t)/n}$. See Figure 4.1 for a sketch of the non-linear expression and of its step-wise constant approximation. The number of steps, *n*, should be chosen so that the step-wise constant approximation remains close to the exact value of the function.

c) For each demand segment $\boldsymbol{DM_i(t)}$ define *n* step-variables (one per grid interval), denoted $s_{1,i}(t), s_{2,i}(t), \ldots, s_{n,i}(t)$. Each *s* variable is bounded below by 0 and above by $ß_i(t)$. One may now replace in equations (4-5)' and (4-6)' each $\boldsymbol{DM_i(t)}$ variable by the sum of the *n*-step variables, and each non-linear term in the objective function by a weighted sum of the *n* step-variables, as follows:

$$DM_i(t) = DM(t)_{\min} + \sum_{j=1}^{n} s_{j,i}(t) \qquad 4-8$$

and

$$DM_i(t)^{1+1/E_i} \cong DM(t)_{\min}^{1+1/E_i} + \sum_{j=1}^{n} A_{j,s,i}(t) \bullet s_{j,i}(t) \qquad 4-9$$

The $\boldsymbol{A_{j,i,t}}$ term is equal to the value of the inverse demand function of the $\boldsymbol{j^{th}}$ demand at the mid-point of the $\boldsymbol{i^{th}}$ interval. The resulting Mathematical Program is now fully linearized.

Since the $\boldsymbol{A_{j,i,t}}$ terms have decreasing values (due to the concavity of the curve), the optimization will always make sure that the $\boldsymbol{s_{j,I}}$ variables are increased consecutively and in the correct order, thus respecting the step-wise constant approximation described above.

Remark: Instead of maximizing the linearized objective function, TIMES minimizes its negative, which then has the dimension of a cost. The portion of that cost representing the negative of the consumer surplus is akin to a *welfare loss.*

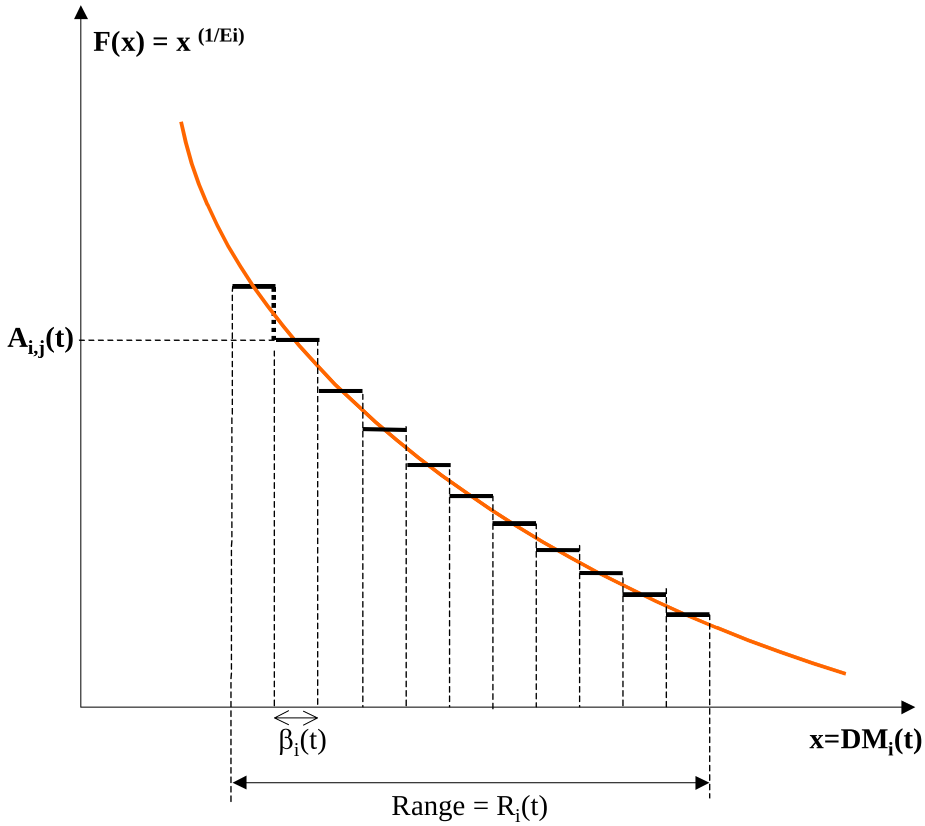

Figure 4.1. Step-wise constant approximation of the non-linear terms in the objective function

4.2.4 Calibration of the demand functions

Besides selecting elasticities for the various demand categories, the user must evaluate each constant $K_i(t)$. To do so, we have seen that one needs to know one point on each demand function in each time period,$\{ p^0_i(t), DM^0_i(t) \}$. To determine such a point, we perform a single preliminary run of the inelastic TIMES model (with exogenous $DM^0_i(t)$), and use the resulting shadow prices $p^0_i(t)$ for all demand constraints, in all time periods for each region.

4.2.5 Computational considerations

Each demand segment that is elastic to its own price requires the definition of as many variables as there are steps in the discrete representation of the demand curve (both upward and down if desired), for each period and region. Each such variable has an upper bound, but is otherwise involved in no new constraint. Therefore, the linear program is augmented by a number of variables, but does not have more constraints than the initial inelastic LP (with the exception of the upper bounds). It is well known that with modern LP codes the number of variables has little or no impact on computational time in Linear Programming, whether the variables are upper bounded or not. Therefore, the inclusion in TIMES of elastic demands has a very minor impact on computational time or on the tractability of the resulting LP. This is an important observation in view of the very large LP's that result from representing multi-regional and global models in TIMES.

4.2.6 Interpreting TIMES costs, surplus, and prices

It is important to note that, instead of maximizing the net total surplus, TIMES minimizes its negative (plus a constant), obtained by changing the signs in expression (4-5). For this and other reasons, it is inappropriate to pay too much attention to the meaning of the *absolute* objective function values. Rather, examining the difference between the objective function values of two scenarios is a far more useful exercise. That difference is of course, the negative of the difference between the net total surpluses of the two scenario runs.

Note again that the popular interpretation of shadow prices as the *marginal costs* of model constraints is inaccurate. Rather, the shadow price of a constraint is, by definition, the incremental value of the objective function per unit of that constraint's right hand side (RHS). The interpretation is that of an amount of *surplus loss* per unit of the constraint's RHS. The difference is subtle but nevertheless important. For instance, the shadow price of the electricity balance constraint is not necessarily the marginal cost of producing electricity. Indeed, when the RHS of the balanced constraint is increased by one unit, one of two things may occur: either the system *produces* one more unit of electricity, or else the system *consumes* one unit less of electricity (perhaps by choosing more efficient end-use devices or by reducing an electricity-intensive energy service, etc.) It is therefore correct to speak of shadow prices as the marginal *system value* of a resource, rather than the marginal *cost* of procuring that resource.

5 Core TIMES Model: A simplified description of the Optimization Program (variables, objective, constraints)

This chapter contains a simplified formulation of the core TIMES Linear Program.

Mathematically, a TIMES instance is a Linear Program, as was mentioned in the previous chapter. A Linear Program (LP for short) consists in the minimization or maximization of an *objective function* (defined as a linear mathematical expression of *decision variables),* subject to linear *constraints,* also called *equations*[26].

Very large instances of Linear Programs involving sometimes millions of constraints and variables may be formulated using modern modeling languages such as GAMS (http://www.gams.com/help/index.jsp), and solved via powerful Linear Programming *optimizers*[27]. The Linear Program described in this chapter is much simplified, since it ignores many exceptions and complexities that are not essential to a basic understanding of the principles of the model. Chapter 14 gives additional details on general Linear Programming concepts. The full details of the parameters, variables, objective function, and constraints of TIMES are given in Part II of this documentation (sections 3, 5, and 6).

A linear optimization problem formulation consists of three types of entities:

- *the decision variables:* i.e. the unknowns, or endogenous quantities, to be determined by the optimization;
- *the objective function*: expressing the criterion to be minimized or maximized; and;
- *the constraints*: equations or inequalities involving the decision variables that must be satisfied by the optimal solution.

5.1 *Indices*

The model data structures (sets and parameters), variables and equations use the following indices:

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[26] This rather improper term includes equality as well as inequality relationships between mathematical expressions.

[27] For more information on optimizers see Brooke et al., 1998

r: indicates the region

t or v: time period; ***t*** corresponds to the current period, and ***v*** is used to indicate the vintage year of an investment. When a process is not vintaged then ***v*** = ***t***.

p: process (technology)

s: time-slice; this index is relevant only for user-designated commodities and processes that are tracked at finer than annual level (e.g. electricity, low-temperature heat, and perhaps natural gas, etc.). Time-slice defaults to “ANNUAL”, indicating that a commodity is tracked only annually.

c: commodity (energy, material, emission, demand).

5.2 *Decision variables*

The decision variables represent the *choices* to be made by the model, i.e. the *unknowns*. All TIMES variables are prefixed with the three letters VAR followed by an underscore.

Important remark: There are two possible choices concerning the very *meaning* of some decision variables, namely those variables that represent yearly flows or process activities. In the original TIMES formulation, the activity of a process during some period ***t*** is considered to be constant in all years constituting the period. This is illustrated in panel M.a of Figure 5.1).In the alternative option the activity variable is considered to represent the value *in a milestone year* of each period, and the values at all other years is linearly interpolated between the consecutive milestone year values, as illustrated in panel M.b). A milestone year is chosen close to the middle of a period. This second option is similar to that of the EFOM and the MESSAGE models. The user is free to choose either option. The constraints and objective function presented below apply to the first option (constant value of activity variables within a period). Appropriate changes in constraints and objective function are made for the alternative option, as explained in section 5.5, and more completely in Part II, section 6.

The main kinds of decision variables in a TIMES model are:

VAR_NCAP(r,v,p)*:** new capacity addition (investment) for technology ***p, in period ***v*** and region ***r***. For all technologies the ***v*** value corresponds to the vintage of the process, i.e. year in which it is invested in. For vintaged technologies (declared as such by the user) the vintage (***v***) information is reflected in other process variables, discussed below. Typical units are PJ/year for most energy technologies, Million tonnes per year (for steel, aluminum, and paper industries), Billion vehicle-kilometers per year (B-vkm/year) or million cars for road vehicles, and GW for electricity equipment (1GW=31.536 PJ/year), etc.

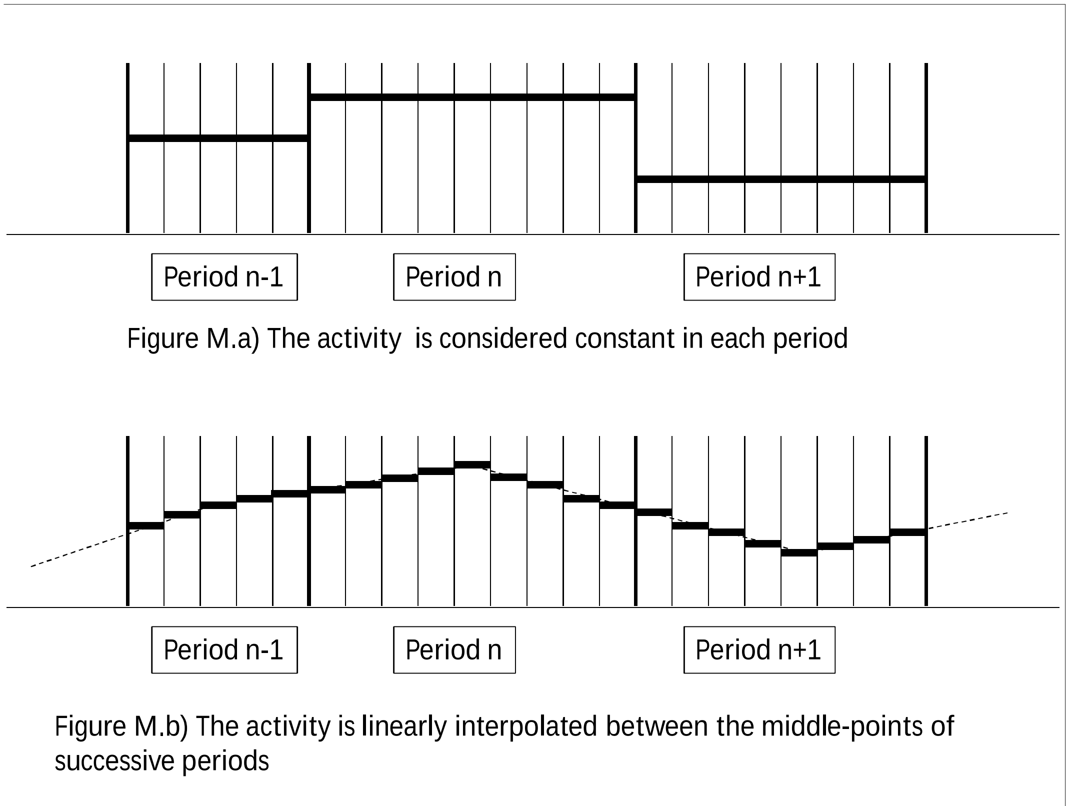

Figure M.a) The activity is considered constant in each period

Figure M.b) The activity is linearly interpolated between the middle-points of successive periods

Figure 5.1.Process activity in the original TIMES formulation (top) and Linear variant (bottom)

VAR_RCAP(r,v,t,p): Amount of capacity that is newly retired at period ***t***. The new retirements will reduce the available capacity of vintage ***v*** in period ***t*** and in all successive periods $t_i > t$ by the value of the variable. This new feature was not available in early versions of TIMES. Note carefully that the feature must be activated by a special switch in order to become effective. Note also that additional a new advanced feature allows the user to specify that capacity retirement may only occur in lump amounts that are either equal to the entire remaining capacity or equal to a multiple of some user defined block. Consult the separate technical note *TIMES Early Retirement of Capacity* for details.

VAR_DRCAP(r,v,t,p,j): Binary variables used in formulating the special early retirement equations. Two variables may be defined, one when retirement must be for the entire remaining capacity **(j=1**), another when retirement must be a multiple of some block defined by the user via parameter RCAP_BLK **(j=2**).

VAR_SCAP(r,v,t,p):Total amount of capacity that has been retired at period ***t*** and periods preceding ***t*** (see above ***VAR_RCAP***paragraph).

CAP(r,v,t,p): installed capacity of process ***p***, in region ***r*** and period ***t,*** optionally with vintage ***v.*** It represents the total capacity available at period ***t***, considering the residual capacity at the beginning of the modeling horizon and adding to it new investments made prior to and including period ***t*** that have not reached their technical lifetime, and subtracting retired capacity. Typical units: same as investments. The ***CAP*** quantity, although convenient for formulation and reporting purposes, is in fact *not explicitly defined in the model*, but is derived from the ***VAR_NCAP*** variables and from data on past investments, process lifetimes, and any retirements.

VAR_CAP(r,t,p): total installed capacity of technology ***p***, in region ***r*** andperiod ***t***, all vintages together. The ***VAR_CAP*** variables are only defined when some bounds or user-constraints are specified for them. They do not enter any other equation.

Remark: The lumpy investment option. There is a TIMES feature that allows the user to impose that new additions to capacity may only be done in predefined blocks. This feature may be useful for technologies that are implementable only in discrete sizes such as a nuclear plant, or a large hydroelectric project. The user should however be aware that using this option voids some of the economic properties of the equilibrium. This feature is described in Chapter 10 of this part of the documentation.

VAR_ACT(r,v,t,p,s): activity level of technology ***p***, in region ***r*** and period ***t*** (optionally vintage ***v*** and time-slice ***s***)***.*** Typical units: PJ for all energy technologies. The ***s*** index is relevant only for processes that produce or consume commodities specifically declared as time-sliced. Moreover, it is the process that determines which time slices prevail. By default, only annual activity is tracked.

VAR_FLO(r,v,t,p,c,s): the quantity of commodity ***c*** consumed or produced by process ***p***, in region ***r*** and period ***t*** (optionally with vintage ***v*** and time-slice ***s***). Typical units: PJ for all energy technologies. The ***VAR_FLO*** variables confer considerable flexibility to the processes modeled in TIMES, as they allow the user to define flexible processes for which input and/or output flows are not rigidly linked to the process activity.

VAR_SIN(r,v,t,p,c,s)/VAR_SOUT(r,v,t,p,c,s): the quantity of commodity ***c*** stored or discharged by storage process ***p,*** in time-slice ***s***, period ***t*** (optionally with vintage ***v)***, and region ***r***.

VAR_IRE(r,v,t,p,c,s,exp) and VAR_IRE(r,v,t,p,c,s,imp)[28]***:*** quantity of commodity ***c*** (PJ per year) sold (***exp***) or purchased (***imp***) by region ***r*** through export (resp. import) process ***p*** in period ***t***(optionally in time-slice ***s***). Note that the topology defined for the exchange process ***p*** specifies the traded commodity ***c***, the region ***r***, and the regions ***r'*** with which region ***r*** is trading commodity ***c***. In the case of bi-lateral trading, if it is desired that region ***r*** should trade with several other regions, each such trade requires the definition of a separate bi-lateral exchange process. Note that it is also possible to define multi-lateral trading relationships between region ***r*** and several other regions ***r'*** by defining one of the regions as the common market for trade in commodity ***c***. In this case, the commodity is 'put on the market' and may be bought by any other region participating in the market. This approach is convenient for global commodities such as emission permits or crude oil. Finally, exogenous trading may also be modeled by specifying the ***r'*** region as an external region. Exogenous trading is required for models that are not global, since exchanges with non-modeled regions cannot be considered endogenous.

VAR_DEM(r,t,d): demand for end-use energy service ***d*** in region ***r*** and period ***t.*** It is a true variable, even though in the reference scenario, this variable is fixed by the user. In alternate scenarios however, ***VAR_DEM(r,t,d)*** may differ from the reference case demand due to the responsiveness of demands to their own prices (based on each service demand's own-price elasticity). Note that in this simplified formulation, we do not show the variables used to decompose ***DEM(r,t,d)*** into a sum of step-wise quantities, as was presented in chapter 4.

Other variables: Several options that have been added to TIMES over the successive versions require the definition of additional variables. They are alluded to in the sections describing the new options, and described more precisely in Part II, and in additional technical notes. Also, TIMES has a number of commodity related variables that are not strictly needed but are convenient for reporting purposes and/or for applying certain bounds to them. Examples of such variables are: the total amount produced of a commodity (***VAR_COMPRD***), or the total amount consumed of a commodity (***VAR_COMCON***).

Important remark: It is useful to know that many variables (for instance the above two accounting variables, but also the flow variables described earlier) add only a moderate computational burden to the optimization process, thanks to the use of a *reduction algorithm* to detect and eliminate redundant variables and constraints before solving the

[28] IRE stands for Inter-Regional Exchange

LP. These variables and constraints are later reinstated in the solution file for reporting purposes.

5.3 *TIMES objective function: discounted total system cost*

5.3.1 The costs accounted for in the objective function

The Surplus Maximization objective is first transformed into an equivalent Cost Minimization objective by taking the negative of the surplus, and calling this value the *total system cost*. This practice is in part inspired from historical custom from the days of the fixed demand MARKAL model. The TIMES objective is therefore to minimize the total 'cost' of the system, properly augmented by the 'cost' of lost demand. All cost elements are appropriately discounted to a user-selected year.

In TIMES, the cost elements are defined at a finer level than the period. While the TIMES constraints and variables are linked to a *period*, the components of the system cost are expressed for each *year* of the horizon (and even for some years outside the horizon). This choice is meant to provide a smoother, more realistic rendition of the stream of cost payments in the energy system, as discussed below. Each year, the total cost includes the following elements:

- *Capital Costs* incurred for *investing* into and/or *dismantling* processes.
- Fixed and variable annual *Operation and Maintenance (O&M) Costs,* and other annual costs occurring during the dismantling of technologies.
- Costs incurred for *exogenous imports* and for domestic resource *extraction* and *production*. An exogenous import is one that imported from a non-specified entity, i.e. not from another modeled region. Exogenous imports are not relevant in global TIMES instances.
- Revenues from exogenous *export.* An exogenous export is one that is exported to a non-specified entity, i.e. not to another modeled region. Exogenous exports are irrelevant in global TIMES instances. Exogenous export earnings are revenues and appear with a negative sign in the cost expressions.
- *Delivery* costs for commodities consumed by the processes. These costs are attached to commodity flows.
- *Taxes* and *subsidies* associated with commodity flows and process activities or investments. A tax is not a cost *per se*. However, since the tax is intended to influence the optimization, it is considered as an integral part of the objective

function. It is however reported separately from regular costs. Similarly for subsidies.

- *Revenues from recuperation of embedded commodities,* accrued when a process's dismantling releases some valuable commodities.
- *Damage costs* (if defined) due to emissions of certain pollutants. Several assumptions are made: the damage costs in region ***r*** result from emissions in ***r*** and possibly in other regions; damage cost is imputed to the emitting region (polluter pays); emissions in period ***t*** entail damages in period ***t*** only; the damage cost from several types of emission is assumed to be the sum of the costs from each emission type (no cross-effect); and the damage function linking cost DAM to emissions EM is a power function of the form:

$$DAM(EM) = MC_0 \cdot \frac{EM^{\beta+1}}{(\beta+1) \cdot EM_0^{\beta}}$$

  Where β is non-negative (i.e. marginal damage costs are non decreasing). Hence, the damage cost function is linear (β=0) or non linear but convex (β >0). Therefore, the same linearization procedure that was used for the surplus may be applied here in order to linearize the damage cost[29]. Appendix B of Part II and Technical note "TIMES Damage", explain how to declare the various parameters required to define the damage functions, to specify the linearization parameters, and to define the switches used to control the optimization. It should be noted that global emissions such as GHG's should not be treated via this feature but rather should make use of the Climate Module option described in chapter 7.
- ***Salvage value*** of processes and embedded commodities at the end of the planning horizon. This revenue appears with a negative sign in the cost expressions. It should also be stressed that the calculation of the salvage value at the end of the planning horizon is very complex and that the original TIMES expressions accounting for it contained some biases (over- or under-estimations of the salvage values in some cases). These biases have been corrected in the present version of TIMES as explained in sections 5.3.4 and 5.5.
- *Welfare loss* resulting from reduced end-use demands. Chapter 4has presented the mathematical derivation of this quantity.

[29] Alternatively, one may use a convex programming code to solve the entire TIMES LP.

