SOLUTION FOR HOMEWORK ASSIGNMENT NO. 12

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Exercise 12.1

In this exercise we had to calculate the upper limit of the Ω-baryon lifetime.
Our given starting points were:

- Estimator: observed decay time $\tau_0 = 0\,7 \cdot 10^{-10}\mathrm{s}$
- Reference sample:
  observations of an exponential decay:

$$f_\tau(\tau_0) = p(\tau_0|\tau) = \frac{1}{\tau} \exp\left(-\frac{\tau_0}{\tau}\right) \tag{1}$$

  Cumulative version:

$$F_\tau(\tau) = 1 - \exp\left(-\frac{\tau_0}{\tau}\right) \tag{2}$$

- Ordering rule: upper limit $F_{\tau_{up}} = \alpha$
- Confidence level: $1 - \alpha = 0.68$

Step 1: The acceptance region is defined for a chosen τ by all τ_0 full filling the condition:

$$F_\tau(\tau_0) = 1 - \exp\left(-\frac{\tau_0}{\tau}\right) \geq \alpha => \tau_0 \geq -\tau \log(1-\alpha) \tag{3}$$

Plotting the acceptance regions of figure 1 into figure 2 with τ plotted against τ_0 results in a red confident belt. The intersection between the blue estimator of tau_0 gives us the upper limit of τ (green). Therefore the confidence interval is given by $\tau \leq 1.815 \cdot 10^{-10}\mathrm{s}$.

Exercise 12.2

In the second exercise we were given a scenario where following binomial statistics k=3 out of n=10 electronic chips fail a functional test.

a) We should calculate equal-tailed 95% Cl intervals for the single-chip fail probability ϵ using the Clopper-Pearson construction.

It says that in such a scenario the upper limit p^u is given by $P(X \leq k; p^u) = 1 - \alpha/2 = 0.025$ while the lower limit p^l is given by $P(X \geq k; p^l) = 1 - \alpha/2 = 0.025$.

For the construction it is needed to know that there is a relation between the binomial distribution and the quantiles of the beta distribution $\beta_\gamma(a;b)$ where a,b are the shape parameters.

$$\sum_{i=0}^{k} \binom{n}{i} \cdot p^i \cdot (1-p)^{(n-i)} = \beta_{1-p}(n-k; k+1) \tag{4}$$

Using the relation of we get for the limits:

