SOLUTION FOR HOMEWORK ASSIGNMENT NO. 02

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Exercise 2.1

We are given a function f which is defined as

$$f(x) = \begin{cases} 0 & \text{for } a \le x \le b \\ \frac{2(x-a)}{(b-a)(c-a)} & \text{for } a \le x < c \\ \frac{2(b-x)}{(b-a)(b-c)} & \text{for } c \le x < b \end{cases} \tag{1}$$

and are asked to calculate its mean, mode, median and variance. We will consider the case where $c = 0$.

Mean μ: $\mu = E[x] = \int\limits_{-\infty}^{\infty} dx \cdot xf(x)$
Median M: $\frac{1}{2} = \int\limits_{-\infty}^{M} dx \cdot f(x)$
Mode m: $\frac{\partial f}{\partial x}(x)|_{x=m} = 0$
Variance σ^2: $\sigma^2 = E[(x - E[x])^2] = \int\limits_{-\infty}^{\infty} dx \cdot (x-\mu)^2 f(x)$

a) For $a = -b$ the function $f(x)$ is defined as

$$f(x) = \begin{cases} 0 & \text{for } x \le -b \text{ and } x \ge b \\ \frac{x+b}{b^2} & \text{for } -b \le x \le 0 \\ \frac{b-x}{b^2} & \text{for } 0 \le x \le b \end{cases}$$

Mean: Since $f(x)$ and x are odd functions their product is an even function. Since we integrate over a symmetric interval the mean is zero.

Median: Because of symmerty we know that $M = 0$.

Mode: Again, due to symmetry and because of the fact that the function is rising for $x \le 0$ and falling for $x \ge 0$, the mode is zero.

Variance:

$$\sigma^2 = \int_{-\infty}^{\infty} dx \cdot (x-\mu)^2 f(x) = \int_{-b}^{0} dx \cdot x^2 \frac{x+b}{b^2} + \int_{0}^{b} dx \cdot x^2 \frac{b-x}{b^2}$$
$$= \frac{1}{b^2}\left[-\frac{b^4}{4} + \frac{b^4}{3} + \frac{b^4}{3} - \frac{b^4}{4}\right] = \frac{b^2}{6}$$

b) For $a = -2b$ the function $f(x)$ is defined as

$$f(x) = \begin{cases} 0 & \text{for } x \leq -2b \text{ and } x \geq b \\ \frac{x+b}{3b^2} & \text{for } -2b \leq x \leq 0 \\ \frac{2}{3}\frac{b-x}{b^2} & \text{for } 0 \leq x \leq b \end{cases}$$

Mean:

$$\mu = \int_{-\infty}^{\infty} dx \cdot x f(x) = \int_{-2b}^{0} dx \cdot x \frac{x+2b}{3b^2} + \int_{0}^{b} dx \cdot x \frac{2}{3}\frac{b-x}{b^2}$$
$$= \frac{1}{3b^2}\left[\frac{8b^3}{3} - 4b^3 + b^3 - \frac{2b^3}{3}\right] = -\frac{b}{3}$$

Median: I suspect that the median lies between $x = -2b$ and $x = 0$.

$$\frac{1}{2} = \int_{-\infty}^{M} dx \cdot f(x) = \int_{-2b}^{M} dx \cdot \frac{x+2b}{3b^2}$$
$$= \frac{1}{3b^2}\left[\frac{M^2}{2} + 2bM + 2b^2\right]$$
$$\Rightarrow M_{1,2} = -2b \pm \sqrt{3}b$$

The only reasonable solution is $M = (-2+\sqrt{3})b$.

Mode: Same argument as before: Since f_1 (function defined for $x \leq 0$) is rising and f_2 is falling the mode lies at zero.

Variance:

$$\sigma^2 = \int_{-\infty}^{\infty} dx \cdot (x-\mu)^2 f(x) = \int_{-2b}^{0} dx \cdot \left(x + \frac{b}{3}\right)^2 \frac{x+b}{3b^2} + \int_{0}^{b} dx \cdot \left(x + \frac{b}{3}\right)^2 \frac{2(b-x)}{3b^2}$$

$$= \ldots$$

$$= \frac{7}{18} b^2$$

Exercise 2.2

The solution for this quesstion can be found in a file named `exercise2_2.C`.

Exercise 2.3

You can find the solution for this exercise in a file named `exercise2_3.C`. The resulting plots will be displayed here and attatched to the email.

Figure 1: This plot summarizes the first two tasks given in this exercise. On the top left corner is the two dimensional histogram of the distribution of the pion energies. The other two histograms show the projection on each axis.

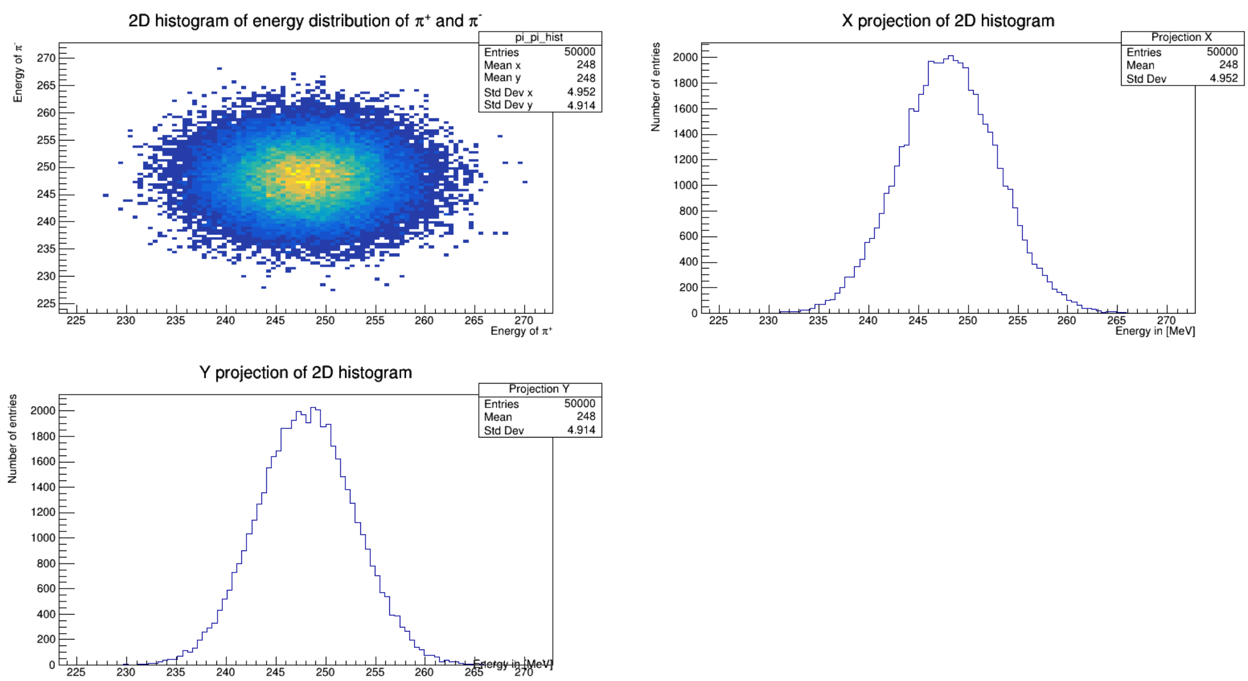

Figure 2: This plot summarizes the last task given in this exercise. The top two histograms show all entries which have a are in bin 50 and 70, respectively. The bottom histogram shows both individual histograms combined in one graph.