5.3.2 Cash flow tracking

As already mentioned, in TIMES, special care is taken to precisely track the cash flows related to process investments and dismantling in each year of the horizon. Such tracking is made complex by several factors:

- First, TIMES recognizes that there may be a lead-time (ILED) between the beginning and the end of the construction of some large processes, thus spreading the investment installments over several years. A recent TIMES feature allows the definition of a negative lead-time, with the meaning that the construction of the technology starts before the year the investment decision is made (this is useful for properly accounting for interest during construction, and is especially needed when using the time-stepped version of TIMES described in chapter 9.)
- Second, TIMES also recognizes that for some other processes (e.g. new cars), the investment in new capacity occurs *progressivel*y over the years constituting the time period (whose length is denoted by D(t)), rather than in one lumped amount.
- Third, there is the possibility that a certain investment decision made at period ***t*** will have to be repeated more than once during that same period. (This will occur if the period is long compared to the process technical life.)
- Fourth, TIMES recognizes that there may be dismantling capital costs at the end-of-life of some processes (e.g. a nuclear plant), and that these costs, while attached to the investment variable indexed by period ***t***, are actually incurred much later.
- Finally, TIMES permits the payment of any capital cost to be spread over an *economic life (ELIFE)* that is different from the *technical life (TLIFE)* of the process. Furthermore it may be annualized at a different rate than the overall discount rate.

To illustrate the above complexities, we present a diagram taken from Part II that pictures the yearly investments and yearly outlays of capital in one particular instance where there is no lead time and no dismantling of the technology, and the technical life of the technology does not exceed the period length. There are 4 distinct such instances, discussed in detail in section 6.2 of Part II.

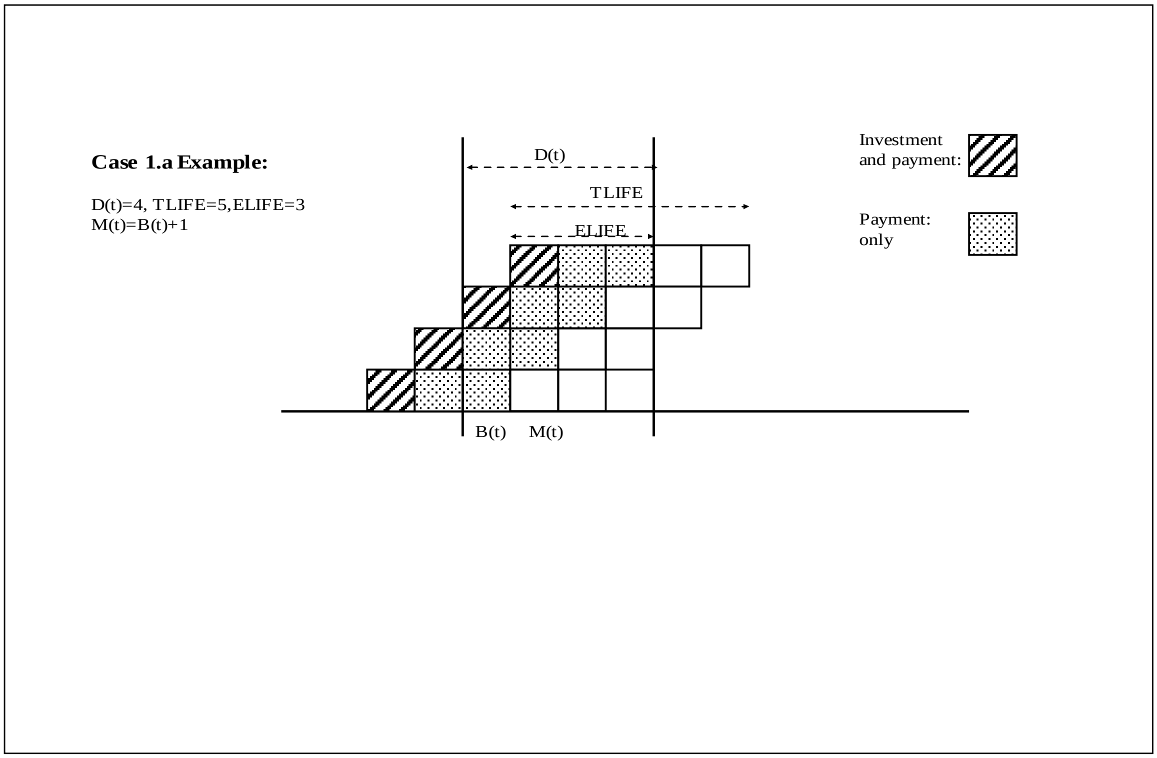

Figure 5.2. Illustration of yearly investments and payments for one of four investment tracking cases

5.3.3 Aggregating the various costs

The above considerations, while adding precision and realism to the cost profile, also introduce complex mathematical expressions into the objective function. In this simplified formulation, we do not provide much detail on these complex expressions, which are fully described in section 6.2 of Part II. We limit our description to giving general indications on the cost elements comprising the objective function, as follows:

- The capital costs (investment and dismantling) are first transformed into streams of annual payments, computed for each year of the horizon (and beyond, in the case of dismantling costs and recycling revenues), along the lines presented above.
- A *salvage value* of all investments still active at the end of the horizon (EOH) is calculated as a lump sum revenue which is subtracted from the other costs and

assumed to be accrued in the (single) year following the EOH.[30]It is then discounted to the user selected reference year.

- The other costs listed above, which are all annual costs, are added to the annualized capital cost payments, to form the ***ANNCOST*** quantity below.

TIMES then computes for each region a total net present value of the stream of annual costs, discounted to a user selected reference year. These regional discounted costs are then aggregated into a single total cost, which constitutes the objective function to be minimized by the model in its equilibrium computation.

$$NPV = \sum_{r=1}^{R} \sum_{y \in YEARS} (1 + d_{r,y})^{REFYR-y} \bullet ANNCOST(r,y)$$

where:

NPV	is the net present value of the total cost for all regions (the TIMES objective function);
ANNCOST(r,y)	is the total annual cost in region ***r*** and year ***y***;
$d_{r,y}$	is the general discount rate;
REFYR	is the reference year for discounting;
YEARS	is the set of years for which there are costs, including all years in the horizon, plus past years (before the initial period) if costs have been defined for past investments, plus a number of years after EOH where some investment and dismantling costs are still being incurred, as well as the Salvage Value; and
R	is the set of regions in the area of study.

As already mentioned, the exact computation of ***ANNCOST*** is quite complex and is postponed until section 6.2 of PART II

5.3.4 Variants for the objective function

There are some cases where the standard formulation described above leads to small distortions in the cost accounting between capacity-related costs and the corresponding activity-related costs. This occurs even without discounting but may be increased by

[30] The salvage value is thus the only cost element that remains lumped in the TIMES objective function. All other costs are annualized.

discounting. These distortions may occur at the end of the model horizon, either due to excessive or deficient salvage value.

In addition to these cost accounting problems at the end of horizon, the investment spreads used in the standard formulation can also lead to other cost distortions, regardless of discounting. In very long periods, the investment spreads are divided into D_t successive steps, each amounting to $1/D_t$ of the total capacity to be invested in the period. Recall that the full capacity must be in place by the milestone year, in order to allow activity to be constant over the period. For example, if the period length D_t is 20 years, the investments start already 19 years before the milestone year, and can thus start *even before the previous milestone year*. *If the investment costs are changing over time, it is clear that in such cases the costs will not be accounted in a realistic way, because the investment cost data is taken from the start year of each investment step.*

Similarly, in short periods the investment costs are spread over only a few years, and if the previous period is much longer, this can leave a considerable gap in the investment years between successive periods. Here again, if the investment costs are changing over time, this would lead to a distortion in the cost accounting.

Unfortunately, it is a well-known fact that the original choice of defining milestone years at or near the middle of each period limits the choice of milestone years, and furthermore tends to induce periods that may be very unequal in length, thus exacerbating the anomalies mentioned above. Such variability in period length can increase the cost distortions under discounting due to the larger differences in the timing of the available capacity (as defined by the investments) and the assumed constant activity levels in each period in the original definition of TIMES variables.

These were remedied by making changes in parts of the **OBJ** cost representation. Four options are now available, three of which apply to the original definition of TIMES variables, the fourth one applying to the alternate definition of TIMES variables. The fourth option (named **LIN**) is discussed separately in section 5.5, since it concerns not only the objective function but also several constraints.

The three options are as follows:

- The original OBJ with minor changes made to it, activated via the **OBLONG** switch.

- The modified objective function (**MOD**). The **MOD** formulation adds only a few modifications to the standard formulation:
    - The model periods are defined in a different way; and
    - The investment spreads in the investment Cases 1a and 1b (see section 6.2 of Part II for a list of all cases) are defined slightly differently.

- The **ALT** formulation includes all the modifications made in the MOD formulation. In addition, it includes the following further modifications that eliminate basically all of the remaining problems in the standard formulation:
    - The investment spreads in the investment Case 1b are defined slightly differently;
    - The capacity transfer coefficients for newly installed capacities are defined slightly differently, so that the effective lifetime of technologies is calculated taking into account discounting;
    - Variable costs are adjusted to be in sync with the available capacity.

It has been observed that these three options yield results that have practically the same degree of accuracy and reliability. There is however an advantage to the MOD and ALT options, as the milestone years need no longer be at the middle of a period.

Additional details and comments are provided on all three options in technical note "TIMES Objective Variants"

<u>Conclusion on the variants</u>: The multiplicity of options may confuse the modeler. Extensive experience with their use has shown that the distortions discussed above remain quite small. In practice, old TIMES users seem to stick to the classical OBJ with the OBLONG switch. And, as mentioned above, using MOD allows the further flexibility of freely choosing milestone years. Finally, using the LIN option (described in section 5.5) is a more serious decision, since it implies a different meaning for the TIMES variables; some modelers are more comfortable with this choice, which has also implications for the reporting of results.

5.4 *Constraints*

While minimizing total discounted cost, the TIMES model must satisfy a large number of constraints (the so-called *equations* of the model) which express the physical and logical relationships that must be satisfied in order to properly depict the associated energy system. TIMES constraints are of several kinds. Here we list and briefly discuss the main types of constraints. A full, mathematically more precise description is given in Part II. If any constraint is not satisfied, the model is said to be *infeasible,* a condition caused by a data error or an over-specification of some requirement.

In the descriptions of the equations that follow, the equation and variable names (and their indexes) are in ***bold italic*** type, and the parameters (and their indexes), corresponding to the input data, are in regular *italic* typeset. Furthermore, some parameter indexes have been omitted in order to provide a streamlined presentation.

5.4.1 Capacity transfer (conservation of investments)

Investing in a particular technology increases its installed capacity for the duration of the physical life of the technology. At the end of that life, the total capacity for this technology is decreased by the same amount. When computing the available capacity in some time period, the model takes into account the capacity resulting from all investments up to that period, some of which may have been made prior to the initial period but are still in operating condition (embodied by the residual capacity of the technology), and others that have been decided by the model at, or after, the initial period, up to and including the period in question.

The total available capacity for each technology *p*, in region *r*, in period *t* (all vintages),is equal to the sum of investments made by the model in past and current periods, and whose physical life has not yet ended, plus capacity in place prior to the modeling horizon that is still available. The exact formulation of this constraint is made quite complex by the fact that TIMES accepts variable time periods, and therefore the end of life of an investment may well fall in the middle of a future time period. We ignore here these complexities and provide a streamlined version of this constraint. Full details are shown in section 6.3.18 of Part II.

EQ_CPT(r,t,p) - Capacity transfer

> ***VAR_CAPT(r,t,p)*** = ***Sum****{over all periods t' preceding or equal to t such that t-t'<LIFE(r,t',p) of* ***VAR_NCAP(r,t',p)****}* + *RESID(r,t,p)* **(5-1)**
>
> where *RESID(r,t,p)* is the (exogenously provided) capacity of technology *p* due to investments that were made prior to the initial model period and still exist in region *r* at time *t.*

5.4.2 Definition of process activity variables

Since TIMES recognizes activity variables as well as flow variables, it is necessary to relate these two types of variables. This is done by introducing a constraint that equates an overall activity variable, ***VAR_ACT(r,v,t,p,s)***, with the appropriate set of flow variables, ***VAR_FLO(r,v,t,p,c,s)***, properly weighted. This is accomplished by first identifying the group of commodities that defines the activity (and thereby the capacity as well) of the process. In a simple process, one consuming a single commodity and producing a single commodity, the modeler simply chooses one of these two flows to define the activity, and thereby the process normalization (input or output). In more complex processes, with several commodities (perhaps of different types) as inputs and/or outputs, the definition of the activity variable requires first to choose the *primary commodity group (pcg)* that will serve as the activity-defining group. For instance, the *pcg* may be the group of energy carriers, or the group of materials of a given type, or the group of GHG emissions, etc. The modeler then identifies whether the activity is defined via inputs or via outputs that belong to the selected *pcg*. Conceptually, this leads to the following relationship:

EQ_ACTFLO(r,v,t,p,s) **–** Activity definition

VAR_ACT(r,v,t,p,s) = **SUM**{*c in pcg of* ***VAR_FLO(r,v,t,p,c,s)*** */ ACTFLO(r,v p,c)*} **(5-2)**

where *ACTFLO(r,v,p,c)* is a conversion factor (often equal to 1) from the activity of the process to the flow of a particular commodity.

5.4.3 Use of capacity

In each time period the model may use some or all of the installed capacity according to the Availability Factor (AF) of that technology. Note that the model may decide to use *less* than the available capacity during certain time-slices, or even throughout one or more whole periods, if such a decision contributes to minimizing the overall cost. Optionally, there is a provision for the modeler to force specific technologies to use their capacity to their full potential.

For each technology *p*, period *t*, vintage *v*, region *r*, and time-slice *s*, the activity of the technology may not exceed its available capacity, as specified by a user defined availability factor.

EQ_CAPACT (r,v,t,p,s) - Use of capacity

$$\boldsymbol{VAR\_ACT\ (r,v,t,p,s)} \le \textbf{or} = AF(r,v,t,p,s) * PRC\_CAPACT(r,p)) * FR(r,s) * \boldsymbol{VAR\_CAP(r,v,t,p)} \tag{5-3}$$

Here *PRC_CAPACT(r,p)* is the conversion factor between units of capacity and activity (often equal to 1, except for power plants). The *FR(r,s)*parameter is equal to the (fractional) duration of time-slice s. The availability factor *AF* also serves to indicate the nature of the constraint as an inequality or an equality. In the latter case the capacity is forced to be fully utilized. Note that the ***CAP(r,v,t,p)***"variable" is not explicitly defined in TIMES. Instead it is replaced in (5-3) by a fraction (less than or equal to 1) of the investment variable ***VAR_NCAP(r,v,p)***[31] sum of past investments that are still operating, as in equation (5-1).

<u>Example</u>**:** *a coal fired power plant's activity in any time-slice is bounded above by 80% of its capacity, i.e.* ***VAR_ACT (r,v,t,p,s)*** *≤0.8*31.536* **** CAP(r,v,t,p),*** *where PRC_CAPACT(r,p) = 31.536 is the conversion factor between the units of the capacity variable (GW) and the activity-based capacity unit (PJ/a) The activity-based capacity unit is obtained from the activity unit(PJ) by division by a denominator of one year.*

The *s* index of the *AF* coefficient in equation (5-3) indicates that the user may specify time-sliced dependency on the availability of the installed capacity of some technologies, if desirable. This is especially needed when the operation of the equipment depends on the availability of a resource that cannot be stored, such as wind and sun, or that can be only partially stored, such as water in a reservoir. In other cases, the user may provide an *AF* factor that does not depend on *s*, which is then applied to the entire year. The operation profile of a technology within a year, if the technology has a sub-annual process resolution, is determined by the optimization routine. The number of ***EQ_CAPACT*** constraints is at least equal to the number of time-slices in which the equipment operates. For technologies with only an annual characterization the number of constraints is reduced to one per period (where *s="ANNUAL"*).

[31] That fraction is equal to 1 if the technical life of the investment made in period ***v*** fully covers period ***t***. It is less than 1 (perhaps 0) otherwise.

5.4.4 Commodity balance equation

In each time period, the production by a region plus imports from other regions of each commodity must balance the amount consumed in the region or exported to other regions. In TIMES, the sense of each balance constraint (≥ or =) is user controlled, via a special parameter attached to each commodity. However, the constraint defaults to an equality in the case of materials (i.e. the quantity produced and imported is *exactly* equal to that consumed and exported), and to an inequality in the case of energy carriers, emissions and demands (thus allowing some surplus production). For those commodities for which time-slices have been defined, the balance constraint must be satisfied in each time-slice.

The balance constraint is very complex, due to the many terms involving production or consumption of a commodity. We present a much simplified version below, to simply indicate the basic meaning of this equation.

For each commodity ***c,*** time period ***t (***vintage ***v)***, region ***r***, and time-slice ***s*** (if necessary or "ANNUAL" if not), this constraint requires that the disposition of each commodity balances its procurement. The disposition includes consumption in the region plus exports; the procurement includes production in the region plus imports.

EQ_COMBAL(r,t,c,s) - Commodity balance

[Sum *{over all* ***p,c*** *∈TOP(r,p,c,"out")of:* **[*VAR_FLO(r,v,t,p,c,s)* + *VAR_SOUT(r,v,t,p,c,s)**** *STG_EFF(r,v,p)*] *}* +**

Sum *{over all* ***p,c*** *∈RPC_IRE(r,p,c,"imp") of :****VAR_IRE(r,t,p,c,s,"imp")****}+*

Sum *{over all* ***p*** *of: Release(r,t,p,c)******VAR_NCAP(r,t,p,c)****}* ***] **** *COM_IE(r,t,c,s)*

$$\geq \textit{ or } = \qquad \textbf{(5-4)}$$

Sum *{over all* ***p,c*** *∈* ***TOP(r,p,c,"in")*** *of:* ***VAR_FLO(r,v,t,p,c,s) + VAR_SIN(r,v,t,p,c,s)****}* **+**

Sum {*over all* ***p,c∈ RPC_IRE(r,p,c,"exp")****} of:* ***VAR_IRE(r,t,p,c,s,'exp") +***

Sum** {over all **p** of: Sink(r,t,p,c)*VAR_NCAP(r,t,p,c)**} +*
*FR(c,s) ****VAR_DEM(c,t)***

where:

The constraint is $\geq$ for energy forms and = for materials and emissions (unless these defaults are overridden by the user, see Part II).

TOP(r,p,c,"in/out") identifies that there is an input/output flow of commodity c into/from process *p* in region *r;*

RPC_IRE(r,p,c,"imp/exp") identifies that there is an import/export flow into/from region *r* of commodity *c* via process *p;*

STG_EFF(r,v,p) is the efficiency of storage process p;

COM_IE(r,t,c) is the infrastructure efficiency of commodity c;

Release(r,t,p,c) is the amount of commodity ***c*** recuperated per unit of capacity of process ***p*** dismantled (useful to represent some materials or fuels that are recuperated while dismantling a facility);

Sink(r,t,p,c) is the quantity of commodity ***c*** required per unit of new capacity of process ***p*** (useful to represent some materials or fuels consumed for the construction of a facility);

FR(s) is the fraction of the year covered by time-slice ***s*** (equal to 1 for non- time-sliced commodities).

***<u>Example</u>**: Gasoline consumed by vehicles plus gasoline exported to other regions must not exceed gasoline produced from refineries plus gasoline imported from other regions.*

5.4.5 Defining flow relationships in a process

A process with one or more (perhaps heterogeneous) commodity flows is essentially defined by one or more input and output flow variables. In the absence of relationships between these flows, the process would be completely undetermined, i.e. its outputs would be independent from its inputs. We therefore need one or more constraints stating in a most general case that the ratio of the sum of some of its output flows to the sum of some of its input flows is equal to a constant. In the case of a single commodity in, and a single commodity out of a process, this equation defines the traditional efficiency of the process. With several commodities, this constraint may leave some freedom to individual output (or input) flows, as long as their sum is in fixed proportion to the sum of input (or output) flows. An important rule for this constraint is that *each sum must be taken over* *commodities of the same type* (i.e. in the same group, say: energy carriers, or emissions, etc.). In TIMES, for each process the modeler identifies the input commodity group *cg*1, and the output commodity group *cg*2, and chooses a value for the efficiency ratio, named *FLO_FUNC(p,cg1,cg2)*. The following equation embodies this:

EQ_PTRANS(r,v,t,p,cg1,cg2,s) **–**Efficiency definition

$$\boldsymbol{SUM}\left\{ c \text{ in } cg2 \text{ of : } \mathbf{VAR\_FLO(r,v,t,p,c,s\ )} \right\} =$$

$$FLO\_FUNC(r,v,cg1,cg2,s) * \boldsymbol{SUM}\left\{ c \text{ within } cg1 \text{ of: } COEFF(r,v,p,cg1,c,cg2,s) * \mathbf{VAR\_FLO(r,v,t,p,c,s)} \right\} \qquad \mathbf{(5\text{-}5)}$$

where *COEFF(r,v,p,cg1,c,cg2,s)* takes into account the harmonization of different time-slice resolution of the flow variables, which have been omitted here for simplicity, as well as commodity-dependent transformation efficiencies.

5.4.6 Limiting flow shares in flexible processes

When either of the commodity groups*cg*1 or *cg*2 contains more than one element, the previous constraint allows a lot of freedom on the values of flows. The process is therefore quite flexible. The flow share constraint is intended to limit the flexibility, by constraining the share of each flow within its own group. For instance, a refinery output might consist of three refined products: *c1*=light, *c2*=medium, and *c3*=heavy distillate. If losses are 9% of the input, then the user must specify *FLO_FUNC* = 0.91 to define the overall efficiency. The user may then want to limit the flexibility of the slate of outputs by means of three *FLO_SHAR(ci) coefficients, say 0.4, 0.5, 0.6,* resulting in three flow share constraints as follows (ignoring some indices for clarity):

$VAR\_FLO(c1) \le 0.4*[VAR\_FLO(c1) + VAR\_FLO(c2) + VAR\_FLO(c3)]$, so that *c*1 is at most 40% of the total output,

$VAR\_FLO(c2) \le 0.5*[VAR\_FLO(c1) + VAR\_FLO(c2) + VAR\_FLO(c3)]$, so that *c*2 is at most 50% of the total output,

$VAR\_FLO(c3) \le 0.6*[VAR\_FLO(c1) + VAR\_FLO(c2) + VAR\_FLO(c3)]$, so that *c*3 is at most 60% of the total output,

The general form of this constraint is:

EQ_INSHR(c,cg,p,r,t,s) and *EQ_OUTSHR(c,cg,p,r,t,s)*

$$\textit{\textbf{VAR\_FLO(c)}} \le, \ge, = \textit{FLO\_SHAR(c)} * \textit{Sum \{over all c' in cg of: } \textit{\textbf{VAR\_FLO(c')}} \} \quad \textbf{(5-6)}$$

The commodity group *cg* may be on the input or output side of the process.

