Laboratory activity 1 - "To do" list

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1 Activity goals

The activity consists of designing a continuous–time infinite–horizon LQR for stabilizing the Furuta's pendulum around its unstable equilibrium (i.e. a configuration with an upward–directed pendulum). The first goal of the activity is to analyze how different choices of the weighting matrices in the quadratic cost yield different system responses. The second goal is to properly design a position tracking controller, for moving the base of the pendulum back and forth. Aside, it is proposed to design a swing–up controller for the pendulum, i.e. a controller capable of bringing the system to its unstable configuration, starting from the stable configuration (i.e. a configuration with a downward–directed pendulum).

2 LQR design and experimental testing

Design a discrete–time infinite–horizon LQR capable of stabilizing the Furuta's pendulum around its unstable equilibrium. Analyze how different choices of the weights $\mathbf{Q} = \mathbf{Q}^T \succeq 0$ and $r > 0$ in the quadratic cost

$$J_\infty = \int_0^{+\infty} \mathbf{x}^T[t]\,\mathbf{Q}\,\mathbf{x}[t] + r\,u^2[t]dt$$

with $\mathbf{x} = \left[\alpha,\, \beta,\, \dot{\alpha}\ \dot{\beta}\right]^T$ yield different (optimal) closed–loop responses.

The controller gains should be obtained by using the MATLAB function $lqr.m$
For any choice, analyze how the system reacts to an impulsive torque disturbance, which is generated by applying an impulsive voltage command to the D.C. motor (impulse height

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NB CAVETTO AGISCE come disturbo esterno
⇒ deflessione della POSIZIONE estremamente visibile

$= 3\,V$, impulse width $0.1\,s$, impulse initial time: after that the pendulum has been stabilized to the upward position).

As starting points for the LQR design, consider the following tentative choices for the weights $\mathbf{Q}$ and r:

1. *expensive vs. cheap control cases*:

non buono non mantiene la posizione motore (rotary) ⇒ ROBUSTO

$$\mathbf{Q} = \mathrm{diag}\{1,\, 1,\, 1,\, 1\}, \qquad r = 2 \tag{1}$$

ROBUSTO → non tiene la posizione come caso 1

$$\mathbf{Q} = \mathrm{diag}\{1,\, 1,\, 1,\, 1\}, \qquad r = 1 \tag{2}$$

$$\mathbf{Q} = \mathrm{diag}\{1,\, 1,\, 1,\, 1\}, \qquad r = 0.5 \tag{3}$$

→ ROBUSTO all'impulso sembra tenere meglio la posizione rispetto caso ① e ②

2. *different weights for* θ:

GOOD! ROBUSTO tiene la posizione

$$\mathbf{Q} = \mathrm{diag}\{10,\, 1,\, 1,\, 1\}, \qquad r = 2 \tag{4}$$

$$\mathbf{Q} = \mathrm{diag}\{100,\, 1,\, 1,\, 1\}, \qquad r = 2 \tag{5}$$

→ MEGLIO del ④ forse

3. *different weights for* α:

RIFARE

non tiene la la posizione ma ROBUSTO all'impulso ↳ nel senso che non cade all'applicazione

$$\mathbf{Q} = \mathrm{diag}\{1,\, 10,\, 1,\, 1\}, \qquad r = 2 \tag{6}$$

$$\mathbf{Q} = \mathrm{diag}\{1,\, 100,\, 1,\, 1\}, \qquad r = 2 \tag{7}$$

→ meglio di ① e ② e ⑥ ma comunque non tiene bene la posizione + ROBUSTO ↳ va verso 0 ma lento

4. *different weights for* $\dot{\theta}$:

vibra non tiene la posizione ROBUSTO

$$\mathbf{Q} = \mathrm{diag}\{1,\, 1,\, 10,\, 1\}, \qquad r = 2 \tag{8}$$

$$\mathbf{Q} = \mathrm{diag}\{1,\, 1,\, 100,\, 1\}, \qquad r = 2 \tag{9}$$

→ non tiene niente ⇒ motore vibra ↔ non sta su ⇒ NON ROBUSTO

5. *different weights for* $\dot{\alpha}$:

sempre il più aggressivo ← motore molto aggressivo

⊕ tiene poco la posizione (va lentamente a 0)

