SOLUTION FOR HOMEWORK ASSIGNMENT NO. 04

Nils Hoyer, Maurice Morgenthaler

Exercise 4.1

We are asked to write our own pseudo random number generator. For this we will use the so called *Blum Blum Shub* generator which uses the equation

$$n_{i+1} = n_i^2 \,\%\, (p \cdot q) \tag{1}$$

to generate new numbers. p and q are large prime numbers.

To generate numbers between zero and one one has to divide by $p \cdot q$.

$$r_i = \frac{n_i}{p \cdot q} \tag{2}$$

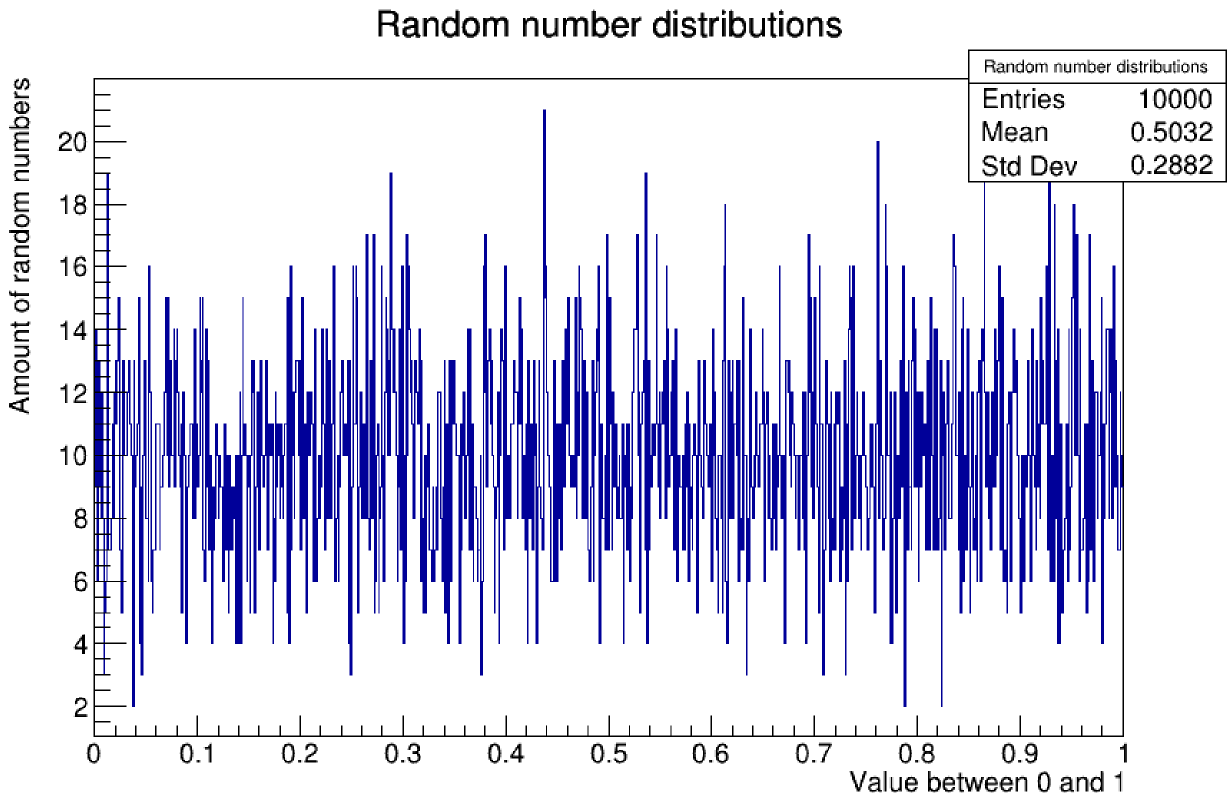
Please find the code in file `exercise4_1.C`. The 20th 'random' numbers of consecitive seeds are listed in table 1. Just by only looking at the twenty numbers below I can not see any correlation.

Table 1: The last twenty random numbers are listed given twenty consecutive seeds starting at 234 509 143.

seed i	**random number** r_i
234 509 143	0.389 45
234 509 144	0.052 98
234 509 145	0.555 13
234 509 146	0.310 14
234 509 147	0.828 52
234 509 148	0.539 09
234 509 149	0.419 62
234 509 150	0.100 40
234 509 151	0.153 32
234 509 152	0.457 29
234 509 153	0.530 53
234 509 154	0.474 94
234 509 155	0.014 97
234 509 156	0.325 98
234 509 157	0.905 85
234 509 158	0.014 88
234 509 159	0.683 29
234 509 160	0.253 34
234 509 161	0.463 13
234 509 162	0.903 14

Next we used the first seed to generate 10 000 random numbers. We filled a histogram with them which you can see in figure 1.

Figure 1: 10 000 random number generated by our own random number generator. The seed which has been used is 234 509 143. We used 200 bins for plotting in the range between zero and one.
Note that the rather large differences in amounts of numbers persists even to higher total numbers of generated 'random' numbers (e.g. 1 000 000).