$$p^l = \beta_{\alpha/2}(k; n-(k-1)) = 0.0667 \tag{5}$$

$$p^u = \beta_{1-\alpha/2}(k+1; n-k) = 0.6524 \tag{6}$$

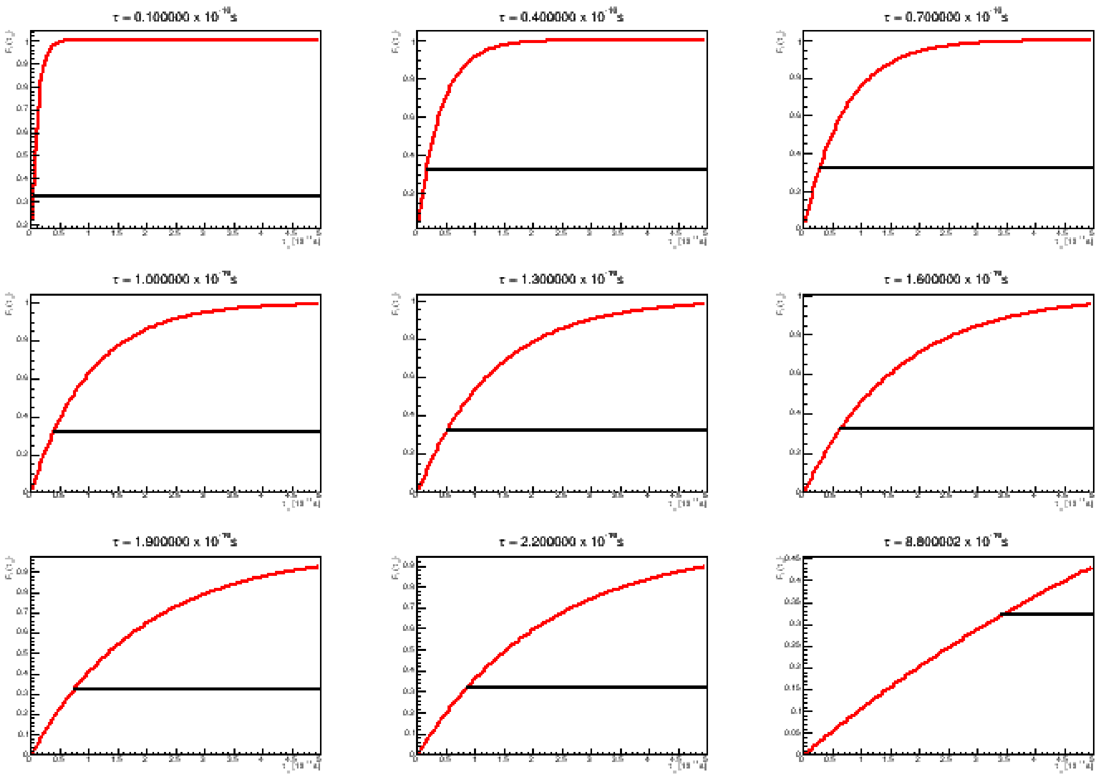

Figure 1: Visualization of the acceptance region for some τ. The red line is the cumulative exponential function while the black line represents the acceptance region

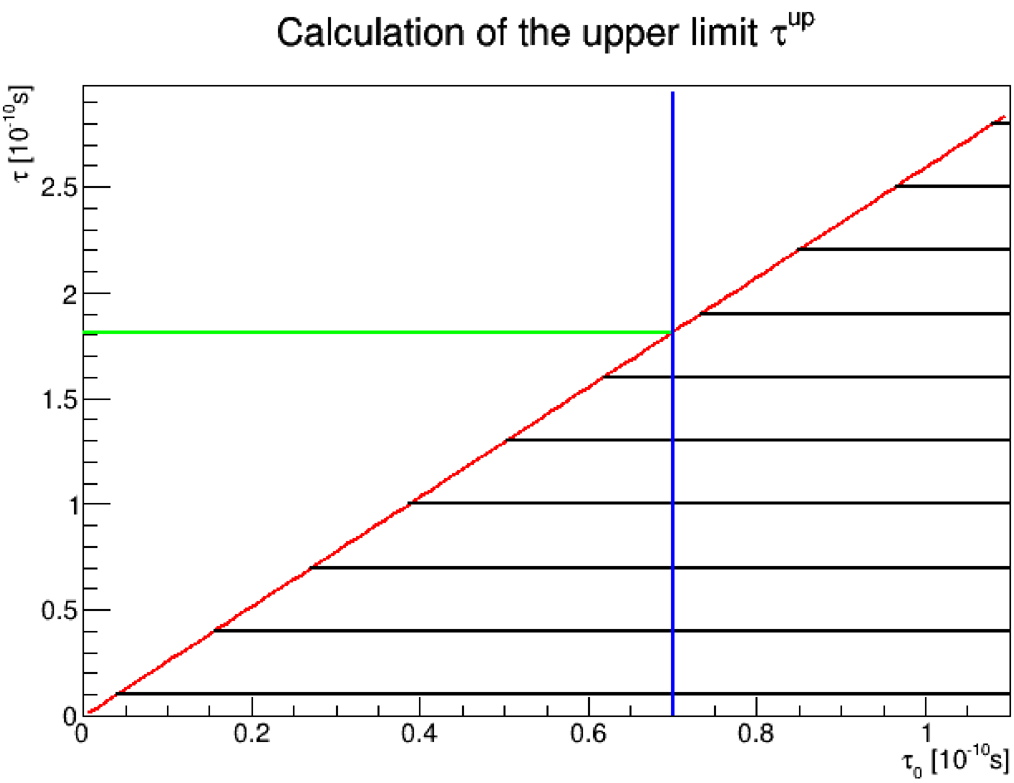

Figure 2: Visualization of confidence interval. The red line shows the confidence belt created from the black acceptance regions. The blue line is the estimator while the green one gives us the value of the upper limit.