SOLUTION FOR HOMEWORK ASSIGNMENT NO. 05

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Exercise 5.1

We are asked to find the position α for a given distance $\beta = 30$. To do this we maximize a likelihood function based on the probability $p(x;\alpha,\beta)$ which is given as

$$p(x;\alpha,\beta) = \frac{\beta}{\pi \cdot \left((x-\alpha)^2 + \beta^2\right)}. \tag{1}$$

Theerefore,

$$\begin{aligned} l(\alpha,\beta;x) &= \ln\left(\prod_{i=1}^{n} \frac{\beta}{\pi \cdot \left((x_i-\alpha)^2 + \beta^2\right)}\right) \\ &= \sum_{i=1}^{n}\left(\ln\left(\frac{\beta}{\pi \cdot \left((x_i-\alpha)^2 + \beta^2\right)}\right)\right). \end{aligned}$$

Please find the results of the maximisation in table 1. The graphical visualisation can be found in figure 1.

Figure 1: *Top*: Distribution of α for fixed β.
Bottom: Three dimensional graphic showind the loglikelihood value for given α in range between 5 and 15 and β in range between 25 and 34.

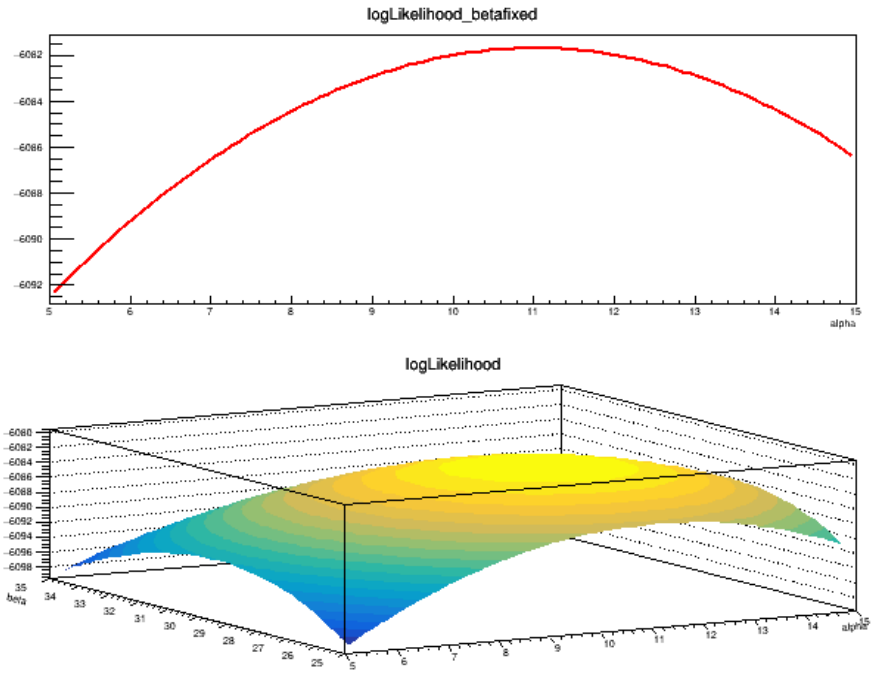

Table 1: Summary of parameters α and β.

Parameter	**Value**	
	fixed β	**variable** β
α	11.017	11.056
β	30	28.844

Exercise 5.2

After importing the dataset we plotted it to confirm that we indeed have a gaussian distribution

$$p(x;\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}. \tag{2}$$

The negative loglikelihood function then looks like

$$
\begin{aligned}
l(\mu,\sigma;x) &= \ln\left(\prod_{i=1}^{n} p(x_i;\mu,\sigma)\right) \\
&= \sum_{i=1}^{n}\left(-\ln\left(\sigma\sqrt{2\pi}\right) - \frac{(x-\mu)^2}{2\sigma^2}\right) \\
&= -\sum_{i=1}^{n}\left(\ln\left(\sigma\sqrt{2\pi}\right) + \frac{(x-\mu)^2}{2\sigma^2}\right).
\end{aligned}
$$

The resulting plot is illustrated in figure 2. The output by the fit is given in table 2.

Figure 2: Distribution of data points given in the file 'data_05.h'. The distribution of points seem to follow a gaussian distribution.

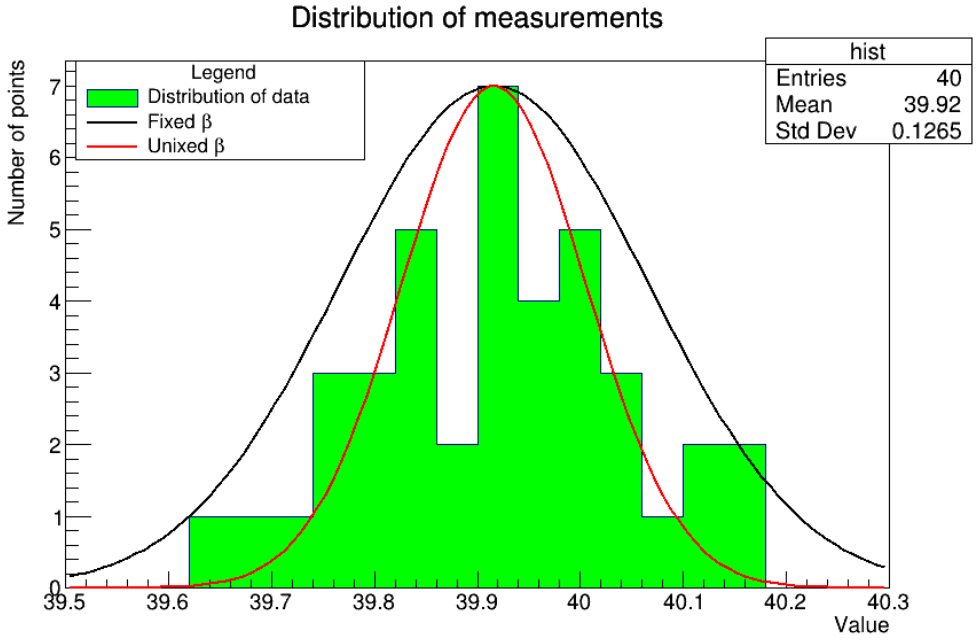

Table 2: Summary of parameters μ and σ found while fitting.

Parameter	**Value**	
	fixed σ	**variable** σ
μ	39.916	39.916
σ	0.15	0.089