SOLUTION FOR HOMEWORK ASSIGNMENT NO. 04

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Exercise 4.1

We are asked to write our own pseudo random number generator. For this we will use the so called *Blum Blum Shub* generator which uses the equation

$$n_{i+1} = n_i^2 \% (p \cdot q) \tag{1}$$

to generate new numbers. p and q are large prime numbers.

To generate numbers between zero and one one has to divide by $p \cdot q$.

$$r_i = \frac{n_i}{M} \tag{2}$$

Please find the code in file exercise4_1.C. The 20th 'random' numbers of consecitive seeds are listed in table

1. Just by only looking at the twenty numbers below I can not see any correlation.

Table 1: The last twenty random numbers are listed given twenty consecutive seeds starting at 234 509 143.

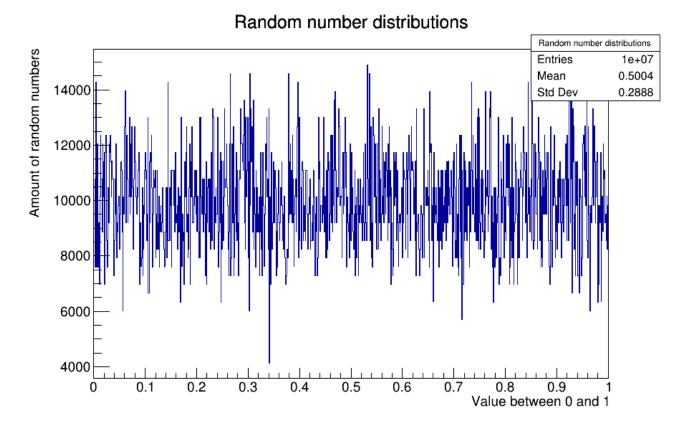
seed i	${\bf random\ number}\ r_i$
234509143	0.38945
234509144	0.05298
234509145	0.55513
234509146	0.31014
234509147	0.82852
234509148	0.53909
234509149	0.41962
234509150	0.10040
234509151	0.15332
234509152	0.45729
234509153	0.53053
234509154	0.47494
234509155	0.01497
234509156	0.32598
234509157	0.90585
234509158	0.01488
234509159	0.68329
234509160	0.25334
234509161	0.46313
234 509 162	0.903 14

Next we used the first seed to generate $10\,000$ random numbers. We filled a histogram with them which you

can see in figure 1.

Figure 1: 10 000 random number generated by our own random number generator. The seed which has been used is 234 509 143. We used 200 bins for plotting in the range between zero and one.

Note that the rather large differences in amounts of numbers persists even to higher total numbers of generated 'random' numbers (e.g. 1000000).



Exercise 4.2

Exercise 4.3

Exercise 4.4

a) We are asked to analytically calulcate the percentage of photons hitting our detector assuming an isotropic distribution of particles.

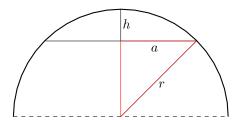
The percentage is the ratio of the surface of the detector divided by the surface of a three dimensional sphere, i.e.

$$p = \frac{O_{\text{segment}}}{O_{\text{sphere}}} = \frac{\pi a^2}{4\pi r^2} = \left(\frac{a}{2r}\right)^2. \tag{3}$$

To be able to find p we need to find r as a is already given. To solve this please have a look at figure 2. From this you can see that

$$(r-h)^2 + a^2 = r^2$$
.

Figure 2: This figure illustrates the setup of the detector and the projection of the two dimensional surface of the sphere. We consider the red triangle to obtain the radius r as only r - h is given in the exercise.



Inserting this in equation 3 yields

$$p = \left(\frac{a}{2\sqrt{a^2 + (r-h)^2}}\right)^2 = \frac{a^2}{4\left(a^2 + (r-h)^2\right)}.$$
 (4)

Inserting $a=0.02\,\mathrm{m}$ and $r-h=0.1\,\mathrm{m}$ yields $p\approx 0.009\,62\approx 0.962\,\%$.

b) Please find the code in the file exercise4_4.C.