Solution for homework assignment no. 08

Nils Hoyer, Maurice Morgenthaler

Exercise 8.1

Given a note on the possibility of combining results from ATLAS and CMS we have to answer the following ten questions:

i. How is the $\mathrm{CL_s}$ method used for the search of the Higgs boson?

The CL_s is used to calculate a confidence level by which the Higgs boson is excluded or included. Here CL_s is defined as

$$\mathrm{CL}_s(\mu) = \frac{p_\mu}{1 - p_b}, \tag{1}$$

where p_μ is the p-value for the *signal + background* hypothesis and p_b the p-value for the *background - only* hypothesis. As an example, if $\mu = 1.0$ and $\mathrm{CL}_s \leq \alpha$ we would exclude the Higgs boson with a $(1 - \alpha) \cdot \mathrm{CL}_s$ confidence level.

ii. What is the shape of a hypothetical Higgs boson signal?

In the paper a simple model with a Gaussian-shaped signal and flat background was considered to estimate that $1\sigma_m$ increments leads to a loss of sensitivity of $5\,\%$.

iii. How is the test statistic constructed?

The test statistic $\tilde{q}_\mu$ was constructed as

$$\tilde{q}_\mu = -2 \cdot \ln \left(\frac{\mathcal{L}\left(\mathrm{data}|\mu, \hat{\theta}_\mu\right)}{\mathcal{L}\left(\mathrm{data}|\hat{\mu}, \hat{\theta}\right)} \right), \quad \text{with} \quad 0 \leq \hat{\mu} \leq \mu. \tag{2}$$

$\mathcal{L}$ is as always the Likelihood and data refers to real observations or toy datasets. μ is a *signal strength modifier* which is applied to the SM Higgs boson cross sections. A hat over the variable signals them the be likelihood estimators. Therefore $\hat{\Theta}_\mu$ is the estimator given μ. The pair $\hat{\mu}$ and $\hat{\theta}$ are together the global maximum of the Likelihood function. $\hat{\mu}$ has to be bigger than zero as the signal is positive.

iv. How is the p-value converted to the significance?

The p-value can be converted to a significance level by using

$$p = \int_Z^\infty \mathrm{d}x \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} = \frac{1}{2} P_{\chi_1^2}\left(Z^2\right) \tag{3}$$

and adopting the convention of a 'one-sided Gaussian tail'. Here $P_{\chi_1^2}\left(Z^2\right)$ stands for the survival function of χ^2 for one degree of freedom.

v. Why is the look-elsewhere effect relevant and how was it estimated?

Since one tests for the *background - only* hypothesis man times as the mass of the Higgs boson is scanned one has to consider the look-elsewhere effect. Because one calculates the *local* p-value for fixed m_H we have to convert to the *global* variables. The p-value therefore becomes

$$p_b^{\mathrm{global}} \leq \langle N_{u_0} \rangle \cdot e^{-\frac{u - u_0}{2}} \tag{4}$$

where $\langle N_{u_0} \rangle$ is the average number of up-crossings of the likelihood ratio scan at level u_0.

Eventually this procedure will give an estimate the factor by which one needs to change the *local* p-value.

vi. Why does the analysis introduce nuisance parameters and how many of them are there for ATLAS and CMS?

Nuisance parameters are introduced such that one split sources of uncertainty to completely correlated or completely uncorrelated. Also, multiple uncertainties might be handled by only one nuisance parameter.
In the paper only one table for the nuisance parameters which are completely correlated between ATLAS and CMS is given. The table features 19 different parameters.

vii. Which shape do these nuisance parameters have?

If a parameter is unconstrained it is considered to be flat.
For systematic uncertainties, for which the values are also allowed to be negative, one uses a standard Gaussian pdf, i.e.

$$p\,(\theta) = \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{-\frac{(\theta-\tilde{\theta})^2}{2\sigma^2}}\,. \tag{5}$$

For parameters which are not allowed to be negative, e.g. the impact parameter, one uses a log-normal pdf,

$$p\,(\theta) = \frac{1}{\sqrt{2\pi}\ln(\kappa)} \cdot \frac{1}{\theta} \cdot e^{-\frac{\left(\ln\left(\frac{\theta}{\tilde{\theta}}\right)\right)^2}{2(\ln(\kappa))^2}}\,, \tag{6}$$

where κ characterises the width.
For uncertainties associated with MC events one uses the gamma distribution, i.e.

$$p(n) = \frac{\left(\frac{n}{\alpha}\right)^N}{N\,!} \cdot \frac{e^{-\frac{n}{\alpha}}}{\alpha}\,. \tag{7}$$

viii. How is the starting point of the Higgs boson mass chosen?

The choise of Higgs boson mass points is driven by the two decays

$$\begin{aligned} &H \to 2\gamma\,, \quad \text{and} \\ &H \to ZZ \to 4l. \end{aligned}$$

Depending on the model of computation it was decided to start searching at values of $110\,\mathrm{GeV}$.

ix. Explain what figures 8, 9 and 10 represent.

Put very nice answer here.

x. Explain how the likelihood of equation 20 is constructed.

The likelihood function is simply constructed by the model given in equation 2 in the paper, i.e.

$$\mathcal{L}\,(\mathrm{data}\,|\,\mu,\theta) = \mathrm{Poisson}\,(\mu \cdot s(\theta) + b(\theta)) \cdot p(\tilde{\theta}\,|\,\theta) \tag{8}$$

where $p(\tilde{\theta}\,|\,\theta)$ is the systematic error pdf.
Of course, one has to sum over all channels which leads to a product after applying the logarithm.