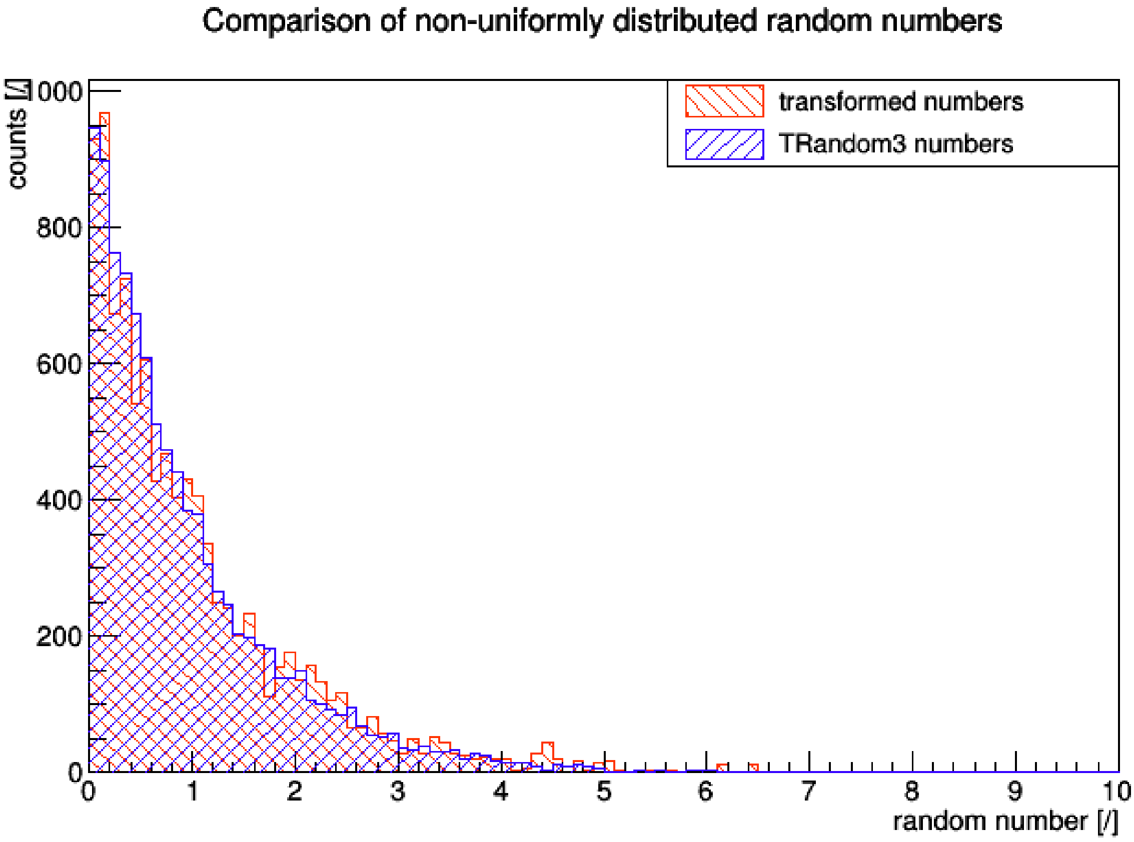


Exercise 4.2

We are asked to generate random numbers according to a specific PDF, in this case an exponential function $f(x) = e^x$. In figure 2 the output of our implementation as well as the ROOT function is given.

As you can see there is no significant difference between both functions.

Figure 2: Comparison between our own implementation and the equivalent ROOT function for 10 000 random variables. You can see no significant difference between both functions.