⊕ ROBUSTO

aggressivo all'impulso

vibra poco

$$\mathbf{Q} = \mathrm{diag}\{1,\, 1,\, 1,\, 10\}, \qquad r = 2 \tag{10}$$

$$\mathbf{Q} = \mathrm{diag}\{1,\, 1,\, 1,\, 100\}, \qquad r = 2 \tag{11}$$

↳ vibra + ROBUSTO + non tiene la POSIZIONE

↳ ~~RIFARE con $\frac{50 s}{s+50}$~~

6. *only relative values are important when defining the weights*:

$$\mathbf{Q} = \mathrm{diag}\{1,\, 1,\, 1,\, 1\}, \qquad r = 2 \tag{12}$$

→ non tiene la posizione bene → non va verso 0 + ROBUSTO

$$\mathbf{Q} = \mathrm{diag}\{0.001,\, 0.001,\, 0.001,\, 0.001\}, \qquad r = 0.002 \tag{13}$$

non tiene per niente la posizione PEGGIO DI 12 ma non vibra + sull'impulso [illegible]

Verify that weights with different absolute values, but with identical relative values (i.e. with the same ratios between the values), yield the same optimal control input and optimal closed–loop response.

non tiene l'impulso e neanche la posizione

oscilla troppo al minimo disturbo e cade

7. *no weighting for angular velocities* $\dot{\theta}$ *and* $\dot{\alpha}$

$$\mathbf{Q} = \text{diag}\{1, 1, 0, 0\}, \qquad r = 2 \tag{14}$$

$$\mathbf{Q} = \text{diag}\{10, 10, 0, 0\}, \qquad r = 2 \tag{15}$$

vibra un casino ma sa su tiene la posizione 0

Compare the results with the choices:

$$\boxed{\mathbf{Q} = \text{diag}\{1, 1, 1, 1\}, \qquad r = 2} \tag{16}$$

$$\mathbf{Q} = \text{diag}\{10, 10, 10, 10\}, \qquad r = 2 \tag{17}$$

impulso lo tiene (forse un po' smadonga) + ROBUSTO all'impulso, sta su non tiene la posizione (non va verso 0)

ROBUSTO (sta su) vibra un po' sta a 0 → tiene pos + o −

8. *correct/wrong choice of the weighting matrix* **Q**:

~~RIFARE~~

sta a 0 neanche il disturbo a 3 s

$$\mathbf{Q} = \text{diag}\{1, 0, 0, 0\}, \qquad r = .5 \tag{18}$$

$$\mathbf{Q} = \text{diag}\{0, 1, 1, 1\}, \qquad r = .5 \tag{19}$$

→ non ∃ soluzione di RICATI } DIMOSTRARE → MATLAB mi dice che non può

mo è instabile (sta su per miracolo)

Explain why the choice (18) yields a stabilizing LQR, while the choice (19) does not produce any result.

3 Discretization - optional

200 → CASO 5 → $\frac{s \cdot 50}{s+50}$

The above design can be developed entirely in a discrete–time setting. For this, the linearized model must be at first discretized with the use of the MATLAB function $c2d$. Then, in order to find the controller that minimize the following cost function:

201 → CASO 5 → $\frac{s \cdot 250}{s+250}$

202 → CASO 8 → $\frac{s \cdot 250}{s+250}$

203 → CASO 8 → $\frac{s \cdot 50}{s+50}$

$$J_\infty = \sum_{k=0}^{+\infty} \mathbf{x}^T[kT]\,\mathbf{Q}\,\mathbf{x}[kT] + r\,u^2[kT]$$

The controller gains should be obtained by using the MATLAB function $dlqr.m$

Take only some selected cases studied in continuous time to compare the performances, differences and analogies.

$W = \frac{2\pi}{T}$ $\quad T = \frac{2\pi}{\omega}$

SINUSOIDALE $\left\{\begin{array}{l} Hz\ 1{,}75 \\ f = 3{,}5 \\ A = -3{,}5 \end{array}\right\}$ TRENO

guardare ANDAMENTI VELOCITÁ errore complicato → annotate andamenti velocitá

CAMBIANO i guadagni $\longrightarrow$ la risposta sembra più bella anche se magari risponde peggio al disturbo

TECNICHE allocazione AUTOVALORI

CASO 5

100 → 550 / (S+50)

CASO 103 è come 5 con 250S / (S+250)

4 Position tracking

Design an experiment for the tracking of a time-varying position reference for the base of the pendulum. The reference to be followed is is a sinusoidal profile, with a frequency of 1 Hz and an amplitude of 45 degrees. → S

5 Swing–up controller

Design a controller (*swing–up controller*) capable of bringing the pendulum in its upward unstable position, starting from the downward stable position. Such controller has to operate only at the system start–up, in order to move the pendulum away from its stable position. Then, when the pendulum is sufficiently close to its unstable equilibrium position, the control authority has to be transferred to the stabilizing LQR designed in the previous step (for a given choice of the weights $\mathbf{Q}$ and r), i.e. the control signal has to be switched (according to a suitable *switching policy*) from the swing–up controller output to the LQR output.