A recent modification of TIMES simplifies the above constraints by allowing the use of the ***VAR_ACT*** variable instead of the sum of ***VAR_FLO*** variables in equation (5-6) or in similar ones. This simplification is triggered when the user defines the new attribute ACT_FLO, which is a coefficient linking a flow to the activity of a process. Furthermore, commodity ***c*** appearing in left-hand-side of the constraint may even be a flow that is not part of the ***cg*** group.

Warning: It is quite possible (and regrettable) to over specify flow related equations such as (5-6), especially when the constraint is an equality. Such an over specification leads to an infeasible LP. A new feature of TIMES consists in deleting some of the flow constraints in order to re-establish feasibility, in which case a warning message is issued.

5.4.7 Peaking reserve constraint (time-sliced commodities only)

This constraint imposes that the total capacity of all processes producing a commodity at each time period and in each region must exceed the average demand in the time-slice where peaking occurs by a certain percentage. This percentage is the Peak Reserve Factor, *COM_PKRSV(r,t,c,s)*, and is chosen to insure against several contingencies, such as: possible commodity shortfall due to uncertainty regarding its supply (e.g. water availability in a reservoir); unplanned equipment down time; and random peak demand that exceeds the average demand during the time-slice when the peak occurs. This constraint is therefore akin to a safety margin to protect against random events not explicitly represented in the model. In a typical cold country the peaking time-slice for electricity (or natural gas) will be Winter-Day, and the total electric plant generating capacity (or gas supply plant) must exceed the Winter-Day demand load by a certain percentage. In a warm country the peaking time-slice may be Summer-Day for electricity (due to heavy air conditioning demand). The user keeps full control regarding which time-slices have a peaking equation.

For each time period ***t*** and for region ***r***, there must be enough installed capacity to exceed the required capacity in the season with largest demand for commodity ***c*** by a safety factor *E* called the *peak reserve factor*.

EQ_PEAK(r,t,c,s) - Commodity peak requirement

> ***Sum*** *{over all* ***p*** *producing* ***c with c=pcg*** *of PRC_CAPACT(r,p)* ******* *Peak(r,v,p,c,s)* ******* *FR(s)* ****VAR_CAP(r,v,t,p) * VAR_ACTFLO(r,v,p,c)*** *}* ***+***
>
> ***Sum*** *{over all* ***p*** *producing* ***c*** *with* ***c≠pcg*** *of NCAP_PKCNT(r,v,p,c,s)* ****VAR_FLO(r,v,t,p,c,s)****} +****VAR_IRE(r,t,p,c,s,i****)*
>
> $$\geq \qquad \textbf{(5-7)}$$
>
> *[1+ COM_PKRSV(r,t,c,s)] * [* ***Sum*** *{over all* ***p consuming c*** *of* ***VAR_FLO(r,v,t,p,c,s) +VAR_IRE(r,t,p,c,s,e)****}]*

where:

COM_PKRSV(r,t,c,s) is the region-specific reserve coefficient for commodity *c* in time-slice *s*, which allows for unexpected down time of equipment, for demand at peak, and for uncertain resource availability, and

NCAP_PKCNT(r,v,p,c,s) specifies the fraction of technology *p*'s capacity in a region *r* for a period *t* and commodity *c* (electricity or heat only) that is allowed to contribute to the peak load in slice *s***;** many types of supply processes are predictably available during the peak and thus have a peak coefficient equal to 1, whereas others (such as wind turbines or solar plants in the case of electricity) are attributed a peak coefficient less than 1, since they are on average only fractionally available at peak (e.g., a wind turbine typically has a peak coefficient of .25 or .3, whereas a hydroelectric plant, a gas plant, or a nuclear plant typically has a peak coefficient equal to 1).

For simplicity it has been assumed in (5-7) that the time-slice resolution of the peaking commodity and the time-slice resolution of the commodity flows (FLO, TRADE) are the same. In practice, this is not the case and additional conversion factors or summation operations are necessary to match different time-slice levels.

Remark: to establish the peak capacity, two cases must be distinguished in constraint *EQ_PEAK*.

– For production processes where the peaking commodity is the only commodity in the primary commodity group (denoted c=pcg), the capacity of the process may be assumed to contribute to the peak.

– For processes where the peaking commodity is not the only member of the pcg, there are several commodities included in the pcg. Therefore, the capacity as such cannot be used in the equation. In this case, the actual production is taken into account in the contribution to the peak, instead of the capacity. For example, in the case of CHP only the production of electricity contributes to the peak electricity supply, not the entire capacity of the plant, because the activity of the process consists of both electricity and heat generation in either fixed or flexible proportions, and, depending on the modeler's choice, the capacity may represent either the electric power of the turbine in condensing or back-pressure mode, or the sum of power and heat capacities in back-pressure mode. There is therefore a slight inconsistency between these two cases, since in the first case, a technology may contribute to the peak requirement without producing any energy, whereas this is impossible in the second case.

Note also that in the peak equation (5-7), it is assumed that imports of the commodity are contributing to the peak of the importing region (thus, exports are implicitly considered to be of the *firm power* type).

5.4.8 Constraints on commodities

In TIMES variables are optionally attached to various quantities related to commodities, such as total quantity produced. Therefore it is quite easy to put constraints on these quantities, by simply bounding the commodity variables in each period. It is also possible to impose cumulative bounds on commodities over more than one period, a particularly useful feature for cumulatively bounding emissions or modeling reserves of fossil fuels. By introducing suitable naming conventions for emissions the user may constrain emissions from specific sectors. Furthermore, the user may also impose global emission constraints that apply to several regions taken together, by allowing emissions to be traded across regions. Alternatively or concurrently a tax or penalty may be applied to each produced (or consumed) unit of a commodity (energy form, emission), via specific parameters.

A specific type of constraint may be defined to limit the share of process (p) in the total production of commodity (c). The constraint indicates that the flow of commodity (c) from/to process (p) is bounded by a given fraction of the total production of commodity (c). In the present implementation, the same given fraction is applied to all time slices.

5.4.9 User constraints

In addition to the standard TIMES constraints discussed above, the user may create a wide variety of so-called User Constraints (UC's), whose coefficients follow certain rules. Thanks to recent enhancements of the TIMES code, user defined constraints may involve virtually any TIMES variable. For example, there may a user-defined constraint limiting investment in new nuclear capacity (regardless of the type of reactor), or dictating that a certain percentage of new electricity generation capacity must be powered by a portfolio of renewable energy sources. User constraints may be employed across time periods, for example to model options for retrofitting existing processes or extending their technical lives. A frequent use of UC's involves cumulative quantities (over time) of commodities, flows, or process capacities or activities. Recent TIMES code changes make the definition of the right-hand-sides of such UC's fairly independent of the horizon chosen for the scenario, and thus make it unnecessary to redefine the RHS's when the horizon is changed.

In order to facilitate the creation of a new user constraint, TIMES provides a *template* for indicating a) the set of variables involved in the constraint, and b) the user-defined coefficients needed in the constraint.

The details of how to build different types of UC are included in section 6.4 of Part II of the documentation.

5.4.10 Growth constraints

These are special cases of UC's that are frequently used to maintain the growth (or the decay) of the capacity of a process within certain bounds, thus avoiding excessive abrupt investment in new capacity. Such bounding of the growth is often justified by the reality of real life constraints on technological adoption and evolution. The user is however advised to exert caution on the choice of the maximum rates of technological change, the risk being to restrict it too much and thus "railroad" the model.

Typically, a growth constraint is of the following generic form (ignoring several indices for clarity:

$$\boldsymbol{VAR\_CAP(t+1)} \leq \left(\boldsymbol{1} + \boldsymbol{GROWTH}^{\boldsymbol{M(t+1)-M(t)}}\right) \times \boldsymbol{VAR\_CAP(t)} + \boldsymbol{K} \qquad (5\text{-}8)$$

The GROWTH coefficient is defined as a new attribute of the technology, and represents the maximum annual growth allowed for the capacity. The quantity M(t+1)-M(t) is the number of years between the milestones of periods t and t+1. The constant K is useful whenever the technology has no capacity initially, in order to allow capacity to build over time (if K were absent and initial capacity is zero, the technology would never acquire any capacity)

Note that the sign of the constraint may also be of the "larger than or equal to" type to express a maximum rate of abandonment, in which case the "+" sign is replaced by a "–" sign in the right-hand-side of the constraint. Equality is also allowed, but must be used only exceptionally in order to avoid railroading of the model.

5.4.11 Early retirement of capacity

With this new TIMES feature the user may allow the model to retire some technologies before the end of their technical lives. The retirement may be continuous or discrete. In the former case, the model may retire any amount of the remaining capacity (if any) at each period. In the latter case, the retirement may be effected by the model either in a single block (i.e. the remaining capacity is completely retired) or in multiples of a user chosen block. Please refer to chapter 10 of this document *The lumpy investment option*, for additional discussion of the mathematical formulation of MIP problems.

This feature requires the definition of three new constraints, as listed and briefly described in table 5.1, as well as the alteration of many existing constraints and the objective function, as described in table 5.2 Part II and the special separate note *TIMES Early Retirement of Capacity* provide additional detail.

The user is advised to use the discrete early retirement feature sparingly, as it implies the use of mixed integer programming optimizer, rather than the computationally much more efficient linear programming optimizer. The user should also be aware that using the discrete option voids some of the economic properties of the equilibrium, as discussed in section 10.3.

New Equation	Description
EQ_DSCRET(r,v,t,p)	Discrete retirement equation for process **p** and vintage **v** in region **r** and period **t**. Plays an analogous role to equation EQ_DSCNCAP in the Discrete Capacity Investment Extension.
EQ_CUMRET(r,v,t,p)	Cumulative retirement equation for process **p** and vintage **v** in region **r** and period **t**.
EQL_SCAP(r,t,p,ips)	Maximum salvage capacity constraint for process **p** in region **r** and period **t**, defined for **ips** = N (unless NCAP_OLIFE is specified).

Table 5.1. The new constraints required to implement early retirement of capacity

Existing Equation	Equation Description	Purpose of Modification
EQ_OBJFIX	Fixed cost component of objective function	To credit back the fixed costs of the capacity that is retired early
EQ_OBJVAR	Variable cost component of objective function	To reflect the effect of capacity that is retired early in the costs of capacity-related flows
EQ_OBJSALV	Salvage cost component of objective function	To subtract the salvage value (if any) of capacity that is retired early
EQ(**l**)_CPT for **l** = L, E, G	Capacity transfer equation	To reflect the effect of capacity that is retired early
EQ(**l**)_CAPACT for **l** = L, E, G	Capacity utilization equation	To reflect the effect of capacity that is retired early
EQ(**l**)_CAFLAC for **l** = L, E	Commodity based availability constraint	To reflect the effect of capacity that is retired early
EQ(**l**)_COMBAL for **l** = G, E	Commodity balance equation	To reflect the effect of capacity that is retired early in capacity-related flows
EQ_PEAK	Commodity peaking constraint	To subtract the peak contribution of capacity that is retired early
EQ(**l**)_UC* for **l** = L, E, G	The FLO component of all user constraints	To reflect the effect of capacity that is retired early in capacity-related flows
EQ(**l**)_MRKCON for **l** = L, E, G	Market share of flow in the consumption of a commodity	To reflect the effect of capacity that is retired early in capacity-related flows
EQ(**l**)_MRKPRD for **l** = L, E, G	Market share of flow in the production of a commodity	To reflect the effect of capacity that is retired early in capacity-related flows

Table 5.2. List of existing constraints that are affected by the early retirement option.

5.4.12 Electricity grid modeling

The electricity sector plays a central role in any energy model, and particularly so in TIMES. The electricity commodity has features that present particular challenges for its representation, in that it is difficult to store, and requires a network infrastructure to be transported and delivered. The considerable development of new renewable electricity generation technologies adds to the complexity, inasmuch as the technical requirements of integrating interruptible generation facilities (such as wind turbines and solar plants) to a set of traditional plants, must be satisfied for the integration to be feasible. Such considerations become even more relevant in large regions or countries, where the distances between potential generation areas and consumption areas are quite large.

Such considerations have led to the introduction of an optional grid modeling feature into the TIMES model's equations. A grid consists in a network of nodes linked by arcs (or branches). Each node may represent a well-defined geographic area that is deemed distinct from other areas of the region, either because of its generation potential (e.g. a windy area suitable for wind farms) and/or because of a concentration of points of consumption of electricity (e.g. a populated area separated from other populated areas or from generation areas.)

The purpose of this section is to indicate the broad principles and characteristics of the grid representation feature in TIMES. The modeler wishing to implement the feature is urged to read to the detailed Technical Note “TIMES Grid modeling feature”, which contains the complete mathematical derivations of the equations, and their implementation in TIMES. What follows is a much streamlined version outlining only the main approach and ignoring the many details of the mathematical equations.

5.4.12.1 A much simplified sketch of the grid constraints

The traditional way to represent the nodes and arcs of a grid is shown in figure 5.3, where each node is shown as a horizontal segment, and the nodes are connected via bi-directional arcs.

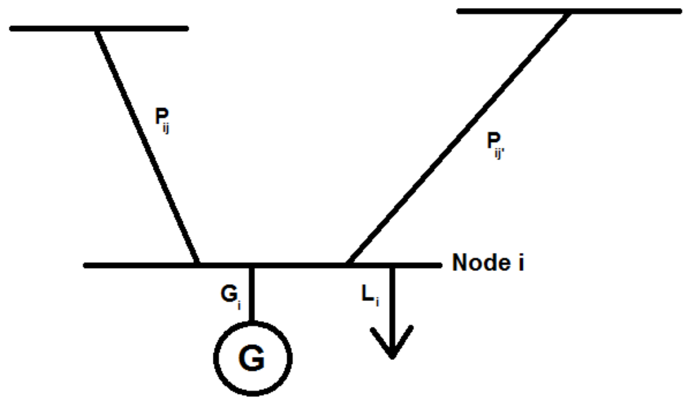

Figure 5.3. Connection of a grid node with other nodes

The basic energy conservation equation of a grid is as follows:

$$G_i - L_i = \sum_{j=1}^{M} P_{i,j} \text{ for each } i=1,2,\ldots,M$$

where:

M = the number of nodes connected with node *i*
G_i = active power injected into node *i* by generators
L_i = active power withdrawn from node *i* by consumer loads
P_{ij} = branch flow from node *i* to node *j*

As mentioned above, these constraints are then modified so as to include important technical requirements on the electrical properties (reactance and phase angle) of each line. Suffice it to say here that the resulting new equations remain linear in the flow and other variables.

5.4.12.2 Integrating grid equations into TIMES

It should be clear that the variables G_i and L_i must be tightly related to the rest of the TIMES variables that concern the electricity commodities. In fact, the modeler must first decide on an allocation of the set of generation technologies into *M* subsets, each subset being attached to a node of the grid. Similarly, the set of all technologies that consume electricity must also be partitioned into *M* subsets, each attached to a node. These two partitions are effected via new parameters specifying the fractions of each generation type to be allocated to each grid node, and similarly for the fractions of each technology consuming electricity to be allocated to grid each node. This indeed amounts to a partial regionalization of the model concerning the electricity sector. Thus, variables G_i and L_i are defined in relation to the existing TIMES variables.

Of course, the introduction of the grid requires modifying the electricity balance equations and peak equations, via the introduction of the net total flow variables the set of grid nodes. The electricity balance equations are modified for each time slice defined for electricity.

Finally, additional a security constraint is added in the case of a multi-regional model, expressing that the total net export or import of electricity from region ***r*** does not exceed a certain (user-defined) fraction of the capacity of the portion of the grid linking region ***r*** to other regions.

5.4.12.3 Costs

New costs attached to the grid are also modeled, and form a new component of the objective function for the region. For this, a new cost coefficient is defined and attached to each node of the grid. TIMES multiplies this cost coefficient by the proper new grid variables and discounts the expression in order to form the new OBJ component.

5.4.13 Reporting "constraints"

These are not constraints proper but expressions representing certain quantities useful for reporting, after the run is completed. They have no impact on the optimization. We have already mentioned ***CAP(r,v,t,p)***, which represents the capacity of a process by vintage.

One sophisticated expression reports the *levelized cost* (LC) of a process. A process's LC is a life cycle quantity that aggregates all costs attached to a process, whether explicit or implicit. It is a useful quantity for ranking processes. However, such a ranking is dependent upon a particular model run, and may vary from run to run. This is so because several implicit costs attached to a process such as the cost of fuels used or produced, and perhaps the cost of emissions, are run dependent.

The general expression for the levelized cost of a process is as follows:

$$LEC = \frac{\sum_{t=1}^{n} \frac{IC_t}{(1+r)^{t-1}} + \frac{OC_t + VC_t + \sum_i FC_{i,t} + FD_{i,t} + \sum_j ED_{j,t}}{(1+r)^{t-0.5}} - \frac{\sum_k BD_{k,t}}{(1+r)^{t-0.5}}}{\sum_{t=1}^{n} \frac{\sum_m MO_{m,t}}{(1+r)^{t-0.5}}} \quad (5\text{-}9)$$

where

- r = discount rate (e.g. 5%)
- IC_t = investment expenditure in (the beginning of) year t
- OC_t = fixed operating expenditure in year t
- VC_t = variable operating expenditure in year t
- FC_{it} = fuel-specific operating expenditure for fuel i in year t
- FD_{it} = fuel-specific acquisition expenditure for fuel i in year t
- ED_{jt} = emission-specific allowance expenditure for emission j in year t (optional)
- BD_{kt} = revenues from commodity k produced by the process in year t (optional;)
- MO_{mt} = output of main product m in year t, i.e. a member of the *pcg*

Comments:

Each cost element listed above is obtained by multiplying a unit cost by the value of the corresponding variable indicated in the run results.

The unit values of the first four costs are simply equal the process input data, i.e. the unit investment cost, the fixed unit O&M cost, the unit variable operating cost, and the unit delivery cost. The last three costs are the shadow prices of the commodities concerned, endogenously obtained as the dual solution of the current model run.

Note also that the user may choose to ignore the last two costs or to include them. Furthermore, concerning the last cost (which is indeed a revenue), the user may decide to ignore the revenue from the main commodities produced by the process and retain only the revenues from the by-products. The choice is specified via the parameter RPT_OPT(‘NCAP’,’1’). Technical note "Levelized costs-TIMES" provides details on the parameter values.

5.5 *The 'Linear' variant of TIMES*

This alternate TIMES formulation (called the LIN variant) assumes a different meaning for the activity and flow variables of TIMES. More precisely, instead of assuming that flows and activities are constant in all years within the same period, the variant assumes that the flow and activity variables apply to only one milestone year within each period.

The variables' values at other years of a period are interpolated between successive milestone years' values. See section 5.2 for a figure depicting the two alternate definitions.

Choosing the LIN formulation affects the variable costs in the objective function as well as all dynamic constraints involving activities or flows. Note also that the LIN variant avoids the cost distortions mentioned in section 5.3.1.

Significant modifications in the LIN formulation concern the variable cost accounting, since the latter are no longer constant in all years of any given period, but evolve linearly between successive milestone years. The objective function components for all variable costs have been modified accordingly.

The following further modifications are done in the LIN formulation:

- The cumulative constraints on commodity production (EQ(l)_CUMNET and EQ(l)_CUMPRD) are modified to include linear interpolation of the commodity variables involved;
- The cumulative constraints on commodity and flow taxes and subsidies (EQ(l)_CUMCST) are modified to include linear interpolation of the commodity and flow variables involved;
- The dynamic equations of the Climate module are modified to include linear interpolation of the variables involved;
- The inter-period storage equations are modified to include linear interpolation of the flow variables involved;
- The cumulative user constraints for activities and flows are also modified in a similar manner.
- Note that in the LIN formulation the activity of ***inter-period storage*** equations is measured at the milestone year (in the standard formulation it is measured at the end of each period). In addition, new EQ_STGIPS equations are added to ensure that the storage level remains non-negative at the end of each period. (Without these additional constraints, the linear interpolation of storage could lead to a negative storage level if the period contains more than a single year.)

6 Parametric analysis with TIMES

Dealing with uncertainty in modeling is a complex endeavour that may be accomplished via a number of (sometimes widely different) approaches. In the case of TIMES, two different features are available: ***Stochastic Programming*** (treated in chapter 8) and ***parametric analysis***, also known as ***sensitivity analysis***, which is the subject of this chapter. In sensitivity analysis, the values of some important exogenous assumptions are varied, and a series of model runs is performed over a discrete set of combinations of these assumptions. Sensitivity analysis is often combined with ***tradeoff analysis***, where the tradeoff relation between several objectives is analyzed.

The uncertain attributes are similar to the corresponding standard TIMES attributes, but they may now have different values according to the different ***states-of-the-world*** (SOW), just as in the case of stochastic programming. The difference between the two approaches is that sensitivity analysis solves a sequence of instances, each assuming different values of the uncertain parameters, whereas stochastic programming solves a single instance that encompasses all potential values of the uncertain parameters simultaneously.

In TIMES, sensitivity analysis and tradeoff analysis facility are implemented using the same setup and some of the attributes of the stochastic mode of TIMES, since both approaches, although conceptually different, use the same state of the world construct.

Here are a few possible set-ups for sensitivity and tradeoff analyses in TIMES, all of which are supported by the model generator:

A. Single phase sensitivity analysis over the set of SOWs. Each run corresponds to a set of values for the uncertain parameters. The runs are mutually independent. This is the most straightforward approach;
B. Two-phase tradeoff analysis, where the model is first run using a user-defined objective function, and then the TIMES objective function is used in phase 2, while the solution from the first phase is used for defining additional constraints in a series of model runs in the second phase.
C. Multiphase tradeoff analysis over N phases, which is a generalization of the two-phase case.

Analyzing tradeoffs between the standard objective function and some other possible objectives (for which the market is not able to give a price) was not possible in an effective way with earlier versions of TIMES.

6.1 *Two-phase tradeoff analysis*

In the ***first phase*** of the TIMES two-phase tradeoff analysis facility, the objective function is user defined as a weighted sum of any number of components, each component being a user constraint's left-hand-side. All UC's must be of the global type, (i.e. aggregated over regions and periods). Optionally, each of the component UCs may also be constrained by upper/lower bounds. The components are defined by the user, via the specification of non-zero weight coefficients for the UC's to be included in the objective. The original objective function (total discounted costs) is automatically pre-defined as a non-constraining user constraint with the name '***OBJZ***', and can therefore always be directly used as one of the component UCs, if desired.

