Solution for homework assignment no. 07

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Exercise 7.1

We are asked to determine the expected standard deviation of λ where $I(\lambda)$ is the Fisher information and $\hat{\lambda}$ an efficient operator. The decay time distributions for $\mathcal{B}^0 \to J/\Psi K_s^0$ and $\overline{\mathcal{B}^0} \to J/\Psi K_s^0$ are

$$N_{\mathcal{B}^0 \to J/\Psi K_s^0} \propto e^{-t}\left[1 + \lambda \sin(0.7t)\right],$$
$$N_{\overline{\mathcal{B}^0} \to J/\Psi K_s^0} \propto e^{-t}\left[1 - \lambda \sin(0.7t)\right].$$

To calculate the standard deviation we shall use a value of $\lambda = 0.3$.

considering a) and b)

As $\hat{\lambda}$ is a proper operator the standard deviation of λ is

$$std(\lambda) = \sqrt{var(\lambda)} = \frac{1}{\sqrt{I(\lambda)}} \tag{1}$$

The Fisher information can be written as

$$\begin{aligned} I(\lambda) &= E\left[\frac{\delta L}{\delta \lambda}^2\right] \\ &= \int_{-\inf}^{\inf} \lambda \sum_{k=1}^{500} \left(\frac{sin(0.7t_k)}{1 \pm \lambda sin(0.7t_k)}\right)^2 d\lambda \end{aligned} \tag{2}$$

Where L are the likelihood functions

$$\begin{aligned} L &= \sum_{k=1}^{500} log\left(e^{-t}\left[1 - \lambda \sin(0.7t)\right]\right) \\ &= \sum_{k=1}^{500} -t_k + log(1 \pm \lambda sin(0.7t_k)) \end{aligned} \tag{3}$$

a) We consider 500 decays of $\mathcal{B}^0$.

b) We consider 500 decays of $\overline{\mathcal{B}^0}$.

c) We consider 250 decays of $\mathcal{B}^0$ and 250 decays of $\overline{\mathcal{B}^0}$.

d) Put very nice explanation here.

Exercise 7.2

Given electron tracks we are asked to first plot five tracks into one image. Afterwards we shall construct a theoretical model for the motion of electrons under the influence of a magnetic field. Using this model we have to fit it to the given data using a self-written χ^2 model. After plotting different values we eventually state a potential isotope from which the electrons come from.

As usual, please find the script in the text file `exercise7_2.C`.

You can find the visualisation of the tracks of the first five electrons in figure 1.

Figure 1: Movement of the first five tracks from the file `exercise07.root`. Please not that the x and z positions are switched to enable fitting.

./tracks.png

The equation we are looking for have the following expressions

$$x = -\frac{v_z m_e \gamma}{c \cdot |q| \cdot |B|} \cdot \cos\left(\frac{|q||B|}{m_e}\right), \tag{4}$$

$$y = +\frac{v_z m_e \gamma}{c \cdot |q| \cdot |B|} \cdot \sin\left(\frac{|q||B|}{m_e}\right), \tag{5}$$

where $x^2 + y^2 = \left(\frac{v_z \cdot m_e \cdot \gamma}{c \cdot |q| \cdot |B|}\right)^2$ and therefore $y = \sqrt{\left(\frac{v_z \cdot m_e \cdot \gamma}{c \cdot |q| \cdot |B|}\right)^2 - x^2}$.