Many solution are available in literature to design a swing–up controller for an inverted pendulum (see also the proposed study material for this laboratory activity). You can try to implement one of the solutions available in literature; however, even a very simple solution (based on intuitive arguments) can yield reasonable results.

diag{100, 1, 25, 1} → CASO 101

↳ vibra → SMORZATO

diag{100, 10, 1, 1} → CASO 102

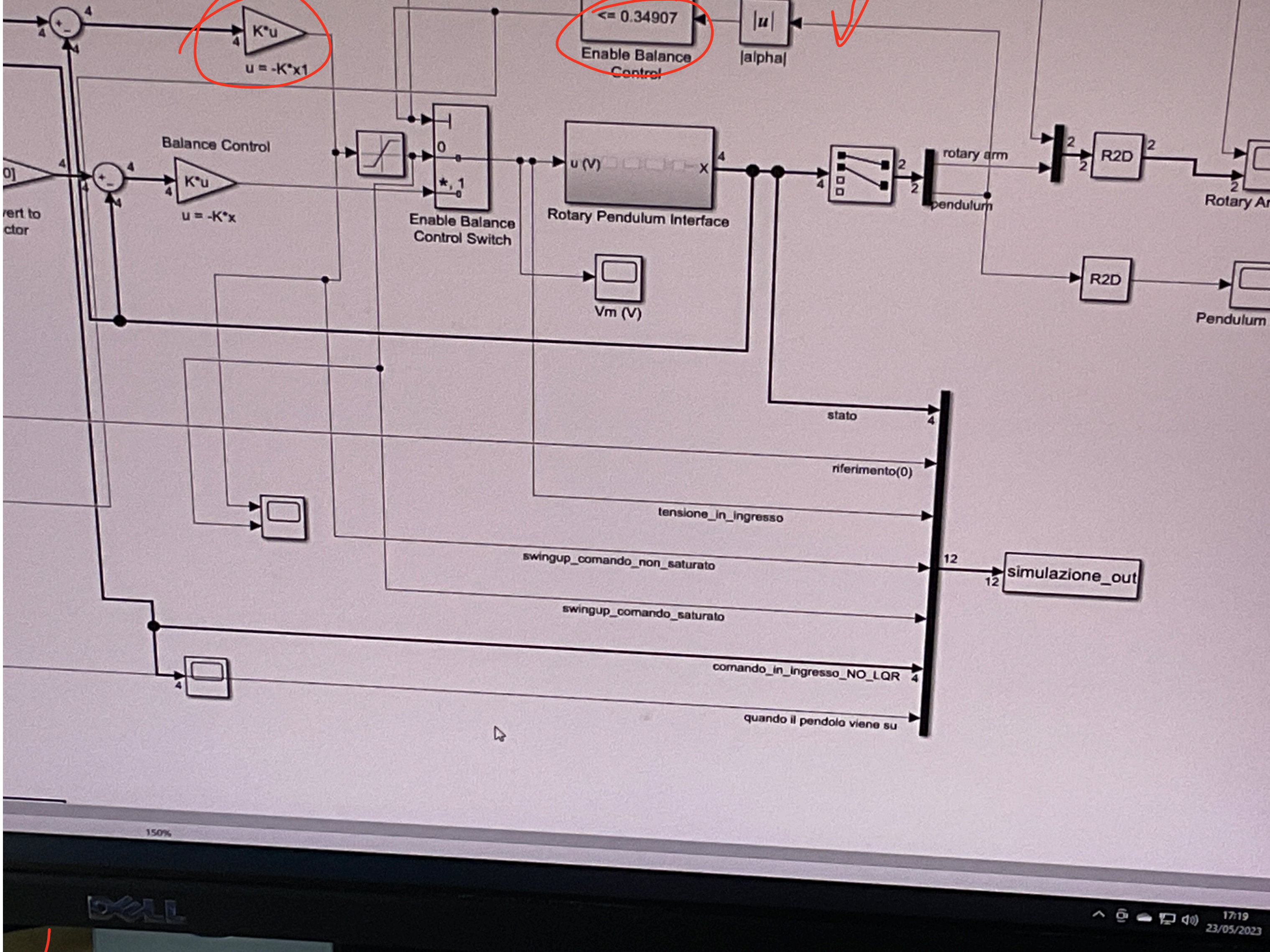
CASO 105 diag{10, 1, 10, 1} → 550 / (S+50)

CASO 106 diag{10, 1, 10, 1} → S250 / (S+250)

SPIEGARE

±30°

±20°



CASO 300 → SWING_MARCO con 20° deg

CASO 301 → SWING_MARCO con 30° deg

CASO 302 → SWING_FORNA con 30° deg

CASO 303 → SWING_FORNA con 20° deg