Consequently, the first phase can be considered as representing a Utility Tradeoff Model, which can also be used as a stand-alone option. If used in a stand-alone manner, it constitutes a case of multi-criterion decision making (see e.g. Weistroffer, 2005). The resulting objective function to be minimized can be written as follows:

$$min\ obj1 = \sum_{uc \in UC\_GLB} W(uc) \bullet LHS(uc)$$

where:

W(uc)	=	weight of objective component *uc* in Phase 1
LHS(uc)	=	LHS expression of user constraint *uc* according to its definition
UC_GLB	=	the set of all global UC constraints (including '***OBJZ***')

In the ***second phase*** of the TIMES two-phase tradeoff analysis facility the objective function is always the ***original objective function*** in TIMES, i.e. the total discounted system cost (this ensures that the second phase solution produce an economically meaningful set of values for the dual variables.)

In addition, in the second phase the user can specify bounds on fractional deviations in the LHS values of any or all user constraints, in comparison to the optimal LHS values obtained in the first phase. Such deviation bounds can be set for both global and non-global constraints, and for both non-constraining and constrained UCs (however, any original absolute bounds are overridden by the deviation bounds). The ***objective function***

used in Phase 1 is also available as an additional pre-defined UC, named '***OBJ1***', so that one can set either deviation bounds or absolute bounds on that as well, if desired. In addition, both the total and regional original objective functions can be referred to by using the predefined UC name '***OBJZ***' in the deviation bound parameters.

The objective function to be minimized in the second phase, and the additional bounds on the LHS values of UCs, can be written as follows:

$$min\ objz = LHS('OBJZ')$$

$$\left.\begin{array}{l} LHS(uc) \leq (1 + maxdev(uc)) \bullet LHS^*(uc) \\ LHS(uc) \geq (1 - maxdev(uc)) \bullet LHS^*(uc) \end{array}\right\} \begin{array}{l} \text{for each } uc \text{ for which} \\ maxdev(uc) \text{ has been specified} \end{array}$$

where:

LHS('OBJZ')	=	the standard objective function (discounted total system costs)
LHS(uc)	=	LHS expression of user constraint *uc* according to its definition
LHS(uc)*	=	optimal LHS value of user constraint *uc* in Phase 1
maxdev(uc)	=	user-specified fraction defining the maximum proportional deviation in the value of LHS(uc) compared to the solution in Phase 1

Remarks:

1. Use of the two-phase tradeoff analysis facility requires that a weight has been defined for at least one objective component in the first phase.
2. If no deviation bounds are specified, the second phase will be omitted.
3. Automatic discounting of any commodity or flow-based UC component is possible by using a new UC_ATTR option 'PERDISC' which could be applied e.g. to the user-defined objective components in Phase 1.
4. The two-phase tradeoff analysis can be carried over a set of distinct cases, each identified by a unique SOW index.

6.2 *Multiphase tradeoff analysis*

The multiphase tradeoff analysis is otherwise similar to the two-phase analysis, but in this case the objective function can be defined in the same way as in the Phase 1 described above also in all subsequent phases. The different objective functions in each phase are distinguished by using an additional phase index (the SOW index). Deviation bounds can be specified in each phase, such that they will be in force over all subsequent phases (any user constraints), or only in some of the succeeding phases (any user constraints excluding OBJ1). The deviation bounds defined on any of the user-defined objectives OBJ1 will thus always be preserved over all subsequent phases.

Remark: Although the multiphase tradeoff analysis allows the use of any user-defined objective functions in each phase, it is highly recommended that the original objective function be used in the last phase, so that the economic meaning is maintained in the final solution.

The procedure was presented in a very general form, in order to let the user exert her ingenuity at will. Typical simple examples of using the feature may be useful.

<u>Example 1</u>: trade-off between cost and risk.

First, a special UC (call it RISK) is defined that expresses a **global risk** measure. The successive phases consist in minimizing the following parameterized objective:

$$Min\ OBJZ + \ \alpha \cdot RISK$$

where α is a user chosen coefficient that may be varied within a range to explore an entire trade-off curve such as illustrated in figure 6.1, where the vertical axis represents the values of the cost objective function, and the horizontal axis the risk measure.

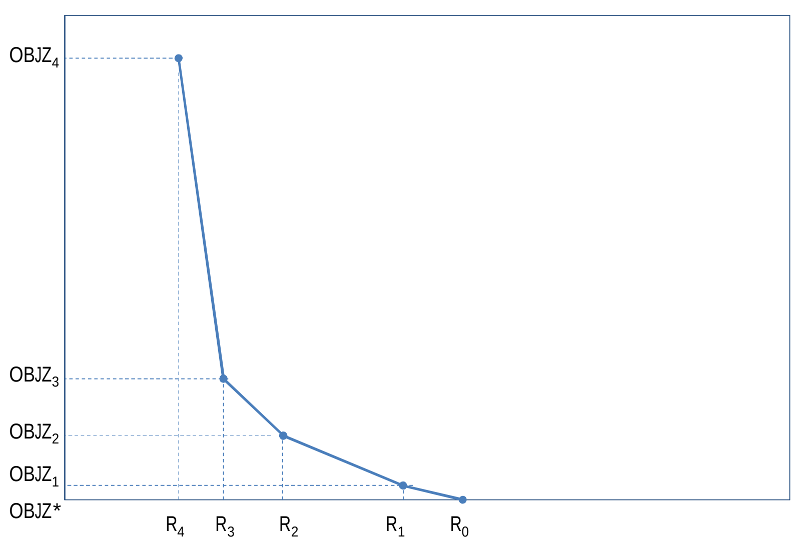

Figure 6.1. Trade-off between Risk and Cost

OBJZ* is the lowest value for OBJZ, corresponding to a relatively large value R_0 for RISK, i.e. when $\alpha = 0$. As α increases, RISK decreases and OBJZ increases. In this example, 4 alternate values of α were chosen until the value of OBJZ becomes very large, at point (R_4,$OBJZ_4$). This would correspond to very large value for α, i.e. a point where RISK is minimized.

An example of such an analysis is fully developed in Kanudia et al (2013), where a risk index is constructed to capture an indicator of energy security for the European Union. A complex (but linear) risk measure was developed to evaluate the risk for a large number of alternative channels of energy imports into the EU, and the trade-off between risk and overall cost was explored.

<u>Example 2</u>: exploring the opportunity cost of the nuclear option

At phase 1, the original *OBJZ* is minimized with the habitual TIMES constraints. This results in an optimal cost *OBJZ**. At phase 2, the objective function is equal to the total nuclear capacity over the entire horizon and over all regions, and a new constraint is added as follows:

$$OBJZ \ \leq (1 + \alpha) \cdot OBJZ^{*}$$

The α parameter may be varied to explore the entire trade-off curve. A last phase may also be added at the end, with OBJZ as objective function, and a user selected value for the maximum level of nuclear capacity.

7 The TIMES Climate Module

This chapter provides a detailed description of the theoretical approach taken to model changes in atmospheric greenhouse gas concentrations, radiative forcing, and global mean temperatures in the TIMES Climate Module. Appendix A of Part II contains a full description of the implementation of the Climate Module in TIMES, including parameters, variables, and equations, as represented in the TIMES code.

The Climate Module starts from global emissions of CO2, CH4, and N2O, as generated by the TIMES global model, and proceeds to compute successively:

- the changes in CO2, CH4, and N2O concentrations via three separate sets of equations;
- the total change (over pre-industrial times) in atmospheric radiative forcing resulting from the three gases plus an exogenously specified additional forcing resulting from other causes (other anthropogenic and/or natural causes, as defined by the user), and
- the temperature changes (over pre-industrial times) in two reservoirs (surface and deep ocean).

The climate equations used to perform these calculations were adapted from Nordhaus and Boyer (1999), who proposed a three reservoir model for the CO2 cycle only[32]. This leads to linear recursive equations for calculating CO2 concentrations in each reservoir. The temperature equations use a two-reservoir model leading also to linear equations. The forcing equation is the one used in most climate models, and is non-linear.

In TIMES, we have modeled separately the life cycles of two other GHG's besides CO2, namely methane and nitrous oxide. These linear equations give results that are good approximations of those obtained from more complex climate models (Drouet et al., 2004; Nordhaus and Boyer, 1999).

The non-linear radiative forcing equation used in virtually all climate models was replaced in TIMES by a linear approximation whose values closely approach the exact ones as long as the useful range is carefully selected. This was done in order to keep the

[32]Other important GHG's such as CH4 and N2O may either be expressed in CO2-equivalent, or a special exogenous forcing term may be added to CO2 forcing. The latter approach is not attractive as it keeps two major GHG's fully exogenous.

entire model linear, and therefore to allow the user to set constraints on forcing and on temperature as well as on concentrations and on emissions.

The temperature equations have been kept as in Nordhaus and Boyer.

We now describe the mathematical equations used at each of the three steps of the climate module.

7.1 *Concentrations (accumulation of CO2, CH4, N2O[33])*

a) CO2 accumulation is represented as the linear three-reservoir model below: the atmosphere, the quickly mixing upper ocean + biosphere, and the deep ocean. CO2 flows in both directions between adjacent reservoirs. The 3-reservoir model is represented by the following 3 equations when the step of the recursion is equal to one year:

$$M_{atm}(y) = E(y) + (1 - \varphi_{atm\text{-}up})\, M_{atm}(y\text{-}1) + \varphi_{up\text{-}atm}\, M_{up}(y\text{-}1) \qquad (7\text{-}1)$$
$$M_{up}(y) = (1 - \varphi_{up\text{-}atm} - \varphi_{up\text{-}lo})\, M_{up}(y\text{-}1) + \varphi_{atm\text{-}up}\, M_{atm}(y\text{-}1) + \varphi_{lo\text{-}up}\, M_{lo}(y\text{-}1) \qquad (7\text{-}2)$$
$$M_{lo}(y) = (1 - \varphi_{lo\text{-}up})\, M_{lo}(y\text{-}1) + \varphi_{up\text{-}lo}\, M_{up}(y\text{-}1) \qquad (7\text{-}3)$$

with

- $M_{atm}(y)$, $M_{up}(y)$, $M_{lo}(y)$: Concentration (expressed in mass units) of CO_2 in atmosphere, in a quickly mixing reservoir representing the upper level of the ocean and the biosphere, and in deep oceans (GtC), respectively, in year y (GtC)
- E(y) = CO_2 emissions in year y (GtC)
- φ_{ij}, transport rate from reservoir i to reservoir j (i, j = atm, up, lo) from year y-1 to y

b) CH4 accumulation is represented by a so-called single-box model in which the atmospheric methane concentration obeys the following equations assuming a constant annual decay rate of the anthropogenic concentrations Φ_{CH4} (whereas the natural concentration is assumed in equilibrium):

$$CH4_{atm}(y) = (1 - \Phi_{CH4}) \cdot CH4_{atm}(y-1) + EA_{CH4}(y) \qquad (7\text{–}4)$$

$$CH4_{up}(y) = CH4_{up}(y-1) \qquad (7\text{–}5)$$

[33] In keeping with the literature, we have expressed all concentrations as masses in megatonnes.

$$CH4_{tot}(y) = CH4_{atm}(y) + CH4_{up}(y) \qquad (7\text{–}6)$$

where

- $CH4_{atm}$,, $CH4_{up}$,$CH4_{tot}$, , and EA_{CH4} are respectively: the anthropogenic atmospheric concentration, the natural atmospheric concentration[34], the total atmospheric concentration (all three expressed in Mt), and the anthropogenic emission of CH4 (expressed in Mt/yr). The anthropogenic emissions EA_{CH4} are generated within the model and enter the dynamic equation (7-4) in order to derive the anthropogenic concentration. Note that the natural concentration $CH4_{up}$is constant at all times. (See initial values for these and other parameters in Part II, Appendix A.)
- $CH4_{tot}$ is then reported and used in the forcing equations. All quantities are indexed by year.
- $1 - \Phi_{CH4}$is the one-year retention rate of CH4 in the atmosphere.
- $d_{CH4} = 2.84$ (the density of *CH4,* expressed in *Mt/ppbv*) is then used to convert concentration in Mt into ppbv for reporting purposes.

c) N2O accumulation is also represented by a single-box model in a manner entirely similar to CH4, although with different parameter values. The corresponding equations are as follows:

$$N2O_{atm}(y) = (1 - \Phi_{N2O}) \cdot N2O_{atm}(y-1) + EA_{N2O}(y)$$

$$N2O_{up}(y) = N2O_{up}(y-1)$$

$$N2O_{tot}(y) = N2O_{up}(y) + N2O_{atm}(y)$$

7.2 *Radiative forcing*

We assume, as is routinely done in atmospheric science, that the atmospheric radiative forcings caused by the various gases are additive (IPCC, 2007). Thus:

$$\Delta F(y) = \Delta F_{CO2}(y) + \Delta F_{CH4}(y) + \Delta F_{N2O}(y) + EXOFOR(y) \qquad (7-7)$$

[34]Note that the subscripts *atm* and *up*, which for the CO2 equations referred to the atmosphere and upper reservoirs, have been reused for the CH4 and N2O equations to stand for anthropogenic and natural concentrations.

We now explain these four terms.

a) The relationship between CO2 accumulation and increased radiative forcing, *$\Delta F_{CO2}(y)$*, is derived from empirical measurements and climate models (IPCC 2001 and 2007).

$$\Delta F_{CO2}(y) = \gamma * \frac{\ln(M_{atm}(y)/M_0)}{\ln 2}$$

where:

- M_0 (i.e.CO2ATM_PRE_IND) is the pre-industrial (circa 1750) reference atmospheric concentration of CO2 = 596.4 GtC
- γ is the radiative forcing sensitivity to atmospheric CO_2 concentration doubling = 3.7 W/m^2

b) The radiative forcing due to atmospheric CH4 is given by the following expression (IPCC 2007), where the subscript *tot* has been omitted

$$\Delta F_{CH4}(y) = 0.036 \cdot \left(\sqrt{CH4_y} - \sqrt{CH4_0}\right) - \left[f(CH4_y, N2O_0) - f(CH4_0, N2O_0)\right] \quad (7–8)$$

c) The radiative forcing due to atmospheric N2O is given by the following expression (IPCC, 2007)

$$\Delta F_{N2O}(y) = 0.12 \cdot \left(\sqrt{N2O_y} - \sqrt{N2O_0}\right) - \left[f(CH4_0, N2O_y) - f(CH4_0, N2O_0)\right] \quad (7–9)$$

where:

$$f(x,y) = 0.47 \cdot ln[1 + 2.01 \cdot 10^{-5} \cdot (xy)^{0.75} + 5.31 \cdot 10^{-15} \cdot x(xy)^{1.52}] \quad (7–10)$$

Note that the f(x,y) function, which quantifies the cross-effects on forcing of the presence in the atmosphere of both gases (CH4 and N2O), is not quite symmetrical in the two gases. As usual, the 0 subscript indicates the pre-industrial times (1750).

d) EXOFOR(y) is the increase in total radiative forcing at period t relative to pre-industrial level due to GHG's that are not represented explicitly in the model. Units = W/m^2. In Nordhaus and Boyer (1999), only emissions of CO2 were explicitly modeled, and therefore EXOFOR(y) accounted for all other GHG's. In TIMES, N_2O and CH_4 are fully accounted for, but some other substances are not (e.g. CFC's, aerosols, ozone, volcanic activity, etc.). Therefore, the values for EXOFOR(y) will differ from those in Nordhaus and Boyer (1999). It is the modeler's responsibility to include in the calculation of EXOFOR(y) the forcing from only those gases and other causes that are not modeled. The careful modeler may also want to adapt the EXOFOR trajectory to particular scenarios. This has been done using alternative trajectories for EXOFOR provided by other models, as was done in a multi-model, multi-scenario study conducted at the Energy Modeling Forum (Clarke et al., 2009)

The parameterization of the three forcing equations (7-8, 7-9, and 7-10) is not controversial and relies on the results reported by Working Group I of the IPCC. IPCC (2001, Table 6.2, p.358) provides a value of 3.7 for γ, smaller than the one used by Nordhaus and Boyer ($\gamma = 4.1$). We have adopted this lower value of 3.7 W/m^2 as default in TIMES. Users are free to experiment with other values of the γ parameter. The same reference provides the entire expressions for all three forcing equations.

7.3 *Linear approximations of the three forcings*

In TIMES, each of the three forcing expressions is replaced by a linear approximation, in order to preserve linearity of the entire model. All three forcing expressions are concave functions. Therefore, two linear approximations are obvious candidates. The first one is an approximation from below, consisting of the chord of the graph between two selected end-points. The second one has the same slope as the chord and is tangent to the graph, thus approximating the function from above. The final approximation is the arithmetic average of the two approximations. These linear expressions are easily derived once a range of interest is defined by the user.

As an example, we derive below the linear approximation for the CO2 forcing expression. The other approximations are obtained in a similar manner.

Linear approximation for the CO2 forcing expression (see technical note “TIMES Climate Module” for similar approximations of the other two forcings):

First, an interval of interest for the concentration M must be selected by the user. The interval should be wide enough to accommodate the anticipated values of the concentrations, but not so wide as to make the approximation inaccurate. We denote the interval (M_1,M_2).

Next, the linear forcing equation is taken as the half sum of two linear expressions, which respectively underestimate and overestimate the exact forcing value. The underestimate consists of the chord of the logarithmic curve, whereas the overestimate consists of the tangent to the logarithmic curve that is parallel to the chord. These two estimates are illustrated in Figure 7.1, where the interval (M_1,M_2) is from 375 ppm to 550 ppm.

By denoting the pre-industrial concentration level as M_0, the general formulas for the two estimates are as follows:

Overestimate:

$$F_1(M) = \frac{\gamma}{ln\,2} \cdot \left[ln\left(\frac{\gamma}{slope \cdot ln(2) \cdot M_0}\right) - 1 \right] + slope \cdot M$$

Underestimate:

$$F_2(M) = \gamma \cdot ln(M_1/M_0)/\,ln\,2 + slope \cdot (M - M_1)$$

where:

$$slope = \gamma \cdot \frac{ln(M_2/M_1)/\,ln\,2}{(M_2 - M_1)}$$

Final approximation: $F_3(M) = \frac{F_1(M)+F_2(M)}{2}$

7.4 *Temperature increase*

In the TIMES Climate Module as in many other integrated models, climate change is represented by the global mean surface temperature. The idea behind the two-reservoir model is that a higher radiative forcing warms the atmospheric layer, which then quickly warms the upper ocean. In this model, the atmosphere and upper ocean form a single layer, which slowly warms the second layer consisting of the deep ocean.

$\Delta T_{up}(y) = \Delta T_{up}(y-1) + \sigma_1\{F(y) - \lambda \Delta T_{up}(y-1) - \sigma_2\,[\Delta T_{up}(y-1) - \Delta T_{low}(y-1)]\}$ (7–11)
$\Delta T_{low}(y) = \Delta T_{low}(y-1) + \sigma_3[\Delta T_{up}(y-1) - \Delta T_{low}(y-1)]$ *(7-12)*

with

- ΔT_{up} = globally averaged surface temperature increase above pre-industrial level,

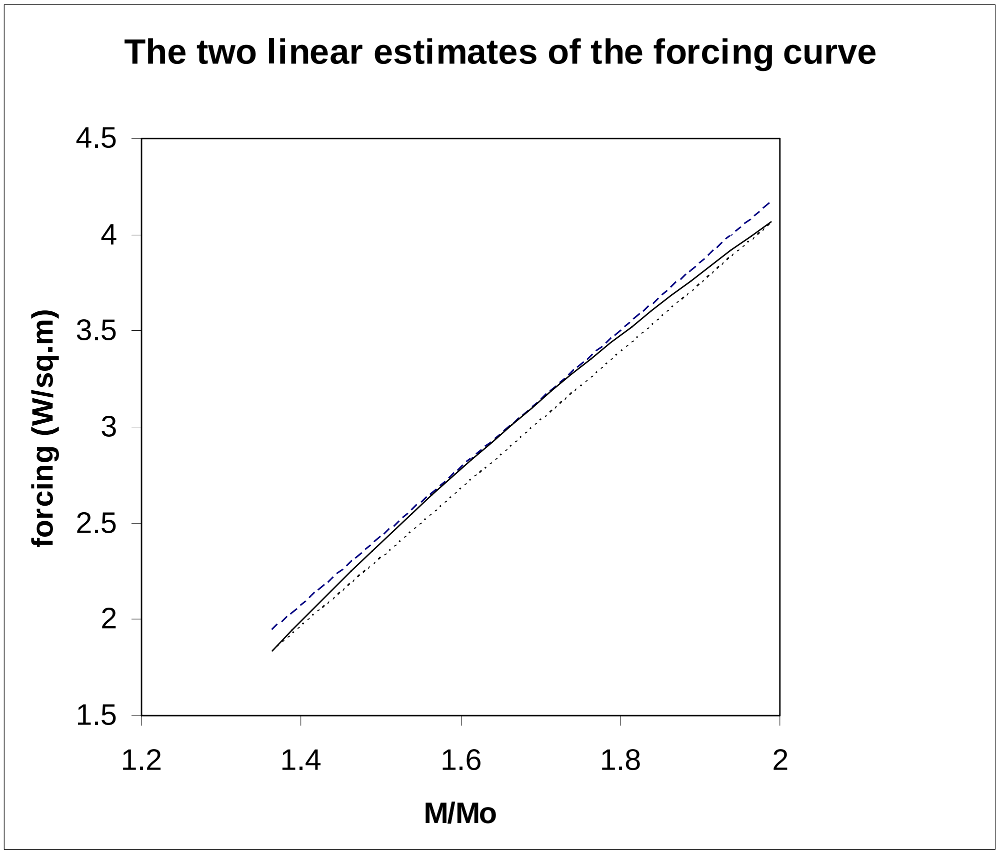

Figure 7.1. Illustration of the linearization of the CO2 radiative forcing function

- ΔT_{low}= deep-ocean temperature increase above pre-industrial level,
- σ_1= 1-year speed of adjustment parameter for atmospheric temperature (also known as the lag parameter),
- σ_2= coefficient of heat loss from atmosphere to deep oceans,
- σ_3 = 1-year coefficient of heat gain by deep oceans,
- λ= feedback parameter (climatic retroaction). It is customary to write λ as $\lambda = \gamma / C_s$, C_s being the climate sensitivity parameter, defined as the change in equilibrium atmospheric temperature induced by a doubling of CO_2 concentration. In contrast with most other parameters, the value of C_s is highly uncertain, with a possible range of values from 1°C to 10°C. This parameter is therefore a prime candidate for sensitivity analysis, or for treatment by probabilistic methods such as stochastic programming.

For more details on the implementation of the Climate Module in TIMES, including parameters, variables, and equations, as represented in the TIMES code, see Appendix A of Part II.

