Solution for homework assignment no. 06

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Exercise 6.1

We are asked to derive the extenden likelihood function for the binned case.

We start by looking at the content in each bin, i.e.

$$n_b(\theta) = n_{\text{tot}} \int\limits_{x_0+h(b-1)}^{x_0+hb} \mathrm{d}x \cdot f(x \,|\, \theta) \tag{1}$$

where h is the width of the bin and b the bin number.
For a Poissonian distribtion we then get

$$f(m, n(\theta)) = \frac{n_{tot}^{m} e^{-n_{tot}}}{m_{tot}!} \cdot \frac{m_{tot}!}{m_1! \ldots m_N!} \left(\frac{n_1}{n_{tot}}\right)^{m_1} \ldots \left(\frac{n_N}{n_{tot}}\right)^{m_N} = \prod_{i=1}^{N} \frac{n_i^{m_i}}{m_i!} e^{-n_i} \tag{2}$$

Therefore

$$\mathcal{L} = \ln\left(\sum_{i=1}^{N} f(m, n(\theta))\right) = \sum_{i=1}^{N} m_i \cdot \ln(n_i) - \ln(m_i\,!) - n_i \approx \sum_{i=1}^{N} m_i \cdot (\ln(n_i) - \ln(m_i)) - (m_i + n_i) \tag{3}$$

where I used Stirling's formula.

Exercise 6.2

Given a PDF

$$f(t \,|\, \tau) = \frac{e^{-t/\tau}}{\tau} \tag{4}$$

we are asked to generate 100 pseudo events for $\tau_{\text{true}} = 1.0\,\text{s}$. Using the unbinned maximum likelihood function we are asked to find the best fit value for τ by computing a local minimum using Minuit. Afterwards we should repeat this process for the binned case with $\Delta t = 0.5\,\text{s}$ where

$$\nu_i(\tau) = n_{\text{tot}} \int_{t_i^{\min}}^{t_i^{\max}} \mathrm{d}t \cdot f(t \,|\, \tau). \tag{5}$$

Eventually we shall use different binning values and see what happens if $\Delta t \to 0$ and $\Delta t \to \infty$.

The resulting plot for the randomly generated values of the exponential function and the overlying function with the fitted value for τ is shown in figure 1.

Figure 1: The plot shows the distribution of randomly generated values x following a exponential distribution with $\tau = 1.0$. The overlying function shows the exponential function together with τ given by Minuit.