SOLUTION FOR HOMEWORK ASSIGNMENT NO. 11

Nils Hoyer, Maurice Morgenthaler

Exercise 11.1

a) Given a Poissonian distribution with mean ν

$$f_n(\nu) = e^{-n}\frac{n^\nu}{\nu\,!} \tag{1}$$

we are asked to list the number of observed events such that there is a $10\,\%$ chance to observe them above, below and outside of the central interval.
As usual, please find the code in file `exercise11_1a.C`. The results are given in table 1.

Exercise	ν	1	2	3	4	5	6	7	8	9	10	11	12
1	**n**	5	5	6	8	9	10	11	13	14	15	16	18
2	**n**	1	1	2	3	3	4	5	6	6	7	8	9
3	**n'**	1	1	2	2	3	3	4	5	5	6	7	8
	n	4	6	7	9	10	11	13	14	15	16	18	19

Table 1: Results obtained for a fixed mean ν.

b) Similar to the first part we are asked to calculatze the $90\,\%$ CL for ν given the total number of observed events n.
As usual, please find the code in file `exercise11_1b.C`. The results are given in table 2.

Exercise	n	0	1	2	3	4	5	6	7	8	9	10	11	12
1	ν	2.30	3.89	5.32	6.68	7.99	9.28	10.5	11.77	13.00	14.21	15.41	16.60	17.78
2	ν	0.11	0.53	1.10	1.75	2.43	3.15	3.90	4.66	5.43	6.22	7.02	7.83	8.65
3	ν'	0.05	0.36	0.82	1.37	1.97	2.61	3.29	3.98	4.70	5.43	6.17	6.93	7.69
	ν	3.00	4.74	6.30	7.75	9.15	10.51	11.84	13.15	14.44	15.71	16.96	18.21	19.44

Table 2: Results obtained for a fixed number of events n.

Exercise 11.2

Given the total number of observed events $n_{\text{tot}} = n_{\text{S}} + n_{\text{B}}$ where n_{S} is the number of signal and n_{B} the number of background events we are asked to derive an upper limit for the mean ν_{S} at the $95\,\%$ confidence limit. We also know that ν_{B} and that n follows a Poissonian distribution.
To answer this question we relate the upper limit with respect to a Poissonian distribution and the quantile of the χ^2 distribution such that

$$s_{\text{up}} = \frac{1}{2}\mathrm{F}^{-1}_{\chi^2}\left[p, 2(n+1)\right] - b. \tag{2}$$

This is possible because of the relation between the cumulative Poissonian distribution and the cumulative χ^2 distribution.

$$\Pr(X = k) = \mathrm{F}_{\chi^2}(2\lambda; 2(k+1)) - \mathrm{F}_{\chi^2}(2\lambda; 2k) \tag{3}$$

The implementation into code is given in file `exercise11_2.C`.
Using $n_{obs} = 5$ yields $\nu_S^{max} \approx 8.730\,96$.

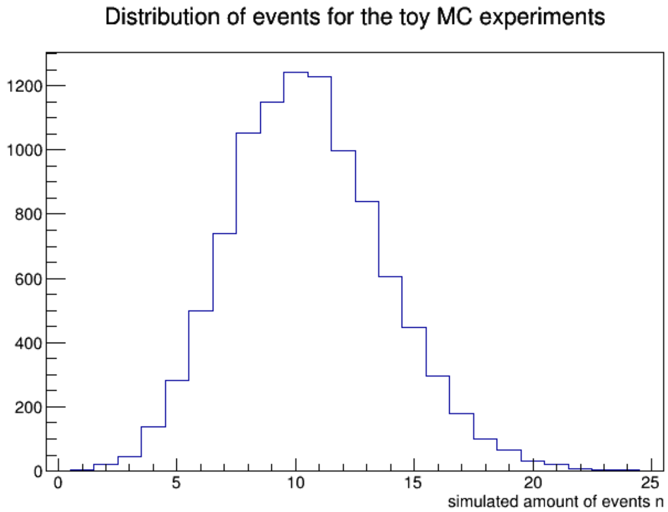

Figure 1: Results from our $10\,000$ Monte Carlo simulation of n.

Exercise 11.3

We are asked to verify the value obtained in exercise . To do this we use $10\,000$ toy Monte Carlo experiments to generate random variables according to a Poissonian distribution with mean $\nu = \nu_\mathrm{B} + \nu_\mathrm{S}^\mathrm{max}$ where $\nu_\mathrm{S}^\mathrm{max}$ is used from exercise 11.2. By construction the number of events $< n_\mathrm{obs}$ should be $5\,\%$.
The implementation into code is given in file `exercise11_3.C`. The plot showing the results is given in figure ??.
Using the value given in exericse we get a fraction of n which is observed below $n_{obs} = 5$ of $4.89\,\%$ which does not disagree with the expected value of $5\,\%$.