8 The Stochastic Programming extension

8.1 *Preamble to chapters 8 to 11*

Recall that the core TIMES paradigm described in chapters 3, 4, and 5 makes several basic assumptions:

- Linearity of the equations and objective function
- Perfect foresight of all agents over the entire horizon
- Competitive markets (i.e. no market power by any agent)

If any or all of these assumptions are violated, the properties of the resulting equilibrium are no longer entirely valid. In the following four chapters, we present four variants of the TIMES paradigm that depart from the core model. Each of these variants (extensions) departs from one or more assumptions above, as follows:

- Stochastic Programming TIMES extension: departs from the perfect foresight assumption and instead assumes that certain key model parameters are random. This extension requires the use of stochastic programming rather than the usual deterministic linear programming algorithm;

- Limited horizon TIMES extension: departs from the perfect foresight assumption and replaces it by an assumption of limited (in time) foresight. This extension requires the use of sequential linear programming rather than a single global linear optimization;

- Lumpy investments extension: departs from the linearity assumption and replaces it by the assumption that certain investments may only be made in discrete units rather than in infinitely divisible quantities. This extension requires the use of mixed integer programming (MIP) instead of Linear programming;

- The endogenous technological learning (ETL) extension: departs from the linearity assumption for the cost of technologies and replaces it by an assumption that the costs of some technologies are decreasing functions of the cumulative amounts of the technologies, i.e. a learning curve is assumed. This entails that some parts of the objective function are non-linear and non-convex, and requires the use of MIP.

Remark: None of these four extensions departs from the competitive market assumption. It is *also* possible to simulate certain types of non-competitive behavior using TIMES. For instance, it has been possible to simulate the behavior of the OPEC oil cartel by assuming that OPEC imposes an upper limit on its oil production in order to increase its long term profit (Loulou et al, 2007). Such uses of TIMES are not embodied in new extensions. Rather, they are left to the ingenuity of the user.

8.2 *Stochastic Programming concepts and formulation*

Stochastic Programming is a method for making optimal decisions under risk. The risk consists of facing uncertainty regarding the values of some (or all) of the LP parameters (cost coefficients, matrix coefficients, RHSs). Each uncertain parameter is considered to be a random variable, usually with a discrete, known probability distribution. The objective function thus becomes also a random variable and a criterion must be chosen in order to make the optimization possible. Such a criterion may be expected cost, expected utility, etc., as mentioned by Kanudia and Loulou (1998). Technical note "TIMES-Stochastic" provides a more complete description of the TIMES implementation

Uncertainty on a given parameter is said to be resolved, either fully or partially, at the *resolution time*, i.e. the time at which the actual value of the parameter is revealed. Different parameters may have different times of resolution. Both the resolution times and the probability distributions of the parameters may be represented on an event tree, such as the one of figure 8.1, depicting a typical energy/environmental situation. In figure 8.1, two parameters are uncertain: mitigation level, and demand growth rate. The first may have only two values (High and Low), and becomes known in 2010. The second also may have two values (High and Low) and becomes known in 2020. The probabilities of the outcomes are shown along the branches. This example assumes that present time is 2000. This example is said to have three stages (i.e. two resolution times). The simplest non-trivial event tree has only two stages (a single resolution time). Each pathway along the event tree, representing a different realization of the uncertain parameters is referred to as a state-of-the-world (SOW).

The **key observation** is that prior to resolution time, the decision maker (and hence the model) does not know the eventual values of the uncertain parameters, but still has to take decisions. On the contrary, after resolution, the decision maker knows with certainty the outcome of some event(s) and his subsequent decisions will be different depending on which outcome has occurred.

For the example shown in figure 8.1, in 2000 and 2010 there can be only one set of decisions, whereas in 2020 there will be two sets of decisions, contingent on which of the mitigation outcomes (High or Low) has occurred, and in 2030, 2040, 2050 and 2060, there will be four sets of contingent decisions.

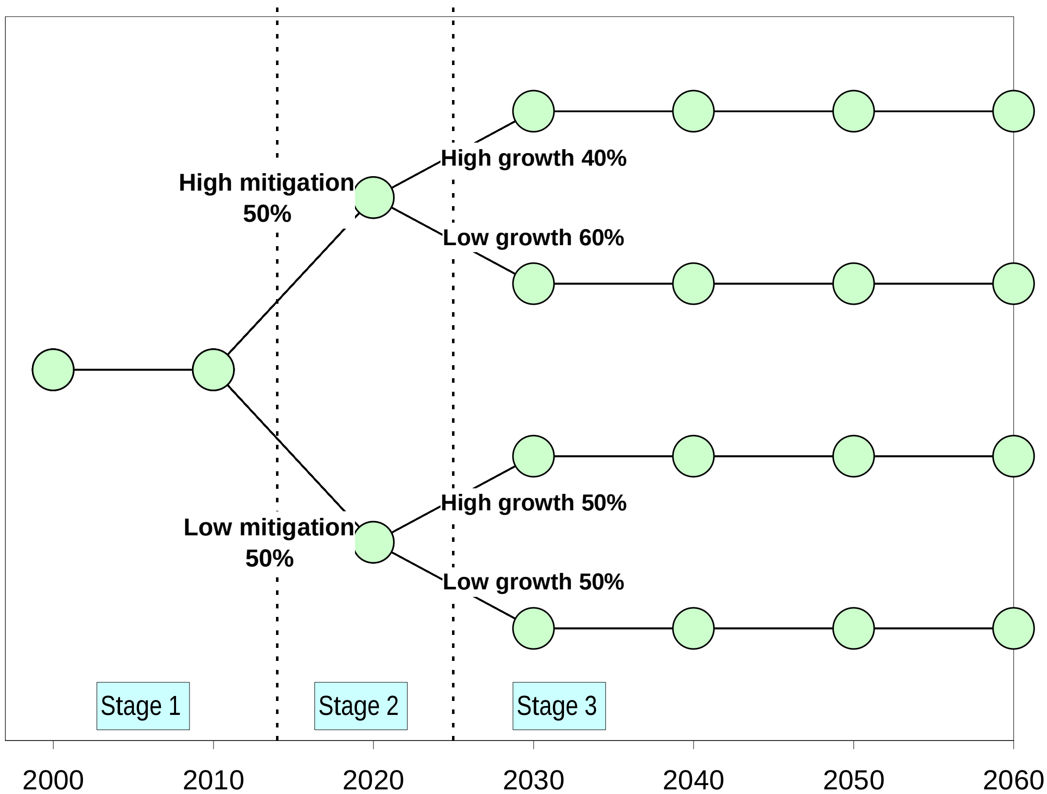

Figure 8.1. Event Tree for a three-stage stochastic TIMES Example.

This remark leads directly to the following general multi-period, multi-stage stochastic program in Equations 8-1 to 8-3 below. The formulation described here is based on Dantzig (1963, Wets (1989), or Kanudia and Loulou (1999), and uses the expected cost criterion. Note that this is a LP, but its size is much larger than that of the deterministic TIMES model.

Minimize

$$Z = \sum_{t \in T} \sum_{s \in S(t)} C(t,s) \times X(t,s) \times p(t,s) \qquad (8\text{–}1)$$

Subject to:

$$A(t,s) \times X(t,s) \geq b(t,s) \forall s \in S(T), t \in T \qquad (8\text{–}2)$$

$$\sum_{t \in T} D(t, g(t,s)) \times X(t, g(t,s)) \geq e(s) \forall s \in S(T) \qquad (8\text{–}3)$$

where

t	=	time period
T	=	set of time periods
s	=	state index
S(t)	=	set of state indices for time period t;

For Figure 8.1, we have: S(2000) = 1; S(2010) = 1; S(2020) = 1,2; S(2030) = 1,2,3,4; S(2040) = 1,2,3,4; S(2050) = 1,2,3,4; S(2060) = 1,2,3,4;

S(*T*)	=	set of state indices at the last stage (the set of *scenarios*). Set *S(T)* is homeomorphic to the set of paths from period 1 to last period, in the event tree.
g(*t*,*s*)	=	a unique mapping from $\{(t,s) \mid s \in S(T)\}$ to *S*(*t*), according to the event tree. *g(t,s)* is the state at period *t* corresponding to scenario *s*.
X(*t*,*s*)	=	the column vector of decision variables in period t, under state s
C(*t*,*s*)	=	the cost row vector
p(*t*,*s*)	=	event probabilities
A(*t*,*s*)	=	the LP sub-matrix of single period constraints, in time period t, under state s
b(*t*,*s*)	=	the right hand side column vector (single period constraints) in time period t, under state s
D(*t*,*s*)	=	the LP sub-matrix of multi-period constraints under state s
e(*s*)	=	the right hand side column vector (multi-period constraints) under scenario s

Alternate formulation: The above formulation makes it a somewhat difficult to retrieve the strategies attached to the various scenarios. Moreover, the actual writing of the cumulative constraints (8-3) is a bit delicate. An alternate (but equivalent) formulation consists in defining one scenario per path from initial to terminal period, and to define distinct variables *X(t,s)* for each scenario and each time period. For instance, in this alternate formulation of the example, there would be four variables *X(t,s)* at every period t, (whereas there was only one variable X(2000,1) in the previous formulation).

Minimize

$$Z = \sum_{t \in T} \sum_{s \in S(t)} C(t,s) \times X(t,s) \times p(t,s) \qquad (8\text{–}1)'$$

Subject to:

$$A(t,s) \times X(t,s) \geq b(t,s) \quad \textit{all } t, \textit{all } \mathrm{s} \qquad (8\text{–}2)'$$

$$\textstyle\sum_{t \in T} D(t,s) \times X(t,s) \geq e(s) \; \textit{all } t, \textit{all } \mathrm{s} \qquad (8\text{–}3)'$$

Of course, in this approach we need to add equality constraints to express the fact that some scenarios are identical at some periods. In the example of Figure 8.1, we would have:

X(2000,1)=X(2000,2)=X(2000,3)=X(2000,4),
X(2010,1)=X(2010,2)=X(2010,3)=X(2010,4),
X(2020,1)=X(2020,2),
X(2020,3)=X(2020,4).

Although this formulation is less parsimonious in terms of additional variables and constraints, many of these extra variables and constraints are in fact eliminated by the pre-processor of most optimizers. The main advantage of this new formulation is the ease of producing outputs organized by scenario.

In the current implementation of stochastic TIMES, the first approach has been used (Equations 8-1 to 8-3). The results are however reported for all scenarios in the same way as in the second approach.

In addition, in TIMES there is also an experimental variant for the modeling of recurring uncertainties with stochastic programming, described in Appendix A of technical note “TIMES-Stochastic”.

8.3 *Alternative criteria for the objective function*

The preceding description of stochastic programming assumes that the policy maker accepts the expected cost as his optimizing criterion. This is equivalent to saying that he is risk neutral. In many situations, the assumption of risk neutrality is only an approximation of the true utility function of a decision maker.

Two alternative candidates for the objective function are:

- Expected utility criterion with linearized risk aversion

- Minimax Regret criterion (Raiffa,1968, applied in Loulou and Kanudia, 1999)

8.3.1 Expected utility criterion with risk aversion

The first alternative has been implemented into the stochastic version of TIMES. This provides a feature for taking into account that a decision maker may be risk averse, by defining a new utility function to replace the expected cost.

The approach is based on the classical E-V model (an abbreviation for Expected Value-Variance). In the E-V approach, it is assumed that the variance of the cost is an acceptable measure of the risk attached to a strategy in the presence of uncertainty. The variance of the cost of a given strategy k is computed as follows:

$$Var(C_k) = \sum_j p_j \bullet (Cost_{j|k} - EC_k)^2$$

where $\boldsymbol{Cost_{j|k}}$ is the cost when strategy $\boldsymbol{k}$ is followed and the $\boldsymbol{j^{th}}$ state of nature prevails, and $\boldsymbol{EC_k}$ is the expected cost of strategy $\boldsymbol{k}$, defined as usual by:

$$EC_k = \sum_j p_j \bullet Cost_{j|k}$$

An E-V approach would thus replace the expected cost criterion by the following utility function to minimize:

$$U = EC + \lambda \cdot \sqrt{Var(C)}$$

where $\lambda>0$ is a measure of the risk aversion of the decision maker. For $\lambda=0$, the usual expected cost criterion is obtained. Larger values of $\boldsymbol{\lambda}$ indicate increasing risk aversion.

Taking risk aversion into account by this formulation would lead to a non-linear, non-convex model, with all its ensuing computational restrictions. These would impose serious limitations on model size.

8.3.2 Utility function with linearized risk aversion

To avoid non-linearities, it is possible to replace the semi-variance by the upper-absolute-deviation, defined by:

$$UpAbsDev(Cost_k) = \sum_j p_j \bullet \{Cost_{j|k} - EC_k\}^+$$

where $\boldsymbol{y} = \{\boldsymbol{x}\}^+$ is defined by the following two *linear* constraints: $\boldsymbol{y} \geq \boldsymbol{x}$, and $\boldsymbol{y} \geq \boldsymbol{0}$, and the utility is now written via the following *linear* expression:

$$U = EC + \lambda \cdot UpsAbsDev(C)$$

This is the expected utility formulation implemented into the TIMES model generator.

8.4 *Solving approaches*

General multi-stage stochastic programming problems of the type described above can be solved by standard deterministic algorithms by solving the deterministic equivalent of the stochastic model. This is the most straightforward approach, which may be applied to all problem instances. However, the resulting deterministic problem may become very large and thus difficult to solve, especially if integer variables are introduced, but also in the case of linear models with a large number of stochastic scenarios.

Two-stage stochastic programming problems can also be solved efficiently by using a Benders decomposition algorithm (Wets, 1989). Therefore, the classical decomposition approach to solving large multi-stage stochastic linear programs has been nested Benders decomposition. However, a multi-stage stochastic program with integer variables does not, in general, allow a nested Benders decomposition. Consequently, more complex decompositions approaches are needed in the general case (e.g. Dantzig-Wolfe decomposition with dynamic column generation, or stochastic decomposition methods).

The current version of the TIMES implementation for stochastic programming is solely based on directly solving the equivalent deterministic problem. As this may lead to very large problem instances, stochastic TIMES models are in practice limited to a relatively small number of branches of the event tree (SOW's).

8.5 *Economic interpretation*

The introduction of uncertainty alters the economic interpretation of the TIMES solution. Over the last two decades, economic modeling paradigms have evolved to a class of equilibria called Dynamic Stochastic General Equilibria (DSGE, see references Chen and

Crucuni, 2012; de Walque et al., 2005; Smets et al., 2007). In the case of Stochastic TIMES, we are in the presence of a Dynamic Stochastic Partial Equilibria (DSPE), with a much less developed literature. The complete characterization of a DSPE is beyond the scope of this documentation, but it is useful to note some of its properties, which derive from the theory of Linear Programming, as follows:

- During the first stage (i.e. before resolution of any uncertainties), the meaning of the primal solution is identical to that of a deterministic TIMES run, i.e. of a set of optimal decisions, whereas the meaning of the shadow prices is that of *expected prices*(resp. expected marginal utility changes) of the various commodities. This is so because the shadow price is the marginal change in objective function when a commodity's balance is marginally altered, and the objective function is an expected cost (resp. an expected utility function).

- During subsequent stages, the primal values of any given branch of the event tree represent the optimal decisions *conditional on the corresponding outcome being true*, and the shadow prices are the *expected*[35] *prices* of the commodities also conditional on the corresponding outcome being true.

[35] The expected prices become deterministic prices if the stage is the last one, so that there is no uncertainty remaining at or after the current period.

9 Using TIMES with limited foresight (time-stepped)

It may be useful to simulate market conditions where all agents take decisions with only a limited foresight of a few years or decades, rather than the very long term. By so doing, a modeler may attempt to simulate "real-world" decision making conditions, rather than socially optimal ones. Both objectives are valid provided the modeler is well aware of each approach's characteristics.

Be that as it may, it is possible to use TIMES in a series of time-stepped runs, each with an optimizing horizon shorter than the whole horizon. The option that enables this mode is named FIXBOH, which freezes the solution over some user chosen years, while letting the model optimize over later years. The FIXBOH feature has several applications and is first described below before a full description of the time-stepped procedure.

9.1 *The FIXBOH feature*

This feature requires that an initial run be made first, and then FIXBOH sets fixed bounds for a subsequent run according to the solution values from the initial run up to the last milestone year less than or equal to the year specified by the FIXBOH control parameter. For instance, the initial run may be a reference case, which is run from 2010 to 2100, and the FIXBOH value might be set at 2015, in which case a subsequent run would have exactly the same solution values as the reference case up to 2015. This is an extremely convenient feature to use in most situations.

As a generalization to the basic scheme described above, the user can also request fixing to the previous solution different sets of fixed years according to region.

Example: Assume that you would like to analyze the 15-region ETSAP TIAM model with some shocks after the year 2030, and you are interested in differences in the model solution only in regions that have notable gas or LNG trade with the EU. Therefore, you would like to fix the regions AUS, CAN, CHI, IND, JPN, MEX, ODA and SKO completely to the previous solution, and all other regions to the previous solution up to 2030.

9.2 ***The time-stepped option (TIMESTEP)***

The purpose of the TIMESTEP option is to run the model in a stepwise manner with limited foresight. The TIMESTEP control variable specifies the number of years that should be optimized in each solution step. The total model horizon will be solved by a series of successive steps, so that in each step the periods to be optimized are advanced further in the future, and all periods before them are fixed to the solution of the previous step (using the FIXBOH feature). It is important that any two successive steps have one or more overlapping period(s), in order to insure overall continuity of the decisions between the two steps (in the absence of the overlap, decisions taken at step ***n*** would have no initial conditions and would be totally disconnected from step ***n-1*** decisions.)

Figure 9.1 illustrates the step-wise solution approach with a horizon of 8 periods and 6 successive optimization steps. Each step has a 2 period sub-horizon, and there is also an overlap of one period between a step and the next. More explicitly: at step 2, all period 2 variables are frozen at the values indicated in the solution of step 1, and period 3 is free to be optimized. At step 3, period 3 variables are frozen and period 4 is optimized, etc.

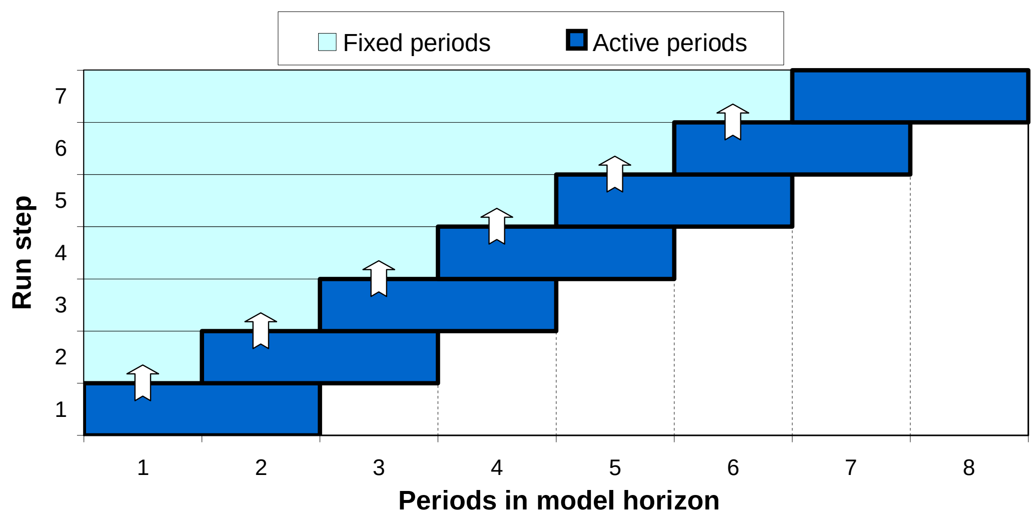

Figure 9.1. Sequence of optimized periods in the stepped TIMES solution approach. Each run includes also the fixed solution of all earlier periods.

The amount of overlapping years between successive steps is by default half of the active step length (the value of TIMESTEP), but it can be controlled by the user.

<u>Important remark</u>: as mentioned above, the user chooses the lengths of the sub-horizons and the length of the overlaps, *both expressed in years*. Because the time periods used in

the model may be variable and may not always exactly match with the step-length and overlap, the actual active step-lengths and overlaps may differ somewhat from the values specified by the user. At each step the model generator uses a heuristic that tries to make a best match between the remaining available periods and the prescribed step length. However, at each step it is imperative that at least one of the previously solved periods must be fixed, and at least one remaining new period is taken into the active optimization in the current step.

10 The Lumpy Investment extension

In some cases, the linearity property of the TIMES model may become a drawback for the accurate modeling of certain investment decisions. Consider for example a TIMES model for a relatively small community such as a city. For such a scope the *granularity* of some investments may have to be taken into account. For instance, the size of an electricity generation plant proposed by the model would have to conform to an implementable minimum size (it would make no sense to decide to construct a 50 MW nuclear plant). Another example for multi-region modeling might be whether or not to build cross-region electric grid(s) or gas pipeline(s) in discrete size increments. Processes subject to investments of only specific size increments are described as “lumpy” investments.

For other types of investments, size does not matter: for instance the model may decide to purchase 10,950.52 electric cars, which is easily rounded to 10,950 without any serious inconvenience, especially since this number is an annual figure. The situation is similar for a number of residential or commercial heating devices; or for the capacity of wind turbines; or of industrial boilers; in short, for any technologies with relatively small minimum feasible sizes. Such technologies would not be candidates for treatment as “lumpy” investments.

This chapter describes the basic concept and mathematics of lumpy investment option, whereas the implementation details are available in Part II, section 6.3.24. We simply note here that this option, while introducing new variables and constraints, does not affect existing TIMES constraints.

It is the user’s responsibility to decide whether or not certain technologies should respect the minimum size constraint, weighing the pros and cons of so doing. This chapter explains how the TIMES LP is transformed into a Mixed Integer Program (MIP) to accommodate minimum or multiple size constraints, and states the consequences of so doing on computational time and on the interpretation of duality results.

The lumpy investment option available in TIMES is slightly more general than the one described above. It insures that investment in technology $\boldsymbol{k}$ is equal to one of a finite number $\boldsymbol{N}$ of pre-determined sizes: $\boldsymbol{0, S_1(t), S_2(t), \ldots, S_N(t)}$. This is useful when several typical plant sizes are feasible in the real world. As implied by the notation, these discrete sizes may be different at different time periods. Note that by choosing the $\boldsymbol{N}$ sizes as the successive multiples of a fixed number ***S***, it is possible to invest (perhaps many times) in a technology with fixed standard size.

