

## SOLUTION FOR HOMEWORK ASSIGNMENT NO. 03

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### Exercise 3.1

We are asked about the probability density function (PDF) of  $f(z)$  where  $z = x \times y$ ,  $x$  and  $y$  are independent variables and  $h(x)$  and  $g(y)$  are their PDF's, respectively.

Results to be added.

### Exercise 3.2

We are asked to determine what the expectation value of  $E[z]$  is where  $z = z(x)$  and

$$E[x] = \int_{-\infty}^{\infty} dx \cdot x f(x). \quad (1)$$

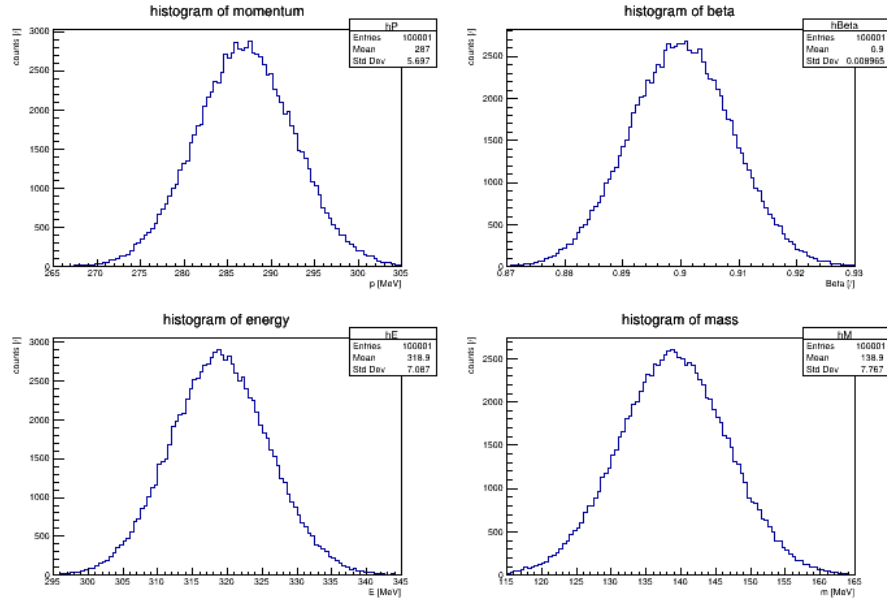
Results to be added.

### Exercise 3.3

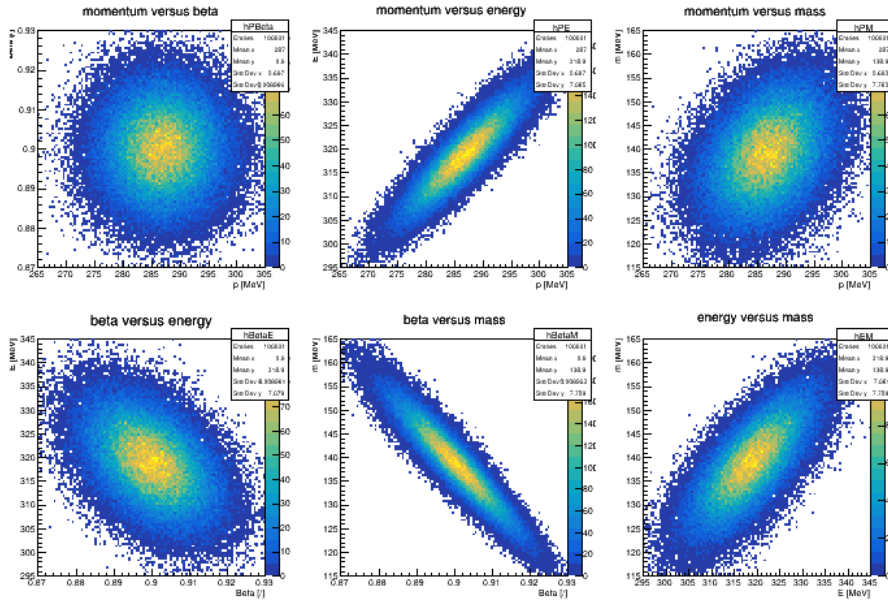
Please note that we will only include graphs and solutions here but not the code itself as it would take up a few pages.

- a) We are asked to read the .txt-file and plot each variable into a separate histogram as well as creating two-dimensional histograms to visualize potential correlations. Please find the plots in figure 1 and 2.

**Figure 1:** Histograms of the momentum  $p$ , energy  $E$ , normed velocity  $\beta$  and mass  $m$ .



**Figure 2:** Histograms of potential correlations between the momentum  $p$ , energy  $E$ , normed velocity  $\beta$  and mass  $m$ .



b) We are asked to explicitly determine the mean and variance for each variable as well as obtaining the covariances for each set of pairs of variables. The results from the explicit determination as well as the results given by functions from ROOT are given in table 1 which you can find below.

**Table 1:** Results obtained for each variable and pair of variables.  
Results to be added.

Set of variables	Mean	Variance	Covariance
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	Code	ROOT	Code	ROOT	Code	ROOT
$p$	0.0	0.0	0.0	0.0	—	—
$E$	0.0	0.0	0.0	0.0	—	—
$\beta$	0.0	0.0	0.0	0.0	—	—
$m$	0.0	0.0	0.0	0.0	—	—
$p \times E$	—	—	—	—	0.0	0.0
$p \times \beta$	—	—	—	—	0.0	0.0
$p \times m$	—	—	—	—	0.0	0.0
$E \times \beta$	—	—	—	—	0.0	0.0
$E \times m$	—	—	—	—	0.0	0.0
$\beta \times m$	—	—	—	—	0.0	0.0

- c) We are asked to determine and explain the correlation coefficients between the pairs of variables. Please find the results for the coefficients in table 2.

**Table 2:** Correlation coefficients between each pair of variables.  
Results to be added.

Pair of variables	Correlation coefficient
$p \times E$	0.0
$p \times \beta$	0.0
$p \times m$	0.0
$E \times \beta$	0.0
$E \times m$	0.0
$\beta \times m$	0.0

- d) We are asked to calculate the covariance  $\text{cov}[m, E]$  analytically and compare the result to the one obtained in part a.

Results to be added.

- e) We are asked to calculate the uncertainty of  $T = E - m$  taken the correlation between  $E$  and  $m$  into account.

Results to be added.