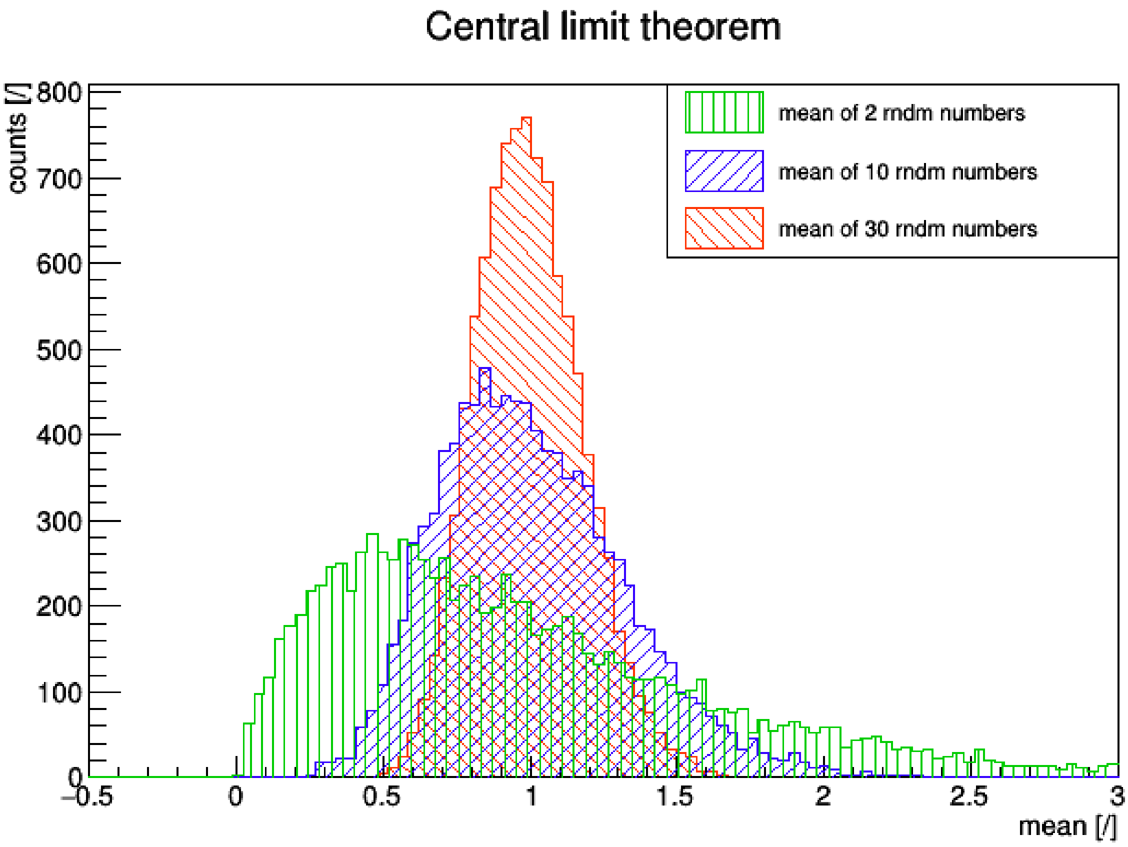


Exercise 4.3

We are asked to numerically verify the *central limit theorem*. To do this we use the exponential function and compute 10 000 averages of of k random numbers. The result can be found in figure 3.

You can see that increasing the total number of random numbers indeed leads to shape similar to that of a normal distribution.

Figure 3: Comparison between three exponential distributions with k random numbers using 10 000 averages. You can see the distribution tends towards a normal distribution with an increasing number of random variables k.



Exercise 4.4

a) We are asked to analytically calulcate the percentage of photons hitting our detector assuming an isotropic distribution of particles.

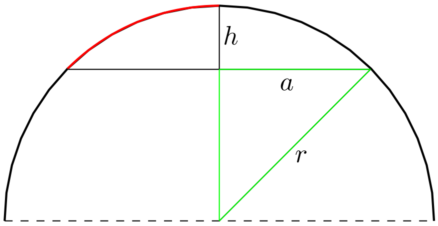
The percentage is the ratio of the surface of the detector divided by the surface of a three dimensional sphere, i.e.

$$p = \frac{O_{\text{segment}}}{O_{\text{sphere}}} \tag{3}$$

To be able to find p we need to find r as only $r - h$ is given. From 4 you can see that

$$(r-h)^2 + a^2 = r^2.$$

Figure 4: This figure illustrates the setup of the detector and the projection of the two dimensional surface of the sphere. We consider the green triangle to obtain the radius r as only $r - h$ is given in the exercise.



The segment can easily be caluclated by switching to spherical coordinates. You integrate the circular segment which the detector covers (indicate dby the red line in figure 4) over ϕ.

After the integration you get

$$
\begin{aligned}
O_{\text{segment}} &= 2\pi \cdot r^2 \cdot (1 - \cos(\theta_0)) \\
&= 2\pi \cdot \left(a^2 + (r-h)^2\right) \cdot \left(1 - \cos\left(\arctan\left(\frac{a}{r-h}\right)\right)\right) \\
&= 2\pi \cdot \left(a^2 + (r-h)^2\right) \cdot \left(1 - \frac{1}{\sqrt{\left(\frac{a}{r-h}\right)^2 + 1}}\right)
\end{aligned}
$$

This results in

$$
p = \frac{O_{\text{segment}}}{O_{\text{sphere}}} = \frac{2\pi \cdot \left(a^2 + (r-h)^2\right) \cdot \left(1 - \frac{1}{\sqrt{\left(\frac{a}{r-h}\right)^2+1}}\right)}{4\pi \cdot (a^2 + (r-h)^2)} = \frac{1}{2} - \frac{1}{2\sqrt{\left(\frac{a}{r-h}\right)^2 + 1}} \tag{4}
$$

Using $a = 0.02\,\mathrm{m}$ and $r - h = 0.10\,\mathrm{m}$ yields $p = 0.971\,\%$.

b) Please find the code in the file `exercise4_4.C`.

The results we obtain lie around $1.25\,\%$. This does not match the calculated value from the previous part.