Imposing such a constraint on an investment is unfortunately impossible to formulate using standard LP constraints and variables. It requires the introduction of *integer variables* into the formulation. The optimization problem resulting from the introduction of integer variables into a Linear Program is called a Mixed Integer Program (MIP).

10.1 *Formulation and solution of the Mixed Integer Linear Program*

Typically, the modeling of a lumpy investment involves Integer Variables, i.e. variables whose values may only be non-negative integers (0, 1, 2, …). The mathematical formulation is as follows:

$$VAR_{NCAP(p,t)} = \sum_{i=1}^{N} S_i(p,t) \times Z_i(p,t) \quad \text{each } t = 1,..,T$$

with

$$Z_i(p,t) = 0 \text{ or } 1$$

and

$$\sum_{i=1}^{N} Z_i(p,t) \leq 1$$

The second and third constraints taken together imply that at most one of the ***Z*** variables is equal to 1 and all others are equal to zero. Therefore, the first constraint now means that ***NCAP*** is equal to one of the preset sizes or is equal to 0, which is the desired result.

Although the formulation of lumpy investments *looks* simple, it has a profound effect on the resulting optimization program. Indeed, MIP problems are notoriously more difficult to solve than LPs, and in fact many of the properties of linear programs discussed in the preceding chapters do not hold for MIPs, including duality theory, complementary slackness, etc. Note that the constraint that *Z(p,t)* should be 0 or 1 departs from the *divisibility* property of linear programs. This means that the *feasibility domain* of integer variables (and therefore of some investment variables) is no longer contiguous, thus making it vastly more difficult to apply purely algebraic methods to solve MIP's. In fact, practically all MIP solution algorithms make use (at least to some degree) of partial enumerative schemes, which tend to be time consuming and less reliable[36] than the algebraic methods used in LP.

The reader interested in more technical details on the solution of LPs and of MIPs is referred to references (Hillier and Lieberman, 1990, Nemhauser et al. 1989). In the next section we shall be content to state one important remark on the interpretation of the dual results from MIP optimization.

10.2 *Discrete early retirement of capacity*

The discrete retirement of capacity that was briefly mentioned in section 5.4.11 requires a treatment quite similar to that of discrete addition to capacity presented here. The complete mathematical formulation mimics that presented above, and is fully described in Part II, section 6.3.26, of the TIMES documentation.

10.3 *Important remark on the MIP dual solution (shadow prices)*

Using MIP rather than LP has an important impact on the interpretation of the TIMES shadow prices. Once the optimal MIP solution has been found, it is customary for MIP solvers to fix all integer variables at their optimal (integer) values, and to perform an additional iteration of the LP algorithm, so as to obtain the dual solution (i.e. the shadow prices of all constraints). However, the interpretation of these prices is different from that of a pure LP. Consider for instance the shadow price of the natural gas balance constraint: in a pure LP, this value represents the price of natural gas. In MIP, this value represents the price of gas *conditional on having fixed the lumpy investments at their optimal integer values.* What does this mean? We shall attempt an explanation via one example: suppose that one lumpy investment was the investment in a gas pipeline; then, *the gas shadow price will not include the investment cost of the pipeline, since that investment was fixed when the dual solution was computed.*

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[36] A TIMES LP program of a given size tends to have fairly constant solution time, even if the database is modified. In contrast, a TIMES MIP may show some erratic solution times. One may observe reasonable solution times (although significantly longer than LP solution times) for most instances, with an occasional *very* long solution time for some instances. This phenomenon is predicted by the theory of complexity as applied to MIP, see Papadimitriou and Stieglitz (1982).

In conclusion, when using MIP, only the primal solution is fully reliable. In spite of this major caveat, modeling lumpy investments may be of paramount importance in some instances, and may thus justify the extra computing time and the partial loss of dual information.

11 The Endogenous Technological Learning extension

In a long-term dynamic model such as TIMES the characteristics of many of the future technologies are almost inevitably changing over the sequence of future periods due to *technological learning*.

In some cases it is possible to forecast such changes in characteristics as a function of time, and thus to define a time-series of values for each parameter (e.g. unit investment cost, or efficiency). In such cases, technological learning is *exogenous* since it depends only on time elapsed and may thus be established outside the model.

In other cases there is evidence that the pace at which some technological parameters change is dependent on the *experience* acquired with this technology. Such experience is not solely a function of time elapsed, but typically depends on the cumulative investment (often global) in the technology. In such a situation, technological learning is *endogenous*, since the future values of the parameters are no longer a function of time elapsed alone, but depend on the cumulative investment decisions taken by the model (which are unknown). In other words, the evolution of technological parameters may no longer be established outside the model, since it depends on the model's results.

Endogenous technological learning (ETL) is also named *Learning-By-Doing* (LBD) by some authors.

Whereas exogenous technological learning does not require any additional modeling, ETL presents a tough challenge in terms of modeling ingenuity and of solution time. In TIMES, there is a provision to represent the effects of endogenous learning on the unit investment cost of technologies. Other parameters (such as efficiency) are not treated, at this time.

11.1 *The basic ETL challenge*

Empirical studies of unit investment costs of several technologies have been undertaken in several countries. Many of these studies find an empirical relationship between the unit investment cost of a technology at time $\boldsymbol{t}$, $\boldsymbol{INVCOST_t}$, and the cumulative investment in that technology up to time $\boldsymbol{t}$, $C_t = \sum_{j=-1}^{t} VAR\_NCAP_j$.

A typical relationship between unit investment cost and cumulative investments is of the form:

$$INVCOST_t = a \cdot C_t^{-b} \qquad (11-1)$$

where

- *INVCOST*[37] is the unit cost of creating one unit of the technology, which is no longer a constant, but evolves as more units of the technology are produced;
- ***a*** is the value of I*NVCOST* for the first unit of the technology (when $\boldsymbol{C_t}$ is equal to 1) and;
- ***b*** is the learning index, representing the speed of learning[38].

As experience builds up, the unit investment cost decreases, potentially rendering investments in the technology more attractive. It should be clear that near-sighted investors will not be able to detect the advantage of investing early in learning technologies, since they will only observe the high initial investment cost and, being near-sighted, will not anticipate the future drop in investment cost resulting from early investments. In other words, tapping the full potential of technological learning requires far-sighted agents who accept making initially non-profitable investments in order to later benefit from the investment cost reduction.

With regard to actual implementation, simply using (11-1) as the objective function coefficient of $\boldsymbol{VAR\_NCAP_t}$ will yield a non-linear, non-convex expression. Therefore, the resulting mathematical optimization is no longer linear, and requires special techniques for its solution. In TIMES, a Mixed Integer Programming (MIP) formulation is used, that we now describe.

11.2 *The TIMES formulation of ETL*

11.2.1 The cumulative investment cost

We follow the basic approach described in Barreto, 2001.

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[37] The notation in this chapter is sometimes different from the standard notation for parameters and variables, in order to conform to the more detailed technical note on the subject.

[38] It is usual to define, instead of ***b***, another parameter, ***pr*** called the *progress ratio*, which is related to ***b*** via the following relationship: $pr = 2^{-b}$. Hence, ***1-pr*** is the cost reduction incurred when cumulative investment is doubled. Typical observed ***pr*** values are in a range of .75 to .95.

The first step of the formulation is to express the total investment cost, i.e. the quantity that should appear in the objective function. The cumulative investment cost $\boldsymbol{TC_t}$ of a learning technology in period $\boldsymbol{t}$is obtained by integrating expression (11-1):

$$TC_t = \int_0^{C_t} a \cdot y^{-b} * dy = \frac{a}{1-b} \cdot {C_t}^{-b+1} \quad (11-2)$$

$\boldsymbol{TC_t}$ is a concave function of $\boldsymbol{C_t}$, with a shape as shown in figure 11.1

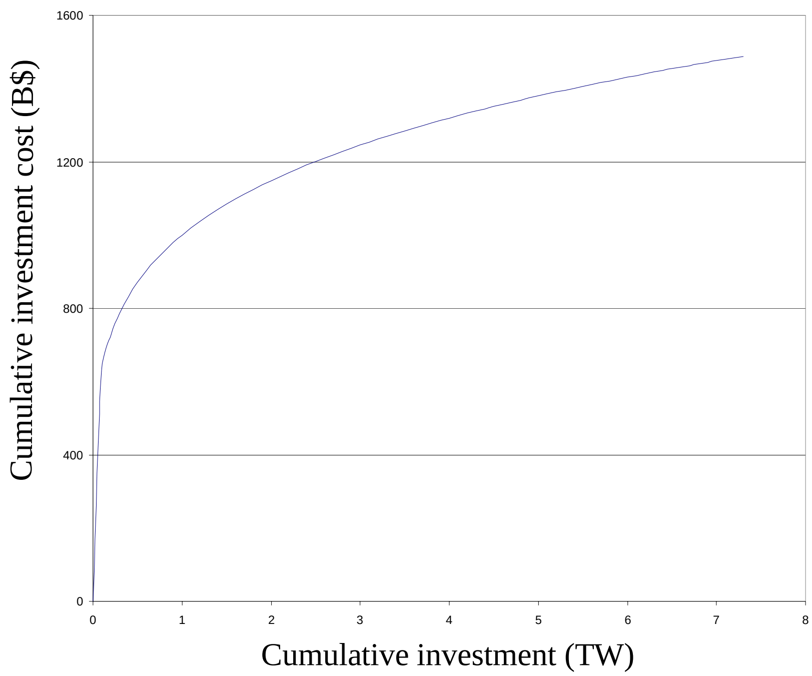

Figure 11.1. Example of a cumulative learning curve

With the Mixed Integer Programming approach implemented in TIMES, the cumulative learning curve is approximated by linear segments, and binary variables are used to represent some logical conditions. Figure 11.2 shows a possible piecewise linear approximation of the curve of Figure 11.1. The choice of the number of steps and of their respective lengths is carefully made so as to provide a good approximation of the smooth cumulative learning curve. In particular, the steps must be smaller for small values than for larger values, since the curvature of the curve diminishes as total investment increases. The formulation of the ETL variables and constraints proceeds as follows (we omit the period, region, and technology indexes for notational clarity):

1. The user specifies the set of learning technologies;
2. For each learning technology, the user provides:
   a) The progress ratio ***pr*** (from which the learning index ***b*** may be inferred)
   b) One initial point on the learning curve, denoted (C_0, TC_0)
   c) The maximum allowed cumulative investment C_{max} (from which the maximum total investment cost TC_{max} may be inferred)
   d) The number ***N*** of segments for approximating the cumulative learning curve over the (C_0, C_{max}) interval.

   Note that each of these parameters, including ***N***, may be different for different technologies.
3. The model automatically selects appropriate values for the ***N*** step lengths, and then proceeds to generate the required new variables and constraints, and the new objective function coefficients for each learning technology. The detailed formulae are shown and briefly commented on below.

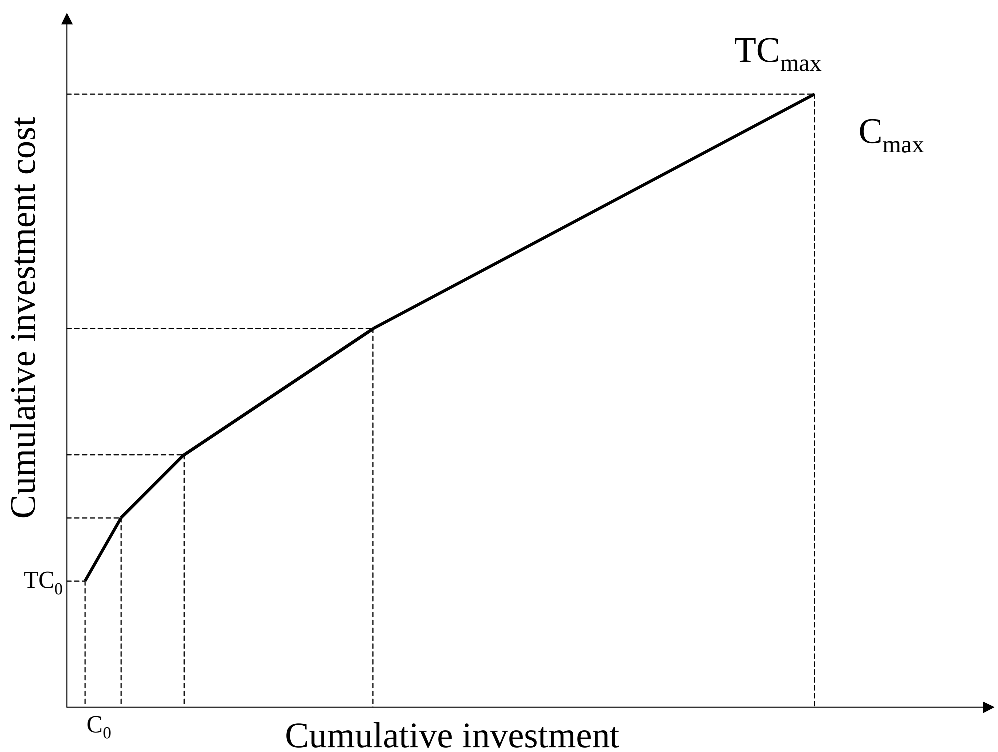

Figure 11.2. Example of a 4-segment approximation of the cumulative cost curve

11.2.2 Calculation of break points and segment lengths

The successive interval lengths on the vertical axis are chosen to be in geometric progression, each interval being twice as wide as the preceding one. In this fashion, the intervals near the low values of the curve are smaller so as to better approximate the curve in its high curvature zone. Let $\{TC_{i-1}, TC_i\}$ be the i^{th} interval on the vertical axis, for $i = 1, \ldots, N-1$. Then:

$$TC_i = TC_{i-1} + 2^{i-N-1}(TC_{max} - TC_0)/(1-0.5^N), \quad i=1,2,\ldots,N$$

Note that TC_{max} is equal to TC_N.

The break points on the horizontal axis are obtained by plugging the TC_i 's into expression (11-2), yielding:

$$C_i = \left(\frac{(1-b)}{a}(TC_i)\right)^{\frac{1}{1-b}}, i = 1,2,\ldots,N$$

11.2.3 New variables

Once intervals are chosen, standard approaches are available to represent a concave function by means of integer (0-1) variables. We describe the approach used in TIMES. First, we define N continuous variables x_i, $i = 1,\ldots,N$. Each x_i represents the portion of cumulative investments lying in the i^{th} interval. Therefore, the following holds:

$$C = \sum_{i=1}^{N} x_i \qquad 11-3$$

We now define N integer (0-1) variables z_i that serve as indicators of whether or not the value of C lies in the i^{th} interval. We may now write the expression for TC, as follows:

$$TC = \sum_{i=1}^{N} a_i z_i + b_i x_i \qquad 11-4$$

where b_i is the slope of the i^{th} line segment, and a_i is the value of the intercept of that segment with the vertical axis, as shown in figure 11.3. The precise expressions for a_i and b_i are:

$$b_i = \frac{TC_i - TC_{i-1}}{C_i - C_{i-1}} \quad i = 1,2,\ldots,N \qquad (11-5)$$

$$a_i = TC_{i-1} - b_i \cdot C_{i-1} \quad i = 1,2,\ldots,N$$

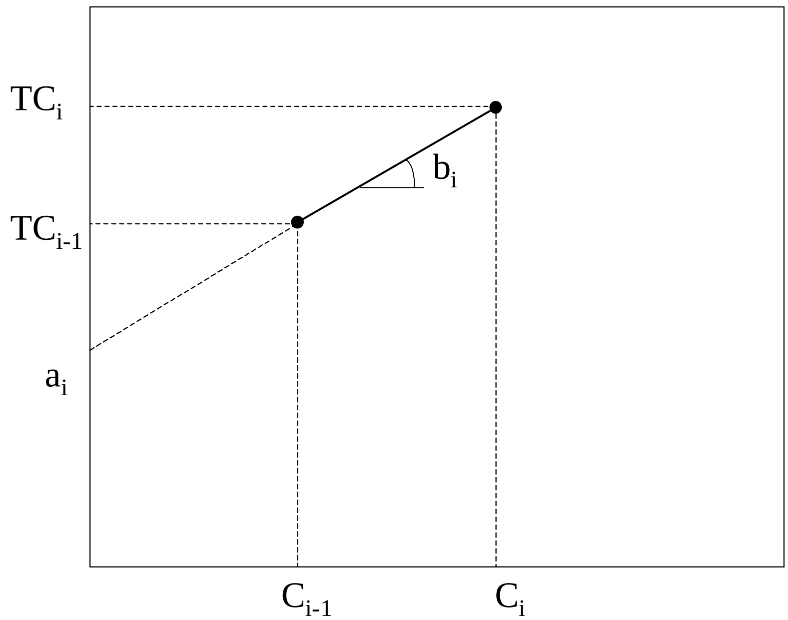

Figure 11.3. The i^{th} segment of the step-wise approximation

11.2.4 New constraints

For (11-4) to be valid we must make sure that exactly one z_i is equal to 1, and the others equal to 0. This is done (recalling that the z_i variables are 0-1) via:

$$\sum_{i=1}^{N} z_i = 1$$

We also need to make sure that each x_i lies within the i^{th} interval whenever z_i is equal to 1 and is equal to 0 otherwise. This is done via two constraints:

$$C_{i-1} \cdot z_i \le x_i \le C_i \cdot z_i$$

11.2.5 Objective function terms

Re-establishing the period index, we see that the objective function term at period ***t***, for a learning technology is thus equal to $\boldsymbol{TC_t - TC_{t-1}}$, which needs to be discounted like all other investment costs.

11.2.6 Additional (optional) constraints

Solving integer programming problems is facilitated if the domain of feasibility of the integer variables is reduced. This may be done via additional constraints that are not strictly needed but that are guaranteed to hold. In our application we know that experience (i.e. cumulative investment) is always increasing as time goes on. Therefore, if the cumulative investment in period t lies in segment i, it is certain that it will not lie in segments *i-1, i-2, .., 1* in time period *t+1*. This leads to two new constraints (re-establishing the period index ***t*** for the ***z*** variables):

$$\sum_{j=1}^{i} z_{j,t} \ge \sum_{j=1}^{i} z_{j,t+1}$$

$$\sum_{j=i}^{N} z_{j,t} \le \sum_{j=i}^{N} z_{j,t+1} \qquad i = 1, 2, \ldots, N-1, \;\; t = 1, 2, \ldots, T-1$$

Summarizing the above formulation, we observe that each learning technology requires the introduction of ***N*T*** integer (0-1) variables. For example, if the model has 10 periods and a 5-segment approximation is selected, 50 integer (0-1) variables are created for that learning technology, assuming that the technology is available in the first period of the model. Thus, the formulation may become very onerous in terms of solution time, if many learning technologies are envisioned, and if the model is of large size to begin with. In section 11.5 we provide some comments on ETL, as well as a word of warning.

11.3 ***Clustered learning***

An interesting variation of ETL is also available in TIMES, namely the case where several technologies use the same key technology (or component), itself subject to learning. For instance, table 11.1 lists 11 technologies using the key Gas Turbine technology. As experience builds up for gas the turbine, each of the 11 technologies in the cluster benefits. The phenomenon of clustered learning is modeled in TIMES via the following modification of the formulation of the previous section.

Let ***k*** designate the key technology and let ***l* = 1, 2, …,*L*** designate the set of clustered technologies attached to ***k***. The approach consists of three steps:

i) Step 1: designate ***k*** as a learning technology, and write for it the formulation of the previous section;

ii) Step 2: subtract from each $INVCOST_l$ the initial investment cost of technology ***k*** (this will avoid double counting the investment cost of ***k***);

iii) Step 3: add the following constraint to the model, in each time period. This ensures that learning on ***k*** spreads to all members of its cluster:

$$VAR\_NCAP_k - \sum_{l=1}^{L} VAR\_NCAP_l = 0$$

Table 11.1: Cluster of gas turbine technologies
(from A. Sebregts and K. Smekens, unpublished report, 2002)

Description
Integrated Coal gasification power plant
Integrated Coal Gasification Fuel Cell plant
Gas turbine peaking plant
Existing gas Combined Cycle power plant
New gas Combined Cycle power plant
Combined cycle Fuel Cell power plant
Existing gas turbine CHP plant
Existing Combined Cycle CHP plant
Biomass gasification: small industrial cog.
Biomass gasification: Combined Cycle power plant
Biomass gasification: ISTIG+reheat

11.4 *Learning in a multiregional TIMES model*

Technological learning may be acquired via global or local experience, depending on the technology considered. There are examples of technologies that were developed and perfected in certain regions of the world, but have tended to remain regional, never fully spreading globally. Examples are found in land management, irrigation, and in household heating and cooking devices. Other technologies are truly global in the sense that the same (or close to the same) technology becomes rather rapidly commercially available globally. In the latter case, global experience benefits users of the technology worldwide. Learning is said to *spillover* globally. Examples are found in large electricity plants, in steel production, wind turbines, and many other sectors.

The first and obvious implication of these observations is that the appropriate model scope must be used to study either type of technology learning. The formulation described in the previous sections is adequate in two cases: a) learning in a single region model, and b) regional learning in a multiregional model. It does *not* directly apply to *global learning* in a multiregional global model, where the cumulative investment variable must represent the sum of all cumulative investments in all regions together. We now describe an approach to global learning that may be implemented in TIMES, using only standard TIMES entities.

The first step in modeling multiregional ETL is to create one additional region, region 0, which will play the role of the Manufacturing Region. This region's RES consists only of the set of (global) learning technologies (LT's). Each such LT has the following specifications:

a) The LT has no commodity input.
b) The LT has only one output, a new commodity ***c*** representing the 'learning'. This output is precisely equal to the investment level in the LT in each period.
c) Commodity ***c*** may be exported to all other regions.

Finally, in each 'real' region, the LT is represented with all its attributes *except the investment cost NCAP_COST.* Furthermore, the construction of one unit of the LT requires an input of one unit of the learning commodity ***c*** (using the *NCAP_ICOM* parameter see chapter 3 of PART II). This ensures that the sum of all investments in the LT in the real regions is exactly equal to the investment in the LT in region 0, as desired.

11.5 *Endogenous vs. exogenous learning: a discussion*

In this section, we formulate a few comments and warnings that may be useful to potential users of the ETL feature.

We start by stating a very important caveat to the ETL formulation described in the previous sections: if a model is run with such a formulation, it is very likely that the model will select some technologies, and *will invest massively at some early period* in these technologies unless it is prevented from doing so by additional constraints. Why this is likely to happen may be qualitatively explained by the fact that once a learning technology is selected for investing, two opposing forces are at play in deciding the optimal timing of the investments. On the one hand, the discounting provides an incentive for postponing investments. On the other hand, investing early allows the unit investment cost to drop immediately, and thus allows much cheaper investments in the learning technologies in the current and all future periods. Given the considerable cost reduction that is usually induced by learning, the first factor (discounting) is highly unlikely to predominate, and hence the model will tend to invest massively and early in such technologies, or not at all. Of course, what we mean by “massively” depends on the other constraints of the problem (such as the extent to which the commodity produced by the learning technology is in demand, the presence of existing technologies that compete with the learning technology, etc.). However, there is a clear danger that we may observe unrealistically large investments in some learning technologies.

