SEARCH AND OPTIMIZATION

Artificial Intelligence

Syed Muhammad Mustafa

Alina Rashid

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HABIB UNIVERSITY

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1. Why is it important to have an admissible heuristic in A* to ensure optimality?

An admissible Heuristic is one in which the heuristic never overestimates the actual cost of the step, i.e.

$$h(n) \leq h^*(n)$$

Where $h^*(n)$ is the actual cost and $h(n)$ is the estimate. An admissible heuristic will expand the front tier nodes in a manner such that it will include the nodes with a lower value of $f(n)$. Although this might result in backtracking, in the case where the heuristic is underestimating the actual cost. And, in the case that the heuristic ideally predicts the cost, there will be no backtracking. But, the path at the end of the program will always be optimal.

In a case of an inadmissible heuristic the front tier nodes **may** (not necessarily) include nodes in which the actual cost from the start node to the current node is optimal, but the heuristic overestimated the cost, due to which that node will not be added in the final path. Hence, the final path will be a sub-optimal path.

2. In addition to admissibility, A* also requires monotonicity in graph based problems. You are required to do some readings to understand monotonicity requirement of A*. Describe it in your own words.

Monotonicity is also called the consistency of a node. It is defined as follows: An A-star algorithm having a heuristic $h(n)$ such that,

$$\forall n \in N \left(\forall n' \in s(n)\right): (h(n) \leq c(n, a, n') + h(n'))$$

Where, n is the current node

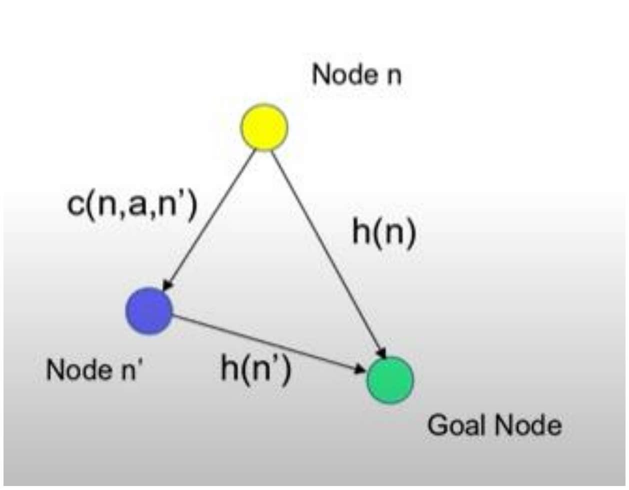
s(n)←Set of successor of n

n' any successor node of n

a is some action performed to get from $n \rightarrow n'$

$c(n, a, n')$ is defined as the cost from node n to n'

What this means is that the estimated cost of node n is lesser than the step cost of travelling from $n \rightarrow n'$ plus the estimated cost of $h(n')$. This is known as the triangle inequality:



We can see from the diagram above that a consistent heuristic will always be an admissible heuristic. Since the actual path will always be more costly than the estimated cost (h(n)). The Actual proof of the previous statement will not be provided here, however, it can be proved using induction. Intuitively:

$c(n, a, n')$ is the actual per step cost. If heuristic is consistent than we know:

$$h(n') \leq c(n', a, n'') + h(n'')$$
$$h(n'') \leq c(n'', a, n''') + h(n''')$$

So,

$$h(n) \leq c(n, a_1, n') + \ c(n', a_2, n'') + \ c(n'', a_3, n''') + h(n'''')$$

And

$$Actual\ Cost = \ c(n, a_1, n') + \ c(n', a_2, n'') + \ c(n'', a_3, n''') + \cdots + c(n^{t-1}, a_t n^t)$$

Where $n^t$is the goal node, so: $h(n) < actual\ cost$

Once Again I state that this is not a formal mathematical proof. Rather it is more intuitive.

References

[1] Russell, Stuart J. (Stuart Jonathan). (2010). Artificial intelligence : a modern approach. Upper Saddle River, N.J. :Prentice Hall,

[2] Admissible and Consistent Heuristics. Doi: https://www.youtube.com/watch?v=0K0H-z7HZ1o