ETL modelers are well aware of this phenomenon, and they use additional constraints to control the penetration trajectory of learning technologies. These constraints may take the form of upper bounds on the capacity of or the investment in the learning technologies in each time period, reflecting what is considered by the user to be realistic penetrations. These upper bounds play a determining role in the solution of the problem, and it is most often observed that the capacity of a learning technology is either equal to 0 or to the upper bound. This last observation indicates that the selection of upper bounds (or capacity/investment growth rates) by the modeler is the predominant factor in controlling the penetration of successful learning technologies.

In view of the preceding discussion, a fundamental question arises: is it worthwhile for the modeler to go to the trouble of modeling e*ndogenous* learning (with all the attendant computational burdens) when the results are to a large extent conditioned by *exogenous* upper bounds? We do not have a clear and unambiguous answer to this question; that is left for each modeler to evaluate.

However, given the above caveat, a possible alternative to ETL would consist in using exogenous learning trajectories. To do so, the same sequence of ‘realistic’ upper bounds on capacity would be selected by the modeler, and the values of the unit investment costs (INVCOST) would be externally computed by plugging these upper bounds into the learning formula (11-1). This approach makes use of the same exogenous upper bounds as the ETL approach, but avoids the MIP computational burden of ETL. Of course, the running of exogenous learning scenarios is not entirely foolproof, since there is no absolute guarantee that the capacity of a learning technology will turn out to be exactly equal to its exogenous upper bound. If that were not the case, a modified scenario would have to be run, with upper bounds adjusted downward. This trial-and-error approach may seem inelegant, but it should be remembered that it (or some other heuristic approach) might prove to be necessary in those cases where the number of learning technologies and the model size are both large (thus making the rigorous ETL formulation computationally intractable).

12 General equilibrium extensions

12.1 *Preamble*

In order to achieve a general (as opposed to partial) equilibrium, the energy system described in TIMES must be linked to a representation of the rest of the economy. The idea of hard-linking an energy model with the economy while still keeping the resulting model as an optimization program, dates back to the ETA-MACRO model (Manne, 1977), where both the energy system and the rest of the economy were succinctly represented by a small number of equations. This approach differs from the one taken by the so-called Computable General Equilibrium (CGE), models (Johanssen 1960, Rutherford 1992), where the calculation of the equilibrium relies on the resolution of simultaneous non-linear equations. In CGE's, the use of (non-linear, non-convex) equation solvers limits the size of the problem and thus the level of detail in the energy system description. This computational difficulty is somewhat (but not completely) alleviated when the computation relies on a single non-linear optimization program. Note however that MACRO is a much simplified representation of the economy as a single producing sector and no government sector, thus precluding the endogenous representation of taxes, subsidies, multi-sector interactions, etc. Therefore, the idea of a linked TIMES-MACRO model is not to replace the CGE's but rather to create an energy model where the feedbacks from the economy goes beyond the endogenization of demands (which TIMES does) to include the endogenization of capital.

Some years after ETA-MACRO, MARKAL-MACRO (Manne-Wene, 1992) was obtained by replacing the simplified ETA energy sub-model by the much more detailed MARKAL, giving rise to a large optimization model where most, but not all equations were linear. The MERGE model (Manne et al., 1995) is a multi-region version of ETA-MACRO with much more detail on the energy side –although not as much as in MARKAL-MACRO. The TIMES-MACRO model (Remme-Blesl, 2006) is based on exactly the same approach as MARKAL-MACRO. Both MARKAL-MACRO and TIMES-MACRO were essentially single-region models, until the multi-region version of TIMES-MACRO (named TIMES-MACRO-MSA, Kypreos-Lettila, 2013) was devised as an extension that accommodates multiple regions.

In this chapter, we describe the single region and the multi-region versions of TIMES-MACRO, focusing on the concepts and mathematical representation, whereas the implementation details are left to Part II of the TIMES documentation and to technical notes.

12.2 *The single-region TIMES-MACRO model*

As was already discussed in chapter 4, the main physical link between a TIMES model and the rest of the economy occurs at the level of the consumption of energy by the end-use sectors. There are however other links, such as capital and labor, which are common to the energy system and the rest of the economy. Figure 12.1 shows the articulation of the three links in TIMES-MACRO. Energy flows from TIMES to MACRO, whereas money flows in the reverse direction. Labor would also flow from MACRO to TIMES, but here a simplification is used, namely that the representation of labor is purely exogenous in both sub-models. Thus, TIMES-MACRO is not suitable for analyzing the impact of policies on labor, or on taxation, etc.

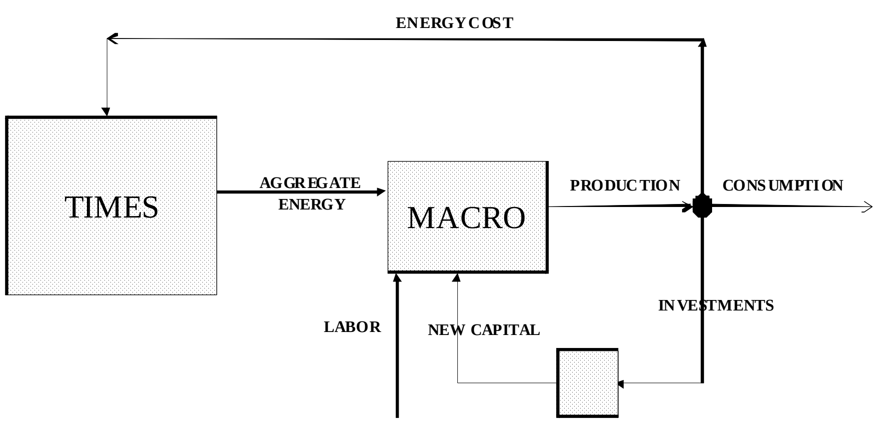

Figure 12.1. Energy, Labor, and Monetary flows between TIMES and MACRO

We now turn to the mathematical description of the above, starting with the MACRO portion of the hybrid model.

12.2.1 Formulation of the MACRO model

We start our description of the hybrid model by stating the MACRO equations (12-1) – (12-6)[39]:

$$Max \quad \sum_{t=1}^{T-1} dfact_t \cdot ln(C_t) + \frac{dfact_{T-1} \cdot dfactcurr_{T-1}^{\frac{d_{T-1}+d_T}{2}}}{1 - dfactcurr_T^{\frac{d_{T-1}+d_T}{2}}} \cdot ln(C_T) \qquad (12\text{-}1)$$

$$Y_t = C_t + INV_t + EC_t \qquad (12\text{-}2)$$

$$Y_t = \left(akl \cdot K_r^{kpvs \cdot \rho} \cdot l_t^{(1-kpvs)\rho} + \sum_{dm} b_{dm} \cdot DEM\_M_{t,dm}^{\rho}\right)^{1/\rho} \qquad (12\text{-}3)$$

$$l_1 = 1 \quad \text{and} \quad l_{t+1} = l_t \cdot (1 + growv_t)^{\frac{d_t + d_{t+1}}{2}} \qquad (12\text{-}4)$$

$$K_{t+1} = tsrv_t \cdot K_t + \frac{1}{2}(d_t \cdot tsrv \cdot INV_t + d_{t+1} \cdot INV_{t+1}) \qquad (12\text{-}5)$$

$$K_T \cdot (growv_T + depr) \le INV_T \qquad (12\text{-}6)$$

with the model **variables**:

C_t: annual consumption in period *t*,
$DEM\_M_{t,dm}$: annual energy demand in MACRO for commodity *dm* in period *t*,
Y_t: annual production in period *t*,
INV_t: annual investments in period *t*,
EC_t: annual energy costs in period *t*,
K_t: total capital in period *t*

and the **exogenous parameters**:

akl: production function constant,
b_{dm}: demand coefficient,
d_t: duration of period *t* in years,
$depr$: depreciation rate,
$dfact_t$: utility discount factor,
$dfactcurr_t$: annual discount rate,
$growv_t$: growth rate in period *t*,

[39] The concrete implementation in the TIMES-MACRO model differs in some points, e.g. the consumption variable in the utility function is substituted by equations (12-2) and (12-3).

$kpvs$	capital value share,
l_t:	annual labor index in period t,
ρ:	substitution constant,
T:	period index of the last period,
$tsrv_t$:	capital survival factor between two periods.

The objective function (12-1) of the MACRO model is the maximization of the summation of discounted utility at each period. The utility is defined as the logarithm of consumption $C_t$of the households. A logarithmic utility function embodies a decreasing marginal utility property (Manne, 1977). Note that the discount factor $dfact_t$for period t must take into account both the length of the period and the time elapsed between the period's start and the base year. Note also that the discount factor of the last period has a larger impact since it is assumed to apply to the infinite time horizon after the last model period (alternatively, the user may decide to limit the number of years in the last term, in those cases where it is deemed important to confer less weight to the indefinite future).

The national accounting equation (12-2) simply states that national production Y_t must cover national consumption C_t , plus investments INV_t , plus energy costs EC_t.

The production function (12-3) represents the entire economy. It is a nested, constant elasticity of substitution (CES) function with the three input factors capital, labor and energy. The production input factors labor $l_t$and capital $K_t$form an aggregate, in which both can be substituted by each other represented via a Cobb-Douglas function. Then, the aggregate of the energy services and the aggregate of capital and labor can substitute each other. Note that labor is not endogenous in MACRO,but is specified exogenously by the user provideda labor growth rate$growv_t$.

The energy in term in (12-3) is a weighted sum of end-use demands in all sectors *dm* of the economy, $DEM\_M_{t,dm}$, raised to the power ρ. We defer the definition of these quantities until the next subsection.

The lower the value of the elasticity of substitution the closer is the linkage between economic growth and increase in energy demand. For homogenous production functions with constant returns to scale[40] the substitution constant ρin (12-3) is directly linked with the user-defined elasticity of substitution σ by the expression $\rho = 1 - 1/\sigma$.

[40] A production function is called homogenous of degree *r*, if multiplying all production factors by a constant scalar leads λ to an increase of the function by λ^r . If *r*= 1, the production function is called linearly homogenous and leads to constant returns to scale.

The capital value share $kpvs$ describes the share of capital in the sum of all production factors and must be specified by the user. The parameter akl is the level constant of the production function. The parameters akl and b_{dm} of the production are determined based on the results from a TIMES model run without the MACRO module.

The capital dynamics equation (12-5) describes the capital stock in the current period K_{t+1} based on the capital stock in the previous period and on investments made in the current and the previous period. Depreciation leads to a reduction of the capital. This effect is taken into account by the capital survival factor$tsrv_t$, which describes the share of the capital or investment in period *t* that still exists in period *t+1*. It is derived from the depreciation rate $depr$ using the following expression:

$$tsrv_t = (1 - depr)^{\frac{(d_{t+1}+d_t)}{2}} \qquad (12\text{-}7)$$

Expression (12-7) calculates the capital survival factor for a period of years beginning with the end of the middle year m_t and ending with the end of the year m_{t+1}. The duration between these two middle years equals the duration $\frac{d_{t+1}+d_t}{2}$. Then, a mean investment in period *t* is calculated by weighting the investments in *t* and *t+1* with the respective period duration: $1/2\,(d_t \cdot tsrv \cdot INV_t + d_{t+1} \cdot INV_{t+1})$.

For the first period it is assumed that the capital stock grows with the labor growth rate of the first period$growv_0$. Thus, the investment has to cover this growth rate plus the depreciation of capital. Since the initial capital stock is given and the depreciation and growth rates are exogenous, the investment in the first period can be calculated beforehand:

$$INV_0 = K_0 \cdot (depr + growv_0) \qquad (12\text{-}8)$$

Since the model horizon is finite, one has to ensure that the capital stock is not fully exhausted (which would maximize the utility in the model horizon.) Therefore a terminal condition (12-6) is added, which guarantees that after the end of the model horizon a capital stock for the following generations exists. It is assumed that the capital stock beyond the end of horizon grows with the labor growth rate$growv_T$. This is coherent with the last term of the utility function.

12.2.2 Linking MACRO with TIMES

TIMES is represented via the following condensed LP

$$Min \sum_t dfact_t \cdot COST\_T_t \quad (x)$$

s.t.

$$E \cdot x = DEM\_T_{dm,t} \qquad (A)$$

$$A \cdot x = b \qquad (B)$$

where

- x is the vector of TIMES variables
- $COST\_T_t \quad (x)$ is the annual undiscounted cost TIMES expression
- $dfact_t$ is the discount factor for period t
- equations *(A)* express the satisfaction of demands in TIMES (and thus defines the $DEM\_T_{dm,t}$ variables), and
- equations *(B)* is the set of all other TIMES constraints

MACRO and TIMES are hard linked via two sets of variables: the energy variables $DEM\_T_{dm,t}$, and the period energy costs $COST\_T_t$.

The aggregate energy input into MACRO (see equation (12-3)), is slightly different from the TIMES variables defined above. In the linked model, each term *DEM_M* is obtained by further applying a factor $aeeifac_{t,dm}$ as shown in equation (12-9).

$$DEM\_T_{t,dm} = aeeifac_{t,dm} \cdot DEM\_M_{t,dm} \qquad (12\text{-}9)$$

Indeed, the energy demand in the TIMES model can be lower than the energy requirement of the MACRO model due to demand reductions, which are caused by autonomous energy efficiency improvements and come in addition to those captured in the energy sector of the TIMES model. The autonomous energy efficiency improvement factor $aeeifac_{t,dm}$ is determined in a calibration procedure described in technical note “Documentation of the TIMES-MACRO model”, which also discusses the weighing coefficientsb_{dm}.

The other link consists in accounting for the monetary flowEC_t, equal to the expenditures made in the energy sector. Precisely,$EC_t$is equal to the annual *undiscounted* energy system cost of the TIMES model,$COST\_T_t$, (as used in the TIMES objective function), augmented with an additional term as shown in equation (12-10):

$$COST\_T_t + \frac{1}{2} qfac \sum_p \frac{cstinv_{t,p}}{expf_t \cdot capfy_p} \cdot XCAP_{t,p}^2 = EC_t \quad (12\text{-}10)$$

with

$XCAP_{t,p}$: portion of the capacity expansion for technology p in period *t* that is penalized. Constraint (12-11) below states that it is the portion exceeding a predefined tolerable expansion rate$expf_t$,

EC_t: costs for the production factor energy in the MACRO model,

$qfac$: trigger to activate penalty term (0 for turning-off penalty, 1 for using penalty term),

$cstinv_{t,p}$: specific annualized investment costs of technology p in period *t*,

$capfy_p$: maximum level of capacity for technology *p*,

$expf_t$: tolerable expansion between two periods.

Just like in the pure MACRO model, the quadratic penalty term added on the left hand side of Eqn. (11) serves to slow down the penetration of technologies. This term plays a somewhat similar role as the growth constraints do in the stand-alone TIMES model. The variable $XCAP_{t,p}$ is the amount of capacity exceeding a predefined expansion level expressed by the expansion factor $expf_t$ and is determined by the following equation:

$$VAR\_CAP_{t+1,p} \leq (1 + expf_t) \cdot VAR\_CAP_{t,p} + XCAP_{t+1,p} \quad (12\text{-}11)$$

with:

$VAR\_CAP_{t,p}$: total installed capacity of technology p in period *t*.

As long as the total installed capacity in period *t+1* is below $(1 + expf_t) \cdot CAP_{t,p}$ no penalty costs are applied. For the capacity amount $XCAP_{t+1,p}$ exceeding this tolerated capacity level penalty costs are added to the regular costs of the TIMES model in Equation (12-10).

The quadratic term in Eqn. (11) introduces a large number of nonlinear terms (one for each technology and period) that may constitute a considerable computational burden for large models. These constraints are therefore replaced in the current implementation of TIMES, by linear piece-wise approximations in a way quite similar to what was done to linearize the surplus in chapter 4.

12.2.3 A brief comment

In spite of the linearization of the penalty terms in equation (12-10), TIMES-Macro still contains non-linearities: its objective function is a concave function, a good property when maximizing, but there are T nonlinear, non convex constraints as per equation (12-3) that introduce a non trivial computational obstacle to large size instances of the model.

Although not discussed here, the calibration of the TIMES-MACRO model is an exceedingly important task, since the model must agree with the initial state of the economy in the dimensions of labor, capital, and the links between the energy sector and the economy at large. Fuller details on calibration are provided in the above-mentioned technical note.

Overall, the experience with TIMES-MACRO has been good, with sizable model instances solved in reasonable time. But the modeler would benefit from carefully weighing the limitation of model size imposed by the non-linear nature of TIMES-MACRO, against the advantage of using a (single sector) general equilibrium model.

12.3 ***The multi-regional TIMES-MACRO model (MSA)***

In this section, we only sketch the generalization of TIMES-MACRO to a multi-regional setting. Full details, including the important calibration step and other implementation issues, appear in technical note "TIMES-Macro: Decomposition into Hard-Linked LP and NLP Problems".

12.3.1 Theoretical background

In a multi-regional setting, inter-regional trade introduces an important new complication in the calculation of the equilibrium[41]. Indeed, the fact that the utility function used in the MACRO module is highly non linear also means that the global utility is not equal to the sum of the national utilities. Also, it would be impractical and conceptually wrong to define a single consumption function for the entire set of regions, since the calibration of

[41] Of course, if no trade between the regions is assumed, the global equilibrium amounts to a series of independent national equilibria, which may be calculated by the single region TIMES-MACRO.

the model may only be done using national statistics, and furthermore, there may be large differences in the parameters of each region's production function, etc.
It follows from the above that it is not possible to use a single optimization step to calculate the global equilibrium. Instead, one must resort to more elaborate approaches in order to compute what is termed a Pareto-optimal solution to the equilibrium problem, i.e. a solution where the utility of any region may not be improved without deteriorating the utility of some other region(s).
Such a situation has been studied in the economics literature, starting with the seminal paper by Negishi (1960) that established the existence of equilibria that are Pareto-optimal in the Welfare functions. Manne (1999) applied the theory to the MACRO model, and Rutherford (1992) proposed a decomposition algorithm that makes the equilibrium computation more tractable. The Rutherford algorithm is used in the TIMES-MACRO model. An interesting review of the applications of Negishi theory to integrated assessment models appeared in Stanton (2010).

12.3.2 A sketch of the algorithm to solve TIMES-MACRO-MSA

Rutherford's procedure is an iterative decomposition algorithm. Each iteration has two steps. The first step optimizes a large TIMES LP and the second step optimizes a stand-alone *reduced* non-linear program which is an alteration of MACRO, and is named MACRO-MSA. These two steps are repeated until convergence occurs.
Because the two steps must be solved repeatedly, the iterative procedure is computationally demanding; furthermore, it is established that the speed of convergence is dependent upon the number of trade variables that link the regions. For this and other reasons, the trade between regions is limited to a single commodity, namely a *numéraire*, expressed in monetary units. The numéraire $\boldsymbol{NTX_{r,t}}$ affects the national account equation (12-2) of each region, as follows:

$$Y_{r,t} = C_{r,t} + INV_{r,t} + EC_{r,t} + NTX_{r,t}$$

and is subject to the conservation constraint: $\sum_r NTX_{r,t} = 0 \ \forall \{t\}$,

which insures that trade is globally balanced.

First step: at each iteration, the first step is the resolution of TIMES using non-elastic demands provided by the previous solution of the non-linear program (except at iteration 1, where demands are either exogenously provided or generated by TIMES[42]).

Second step: once the TIMES solution is obtained, it is used to form a quadratic expression representing an approximation of the aggregate energy cost, to be used in MACRO-MSA. Defining this approximation is the crux of Rutherford decomposition idea. It replaces the entire TIMES model, thus greatly simplifying the resolution of Step 2. The global objective function of MACRO-MSA is a weighted sum (over all regions) of the regional MACRO welfare functions, where the weights are the Negishi weights for each region. The thus modified global objective function is maximized. Then, a convergence criterion is checked. If convergence is not observed, the new demands are fed into TIMES and a new iteration is started. The Negishi weights are also updated at each iteration, leading to a new version of the objective, until the algorithm converges to the Pareto-optimal equilibrium.

The adaptation of Rutherford algorithm to TIMES-MACRO was formalized by Kypreos (2006) and implemented by Kypreos and Lettila as the above-mentioned technical note.

[42]It may be desirable, although not required to use non-zero demand elasticities at the very first iteration.

13 Appendix A: History and comparison of MARKAL and TIMES

13.1 *A brief history of TIMES[43] and MARKAL*

The TIMES (The Integrated Markal-EfomSystem) and the MARKAL (MARket ALlocation) models have a common history beginning in the 1970's when a formal decision of the International Energy Agency (IEA) led to the creation of a common tool for analyzing energy systems, to be shared by the participating OECD nations. MARKAL became a reality by the year 1980 and became a common tool of the members of the Energy Technology Systems Analysis Programme (ETSAP), an IEA Implementing Agreement (IA).

Development of the new modeling paradigm was undertaken over a period of three years. First a team of national experts from more than sixteen countries met numerous times to define the data requirements and mathematics that were to underpin MARKAL. Then the actual coding and testing of the model formulation proceeded on two parallel tracks. One team at Brookhaven National Laboratory (BNL) embarked on the undertaking employing OMNI[44], a specialized programming language specifically designed for optimization modeling, that was widely used for modeling oil refinery operations. The second team at KFA Julich chose to use Fortran to code the model. While both teams initially succeeded, changes were quickly necessary that proved to be more manageable in the BNL OMNI version of MARKAL than in the KFA Fortran version – leading to the decision to formally adopt only the BNL OMNI version for general use. A full description of this initial incarnation of the model maybe be found in the MARKAL User's Guide (Fishbone, 1983).

MARKAL was used intensively by ETSAP members throughout the two decades after 1980 and beyond, undergoing many improvements. The initial mainframe OMNI version of MARKAL was in use until 1990, when BNL ported the model to the person computer that was just becoming a viable alternative. At the same time, as part of this move of MARKAL to the PC, the first model management system for MARKAL databases and model results was developed at BNL which greatly facilitated working with MARKAL and opened it up to a new class of users. This PC based shell, MUSS (MARKAL User Support System, Goldstein, 1991), provided spreadsheet-like browse/edit facilities for

[43] With the kind permission of Professor Stephen Hawking

[44]A product of Haverly Systems Incorporated, http://www.haverly.com/.

managing the input data, Reference Energy System (RES) network diagramming to enable viewing the underlying depiction of the energy system, scenario management and run submission, and multi-case comparison graphics that collectively greatly facilitated the ability to work effectively with MARKAL.

The next big step in the evolution of MARKAL/TIMES arose from the BNL collaboration with Professor Alan Manne of Stanford University resulting in the porting of MARKAL to the more flexible General Algebraic Modeling System (GAMS), still used for TIMES today. The driving motivation for this move to GAMS was to enable the creation of MARKAL-MACRO (see chapter 12), a major model variant enhancement resulting in a General Equilibrium version of the model. One drawback of MARKAL-MACRO is that it was implemented as a non-linear programming (NLP) optimization model, which limits its usability for large energy system models.

To overcome this shortcoming while embracing one of the main benefits arising from MARKAL-MACRO, another major model enhancement was implemented in 1995 from a proposal made in 1980 by Giancarlo Tosato (1980), to allow end-use service demands to be price sensitive, thus transforming MARKAL from a supply cost optimization model to a system computing a supply demand partial equilibrium, named MARKAL-ED (Loulou and Lavigne, 1996) while retaining its linear form. An alternative formulation using non-linear programming, MARKAL-MICRO (Van Regemorter, 1998) was also implemented. Many other enhancements were made in the late 1990's and early 2000's and are described in the second comprehensive version of the MARKAL model documentation (Loulou et al., 2004).

The development of ANSWER, the first Windows interface for MARKAL, commenced at the Australian Bureau of Agricultural and Resource Economics (ABARE) in Canberra in early 1996 with primary responsibility taken by then ABARE staff member Ken Noble. By early 1998 the first production version of ANSWER-MARKAL was in use, including by most ETSAP Partners. In late 2003 Ken Noble retired from ABARE, established Noble-Soft Systems and became the owner of the ANSWER-MARKAL software, thereby ensuring its continuing development and support.

By the late 1990's, the need to gather all the existing MARKAL features and to create many new ones was becoming pressing, and an international group of ETSAP researchers was formed to create what became the TIMES model generator. The main desired new features were as follows:

- To allow time periods to be of unequal lengths, defined by the user;

- To allow the user data to control the model structure;
- To make data as independent as possible of the choice of the model periods (data decoupling), in particular to facilitate the recalibration of the model when the initial period is changed, but also to avoid having to redefine the data when period lengths are altered;
- To formally define commodity flows as new variables (as in the EFOM model), thus making it easier to model certain complex processes;
- To define vintaged processes that allow input data to change according to the investment year;
- To enable the easy creation of flexible processes, a feature that was feasible with MARKAL only by creating multiple technologies;
- To permit time-slices to be entirely flexible with a tiered hierarchy of year/season/week/time-of-day to permit much more robust modeling of the power sector;
- To improve the representation and calculation of costs in the objective function;
- To formally identify trade processes in order to facilitate the creation of multi-regional models;
- To define storage processes that carry some commodities from one time-slice to another or from some period to another; and
- To implement more dynamic and inter-temporal user-defined constraints.

Definition and development of TIMES began in late 1998, resulting in a beta version in 1999, and the first production version in year 2000, initially used by only a small number of ETSAP members. The transition from MARKAL to TIMES was slower than anticipated, mainly because ETSAP modellers already had mature MARKAL databases that required serious time and effort to be converted into TIMES databases.

Furthermore there was a need for a TIMES specific model shell to manage the new model. Two data handling shells were created during the 2000's by two private developers closely associated with ETSAP and with partial support from ETSAP: In the early 2000’s, VEDA_FE (VErsatile Data Analysis -Front End) (http://www.kanors.com/Index.asp) and in 2008, ANSWER-TIMES (Noble-Soft Systems, 2009). Even before that, a back-end version of VEDA (VEDA_BE, Kanudia http://www.kanors.com/Index.asp) had been created to explore and exploit the results and create reports.

Following these developments, and as the merits of TIMES over MARKAL became increasingly evident, TIMES became the preferred modeling tool for most ETSAP members, old and new, as well as for energy system modellers who were not formal ETSAP members, but were either associated with ETSAP as partners in several outreach projects or on their own.

The first complete documentation of the TIMES model generator was written in 2005 and made available on the ETSAP website (http://www.iea-etsap.org/web/index.asp). It has since been replaced by this documentation.

As the number of modellers increased and they gained experience with TIMES, the model underwent many new additions and enhancements, and the number of publications based on TIMES rose sharply. One development started in 2000 and achieved by 2005 was the creation of the first world multi-regional TIMES model (Loulou, 2007) and the simultaneous creation of a Climate Module (chapter 7). Together, these two realizations allowed ETSAP to participate in the Stanford Energy Modeling Forum (EMF, https://emf.stanford.edu/) and conduct global climate change analyses alongside other modellers who were mostly using general equilibrium models. Following these developments, several ETSAP teams created multiple versions of global TIMES models.

At the same time other major new features were implemented, some of them found in MARKAL though often further advanced in TIMES, such as the Endogenous Technological Learning feature (chapter 11), the lumpy investment feature (chapter 10), both of which required the use of mixed integer programming, and the multi-stage Stochastic Programming option (chapter 8) allowing users to simulate uncertain scenarios. A particularly challenging development was to enable the computation of general equilibria in a multi-regional setting, since doing so required a methodology beyond simple optimization (chapter 12).

Increasingly as TIMES benefitted from many enhancements and gained prominence in the community of modellers, and while some features found their way into the MARKAL model, in order to provide similar capabilities to the large existing MARKAL user base, ETSAP decided that there would be no further development of MARKAL though support would continue to be provided to the existing users. By the early 2010's, TIMES (and MARKAL) models were recognized as major contributors within the community of energy and climate change researchers, and the number of outreach projects increased tremendously. Today it is estimated that MARKAL/TIMES has been introduced to well over 300 institutions in more than 80 countries, and is generally considered the benchmark integrated energy system optimization platform available for use around the world.

13.2 ***A comparison of the TIMES and MARKAL models***

This section contains a point-by-point comparison of highlights of the TIMES and MARKAL models. It is of interest primarily to modelers already familiar with MARKAL, and to provide a sense of the advancements embodied in TIMES. The descriptions of the features given below are not detailed, since they are repeated elsewhere in the documentation of both models. Rather, the function of this section is to guide the reader, by mentioning the features that are present or improved in one model that are not found or only in a simplified form in the other.

13.2.1 Similarities

The TIMES and the MARKAL models share the same basic modeling paradigm. Both models are technology explicit, dynamic partial equilibrium models of energy markets[45]. In both cases the equilibrium is obtained by maximizing the total surplus of consumers and suppliers via Linear Programming, while minimizing total discounted energy system cost. Both models are by default clairvoyant, that is, they optimize over the entire modeling horizon, though partial look-ahead (or myopic) may also be employed. The two models also share the multi-regional feature, which allows the modeler to construct geographically integrated (even global) instances, though in MARKAL there are no inter-regional exchange process making the representation of trade (much) more cumbersome. These fundamental features were described in Chapter 3 of this documentation, and Section 1.3, PART I of the MARKAL documentation, and constitute the backbone of the common paradigm. However, there are also significant differences in the two models, which we now outline. These differences do not affect the basic paradigm common to the two models, but rather some of their technical features and properties.

13.2.2 TIMES features not in MARKAL

13.2.2.1 Variable length time periods

MARKAL has fixed length time periods, whereas TIMES allows the user to define period lengths in a completely flexible way. This is a major model difference, which indeed required a complete re-definition of the mathematics of most TIMES constraints and of the TIMES objective function. The variable period length feature is very useful in

[45] But recall that some extensions depart from the classical equilibrium properties, see chapters 8-12.

two instances: first if the user wishes to use a single year as initial period (quite useful for calibration purposes), and second when the user contemplates long horizons, where the first few periods may be described in some detail by relatively short periods (say 5 years), while the longer term may be regrouped into a few periods with long durations (perhaps 20 or more years).

13.2.2.2 Data decoupling

This somewhat misunderstood feature does not confer additional power to TIMES, but it greatly simplifies the maintenance of the model database and allows the user great flexibility in modifying the new definition of the planning horizon. In TIMES all input data are specified by the user independently from the definition of the time periods employed for a particular model run. All time-dependent input data are specified by the year in which the data applies. The model then takes care of matching the data with the periods, wherever required. If necessary the data is interpolated (or extrapolated) by the model preprocessor code to provide data points at those time periods required for the current model run. In addition, the user has control over the interpolation and extrapolation of each time series.

The general rule of data decoupling applies also to past data: whereas in MARKAL the user had to provide the residual capacity profiles for all existing technologies in the initial period, and over the periods in which the capacity remains available, in TIMES the user may provide technical and cost data at those past years when the investments actually took place, and the model takes care of calculating how much capacity remains in the various modeling periods. Thus, past and future data are treated essentially in the same manner in TIMES.

One instance when the data decoupling feature immensely simplifies model management is when the user wishes to change the initial period, and/or the lengths of the periods. In TIMES, there is essentially nothing to do, except declaring the dates of the new periods. In MARKAL, such a change represents a much larger effort requiring a substantive revision of the database.

13.2.2.3 Flexible time slices and storage processes

In MARKAL, only two commodities have time-slices: electricity and low temperature heat, with electricity having seasonal and day/night time-slices, and heat having seasonal time-slices. In TIMES, any commodity and process may have its own, user-chosen time-slices. These flexible time-slices are segregated into three groups, seasonal (or monthly), weekly (weekday vs. weekend), and daily (day/night or hourly), where any level may be expanded (contracted) or omitted.

The flexible nature of the TIMES time-slices supports storage processes that ’consume’ commodities at one time-slice and release them at another. MARKAL only supports night-to-day (electricity) storage.

Note that many TIMES parameters may be time-slice dependent (such as availability factor (AF), basic efficiency (ACT_EFF), etc.

13.2.2.4 Process generality

In MARKAL processes in different RES sectors are endowed with different (data and mathematical) properties. For instance, end-use processes do not have activity variables (activity is then equated to capacity), and resource processes have no investment variables. In TIMES, all processes have the same basic features, which are activated or not solely via data specification, with some additional special features relevant to trade and storage processes.

13.2.2.5 Flexible processes

In MARKAL processes are by definition rigid, except for some specialized processes which permit flexible output (such as limit refineries or pass-out turbine CHPs), and thus outputs and inputs are in fixed proportions with one another. In TIMES, the situation is reversed, and each process starts by being entirely flexible, unless the user specifies certain attributes to rigidly link inputs to outputs. This feature permits better modeling of many real-life processes as a single technology, where MARKAL may require several technologies (as well as dummy commodities) to achieve the same result. A typical example is that of a boiler that accepts any of 3 fuels as input, but whose efficiency depends on the fuel used. In MARKAL, to model this situation requires four processes (one per possible fuel plus one that carries the investment cost and other parameters), plus one dummy fuel representing the output of the three “blending” process. In TIMES one process is sufficient, and no dummy fuel is required. Note also that TIMES has a number of parameters that can limit the input share of each fuel, whereas in MARKAL, imposing such limits requires that several user constraints be defined.[46]

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[46]In the end the two models use equivalent mathematical expressions to represent a flexible process. However, TIMES reduces the user’s effort to a minimum, while MARKAL requires the user to manually define the multiple processes, dummy fuels and user constraints.

13.2.2.6 Investment and dismantling lead-times and costs

New TIMES parameters allow the user to model the construction phase and dismantling of facilities that have reached their end-of-life. These are: lead times attached to the construction or to the dismantling of facilities, capital costs for dismantling, and surveillance costs during dismantling. Like in MARKAL, there is also the possibility to define flows of commodities consumed at construction time, or released at dismantling times, thus allowing the representation of life-cycle energy and emission accounting.

13.2.2.7 Vintaged processes and age-dependent parameters

The variables associated with user declared vintaged processes employ both the time period *p* and vintage period *v* (in which new investments are made and associated input data is obtained). The user indicates that a process is to be modeled as a vintaged process by using a special vintage parameter. Note that in MARKAL vintaging is possible only for end-use devices (for which there is no activity variable) and only applies to the device efficiency (and investment cost, which is always vintaged by definition for all technologies) or via the definition of several replicas of a process, each replica being a different vintage. In TIMES, the same process name is used for all vintages of the same process.[47]

In addition, some parameters may be specified to have different values according to the age of the process. In the current version of TIMES, these parameters include the availability factors, the in/out flow ratios (equivalent to efficiencies), and the fixed cost parameters only. Several other parameters could, in principle, be defined to be age-dependent, but such extensions have not been implemented yet.

13.2.2.8 Commodity related variables

MARKAL has very few commodity related variables, namely exports/imports, and emissions. TIMES has a large number of commodity-related variables such as: total production, total consumption, but also (and most importantly) specific variables representing the flows of commodities entering or exiting each process. These variables

[47]The representation of vintage as a separate index helps eliminate a common confusion that existed in MARKAL, namely the confusion of *vintage* with the *age* of a process. For instance, if the user defines in MARKAL an annual O&M cost for a car, equal to 10 in 2005 and only 8 in 2010, the decrease would not only apply to cars purchased in 2010, but also to cars purchased in 2005 and earlier when they reach the 2010 period.

provide the user with many “handles” to define bounds and costs on commodity flows, and foster easier setup of user constraints looking to impose shares across technology groups (e.g., renewable electricity generation targets, maximum share of demand that can be met by a (set of) devices).

13.2.2.9 More accurate and realistic depiction of investment cost payments

In MARKAL each investment is assumed to be paid in its entirety at the beginning of thetime period in which it becomes available. In TIMES the timing of investment payments is quite detailed. For large facilities (e.g. a nuclear plant), capital is progressively laid out in yearly increments over the facility’s construction time, and furthermore, the payment of each increment is made in installments spread over the economic life (which may differ from the technical lifetime) of a facility. For small processes (e.g. a car) the capacity expansion is assumed to occur regularly each year rather than in one large lump, and the payments are therefore also spread over time. Furthermore, when a time period is quite long (i.e. longer that the life of the investment), TIMES has an automatic mechanism to repeat the investment more than once over the period. These features allow for a much smoother (and more realistic) representation of the stream of capital outlays in TIMES than in MARKAL.

Moreover, in TIMES all discount rates can be defined to be time-dependent, whereas in MARKAL both the general and technology-specific discount rates are constant over time.

13.2.2.10 Stochastic Programming

Both MARKAL and TIMES support stochastic programming (SP, Chapter 8) as a means for examining uncertainty and formulating hedging strategies to deal with same. In MARKAL only 2-stage SP was implemented, and thus the resolution of the uncertainty could only occur at one particular time period, whereas in TIMES uncertainty may be resolved progressively at different successive periods (e.g., mitigation level at one period and demand level at another).

13.2.2.11 Climate module

TIMES possesses a set of variables and equations that endogenize the concentration of CO_2, CH4, and N2O, and also calculate the radiative forcing and global temperature changes resulting from GHG emissions and accumulation here. This new feature is described in Chapter 7.

14 Appendix B: Linear Programming complements

This section is not strictly needed for a basic understanding of the TIMES model and may be skipped a first reading. However, it provides additional insight into the microeconomics of the TIMES equilibrium. In particular, it contains a review of the theoretical foundation of Linear Programming and Duality Theory. This knowledge may help the user to better understand the central role shadow prices and reduced costs play in the economics of the TIMES model. More complete treatments of Linear Programming and Duality Theory may be found in several standard textbooks such as Chvátal (1983) or Hillier and Lieberman (1990 and subsequent editions). Samuelson and Nordhaus (1977) contains a treatment of micro-economics based on mathematical programming.

14.1 *A brief primer on Linear Programming and Duality Theory*

14.1.1 Basic definitions

In this subsection, the superscript *t* following a vector or matrix represents the transpose of that vector or matrix. A Linear Program may always be represented as the following *Primal Problem* in canonical form:

	$Max\ c^t x$	(14-1)
s.t.	$Ax \le b$	(14-2)
	$x \ge 0$	(14-3)

where x is a vector of *decision variables*, $c^t x$ is a linear function representing the *objective* to maximize, and $Ax \le b$ is a set of inequality *constraints*. Assume that the LP has a finite optimal solution, x^*.

Then each decision variable, x^*_j falls into one of three categories. x^*_j may be:

- equal to its lower bound (as defined in a constraint), or
- equal to its upper bound, or
- strictly between the two bounds.

In the last case, the variable x^*_j is called *basic*. Otherwise it is *non-basic*.

For each primal problem, there corresponds a *Dual problem* derived as follows:

$$Min\ \ b^t y \qquad (14\text{-}4)$$

$$\text{s.t.} \quad A^t y \geq c \qquad (14\text{-}5)$$

$$y \geq 0 \qquad (14\text{-}6)$$

Note that the number of dual variables equals the number of constraints in the primal problem. In fact, each dual variable y_i may be assigned to its corresponding primal constraint, which we represent as: $A_i x \leq b_i$, where A_i is the i^{th} row of matrix A.

14.1.2 Duality Theory

Duality theory consists mainly of three theorems[48]: weak duality, strong duality, and complementary slackness.

Weak Duality Theorem
If x is any feasible solution to the primal problem and y is any feasible solution to the dual, then the following inequality holds:

$$c^t x \leq b^t y \qquad (14\text{-}7)$$

The weak duality theorem states that the value of a feasible dual objective is never smaller than the value of a feasible primal objective. The difference between the two is called the *duality gap* for the pair of feasible primal and dual solutions (x,y).

Strong duality theorem
If the primal problem has a *finite, optimal* solution x^*, then so does the dual problem (y^*), and both problems have the same optimal objective value (their duality gap is zero):

$$c^t x^* = b^t y^* \qquad (14\text{-}8)$$

Note that the optimal values of the dual variables are also called the *shadow prices* of the primal constraints.

[48] Their proofs may be found in the textbooks on Linear Programming already referenced.

Complementary Slackness theorem

At an optimal solution to an LP problem:

- If y^*_i is > 0 then the corresponding primal constraint is satisfied at equality (i.e. $A_i x^* = b_i$ and the i^{th} primal constraint is called *tight*. Conversely, if the i^{th} primal constraint is *slack* (not tight), then $y^*_i = 0$,
- If x^*_j is basic, then the corresponding dual constraint is satisfied at equality, (i.e. $A^t_j{}^* y = c_j$, where A^t_j is the j^{th} row of A^t, i.e. the j^{th} column of A. Conversely, if the j^{th} dual constraint is slack, then x^*_j is equal to one of its bounds.

Remark: Note however that a primal constraint may have zero slack and yet have a dual equal to 0. And, a primal variable may be non basic (i.e. be equal to one of its bounds), and yet the corresponding dual slack be still equal to 0. These situations are different cases of the so-called degeneracy of the LP. They often occur when constraints are over specified (a trivial case occurs if a constraint is repeated twice in the LP)

14.2 ***Sensitivity analysis and the economic interpretation of dual variables***

It may be shown that if the j^{th} RHS b_j of the primal is changed by an infinitesimal amount d, and if the primal LP is solved again, then its new optimal objective value is equal to the old optimal value plus the quantity $y_j^* \bullet d$, where y_j^* is the optimal dual variable value.

Loosely speaking[49], one may say that the partial derivative of the optimal primal objective function's value with respect to the RHS of the i^{th} primal constraint is equal to the optimal shadow price of that constraint.

14.2.1 Economic interpretation of the dual variables

If the primal problem consists of maximizing the surplus (objective function $c^t x$), by choosing an activity vector x, subject to upper limits on several resources (the b vector) then:

- Each a_{ij} coefficient of the dual problem matrix, A, then represents the consumption of resource b_j by activity x_i;
- The optimal dual variable value y^*_j is the unit price of resource j, and

[49] Strictly speaking, the partial derivative may not exist for some values of the RHS, and may then be replaced by a directional derivative (see Rockafellar 1970).

- The total optimal surplus derived from the optimal activity vector, x^*, is equal to the total value of all resources, b, priced at the optimal dual values y^* (strong duality theorem).

Furthermore, each dual constraint $A^t_j{}^*y \geq c_j$ has an important economic interpretation. Based on the Complementary Slackness theorem, if an LP solution x^* is optimal, then for each x^*_j that is not equal to its upper or lower bound (i.e. each basic variable x^*_j), there corresponds a *tight* dual constraint $y^*A'_j = c_j$, which means that the revenue coefficient c_j must be exactly equal to the cost of purchasing the resources needed to produce one unit of $x_{j.}$ In economists' terms, *marginal cost equals marginal revenue, and both are equal to the market price of* $\boldsymbol{x^*_j}$. If a variable is not basic, then by definition it is equal to its lower bound or to its upper bound. In both cases, the unit revenue c_j need not be equal to the cost of the required resources. The technology is then either non-competitive (if it is at its lower bound) or it is super competitive and makes a surplus (if it is at its upper bound).

Example: The optimal dual value attached to the balance constraint of commodity c represents the change in objective function value resulting from one additional unit of the commodity. This is precisely the internal unit price of that commodity.

14.2.2 Reduced surplus and reduced cost

In a maximization problem, the difference $y^*A'_j - c_j$ is called the *reduced surplus* of technology j, and is available from the solution of a TIMES problem. It is a useful indicator of the competitiveness of a technology, as follows:

- If x^*_j is at its lower bound, its unit revenue c_j is *less* than the resource cost (i.e. its reduced surplus is positive). The technology is not competitive (and stays at its lower bound in the equilibrium);
- If x^*_j is at its upper bound, revenue c_j is *larger* than the cost of resources (i.e. its reduced surplus is negative). The technology is super competitive and produces a surplus; and
- If x^*_j is basic, its reduced surplus is equal to 0. The technology is competitive but does not produce a surplus.

We now restate the above summary in the case of a Linear Program that minimizes cost subject to constraints:

$$Min\ c^t x$$

$$\text{s.t.} \quad Ax \geq b$$

$$x \geq 0$$

In a minimization problem (such as the usual formulation of TIMES), the difference c_j - $y{*}A'_j$ is called the *reduced cost* of technology *j*. The following holds:

- If $x{*}_j$ is at its lower bound, its unit cost c_j is *larger* than the value created (i.e. its reduced cost is positive). The technology is not competitive (and stays at its lower bound in the equilibrium);
- if $x{*}_j$ is at its upper bound, its cost c_j is *less* than the value created (i.e. its reduced cost is negative). The technology is super competitive and produces a profit; and
- if $x{*}_j$ is basic, its reduced cost is equal to 0. The technology is competitive but does not produce a profit

The reduced costs/surpluses may thus be used to rank all technologies, *including those that are not selected by the model.*

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